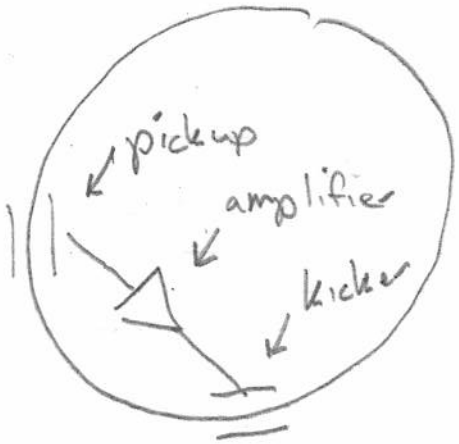
Prebys

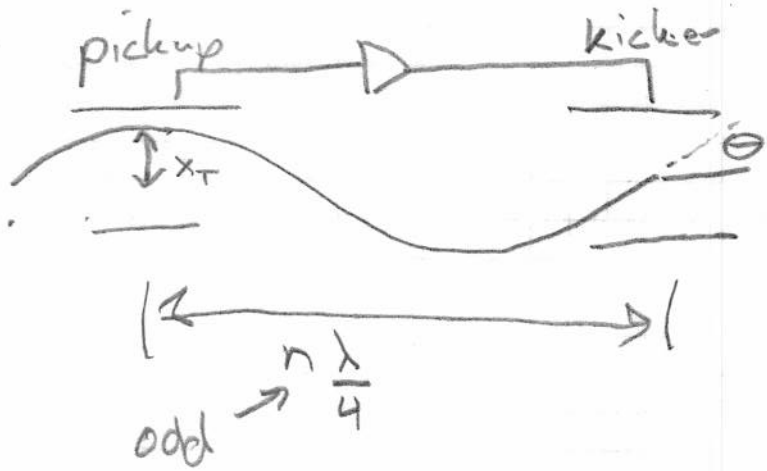
Cooling

Stochastic Cooling (antiprotons or ions)

Basis of stochastic cooling is to detect the displacement at one point and feed it into a restoring kick at a second point



Single particle

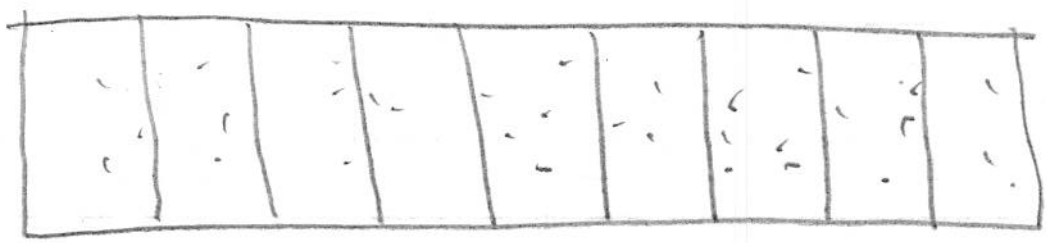


$$X_{T+1} = X_T - g X_T$$

$\uparrow$ gain

$g = 1$ would remove the oscillation in one turn.

Now consider an ensemble of particles



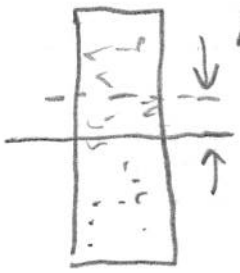
$|T_s|$

$\hookrightarrow T_s \ll T$

$\uparrow$ sampling period $= \dfrac{1}{2W}$

bandwidth of system

Mean of sample



$$\Delta \bar{x} = \frac{1}{N_s} \sum_{j=1}^{N_s} x_j$$

$$\langle x^2 \rangle = \frac{\sigma_x^2}{N}$$

σ_x^2 ← RMS size of the beam

For the ith particle, the change in one turn is

$$x_i \to x_i - g\bar{x}$$

$$x_i^2 = (x_i - g\bar{x})^2 = x_i^2 - 2g x_i \bar{x} + g^2 \bar{x}^2$$

$$\Delta x_i^2 = -g x_i \bar{x} + g^2 \bar{x}^2$$

Write $\bar{x} = \frac{1}{N_s}\left[x_i + \sum_{\substack{j=1 \\ j \neq i}}^{N_s - 1} x_j\right]$

$$= \frac{x_i}{N_s} + \frac{N_s - 1}{N_s}\bar{x}^*$$

one particle

mean position of all other particles

Plugging this back in, we get

$$\Delta x_i^2 = -\frac{2g x_i}{N_s}\left(x_i + (N_s - 1)\bar{x}^*\right) + \frac{g^2}{N_s^2}\left(x_i + (N_s - 1)\bar{x}^*\right)^2$$

