Supersymmetry: Motivation, Algebra, Models and Signatures

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Outline

- Lecture 1:
  Motivation
  Introduction to Supersymmetry
- Lecture 2 :
  Supersymmetric Interactions
  The Minimal Supersymmetric extension of the Standard Model
- Lecture 3 :
  Soft Supersymmetry Breaking
  Higgs and Super-particle masses
- Lecture 4 :
  Coupling Unification
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- Appendix
  Notes on Soft SUSY Breaking
  MSSM Higgs signals at Colliders

Lecture 3:

Soft Supersymmetry Breaking

Higgs and Super-particle Masses

Last lecture:

We started from the SUSY Lagrangian in terms of:

Standard fermions ψ_i and their superpartners, the scalar fields A_i
SM gauge bosons and gauginos λ^a, and the Higgs supermultiplets

We discuss the need for two Higgs doublets,
with the Higgsinos superpartners
to cancel U(1) associated anomalies
and give mass to both, up-quarks and down-quarks/leptons.

We defined the interactions among fields in the MSSM:
No new couplings!!
All interactions in terms of gauge couplings and parameters in the superpotential

Last lecture (continued)

We showed that if SUSY is preserved up to energies of order a TeV, the equality of fermion and boson masses and couplings assures the cancellation of quadratic divergences in the Higgs mass quantum corrections

We discussed advantages of imposing R-parity $P_R = (-1)^{3(B-L)+2S}$.

$\longrightarrow$ forbids Proton Decay

$\longrightarrow$ the LSP is stable and a good Dark Matter candidate

The Minimal Supersymmetric Standard Model

Chiral Supermultiplets

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \quad \widetilde{d}_L)$	$(u_L \quad d_L)$	$(\mathbf{3},\ \mathbf{2}\ ,\ \frac{1}{6})$
($\times 3$ families)	U	$\widetilde{u}_R^*$	$(u^C)_L$	$(\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	D	$\widetilde{d}_R^*$	$(d^C)_L$	$(\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \quad \widetilde{e}_L)$	$(\nu \quad e_L)$	$(\mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$
($\times 3$ families)	E	$\widetilde{e}_R^*$	$(e^C)_L$	$(\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+ \quad H_u^0)$	$(\widetilde{H}_u^+ \quad \widetilde{H}_u^0)$	$(\mathbf{1},\ \mathbf{2}\ ,\ +\frac{1}{2})$
	H_d	$(H_d^0 \quad H_d^-)$	$(\widetilde{H}_d^0 \quad \widetilde{H}_d^-)$	$(\mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$

Gauge Supermultiplets

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^\pm \quad \widetilde{W}^0$	$W^\pm \quad W^0$	$(\mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\mathbf{1},\ \mathbf{1}\ ,\ 0)$

Scalar Interactions and the preservation of SUSY

As we said before: $$V(A_i, A_i^*) = \sum_i |\partial P(A)/\partial A_i|^2 + \frac{1}{2}\sum_a \left(A_i^* T_{ij}^a g A_j\right)^2$$

and given the MSSM superpotential:

$$P[A] = \epsilon_{ij}[\mu H_1^i H_2^j + h_u H_2^i \tilde{Q}^j \tilde{U} + h_d H_1^i \tilde{Q}^j \tilde{D} + h_l H_1^i \tilde{L}^j \tilde{E}]$$

we get for the up-squark potential terms like: $$V_{\tilde{U}} \to h_t \mu^* H_1^* \tilde{Q} \tilde{U} + h.c.$$

Once the Higgs acquire a v.e.v., this induces a mixing between the right-handed and left-handed stops

$$-h_t \mu^* \mathrm{v}_1 \tilde{\mathrm{t}}_\mathrm{L} \tilde{\mathrm{t}}_\mathrm{R}^*$$

This will affect the stop quark masses, which, however, should be equal to the top-quark mass, if SUSY is preserved! What is happening?

- Let's look at the potential for the neutral Higgs bosons

$$V_{H^0} = |\mu|^2 \left(|H_1^0|^2 + |H_2^0|^2\right) + \frac{(g_1^2 + g_2^2)}{8} \left(|H_1^0|^2 - |H_2^0|^2\right)^2$$

- To preserve supersymmetry, we need the vacuum state to have zero energy.
- This may be only obtained, once the Higgs acquire v.e.v., if :

$$\mu = 0, \qquad \tan\beta = 1$$

- The potential presents a flat direction under these conditions.

Supersymmetry Breakdown

If SUSY were an exact symmetry, the SM particles and their superpartners would have the exactly same masses

$m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e = 0.511$ GeV

$m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_u$

$m_{\tilde{g}} = m_{\text{gluon}} = 0 +$ QCD-scale effects

etc.

