Beam Loss and Emittance Growth Mechanisms

Beam Loss and Emittance Growth

Lifetime:

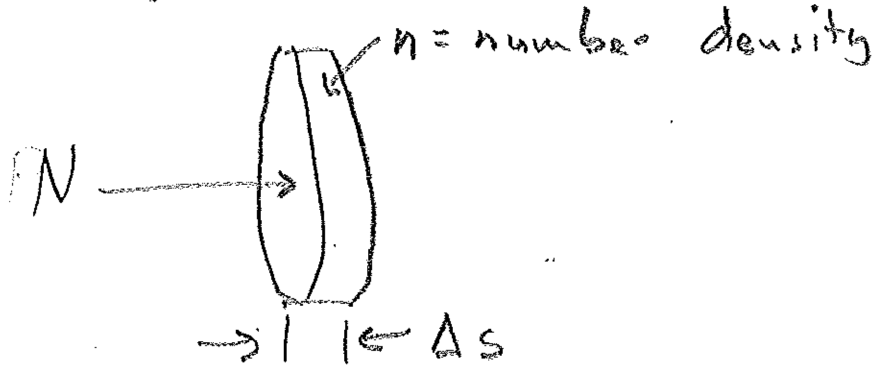
- Residual gas interactions
- Touschek effect
- quantum lifetimes (electron machines)
- beam-beam collisions

Emittance Growth

Residual gas interactions
Intra beam scattering
Random Noise sources

Residual Gas Interactions

Probability of interaction



Thin target approximation

Probability of an interaction

$$P = \underset{\substack{\text{beam}\\\text{particles}}}{N}\ \underset{\substack{\text{number}\\\text{density}}}{n}\ \underset{\text{cross-section}}{\sigma}\ \Delta s$$

Assume all lost $\Rightarrow \dfrac{dN}{ds} = -Nn\sigma$

For relativistic particles $ds = c\,dt$

$$\frac{dN}{dt} = -Nnc\sigma = -\frac{N}{\tau}$$

$$\boxed{\tau = nc\sigma}$$

Look at particular examples

Coulomb Scattering

classical e radius

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{2\pi\Theta d\Theta} = \frac{4Z^2 r_0^2}{(\Theta+\Theta_1)^2}\left(\frac{m_e c}{p}\right)^2$$

Rutherford Scattering

$$\Theta_1 = \underset{\frac{1}{137}}{\alpha} Z^{1/3}\left(\frac{m_e c}{p}\right)$$

If a particle is scattered into a polar angle Θ at a point where lattice function is β, then

$$X, y_{max} = \Theta \sqrt{\underset{\text{scatter}}{\beta}\ \beta_{lim}} / \sqrt{2}$$

where β_{lim} is the β function at the limiting aperture; that is, the minimum value of

$$\frac{b}{\sqrt{\beta}} \leftarrow \text{aperture}$$

so all particles with

$$\Theta > \Theta_{lim} = \frac{\sqrt{2}\, b}{\sqrt{\beta \beta_{lim}}}$$

will be lost

$$\sigma_{coulomb} = 2\pi \int_{\Theta_{lim}}^{\pi} \frac{d\sigma}{d\Omega} \Theta \, d\Theta = \frac{4\pi Z^2 r_0^2}{\Theta_{lim}^2} \left(\frac{m_e c}{p}\right)^2$$

$\leftarrow \Theta_{lim} \gg \Theta_1$

$$= \frac{\beta_{lim} \langle\beta\rangle}{b^2} 2\pi Z^2 r_0^2 \left(\frac{m_e c}{p}\right)^2$$

Example: Nitrogen ($Z=7$)

$b = 30\,mm$; $\beta_{lim} = 30\,m$

$\langle\beta\rangle = 15\,m$

$p = 5\,GeV/c$

$$\Rightarrow \boxed{\begin{aligned}\sigma_{coulomb} &= 1.3\times10^{-25}\,cm^2 \\ &= .13\ barn\end{aligned}}$$

<u>Bremstrahlung</u> (electrons only)