If the samples on each turn are statistically independent (not true in general), then over many turns

$$\langle \bar{x}^* \rangle = 0$$

$$\langle (\bar{x}^*)^2 \rangle = \langle \bar{x}^2 \rangle = \frac{\sigma_x^2}{N_s}$$

$$\rightarrow \Delta x_i^2 = \underbrace{\frac{-2g x_i^2}{N_s} + \frac{g^2 x_i^2}{N_s^2}}_{\text{cooling terms}} + \underbrace{\frac{g^2 (N_s - 1)}{N_s^3} \sigma_x^2}_{\text{statistical "heating" = "Schottky Noise"}}$$

Average over all particles assuming $N_s >> 1$

$$\rightarrow \langle \Delta x^2 \rangle = \frac{-2g}{N_s} \langle x^2 \rangle + \frac{g^2}{N_s} \sigma_x$$

$$\rightarrow \Delta \sigma_x^2 = (-2g + g^2) \frac{\sigma_x^2}{N_s}$$

This is the change <u>per turn</u>, so

$$\frac{1}{\tau_{x^2}} = -\frac{1}{T}\frac{\Delta\sigma_x^2}{\sigma_x^2}$$

period

$$= \frac{1}{N_s T}\left(2g - g^2\right)$$

Since $\frac{N_s}{N} = \frac{T_s}{T} = \frac{1}{2WT}$

$$\rightarrow \frac{1}{\tau_{x^2}} = \frac{2W}{N}\left(2g - g^2\right)$$

$$\frac{1}{\tau_x} = \frac{1}{2\tau_{x^2}} = \frac{W}{N}\left(2g - g^2\right)$$

$$\tau_x = \frac{N}{W\left(2g - g^2\right)}$$

high bandwidth systems

Note, electronic thermal noise will heat the system!

$$\rightarrow \frac{1}{\tau_x} = \frac{W}{N}\left(2g - g^2(1+U)\right)$$

thermal power
Schottky noise

We assumed that the sample was statistically independent on each turn. If the sample is the same, then cooling stops after <u>one turn</u>. This technique works via "mixing", the fact that particles of different momenta have different periods

In general, it will take M turns to completely renew the sample

$$M = \frac{T_s}{\Delta T} = \frac{1}{2W\Delta T}$$

variation in periods

Remember that

$$\frac{T}{T_0} = 1 - \eta\delta$$

$\delta \leftarrow \frac{\Delta p}{p}$

$$\Delta T = T_0 \eta_c \delta$$

$$\rightarrow M = \frac{1}{2W\Delta T} = \frac{f_0}{2W\eta\delta}$$

$f_0 \leftarrow$ revolution frequency

$M \leftarrow$ number of turns to renew sample.

