Supersymmetry: Motivation, Algebra, Models and Signatures

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Outline

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  Motivation

  Introduction to Supersymmetry

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  Supersymmetric Interactions

  The Minimal Supersymmetric extension of the Standard Model

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Lecture 2:

Supersymmetric Interactions

The Minimal Supersymmetric extension of the Standard Model

The Supersymmetric Lagrangian

The total Lagrangian is given by:

$$\begin{aligned}\mathcal{L}_{\rm SUSY} &= \frac{1}{4g^2}\left(Tr[W^\alpha W_\alpha]_F + h.c.\right) + \sum_i \left(\bar{\Phi}\exp(gV)\Phi\right)_D \\ &+ \left([P(\Phi)]_F + h.c.\right)\end{aligned}$$

where $P(\Phi)$ is the most generic dimension-three, gauge invariant, polynomial function of the chiral fields Φ, and it is called Superpotential. It has the general expression

$$P(\Phi) = \frac{m_{ij}}{2}\Phi_i\Phi_j + \frac{\lambda_{ijk}}{3!}\Phi_i\Phi_k\Phi_k$$

and

$$V(x,\theta,\bar{\theta}) = -\left(\theta\sigma^\mu\bar{\theta}\right)V_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D$$

$$W^\alpha(x,\theta,\bar{\theta}=0) = -i\lambda^\alpha + \left(\theta\sigma_{\mu\nu}\right)^\alpha F^{\mu\nu} + \theta^\alpha D - \theta^2\left(\bar{\sigma}^\mu\mathcal{D}_\mu\bar{\lambda}\right)^\alpha$$

$$[\Phi] = 1, \qquad [W_\alpha] = 3/2, \qquad [V] = 0.$$

and one should remember that $[V]_D = [V] + 2$; $[\Phi]_F = [\Phi] + 1$.

SUSY Lagrangian in term of Component Fields

- The above Lagrangian has the usual kinetic terms for the boson and fermion fields. It also contain generalized Yukawa interactions and contain interactions between the gauginos, the scalar and the fermion components of the chiral superfields.

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SUSY}} &= (\mathcal{D}_\mu A_i)^\dagger \mathcal{D} A_i + \left(\frac{i}{2}\bar{\psi}_i \bar{\sigma}^\mu \mathcal{D}_\mu \psi_i + \mathrm{h.c.}\right) \\
&- \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left(\frac{i}{2}\bar{\lambda}^a \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \mathrm{h.c.}\right) \\
&- \left(\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i\psi_j - i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c.\right) \\
&- V(F_i, F_i^*, D^a)
\end{aligned}
$$

The last term is a scalar potential term that depends only on the auxiliary fields

Notation bookkeeping

- All standard matter fermion fields are described by their left-handed components (using the charge conjugates for right-handed fields) ψ_i
- All standard matter fermion superpartners are described **by the scalar** fields A_i. There is one complex scalar for each chiral Weyl fermion
- Gauge bosons are inside covariant derivatives and in the $G_{\mu\nu}$ terms.
- Gauginos, the superpartners of the gauge bosons are described by the fermion fields λ_a. There is one Weyl fermion for each massless gauge boson.
- Higgs bosons and their superpartners are described as regular chiral fields. Their only distinction is that their scalar components acquire a v.e.v. and, as we will see, they are the only scalars with positive R-Parity.

The Scalar Potential

$$V(F_i, F_i^*, D^a) \;=\; \sum_i F_i^* F_i \;+\; \frac{1}{2}\sum_a (D^a)^2$$

where the auxiliary fields may be obtained from their equation of motion, as a function of the scalar components of the chiral fields:

$$F_i^* = -\frac{\partial P(A)}{\partial A_i}, \qquad\qquad D^a = -g\sum_i (A_i^* T^a A_i)$$

Observe that the quartic couplings are governed by the gauge couplings and that scalar potential is positive definite ! The latter is not a surprise. From the supersymmetry algebra, one obtains,

$$H = \frac{1}{4}\sum_{\alpha=1}^{2}\left(Q_\alpha^\dagger Q_\alpha + Q_\alpha Q_\alpha^\dagger\right)$$

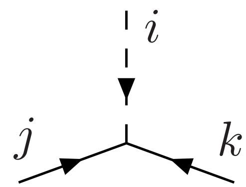
- If for a physical state the energy is zero, this is the ground state.
- Supersymmetry is broken if the vacuum energy is non-zero !