Cross-section of beam lost to bremstrahlung is

$$\frac{d\sigma}{du} \approx \frac{16\alpha r_0^2}{3} Z(Z+1) \ln\left(\frac{184}{Z^{1/3}}\right) \frac{1-u+.75u^2}{u}$$

$u = \frac{\Delta E}{E} \leftarrow$ energy lost

If ΔE_a is the energy aperture of the machine, then

$$\sigma_{brem} = \int_{u_a}^{\infty} \frac{d\sigma}{du} \approx \frac{16\alpha r_0^2}{3} Z(Z+1) \ln\left(\frac{184}{Z^{1/3}}\right)\left[\ln\frac{1}{u_a} - \frac{5}{8}\right]$$

$\leftarrow \frac{\Delta E_a}{E}$

$Z = 7,\ u_a = .003 \rightarrow \sigma_{brem} = 4$ barn

$\gg \sigma_{coulomb}$

Nuclear Scattering (protons only)

No simple formula, but for Nitrogen $\sigma_{nuclear} \approx .4$ barn

Beam Lifetimes (protons)

For protons

$$\sigma = \sigma_{coulomb} + \sigma_{nuclear}$$

$$\frac{1}{\tau} = n c \sigma$$

For an ideal gas

$$n_{mol} = \frac{\overset{\text{pressure}}{P}}{\underset{\text{Boltzman constant}}{k}\,\underset{\text{temp}}{T}}$$

$$= 9.66 \times 10^{24} \frac{P[nTorr]}{T[°K]}$$

For diatomic molecule

$$n = 2 n_{mol}$$

$$\rightarrow \tau[hr] = \frac{.474\, T[°K]}{P[nTorr]\,\sigma[barn]}$$

Example: FNAL Tevatron

$p = 1000 \rightarrow \sigma_{Coulomb}$ negligible

$\sigma_{Nuclear} = .4$ barn $\quad T = 4K$

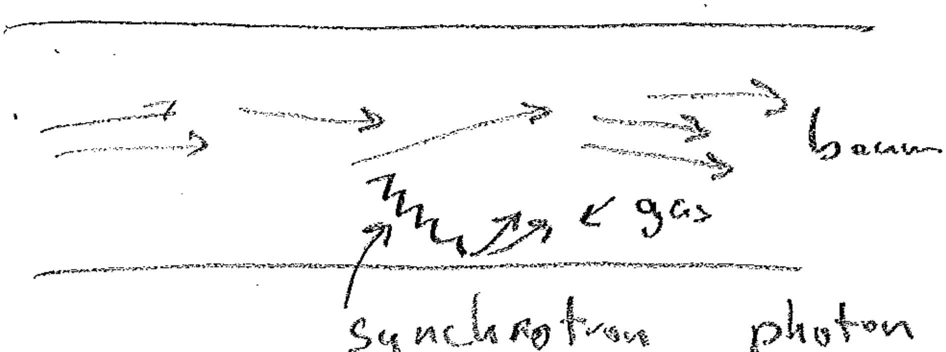
20 hour lifetime $\Rightarrow \boxed{p < 2.3\times10^{-10}\ \text{Torr}}$

Residual Gas Interaction (electrons)

Use

$$\sigma = \sigma_{Coulomb} + \sigma_{Brem}$$

Gets complicated because of "photodesorption"



probability of a desorption by one electron

$$n = n_0 + GN$$

intensity

initial density

gas density

$\rightarrow \dfrac{dN}{dt} = -Nnc\sigma = -Nc\sigma(\underbrace{n_0 + GN_0}_{n_{eff}})$

$\rightarrow P_{eff}\,[nTorr] \leq \dfrac{.474\ T[°K]}{\tau[hr]\ \sigma[barn]}$

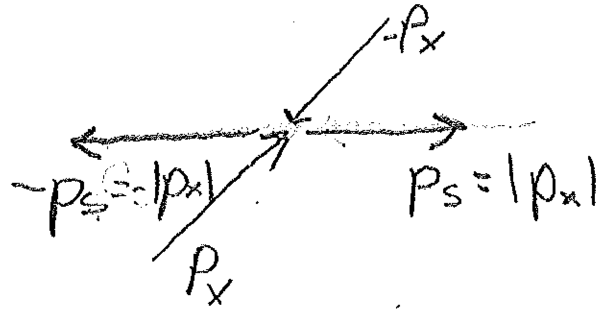
$P_{eff} = \dfrac{kTn_{eff}}{2}$ ← diatomic