The effect is to increase the Schottky heating term by M

$$\rightarrow \frac{1}{\tau_x} = \frac{W}{N}\left(2g - g^2(M+U)\right)$$

$U \leftarrow$ electronic noise

$M \leftarrow$ mixing time

Optimum gain is max of

$$2g - g^2(M+U)$$

$$\rightarrow g = \frac{1}{(M+U)}$$

$$\rightarrow \frac{1}{\tau_x} = \frac{W}{N}\frac{1}{(M+U)}$$

Example: Fermilab Debuncher

rotation period: 1.7 μsec

$W = 2\text{-}4$ GHz

$\delta = .002$

$\eta = .006$

$N = 2 \times 10^7$

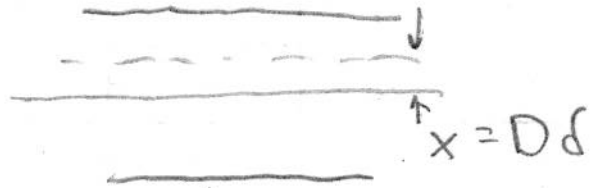
$U \approx 2$ ← noise twice beam signal

$$\rightarrow M = \frac{1}{2 W T_0 \eta_c \delta} = 12$$

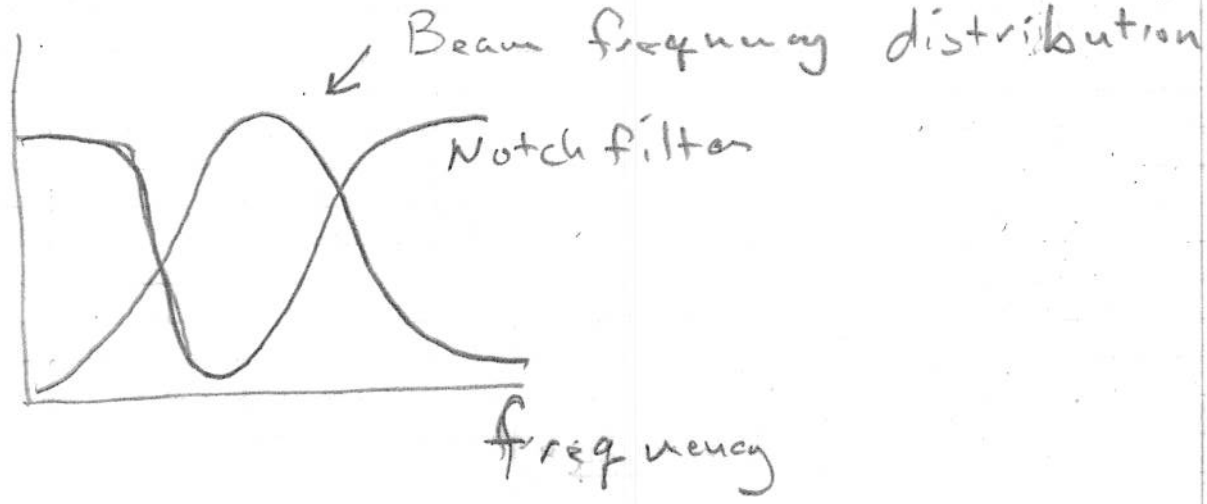
$$\frac{1}{\tau_x} = \frac{W}{N}\left(\frac{1}{M+U}\right) = \frac{2 \times 10^9}{2 \times 10^7} \sim 7\,s^{-1}$$

Longitudinal cooling

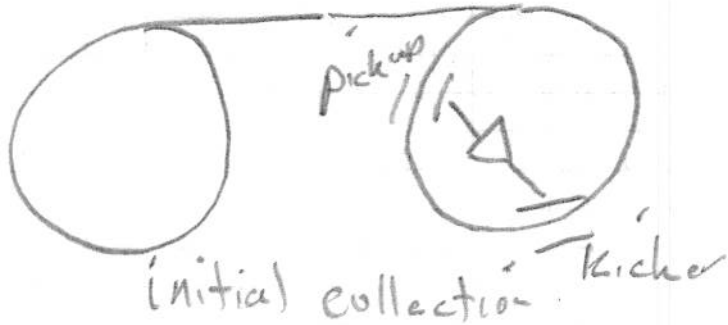
Place a position sensitive pickup in a dispersive region