Couplings

From the scalar part of the superpotential $P(A) = \frac{m_{ij}}{2} A_i A_j + \frac{\lambda_{ijk}}{6} A_i A_j A_k$

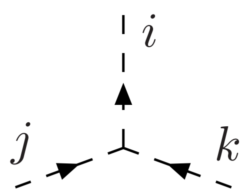
- The Yukawa couplings between scalar and fermion fields

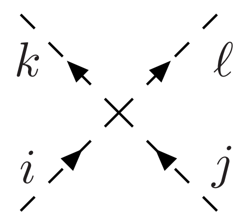
$$\frac{1}{2} \frac{\partial^2 P(A)}{\partial A_i \partial A_j} \psi_i \psi_j + h.c. \quad \longrightarrow \quad \lambda_{ijk}\ \psi_i\ \psi_j\ A_k$$



are governed by the same couplings as the scalar interactions coming from

$$\left(\frac{\partial P(A)}{\partial A_i} \right)^2 \longrightarrow m^*_{ml} \lambda_{mjk}\ A_i^*\ A_j A_k \text{ and } \lambda_{mjk}\ \lambda^*_{mil} A_j A_k A_i^* A_l^*$$





The superpotential parameters determine all non-gauge interactions

- Similarly, the gaugino-scalar-fermion interactions coming from

$$-i\sqrt{2}gA_i^*T_a\psi_i\lambda^a + h.c.$$

are governed by the gauge couplings

No new Couplings!
same couplings are obtained by replacing particles by their superpartners and changing the spinorial structure

Masses

The superpotential parameters determine also the matter field masses and give equal masses to fermions and scalars when the Higgs acquires a v.e.v

$$m_f^2 = m_s^2 = \lambda_{ffh}^2 \; \mathrm{v}^2$$

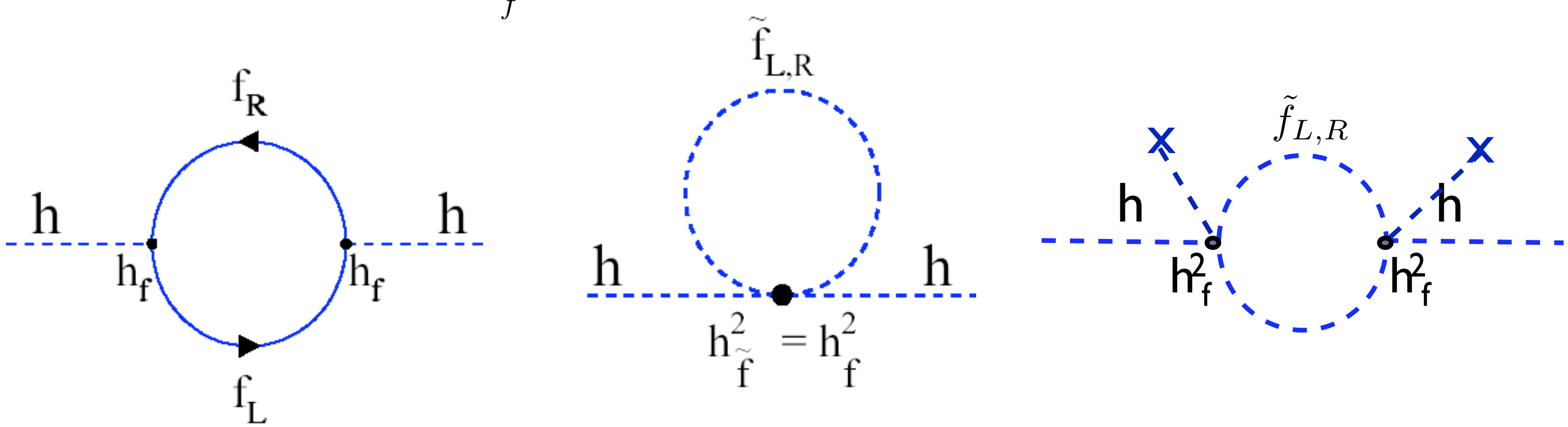
SUSY corrections to the Higgs mass parameter:

Self energy of an elementary scalar related by SUSY to the self energy of a fermion