8

Touschek Effect

= scattering of one particle by another in a bunch

If we look in the rest frame of the bunch (maximum effect)



In the lab frame

$$\Delta p_{max} = \gamma p_s$$

$$= \gamma p_x$$

$$= \gamma x' p$$

If $\dfrac{\Delta E_{max}}{E} = \beta^2 \dfrac{\Delta p_{max}}{p} = \beta^2 \gamma x'$

If $\Delta E_{max} > \Delta E_a$ ← energy acceptance

particle will be lost

The formula is extremely complicated, but the basic structure is

$$\frac{1}{\tau_{Touschek}} \propto \frac{r_0^2 c N_b}{\gamma^4 \epsilon_x \epsilon_y \epsilon_L} \langle f(\ ) \rangle$$

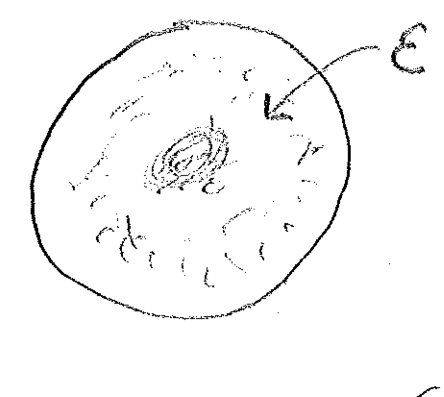
where $f = f(\beta_x, \beta_y, \epsilon_x, \epsilon_y, \delta, D_x, \frac{\Delta E_a}{E})$

Quantum Lifetimes (Electron Machines)

Quantum fluctuations due to photon radiation may cause a particle to exceed the energy acceptance of the machine (extreme case of emittance growth discussed earlier).

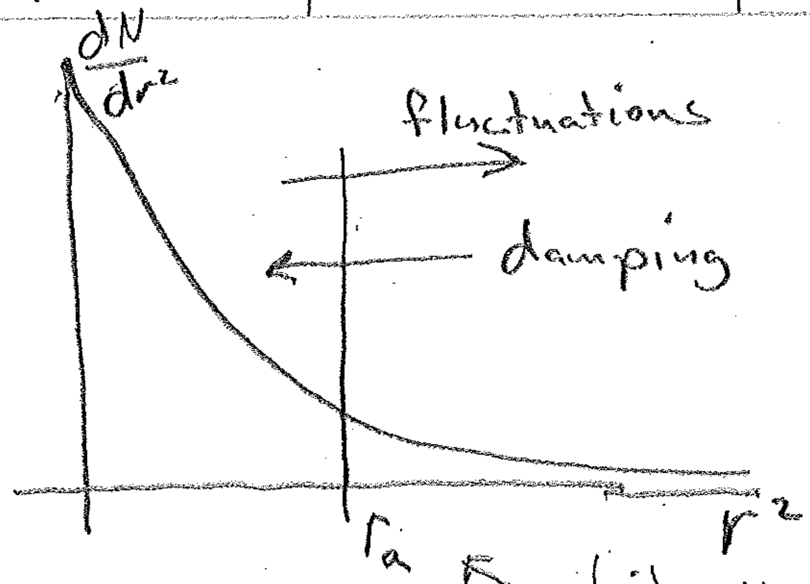
For a gaussian distribution of particles, we can write

$$\frac{dN}{r\,dr\,d\phi} = \frac{N}{2\pi\epsilon} e^{-(r^2/2\epsilon)}$$



$\frac{1}{2} d(r^2)$ $\qquad$ $\int d\phi = 2\pi$

$$\rightarrow \frac{dN}{dr^2} = \frac{N}{2\epsilon} e^{-r^2/2\epsilon}$$



$$\left.\frac{dN}{dt}\right|_{damping} = \left.\frac{dN}{dr^2}\right|_{r_a^2} \left.\frac{dr^2}{dt}\right|_{damping}$$