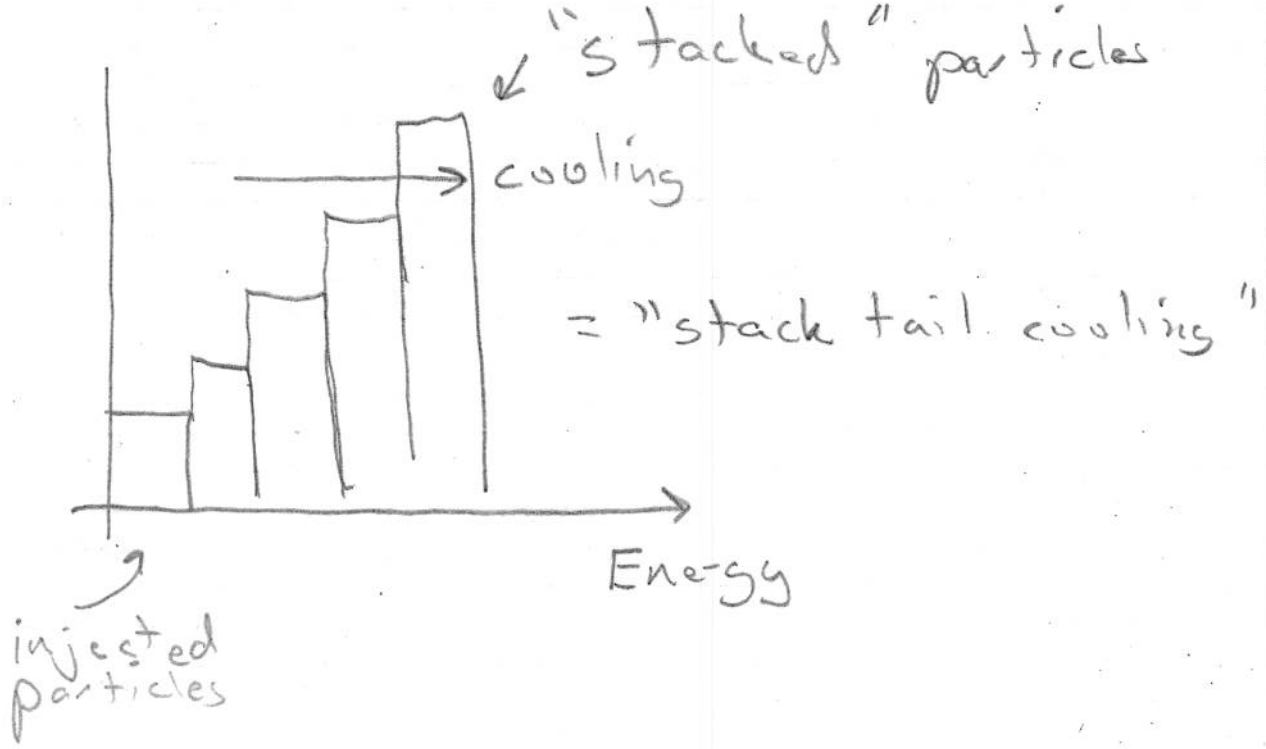
Use a filter to provide a frequency dependent gain