Cancellation of quadratic divergences in Higgs mass quantum corrections has to do with SUSY relation between couplings and bosonic and fermionic degrees of freedom

$$\Delta\mu^2 \approx g^2_{hf\bar{f}}[m_f^2 - m_{\tilde{f}}^2]\ln(\Lambda^2_{eff}/m_h^2)$$

SUSY must be broken in nature



In low energy SUSY: quadratic sensitivity to Λ_{eff} replaced by quadratic sensitivity to SUSY breaking scale

The scale of SUSY breakdown must be of order 1 TeV, if SUSY is associated with scale of electroweak symmetry breakdown

Properties of Supersymmetric theories

- To each complex scalar A_i (two degrees of freedom) there is a Weyl fermion ψ_i (two degrees of freedom)
- To each gauge boson V_μ^a, there is a gauge fermion (gaugino) λ^a.
- The mass eigenvalues of fermions and bosons are the same !
- Theory has only logarithmic divergences in the ultraviolet associated with wave-function and gauge-coupling constant renormalizations.
- Couplings in superpotential $P[\Phi]$ have no counterterms associated with them.
- The equality of fermion and boson couplings are essential for the cancellation of all quadratic divergences, at all oders in perturbation theory.

Types of supermultiplets

Chiral (or “Scalar” or “Matter” or “Wess-Zumino”) supermultiplet:

1 two-component Weyl fermion, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

2 real spin-0 scalars = 1 complex scalar. ($n_B = 2$)

The Standard Model quarks, leptons and Higgs bosons must fit into these.

Gauge (or “Vector”) supermultiplet:

1 two-component Weyl fermion gaugino, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

1 real spin-1 massless gauge vector boson. ($n_B = 2$)

The Standard Model $\gamma, Z, W^\pm, g$ must fit into these.

The SUSY extension of the Standard Model (MSSM)

- Apart from the superpotential $P[\Phi]$, all other properties are directly determined by the gauge interactions of the theory.
- To construct the superpotential, one should remember that chiral fields contain only left-handed fields, and right-handed fields should be represented by their charge conjugates.
- SM right-handed fields are singlet under $SU(2)$. Their complex conjugates have opposite hypercharge to the standard one.
- There is one chiral superfield for each chiral fermion of the Standard Model.
- In total, there are 15 chiral fields per generation, including the six left-handed quarks, the six right-handed quarks, the two left-handed leptons and the right-handed charged leptons.

The Minimal Supersymmetric Standard Model

Chiral Supermultiplets

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \quad \widetilde{d}_L)$	$(u_L \quad d_L)$	$(\ \mathbf{3},\ \mathbf{2}\ ,\ \frac{1}{6})$
($\times 3$ families)	U	$\widetilde{u}^*_R$	$(u^C)_L$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	D	$\widetilde{d}^*_R$	$(d^C)_L$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \quad \widetilde{e}_L)$	$(\nu \quad e_L)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$
($\times 3$ families)	E	$\widetilde{e}^*_R$	$(e^C)_L$	$(\ \mathbf{1},\ \mathbf{1},\ 1)$

The superpartners of the Standard Model particles are written with a ~

Scalar Superpartners are generically called **s**quarks and **s**leptons
short for scalar quarks and scalar leptons

Gauge Supermultiplets

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8},\ \mathbf{1},\ 0)$
winos, W bosons	$\boxed{\widetilde{W}^{\pm} \quad \widetilde{W}^{0}}$	$W^{\pm} \quad W^{0}$	$(\mathbf{1},\ \mathbf{3},\ 0)$
bino, B boson	$\boxed{\widetilde{B}^{0}}$	B^{0}	$(\mathbf{1},\ \mathbf{1},\ 0)$

The spin-$1/2$ **gauginos** transform as the adjoint representation of the gauge group. Each gaugino carries a $\sim$. The color-octet superpartner of the gluon is called the **gluino**. The $SU(2)_L$ gauginos are called **winos**, and the $U(1)_Y$ gaugino is called the **bino**.