$$\left.\frac{dr^2}{dt}\right|_{damping} = -\left.\frac{r^2}{\tau_{r^2}}\right|_{r^2=r_a^2}$$

synch. lecture

but $\tau_{r^2} = \frac{1}{2}\tau_x$; $\tau_x = \frac{2\tau_s E_s}{U_s}(1-\mathcal{D})$

damping time in x

$$\rightarrow -\left.\frac{r^2}{\tau_{r^2}}\right|_{r_w^2} = \frac{2r_w^2}{\tau_x}$$

$$\rightarrow \left.\frac{dN}{dt}\right|_{r=r_a} = \left(-\frac{2r_a^2}{\tau_x}\right)\left(\frac{N}{2\varepsilon}\right)e^{-r_a^2/2\varepsilon}$$

$$= -N\frac{r_a^2}{\varepsilon\,\tau_x}e^{-r_a^2/2\varepsilon}$$

but at equilibrium

$$\frac{dN}{dt} = \frac{dN}{dt}\bigg|_{\text{fluctuations}} + \frac{dN}{dt}\bigg|_{\text{damping}} = 0$$

$$\rightarrow \frac{dN}{dt}\bigg|_{\text{fluctuations}} = + \frac{N r_a^2}{\varepsilon \tau_x} e^{-\left(-r_a^2/2\varepsilon\right)}$$

Imagine there is an <u>aperture restriction</u>, then $\left.\frac{dN}{dt}\right|_{\text{damping}} = 0$

and $\frac{dN}{dt} = - \left.\frac{dN}{dt}\right|_{\text{fluctuations}}$

If this rate is small, then the shape will not change, so

$$\frac{dN}{dt} = - \frac{N r_a^2}{\varepsilon_x \tau_x} e^{-r_a^2/2\varepsilon_x} = - \frac{N}{\tau_q}$$

quantum lifetime

$$\rightarrow \tau_q = \tau_x \frac{\varepsilon_x}{r_a^2} e^{+r_a^2/2\varepsilon_x}$$

but r_a^2 is just $\frac{1}{\pi}$ area of ellipse at limiting aperture = admittance A, so

$$\tau_q = \tau_x \frac{\varepsilon_x}{A_x} e^{A/2\varepsilon_x}$$

Since $\sigma_x = \sqrt{\beta\varepsilon}$; $d = \sqrt{\beta A}$

limiting (half) aperture

$$\tau_q = \tau_x \frac{\sigma_x^2}{d^2} e^{d^2/2\sigma^2}$$ very rapid dependence

Since $\tau_x \sim 10\text{ms}$ typically

we would need $d = 5\sigma$ to get

a 10 hour lifetime

Can do the same analysis in longitudinal plane

$$\rightarrow \tau_q = \tau_\varepsilon \frac{\sigma_\varepsilon^2}{\Delta E_a^2} e^{\Delta E_a^2 / 2\sigma_\varepsilon^2}$$

energy aperture

Once we know the loss mechanism, we have

$$\frac{dN_b}{dt} = \mathcal{L}\,\sigma_{loss}$$

recall $\mathcal{L} \propto N_b^2 = k\,N_b^2$
constant

$$\rightarrow \frac{d\mathcal{L}}{dt} = 2\mathcal{L}\,\frac{dN_b}{dt} = -2kN_b\mathcal{L}\sigma_{loss}$$

$$= -\frac{\mathcal{L}}{\tau_{\mathcal{L}}}$$

$\leftarrow$ luminosity lifetime

$$\rightarrow \tau_{\mathcal{L}} = \frac{1}{2kN_b\,\sigma_{loss}} = \frac{N_b}{2\mathcal{L}\sigma_{loss}}$$

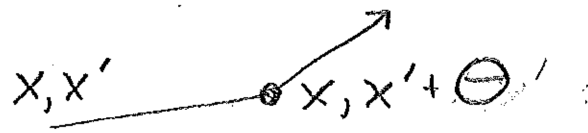
$kN_b = \frac{\mathcal{L}}{N_b}$

not constant!

