SLAC ν Day
April 18, 2003

The Neutrino Mass/Mixing Landscape
Questions and Pathways

A talk related to the
July 24 HEPAP talk

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What Is There To Know?

What physics is responsible for neutrino masses and mixing?

What is the scale of neutrino mass?

Are neutrinos Majorana particles ($\overline{\nu} = \nu$)?

Do neutrino interactions violate CP?

Is leptonic ~~CP~~ responsible for the baryon asymmetry in the universe?

How many neutrino species are there?

What is the neutrino mass spectral pattern?

What is the leptonic mixing matrix?

The Plan

Emphasize what we do not know.

Focus on interesting questions to be answered in the future, explaining why they are interesting.

Describe pathways to answering them.

Only brief summary of what has already been learned, since that was covered in seminar of March 11 (on the web).

What We Already Do Know

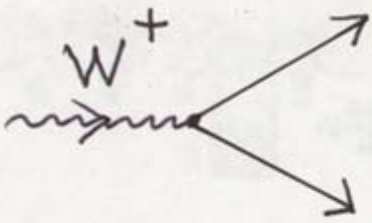
We do not know how many neutrino mass eigenstates ν_i there are.

Confirmation of LSND by MiniBooNE would imply there are 4 or more.

One linear combination of the 4 must be sterile — noninteracting.

Neutrino physics would be greatly affected.

We will assume there are only 3 mass eigenstates ν_i, and 3 corresponding flavor eigenstates ν_α:



$(l_e \equiv e, l_\mu \equiv \mu, l_\tau \equiv \tau)$

$$\nu_\alpha = \sum_i U_{\alpha i}^* \nu_i$$

Neutrino of flavor α (ν_e, ν_μ, or ν_τ)

Unitary 3×3 Leptonic Mixing Matrix

Inverting,

$$\nu_i = \sum_\alpha U_{\alpha i} \nu_\alpha$$

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

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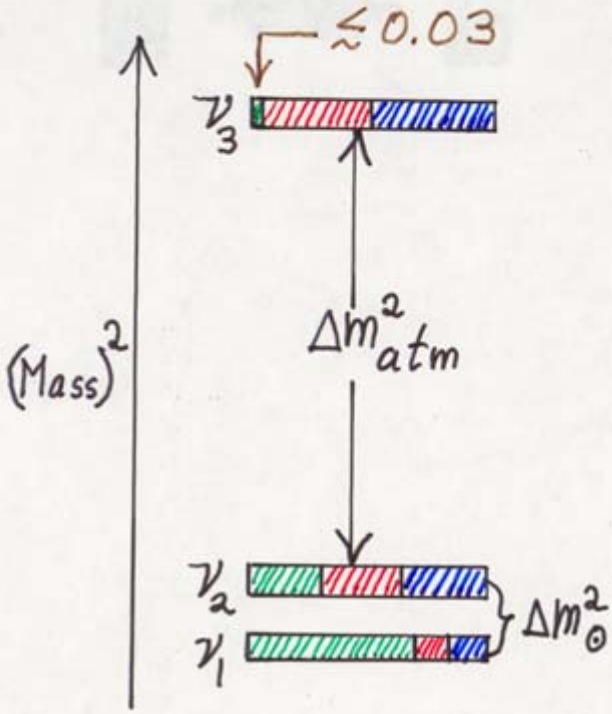
The 3-ν spectrum is $\begin{matrix}-\\=\end{matrix}$ or $\begin{matrix}=\\-\end{matrix}$:



$1.6 \times 10^{-3} < \Delta m^2_{atm} < 3.9 \times 10^{-3}\,\text{eV}^2$

(Super-K; 90% CL)

$5.6 \times 10^{-5} < \Delta m^2_\odot < 9.0 \times 10^{-5}\,\text{eV}^2$

— or —

$13 \times 10^{-5} < \Delta m^2_\odot < 18 \times 10^{-5}\,\text{eV}^2$

(Fogli et al.; 95% CL)

$\nu_e\ [|U_{ei}|^2]$ $\nu_\mu\ [|U_{\mu i}|^2]$ $\nu_\tau\ [|U_{\tau i}|^2]$

B.8

A Cosmic Connection

WMAP + Other Cosmological Data

+ Cosmological Assumptions

$$\Rightarrow \quad \sum_i m_i < 0.71 \text{ eV}.$$

m_i: Mass(ν_i)

(95% CL, Spergel et al.)

Tension between this bound and 4-neutrino spectra.