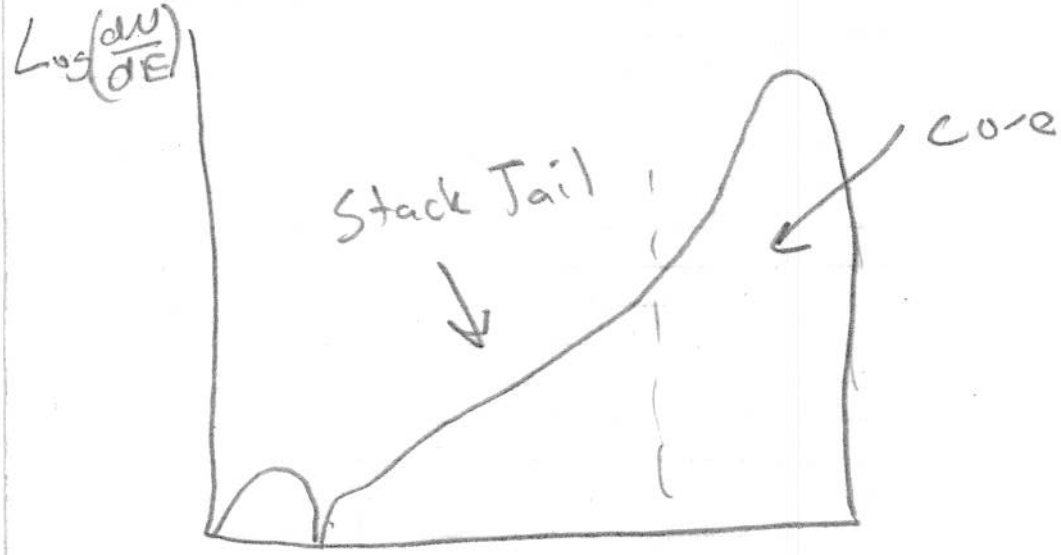
Stochastic energy stacking



Detect ΔN particles at energy $E - \Delta E_k$ and kick the energy up



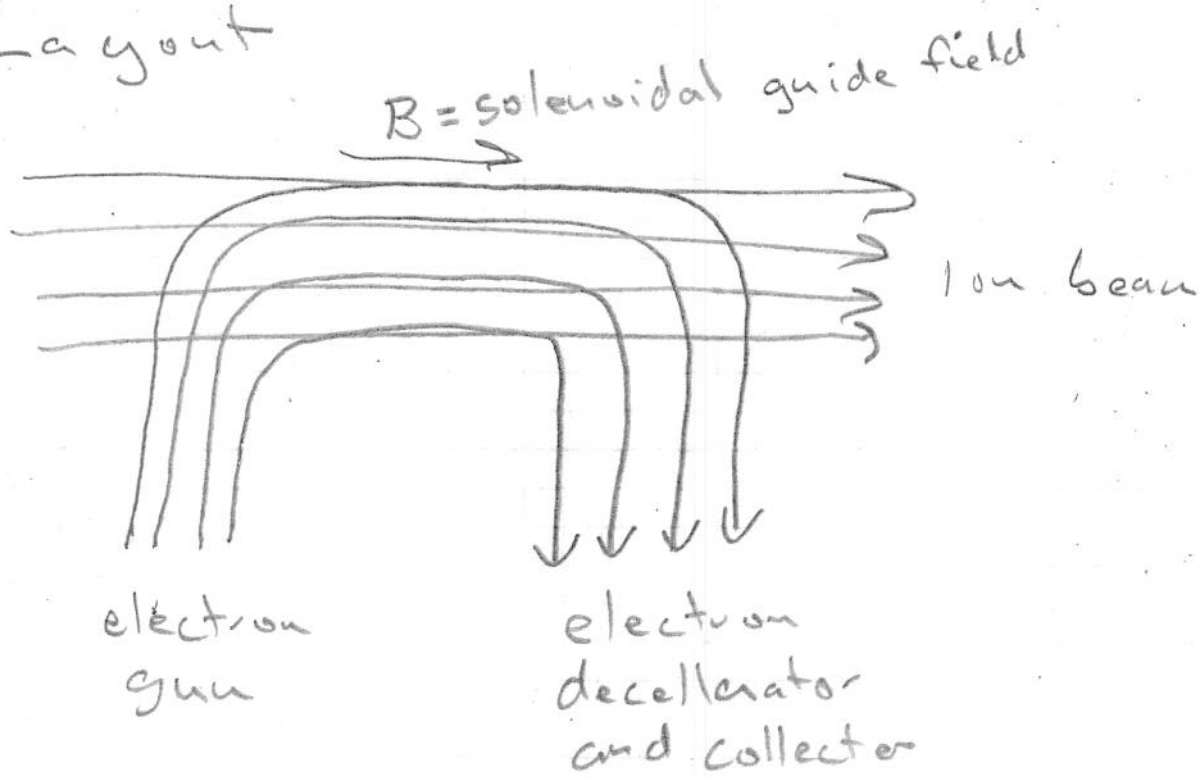
Example: FNAL Accumulator Ring



Electron Cooling

Electron cooling works by injecting "cold electrons" into a beam of negative ions (antiprotons or other) and cooling them through momentum exchange