The winos and bino are not mass eigenstates, they mix with each other and with the Higgs superpartners, called higgsinos, of the same charge

The Higgs Problem

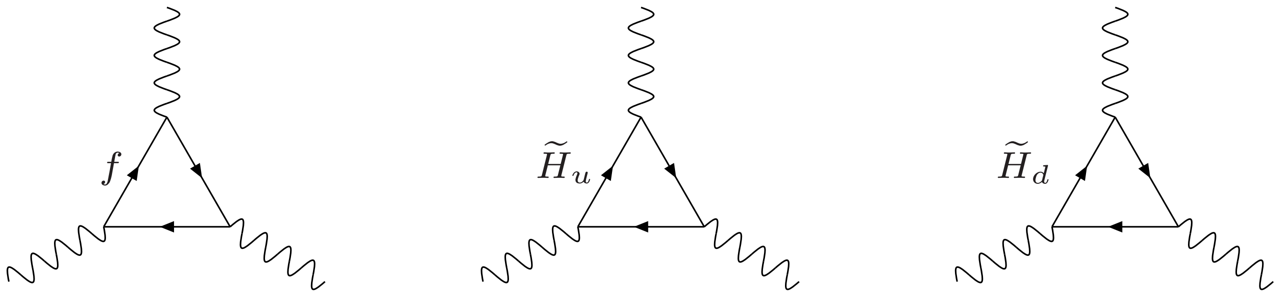
- Problem: What to do with the Higgs field ?
- In the Standard Model masses for the up and down (and lepton) fields are obtained with Yukawa couplings involving H and $H^\dagger$ respectively.
- Impossible to recover this from the Yukawas derived from $P[\Phi]$, since no dependence on $\bar{\Phi}$ is admitted.
- Another problem: In the SM all anomalies cancel,

$$\begin{aligned} \sum_{quarks} Y_i &= 0; \qquad \sum_{left} Y_i = 0; \\ \sum_i Y_i^3 &= 0; \qquad \sum_i Y_i = 0 \end{aligned}$$

- In all these sums, whenever a right-handed field appear, its charge conjugate is considered.
- A Higgsino doublet spoils anomaly cancellation !

Solution: two Higgs Supermultiplets with opposite hypercharges

1) Anomaly Cancellation



$$\sum_{\text{SM fermions}} Y_f^3 = 0 \qquad + \; 2\left(\frac{1}{2}\right)^3 \qquad + \; 2\left(-\frac{1}{2}\right)^3 \quad = \quad 0$$

This anomaly cancellation occurs if and only if **both** $\widetilde{H}_u$ and $\widetilde{H}_d$ higgsinos are present. Otherwise, the electroweak gauge symmetry would not be allowed!

2) Quark and Lepton masses

Only the H_u Higgs scalar can give masses to charge $+2/3$ quarks (top).

Only the H_d Higgs scalar can give masses to charge $-1/3$ quarks (bottom) and the charged leptons. We will show this later.

The Higgs Sector: two Higgs fields with opposite hypercharges

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

- Both Higgs fields acquire v.e.v. New parameter, $\tan\beta = v_2/v_1$.

Both Higgs fields contribute to the superpotential and give masses to up and down/lepton sectors, respectively

$$P[\phi] = h_u Q U H_2 + h_d Q D H_1 + h_l L E H_1$$

$$\boxed{\begin{array}{l} H_1 \equiv H_d \\ H_2 \equiv H_u \end{array}}$$

Once two Higgs doublets are there, a mass term may be written

$$\delta P[\phi] = \mu H_1 H_2$$

μ is only renormalized by wave functions of H_1 and H_2

<u>Interesting to observe:</u>

The quantum numbers of H_1 are the same as those of the lepton superfield L.
One can add terms in the superpotential replacing H_1 by L

Baryon and Lepton Number Violation

- General superpotential contains, apart from the Yukawa couplings of the Higgs to lepton and quark fields, new couplings:

$$P[\Phi]_{new} \rightarrow \begin{array}{l} P_{\Delta L=1} = \frac{1}{2}\lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \mu'_i L_i H_u \\ P_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk} U_i D_j D_k \end{array}$$

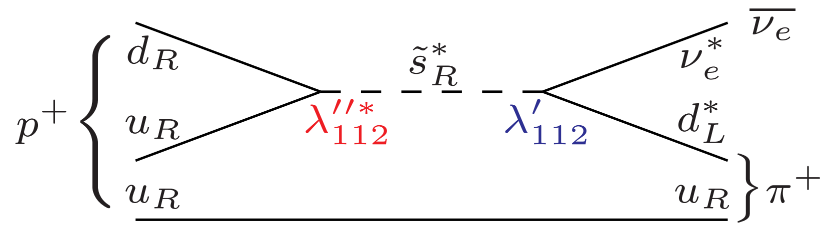
- Assigning every lepton chiral (antichiral) superfield lepton number 1 (-1) and every quark chiral (antichiral) superfield baryon number 1/3 (-1/3) one obtains :
  - Interactions in $P[\Phi]$ conserve baryon and lepton number.
  - Interactions in $P[\Phi]_{\rm new}$ violate either baryon or lepton number.