(Pierce & Murayama, Giunti, Strumia)

If there are only 3 neutrinos,

$$0.05\text{ eV} \lesssim \text{Mass}[\text{Heaviest } \nu_i] < 0.23\text{ eV}$$

0.05 eV: $\sqrt{\Delta m^2_{atm}}$

0.23 eV: Cosmology

The Mixing Matrix

The flavor content picture shows the $|U_{\alpha i}|^2$, but not the signs or phases of the $U_{\alpha i}$.

For 3 neutrinos —

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}$$

$$\times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} e^{i\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ [27]

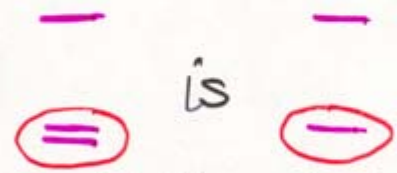
$\theta_{12} \simeq \theta_{\odot} \simeq 34^\circ$, $\theta_{23} \simeq \theta_{atm} \simeq 45^\circ$

$\theta_{13} \lesssim 10^\circ$

The "Two" Neutrino Case

- Only 2 ν_i couple significantly to the initial l_α. (Solar ν case)

or

- The experiment thinks  is

because its L/E is too small for it to see the small splitting. (Atm ν case)

Then—

$$P(\nu_\alpha \to \nu_\alpha) \cong 1 - \sin^2 2\theta \, \sin^2\left(\Delta m^2 \frac{L}{4E}\right)$$

The mixing angle

The sole (mass)^2 splitting

6 The 90% CL ranges —

$0.24 \lesssim \sin^2\theta_{\odot} \lesssim 0.38$

(Lisi)

$0.36 < \sin^2\theta_{atm} < 0.64$

(Super-K)

$\sin^2\theta_{13} \lesssim 0.03$

(CHOOZ, Palo Verde)

[Maximal mixing $\Longrightarrow \sin^2\theta = 1/2$]

We must remember —

LSND and KARMEN $\Longrightarrow$

$0.2 \lesssim \Delta m^2_{LSND} \lesssim 1\,eV^2$ & $.003 \lesssim \sin^2 2\theta_{LSND} \lesssim .03$

— or —

$\Delta m^2_{LSND} \simeq 7\,eV^2$ & $\sin^2 2\theta_{LSND} \simeq .004$

(Church et al.)

– INTERESTING QUESTIONS –

Why Are ν Masses $\ll$ Quark, Charged Lepton Masses?

Is $\overline{\nu} = \nu$?

Suppose $m[\text{Heaviest } \nu_i] \simeq 0.05$ eV.

Suppose we extend the Standard Model to accommodate this via a Dirac mass term:

$$f_i\,(\text{Higgs})\,\overline{\nu}_R\,\nu_L \longrightarrow \underbrace{f_i \overbrace{\langle \text{Higgs} \rangle_0}^{174\,\text{GeV}}}_{m_i = 0.05\,\text{eV}}\,\overline{\nu}_R\,\nu_L$$

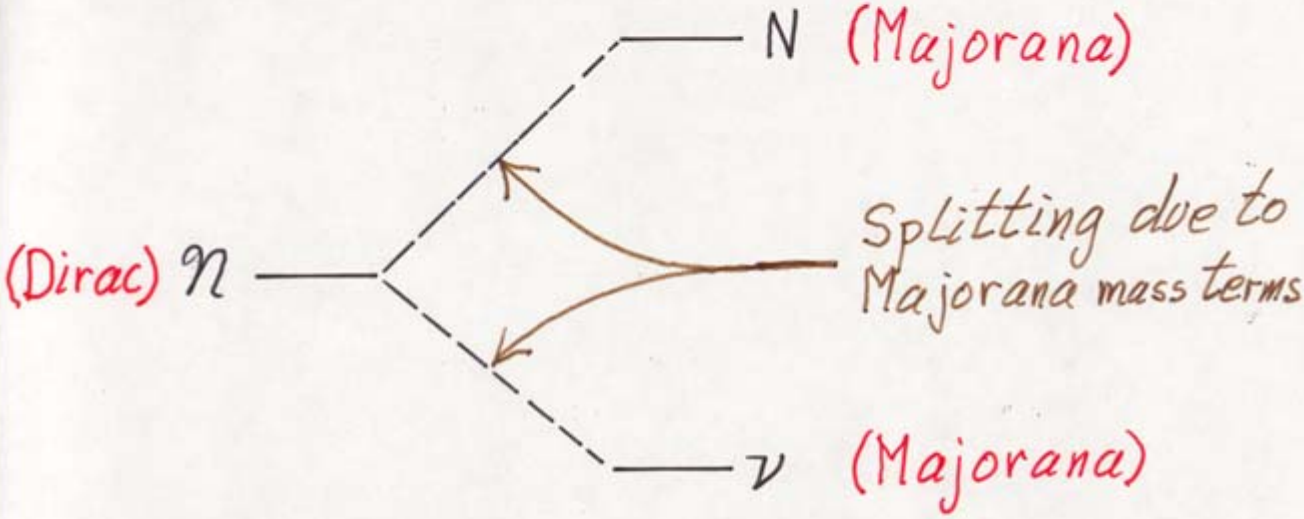
$$\Longrightarrow \quad f_i \sim 10^{-13}$$