Initial $\tau_{\mathcal{L}} = \dfrac{N_0}{2\mathcal{L}_0\,\sigma_{loss}}$

Emittance Growth

Elastic Scattering



Recall, in Floquet coords

$$\Delta \dot{\xi} = \nu \sqrt{\beta}\, \Delta x' = \nu \sqrt{\beta}\, \Theta$$

We write the change in amplitude around one turn as

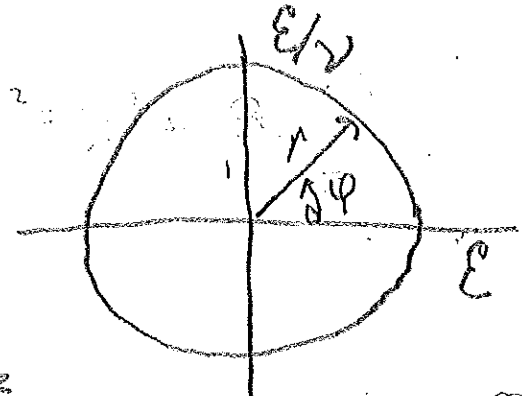
$$\begin{pmatrix} \xi \\ \dot{\xi} \end{pmatrix}_{n+1} = \begin{pmatrix} \cos 2\pi\nu & \frac{\sin 2\pi\nu}{\nu} \\ -\nu \sin 2\pi\nu & \cos 2\pi\nu \end{pmatrix} \begin{pmatrix} \xi \\ \dot{\xi} + \nu\sqrt{\beta}\,\Theta \end{pmatrix}_n$$

In phase space coords, we have

$$\xi = r\cos\varphi$$

$$\dot{\xi} = -\nu r \cos\varphi$$

note different symbol $\neq \Theta$



$$r^2 = \xi^2 + \left(\frac{\dot{\xi}}{\nu}\right)^2$$

$$\rightarrow r^2(\dot{\xi} + \Delta\dot{\xi})$$

$$= \xi^2 + \left(\frac{\dot{\xi}}{\nu}\right)^2 + \frac{2\dot{\xi}\Delta\dot{\xi}}{\nu^2} + \frac{(\Delta\dot{\xi})^2}{\nu^2}$$

$$\rightarrow \frac{\Delta r^2}{dn} = \frac{1}{\nu^2}\left(2\dot{\xi}\Delta\dot{\xi} + (\Delta\dot{\xi})^2\right) \quad (*)$$

$$= -2\sqrt{\beta}\, r \sin\varphi\, \Theta + \beta\Theta^2$$

averages to 0

$$\rightarrow \frac{d\langle r^2\rangle}{dn} = \beta\langle\Theta^2\rangle$$

The emittance from Coulomb scattering

$$\langle\Theta^2\rangle = \left(\frac{13.6\ \text{MeV}}{\beta pc}\right)^2 \frac{s}{X_0}$$

radiation Length

So for residual gas, it would

be $\frac{d\langle r\rangle^2}{dn} = \langle\beta\rangle \left(\frac{13.6\text{ MeV}}{\beta pc}\right)^2 \frac{C}{X_0}$ ← Circumference

For electron machines, $\tau_{scatt} \gg \tau_{damp}$

so not a factor

Only a problem for hadron machines

<u>Other sources of $\langle\Theta^2\rangle$</u>

1. dipole power supplies

$$\langle\Theta^2\rangle = \left\langle \frac{(\Delta B)^2 L^2}{(B\rho)^2} \right\rangle$$

2. Ground motion or vibration
→ random quad motion

$$\langle\Theta^2\rangle = \frac{\langle(\Delta x)^2\rangle}{f^2}$$

Recall $\varepsilon_{RMS} = \frac{\langle r^2 \rangle}{2}$

$$\rightarrow \frac{d\varepsilon}{dt} = \frac{1}{\tilde{\tau}_s} \frac{d\langle r^2 \rangle}{dn} = \frac{1}{2} \langle \beta \Theta^2 \rangle$$

Intrabeam scattering

Very important, but too complicated to treat here