- One of the most dangerous consequences of these new interaction is to induce proton decay, unless couplings are very small and/or sfermions are very heavy.

Proton Decay

In P_{new} there are two type of couplings which violate either lepton number ($\Delta L = 1$) or baryon number ($\Delta B = 1$).

If both types of couplings were present, and of order 1, then the proton would decay in a tiny fraction of a second through diagrams like this:



Many other proton decay modes, and other experimental limits on B and L violation, give strong constraints on these terms in the superpotential.

One cannot require B and L conservation since they are already known to be violated at the quantum number in the SM.
Instead, one postulates a new discrete symmetry called R-parity.

$$P_R = (-1)^{3(B-L)+2S}$$

All SM particles have $P_R = 1$

All Supersymmetric partners have $P_R = -1$

Important Consequences of R-Parity Conservation

Since SUSY partners are R-parity odd (have $P_R = -1$)

every interaction vertex must contain an even number of SUSY particles

- All Yukawa couplings induced by $P(\Phi)_{new}$ are forbidden (have and odd number of SUSY particles)

- The Lightest SUSY Particle (LSP) must be absolutely stable

  If electrically neutral, interacts only weakly with ordinary

  LSP is a good Dark Matter candidate

- In collider experiments SUSY particles can only be produced in even numbers (usually in pairs)

- Each sparticle eventually decays into a state that contains an LSP
  ==> Missing Energy Signal at colliders

SUSY Interactions in the MSSM

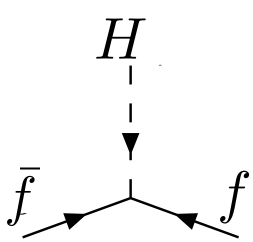
1) Self-interactions of Matter fields

Yukawa interactions + fermion masses

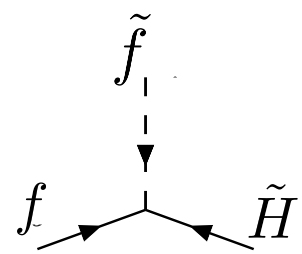
$$\mathcal{L} \to -\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i\psi_j + h.c.$$ with the general form of P(A) in the MSSM

$$P[A] = \epsilon_{ij}[\mu H_1^i H_2^j + h_u H_2^i \tilde{Q}^j \tilde{U} + h_d H_1^i \tilde{Q}^j \tilde{D} + h_l H_1^i \tilde{L}^j \tilde{E}]$$

Usual SM Yukawa interactions



Sfermion-fermion Higgsino interactions

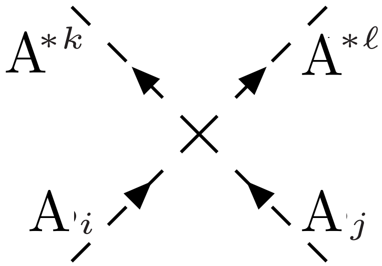


Once the Higgs bosons acquire vev:

$$m_u = h_u\ \mathrm{v}_2 \qquad m_d = h_d\ \mathrm{v}_1 \qquad m_l = h_l\ \mathrm{v}_1 \qquad \tan\beta = \mathrm{v}_2/\mathrm{v}_1$$

Scalar- self interactions

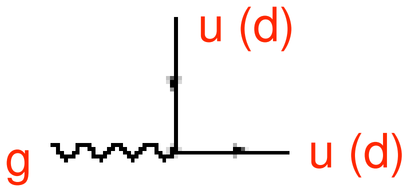
$$V \rightarrow \frac{1}{2}\sum_a (D^a)^2 \quad \text{with} \qquad D^a = -g\sum_i (A_i^* T^a A_i)$$



$$-ig_a^2(T_i^{ak}T_j^{a\ell}+T_i^{a\ell}T_j^{ak})$$

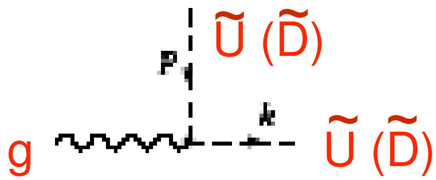
quartic couplings determined
as a function of gauge couplings