Surely such infinitesimal couplings are not the ultimate explanation of light m_ν.

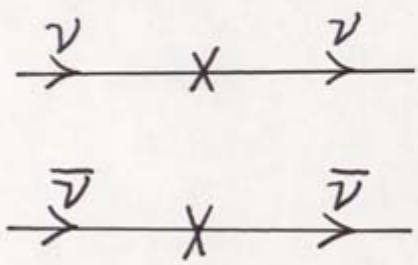
See-Saw Mechanism

A Dirac neutrino $[\bar{\nu} \neq \nu]$: 4 states

A Majorana neutrino $[\bar{\nu} = \nu]$: 2 states

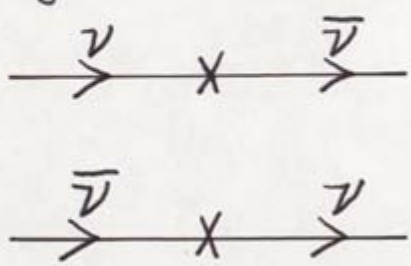


Dirac mass term



Conserves L

Majorana mass term



Violates L

9 Nothing forbids large Majorana mass terms.

In the see-saw,

$$\text{Mass terms} = [\bar{\nu}, \nu] \begin{bmatrix} 0 & M_D \\ M_D & M_M \end{bmatrix} \begin{bmatrix} \nu \\ \bar{\nu} \end{bmatrix},$$

(M_D: Dirac; M_M: Majorana)

with

$$M_M >> M_D .$$

We expect $M_D \sim$ Quark or Charged Lepton Mass

The eigenvalues are approximately —

$$M_M \equiv m_N \quad , \quad \frac{M_D^2}{M_M} \equiv m_\nu .$$

$$m_\nu m_N = M_D^2 \sim (\text{Quark/Chg'd Lepton Mass})^2$$

See-Saw Relation (Gell-Mann, Ramond, Slansky; Yanagida; Mohapatra + Senjanovic)

Predictions

- Each $\overline{\nu_i} = \nu_i$ (Majorana neutrinos)
- The light neutrinos have heavy partners N

How heavy ??

$$m_N \sim \frac{m_{top}^2}{m_\nu} \sim \frac{m_{top}^2}{0.05\,eV} \sim 10^{15}\,GeV$$

Near the GUT scale.

Review of see-saw: hep-ph/0211134

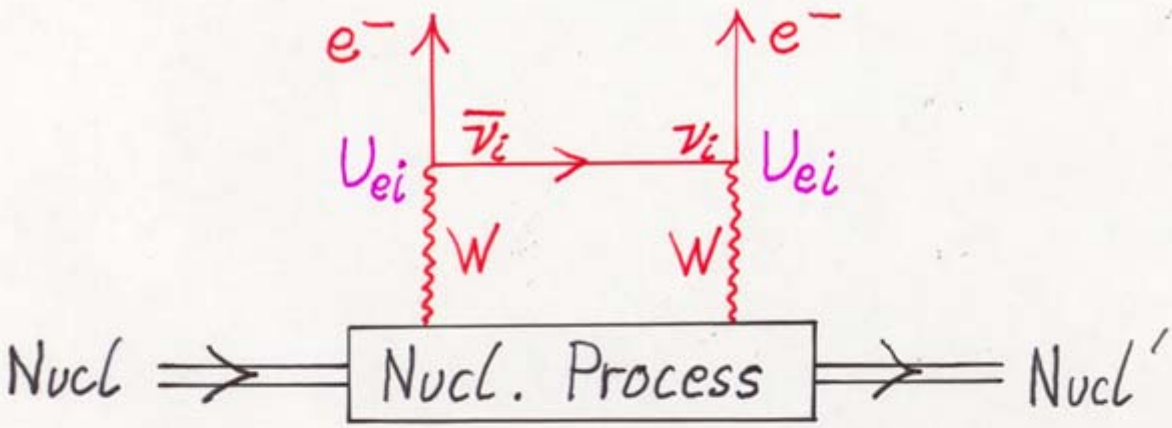
How can we confirm that $\overline{\nu_i} = \nu_i$?