Layout

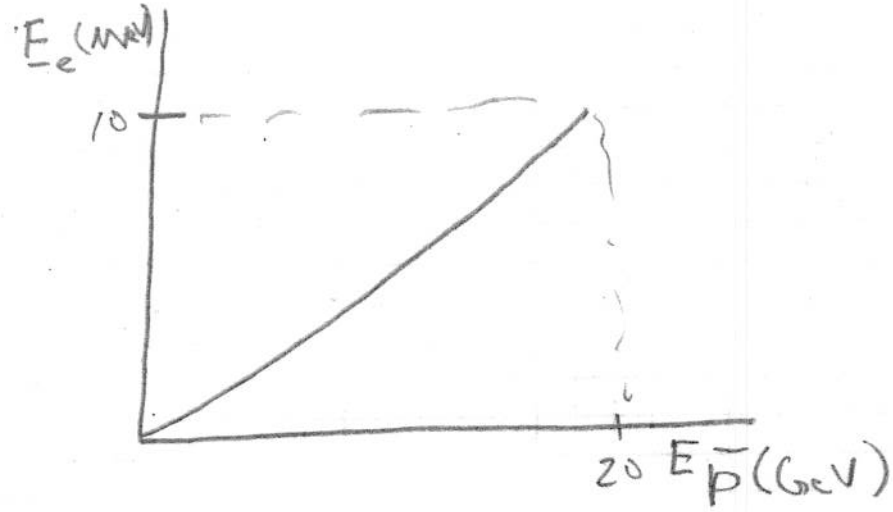


Note, we want $\beta_{electrons} = \beta_{ions}$

$\rightarrow p_{electrons} << p_{ions}$

$$\frac{p_e}{m_e c} = (\beta\gamma)_e = \frac{p_{ion}}{m_{ion} c} = (\beta\gamma)_{ion}$$

$$\rightarrow \quad p_e = \frac{m_e}{m_{ion}} p_{ion}$$



Substantial currents are required, so it's important to recover the electron energy

The electrons act like a drag force on the ions. At low velocities, the ionization energy loss varies as

$$\frac{1}{(v^*)^2}$$

↑ relative velocity

The velocity spread of the electrons is dominated by the energy distribution out of the cathode. In the rest frame, motion is non relativistic, so

$$K = \frac{1}{2}(mc^2)\beta^{*2}$$

$$\rightarrow \delta\beta^* = \sqrt{\frac{2\,\delta K}{mc^2}}$$ ← energy spread, typically ~.5eV

$$\rightarrow \delta\beta_e \sim 1.4 \times 10^{-3}$$

Longitudinal Cooling

the momentum and energy between the rest and lab frames are related by

$$p_{lab} = \gamma\left(p^* - \frac{\beta}{c}E^*\right) \approx \gamma\left(p^* - \beta mc\right)$$ ← β of frame

$$\delta p_{lab} \rightarrow \delta p^* \approx \frac{\delta p_{lab}}{\gamma}$$

$$\rightarrow \delta\beta_{\parallel}^* = \frac{\delta p^*}{mc} = \frac{\delta p}{\gamma mc} = \frac{p}{\gamma mc}\frac{\delta p}{p} = \beta\frac{\delta p}{p}$$

For efficent cooling, we want

$$\delta\beta_{\parallel}^{*} \approx \delta\beta_e = 1.4\times 10^{-3}$$

$$\rightarrow \frac{\delta p}{p} = \frac{1.4\times 10^{-3}}{\beta}$$

→ for low energy beams, large $\frac{\delta p}{p}$ can be tolerated, but as $\beta \rightarrow 1$, only small momentum spreads can be efficiently cooled.

<u>Transvers Cooling</u>

Again, we want

$$\delta\beta_{\perp}^{*} \sim \delta\beta_e^{*}$$

We have

$$\frac{\delta p_{\perp}}{p} = x' \quad ; \quad \delta p_x = \delta p_x^{*}$$

We want

$$\delta\beta_\perp^* = \frac{\delta p_\perp^*}{mc} = \frac{x' p}{mc} = \beta\gamma x'$$

$$\rightarrow \delta\beta_\perp^* \sim \delta\beta_a^* \rightarrow x' \leq \frac{1.4\times10^{-3}}{\beta\gamma}$$