2) Interactions of Gauge and Matter fields



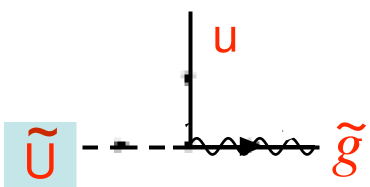
$$-ig_3 T^a \gamma^\mu$$

→ Regular SM interactions

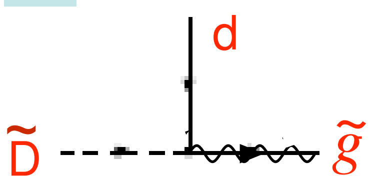


$$-ig_3 T^a (p+k)^\mu$$

→ Sparticle-gauge interactions:
change fermions by scalars
(and gammas by momentum)



$$-ig_3 T^a (c_U P_L + s_U P_R)$$



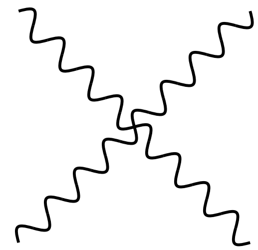
$$-ig_3 T^a (c_D P_L + s_D P_R)$$

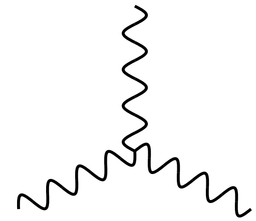
→ Sfermion-gaugino-fermion novel interactions
change gluon by gluino and one fermion by scalar

extra factors are mixing angles to project mass eigenstates into gauge eigenstates

2) <u>Interactions of Gauge Supermultiplets</u>

Usual 3 and 4 gauge bosons vertices
from kinetic term

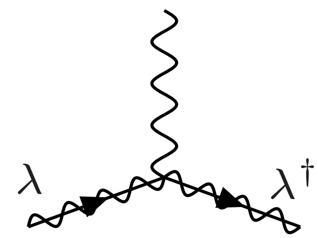




Gaugino- gauge boson interactions

proportional to the gauge group structure constant

$$ig f_{abc} \lambda^a \sigma^\mu \bar{\lambda}^b V_\mu^c$$



for gluon-gluinos ~ $g_3 \ f_{abc} \ \gamma_\mu$

No interaction between U(1) gaugino and gauge field f_{abc} =0

Scalar Interactions and the preservation of SUSY

- As we said before, the scalar potential may be obtained by adding the D-terms, that depend only on the gauge structure, with terms that depend on the square of the derivative of the superpotential.
- For the given superpotential, we get terms like

$$V_{\tilde{U}} = h_t \mu^* H_1^* \widetilde{Q}\widetilde{U} + h.c.$$

- Once the Higgs acquire a v.e.v., this induces a mixing between the right handed stop and left handed stop

$$- h_t \mu^* v_1 \tilde{t}_L \tilde{t}_R^*$$

- This will affect the masses, that, however, should be equal to the top-quark masses if supersymmetry is to be preserved ! What is going on ?

- Let's look at the potential for the neutral Higgs bosons

$$V_{H^0} = |\mu|^2 \left(|H_1^0|^2 + |H_2^0|^2\right) + \frac{(g_1^2 + g_2^2)}{8} \left(|H_1^0|^2 - |H_2^0|^2\right)^2$$

- To preserve supersymmetry, we need the vacuum state to have zero energy.
- This may be only obtained, once the Higgs acquire v.e.v., if :

$$\mu = 0, \qquad\qquad \tan\beta = 1$$

- The potential presents a flat direction under these conditions.

Supersymmetry Breakdown

If SUSY were an exact symmetry, the superpartners would have the exactly same masses

$m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e = 0.511$ GeV

$m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_u$

$m_{\tilde{g}} = m_{\text{gluon}} = 0 +$ QCD-scale effects

etc.

- No supersymmetric particle have been seen: Supersymmetry is broken in nature
- Unless a specific mechanism of supersymmetry breaking is known, no information on the spectrum can be obtained.
- Cancellation of quadratic divergences:
  - Relies on equality of couplings and not on equality of the masses of particle and superpartners.
- Soft Supersymmetry Breaking: Give different masses to SM particles and their superpartners but preserves the structure of couplings of the theory.