Assuming CPT invariance —

$0\nu\beta\beta\ [Nucl \to Nucl' + 2e^-] \Rightarrow \overline{\nu_i} = \nu_i$.

(Schechter + Valle)

The dominant mechanism is expected to be —



$\overline{\nu_i}$ is emitted [RH + $\mathcal{O}(\frac{m_i}{E})$ LH]

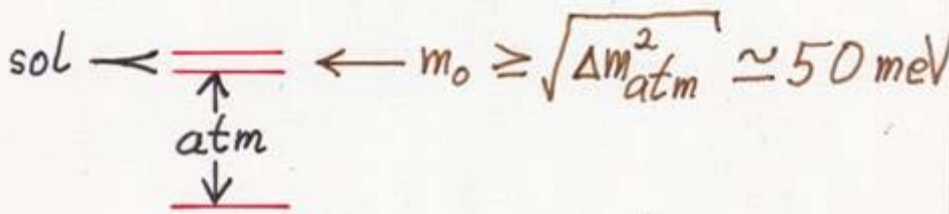
$\therefore$ Amp[ν_i contribution] $\propto m_i$

$$Amp[0\nu\beta\beta] \propto \left| \sum_i m_i U_{ei}^2 \right| \equiv m_{\beta\beta}$$

$m_{\beta\beta}$ is a measure of the ν mass scale.

Desirable sensitivity: $m_{\beta\beta} \lesssim 50$ meV.

If the spectrum looks like —



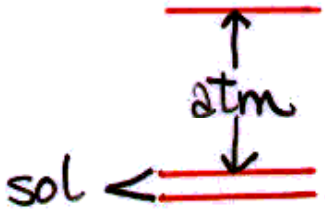
then —

$$m_{\beta\beta} \cong m_0 \sqrt{1 - \sin^2 2\theta_\odot \sin^2\left(\frac{\alpha_2 - \alpha_1}{2}\right)}$$

$$\geq m_0 \cos 2\theta_\odot .$$

$\cos 2\theta_\odot \gtrsim 0.2$ @ 90% CL.

If the spectrum looks like —



then

$$0 < m_{\beta\beta} < \text{Present Bound } [(0.3\text{–}1.0)\ \text{eV}] .$$

(Petcov et al.)

Are There Other Ways to Confirm $\overline{\nu_i} = \nu_i$?

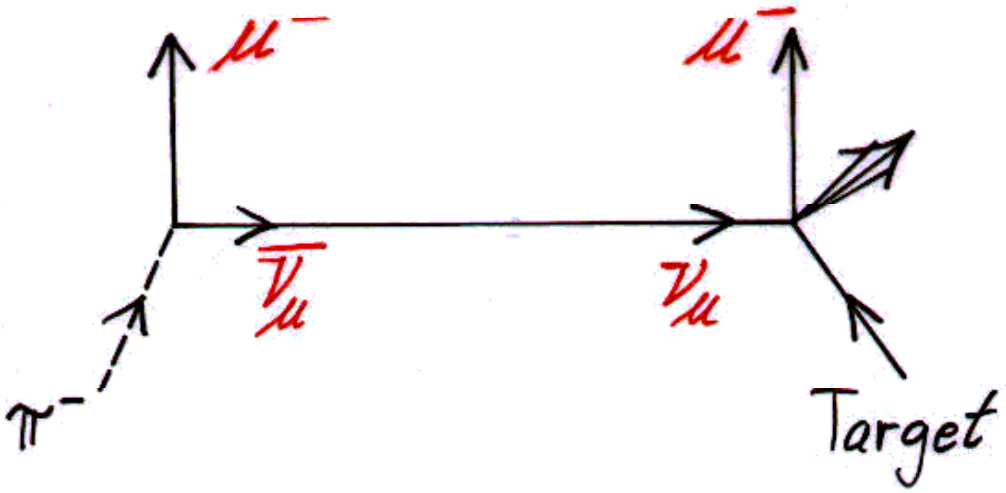
So far, no .