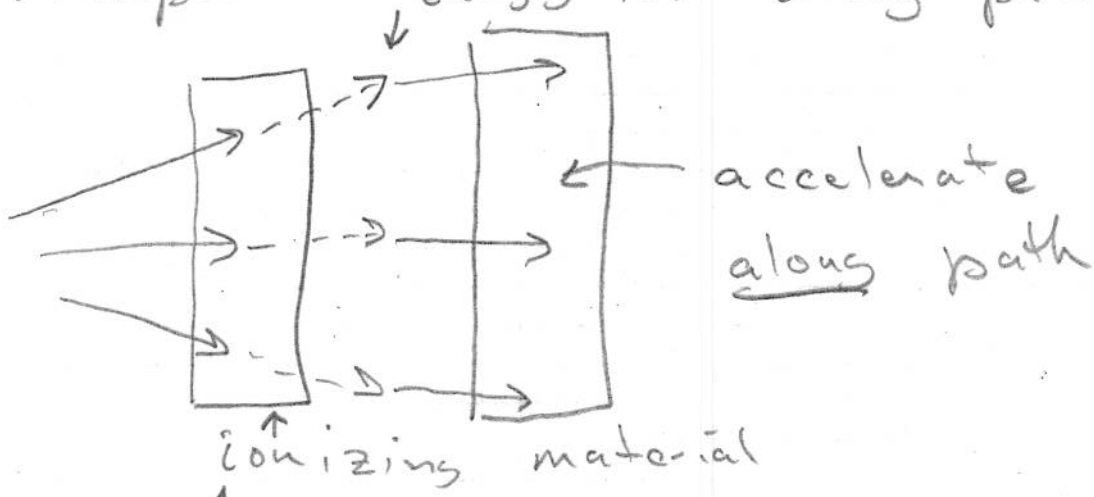
gets less effective for large γ

Ionization Cooling (muons)

For muons:

- radiative cooling won't work, because the muons don't radiate
- stochastic or electron cooling are too slow

Principal



go to a lower energy with the same emittance

→ adiabatically cool as we reaccelerate,

From our discussion of multiple scattering, we have a heating term

$$\frac{d\epsilon}{ds} = \frac{\beta x}{2}\frac{d\langle\theta^2\rangle}{ds}$$

so the change in normalized emittance

$$\frac{d\epsilon_N}{ds} = \frac{d}{ds}(\beta\gamma\epsilon) = \epsilon\frac{d}{ds}(\beta\gamma) + \beta\gamma\frac{d\epsilon}{ds}$$

only from heating

Use

$$\frac{dp}{p} = \frac{m\,d(\gamma\beta)c}{m\gamma\beta c} = \frac{1}{\beta^2}\frac{dE}{E}$$

$$\rightarrow \frac{d(\gamma\beta)}{ds} = \overset{\text{loss}}{-}\frac{\gamma\beta}{\beta^2 E}\frac{dE}{ds}$$

$$\rightarrow \epsilon\frac{d(\gamma\beta)}{ds} = \frac{\overbrace{\epsilon\gamma\beta}^{\epsilon_N}}{\beta^2 E}\frac{dE}{ds}$$

$$\rightarrow \frac{d\epsilon_N}{ds} = \underbrace{-\frac{\epsilon_N}{\beta^2 E}\frac{dE}{ds}}_{\text{cooling}} + \underbrace{\beta\gamma\frac{\beta_x}{2}\frac{d\langle\theta^2\rangle}{ds}}_{\text{heating}}$$

From Coulomb scattering discussion, we have

$$\frac{d\langle\theta^2_{mcs}\rangle}{ds} = \frac{1}{X_0}\left(\frac{13.6\ \text{MeV}}{\beta pc}\right)^2$$

$$\rightarrow \frac{d\epsilon_N}{ds} = -\frac{\epsilon_N}{\beta^2 E}\frac{dE}{ds} + \frac{\beta_x\beta\gamma}{2X_0}\left(\frac{13.6\ \text{MeV}}{\beta pc}\right)^2$$

$$= -\frac{\varepsilon_n}{\beta^2 E}\frac{dE}{ds} + \frac{\beta_x}{2X_0}\frac{(13.6\,\text{MeV})^2}{\beta^3 E m c^2}$$

$$\text{"}\gamma\text{"} = \frac{\beta^2 E}{\left(\frac{dE}{ds}\right)}$$

$$\rightarrow \varepsilon_{n,min} = \text{"}\gamma\text{"}\,\frac{\beta_x}{2X_0}\frac{(13.6\,\text{MeV})^2}{\beta^3 E m c^2}$$

$$\boxed{= \frac{\beta_x}{2X_0\left(\frac{dE}{ds}\right)}\frac{(13.6\,\text{MeV})^2}{\beta m c^2}}$$

Want

- β_x small

- X_0 large $\rightarrow$ low Z material
  - H_2 ?
  - Li ?