ν oscillation ?

$$A(\nu_\alpha \to \nu_\beta) = \sum_i U^*_{\alpha i}\, e^{-i m_i^2 \frac{L}{2E}}\, U_{\beta i}$$

Only size, not origin, matters

$\bar{\nu} \rightarrow \nu$ oscillation?



$$\text{Amp} \propto \frac{m_\nu}{E}$$

Invisible

$0\nu\beta\beta$?

$$\text{Amp} \propto \frac{m_{\beta\beta}}{E}$$

But the competing decays are highly suppressed.

Need a big source, low backgrounds, and patience.

[13] # Major Questions for Future Accelerator ν Experiments

* How big is θ_{13} ?

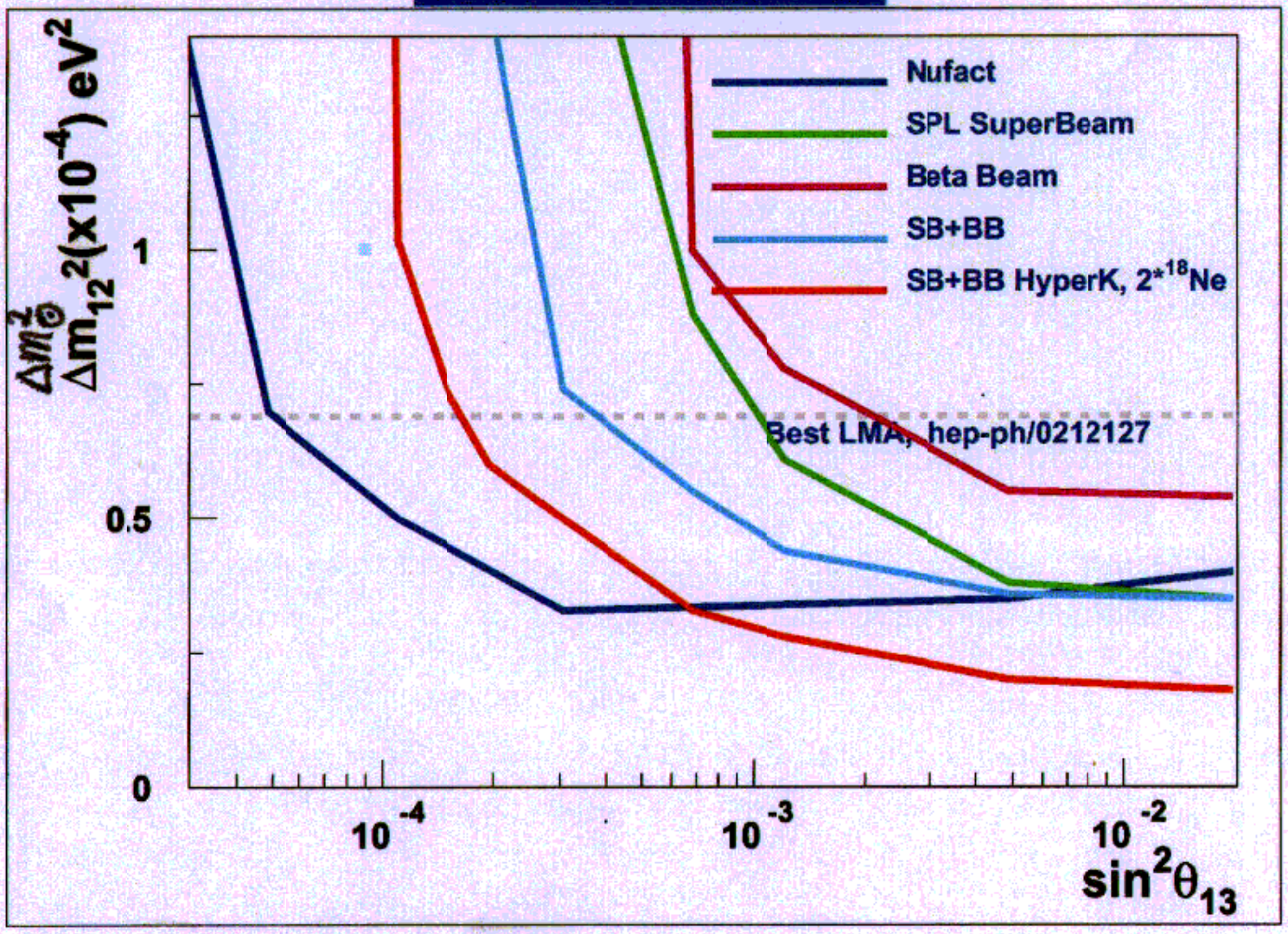
* Does neutrino oscillation violate CP?
  If there are only 3 neutrinos,
  $P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \propto \sin\theta_{13}$.

* If there are only 3 neutrinos,
  is the spectral pattern like $\begin{matrix}-\\=\end{matrix}$ or like $\begin{matrix}=\\-\end{matrix}$?

  Answering this depends on θ_{13}.

* Is atmospheric ν mixing truly maximal?
  Is a symmetry, like CP in $K^0 \leftrightarrow \bar{K}^0$, involved?

Final CP sensitivity.



V.16] How big do we expect θ_{13} to be??

A prejudice

In gauge theory,

$$U = X_\ell X_\nu = \begin{bmatrix} B & B & \theta_{13} \\ B & B & B \\ B & B & B \end{bmatrix} ; \; B \equiv \text{Big}$$

Diagonalizes ℓ mass matrix

Diagonalizes ν mass matrix

Except for $U_{e3} \sim \theta_{13}$,

$$\text{all } U_{\alpha i} = \sum_j (X_\ell)_{\alpha j} (X_\nu)_{ji} \text{ are Big}$$

It would take a special cancellation (caused by a symmetry??) for U_{e3} to be very much smaller than all other $U_{\alpha i}$.

How θ_{13} May Be Measured

$\sin^2\theta_{13} = |U_{e3}|^2$ is the small ν_e piece of ν_3. ν_3 is at one end of Δm^2_{atm}.

∴ We need an experiment with L/E sensitive to Δm^2_{atm}, and involving ν_e.

In a Long Base Line (LBL) experiment, neglecting matter effects,

$$P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) \cong P(\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu) \cong \frac{1}{2}\sin^2 2\theta_{13}\,\sin^2\left(\Delta m^2_{atm}\frac{L}{4E}\right)$$

[R]

How $\overline{=}$ vs. $=\!\!=$ May Be Determined

We determined that $m(K_L) > m(K_S)$ by —

- Passing kaons through matter (Regenerator)
- Beating the unknown Sign $[m(K_L) - m(K_S)]$ against the known Sign [Regeneration Amp.]

[2] θ_{13} with Reactor $\overline{\nu}_e$

For $\Delta m^2_{atm} \frac{L}{4E} \sim 1 \quad \left[\frac{L}{E} \sim \frac{1\ km}{1\ MeV}\right]$,

$$P(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 (\Delta m^2_{atm} \frac{L}{4E}).$$

Look for a very small disappearance of $\overline{\nu}_e$ flux. [Present bound: $\sin^2 2\theta_{13} < 0.1$]

Systematics?

F.15] To determine—

$$\text{Sign}[m^2(—) - m^2(=)] \equiv S,$$

- Pass neutrinos through matter
- Beat the unknown sign S against the known Sign [Extra Energy of ν_e in Matter]

From $\nu_e e \to \nu_e e$

Coherent forward

in matter

— raises ν_e energy.

— lowers $\bar{\nu}_e$ energy.

In earth matter of ~ constant density,

$$P(\overrightarrow{\nu}_\mu \to \overrightarrow{\nu}_e) \cong P(\overrightarrow{\nu}_e \to \overrightarrow{\nu}_\mu) \cong \frac{1}{2} \sin^2 2\overleftarrow{\theta}_M \sin^2\left(\overrightarrow{\Delta m^2_M} \frac{L}{4E}\right)$$

Same formula as in vacuum, but with θ_{13} and Δm^2_{atm} replaced by their effective values in matter.

[3ν spectrum]

If $\overrightarrow{\Delta m^2_M} \lessgtr \Delta m^2_{atm}$.

If $\overrightarrow{\Delta m^2_M} \gtrless \Delta m^2_{atm}$.

One can easily show that –

$$\overrightarrow{\Delta m^2_M} \sin 2\overleftarrow{\theta}_M = \Delta m^2_{atm} \sin 2\theta_{13}.$$

If the (mass)^2 gap shrinks, the mixing grows.

How May We Seek ~~CP~~ in Oscillation?

Compare ν and $\bar{\nu}$ oscillation.

If there are $\geq 4\nu$, the ~~CP~~ pattern may be very rich.

If there are only 3ν, <u>in principle</u> the pattern is very simple —

Let $P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \equiv \Delta_{CP}(\alpha\beta)$.

If there are only 3 neutrinos,

$$\Delta_{CP}(e\mu) = \Delta_{CP}(\mu\tau) = \Delta_{CP}(\tau e)$$

$$= 16\, J\, k_{12} k_{23} k_{31} ,$$

where

$$J \equiv Im(U_{e1}^* U_{e3} U_{\mu 1} U_{\mu 3}^*) \cong \frac{1}{4} \sin 2\theta_0 \sin\theta_{13} \sin\delta ,$$

and

$$k_{ij} \equiv \sin\left(\Delta m_{ij}^2 \frac{L}{4E}\right) .$$

In experiments being considered, perhaps

$$\Delta_{CP} \sim 1\% .$$

Leptonic $\not{CP}$ vs. Quark $\not{CP}$

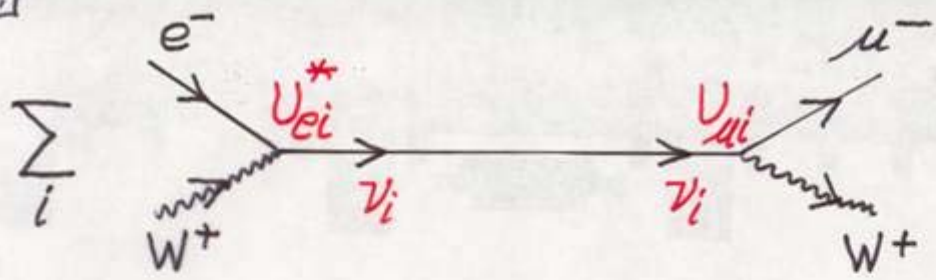
The $\not{CP}$ Phases

The 3x3 Quark mixing matrix contains only one $\not{CP}$ phase.

The 3x3 Lepton mixing matrix may contain three $\not{CP}$ phases.

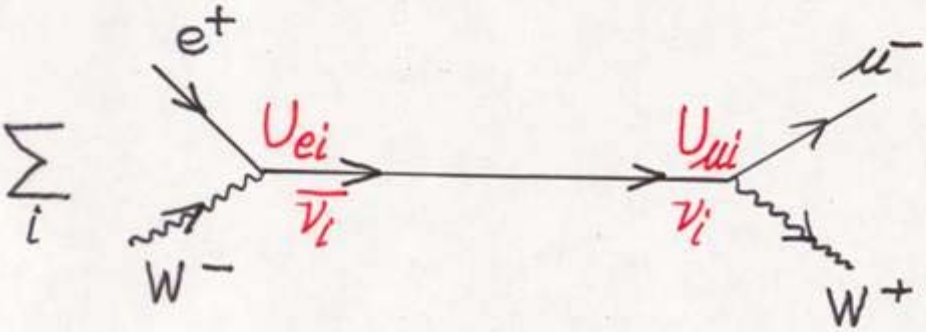
Each extra phase factor $e^{i\frac{\alpha_i}{2}}$ multiplies all $U_{\alpha i}$ for a given i: $U_{\alpha i} = U^{(0)}_{\alpha i} e^{i\frac{\alpha_i}{2}}$.

The α_i have no consequences unless L is violated ($\overline{\nu}_i \leftrightarrow \nu_i$):



The α_i cancel

but —



The α_i matter

We cannot have antiquark $\leftrightarrow$ quark (electric charge conservation), so the quark analogues of $\alpha_{1,2}$ have no consequences.

Neutrinos	B Decays
No hadronic uncertainties	No hadronic uncertainties in selected decays
Same $\not{CP}$ difference in all channels; $\propto J$	Different $\not{CP}$ differences in different decays; α vs. β vs. γ

In Practice

Individual oscillation probabilities depend on the ~~CP~~ phase δ, on CP-conserving variables like θ_{13}, and on matter effects.

Complementary measurements will be needed to disentangle the parameters.

Barenboim, Barger, de Gouvea, Geer, Gomez-Cadenas, Harris, Huber, Lindner, Marfatia, Minakata, Nunokawa, Para, Parke, Szleper, Velasco, Whisnant, Winter, Gavela, Mena, Raja

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Correlations & Ambiguities

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Exracting precise & unambiguous values for all of the three-flavor oscillation parameters (Δm_{32}^2, Δm_{21}^2, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, $\sin^2 2\theta_{12}$, $\delta = 0$) will be challenging :

$$P(\nu_e \to \nu_\mu) \approx \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2}$$
$$\pm \sin\delta_{CP}\, \alpha \sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\sin(\Delta)\frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})}$$
$$+ \cos\delta_{CP}\, \alpha \sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\cos(\Delta)\frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})}$$
$$+ \alpha^2 \sin^2 2\theta_{12}\cos^2\theta_{23}\frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}$$

Fits prone to correlations between the parameters & to degenerate (false) solutions

$\alpha = \Delta m^2_{21}/\Delta m^2_{31}$, $\Delta = \Delta m^2_{31}$ L/4E, $A = 2VE/\Delta m^2_{31}$, $V = \sqrt{2}G_F n_e$

Note: (i) For $\nu \to \bar{\nu}$, $A \to -A$, (ii) The sign of A depends on the sign of Δm_{32}^2

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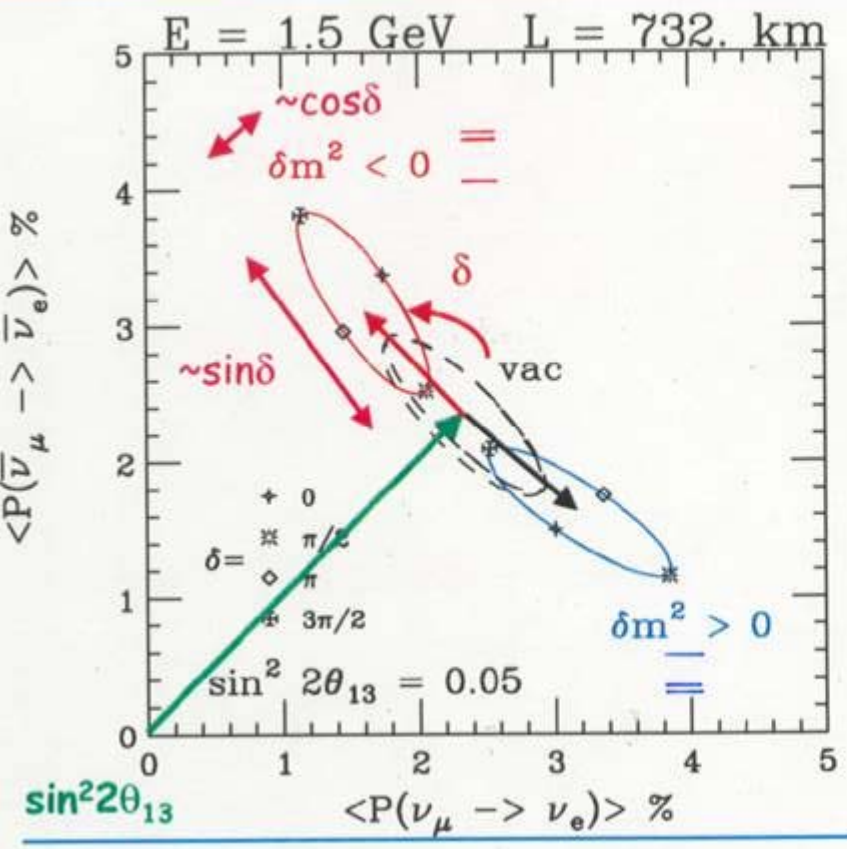
Anatomy of Bi-probability ellipses



Minakata and Nunokawa, hep-ph/0108085

Observables are:

- P
- P

Interpretation in terms of $\sin^2 2\theta_{13}$, δ and sign of Δm^2_{23} depends on the value of these parameters and on the conditions of the experiment: L and E

Why Is $\not{CP}$ in ν Oscillation So Interesting?

Demonstrating that $\not{CP}$ in oscillation is nonzero would establish that $\not{CP}$ is not a peculiarity of quarks.

Leptonic $\not{CP}$ might have been the $\not{CP}$ that made baryogenesis possible.

In the See-Saw mechanism, the light neutrinos have very heavy neutral lepton partners N, with

$$m_N \sim \frac{m_{top}^2}{m_\nu} \sim 10^{15}\ \text{GeV}.$$

Each N is a Majorana particle ($\bar{N}=N$).

Perhaps in the early universe there was

$$\Gamma[N \to \ell^+ + \text{Higgs}^-] > \Gamma[N \to \ell^- + \text{Higgs}^+].$$

(CP)

Leptogenesis

Standard Model (B−L)− conserving, but B- and L-violating, processes would then have converted some of this antilepton excess into a baryon excess.

(Fukugita & Yanagida)

How Is ~~CP~~ in ν Oscillation Related To ~~CP~~ in Leptogenesis?

The relation is model-dependent.

However —

It is not likely that we have one without the other.

(Davidson, Pascoli, Petcov, Rodejohann, Yanagida)

C

Conclusion

There are very interesting open questions about neutrinos.

Answering them will be challenging and very rewarding