- No supersymmetric particle have been seen: Supersymmetry is broken in nature
- Unless a specific mechanism of supersymmetry breaking is known, no information on the spectrum can be obtained.
- Cancellation of quadratic divergences:
  - Relies on equality of couplings and not on equality of the masses of particle and superpartners.
- Soft Supersymmetry Breaking: Give different masses to SM particles and their superpartners but preserves the structure of couplings of the theory.

For a clue on the nature/scale of the SUSY breakdown

let's return to our motivation in the Hierarchy problem:

$$\Delta\mu^2 \approx g^2_{hff}[m^2_f - m^2_{\tilde{f}}]ln(\Lambda^2_{eff}/m^2_h)$$

If we want supersymmetry to be a solution we need to demand that the difference between SM and SUSY particles is not far beyond the TeV scale
==> SUSY partners have to be at/below the TeV scale

This is the best reason to be optimistic that SUSY will be discovered at the TeVatron collider or the LHC in the next few years.

The breakdown of SUSY must be "soft"
This means it does not change the dimensionless terms in the Lagrangian.

If SUSY is realized in nature, why have we seen none of the SUSY particles while we have already seen all the SM particles?

Standard Model quark, leptons and gauge boson masses are protected by chiral and gauge symmetries
==> they acquire mass through EWSB, hence their masses are at most of order v~ 175 GeV

SUSY particles can acquire gauge invariant masses, same as SM Higgs
==> this explains why it is possible that we have not seen SUSY particles at high energy colliders yet

One can probe that after adding
Gaugino masses, Squark and Slepton squared mass terms, and trilinear and bilinear terms proportional to the scalar parts of the superpotential,
the cancellation of quadratic divergences is not spoiled

The Soft SUSY-breaking Lagrangian for the MSSM

$$\mathcal{L}_{soft} = -\frac{1}{2}(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B})$$
$$-m_Q^2\tilde{Q}^\dagger\tilde{Q} - m_U^2\tilde{U}^\dagger\tilde{U} - m_D^2\tilde{D}^\dagger\tilde{D} - m_L^2\tilde{L}^\dagger\tilde{L} - m_E^2\tilde{E}^\dagger\tilde{E}$$
$$-m_{H_1}^2 H_1^* H_1 - m_{H_2}^2 H_2^* H_2 - (\mu B H_1 H_2 + cc.)$$
$$-(A_u h_u \tilde{U}\tilde{Q}H_2 + A_d h_d \tilde{D}\tilde{Q}H_1 + A_l h_l \tilde{E}\tilde{L}H_1) + c.c.$$

Trilinear terms are proportional to the Yukawa couplings

$\longrightarrow$ induce L-R mixing on the squark sector once the Higgs acquire v.e.v.
mixing proportional to fermion masses: relevant for 3rd generation

B $\longrightarrow$ soft SUSY breaking parameter will be determined from condition of proper EWSB

The squark and slepton squared masses and (scalar)^3 couplings are 3×3 matrices in family space. The soft SUSY-breaking Lagrangian of the MSSM contains 105 new parameters not found in the Standard Model.

Most of what we do not already know about SUSY is expressed by the question: "How is supersymmetry broken?"

Many proposals exists but none is completely satisfactory

One could shed light to the mechanism of SUSY-breaking through experiments, by discovering the pattern of Higgs, slepton, squark and gaugino masses

The Higgs Potential

- After supersymmetry breaking effects are considered, the Higgs potential reads

$$V(H_1, H_2) = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 (H_1^T i\tau_2 H_2 + h.c.) + \frac{\lambda_1}{2}\left(H_1^\dagger H_1\right)^2 + \frac{\lambda_2}{2}\left(H_2^\dagger H_2\right)^2 + \lambda_3\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4\left|\left(H_1^T i\tau_2 H_2\right)\right|^2$$

where

$$\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{4}, \qquad \lambda_3 = \frac{g_2^2 - g_1^2}{4}, \qquad \lambda_4 = -\frac{g_2^2}{2}$$

$$m_i^2 = m_{H_i}^2 + |\mu|^2$$

$$m_3^2 = \mu B$$

$$H_1 = \begin{pmatrix} \mathrm{v}_1 + (H_1^0 + iA_1)/\sqrt{2} \\ H_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^+ \\ \mathrm{v}_2 + (H_2^0 + iA_2)/\sqrt{2} \end{pmatrix}$$

This effective potential is valid at the scale of the SUSY particle masses

At low energies the quartic couplings evolve with their Renormalization Group (RG) equations

Neutral Higgs potential

Analyzing first the neutral part of $V[H_i]$

$$V[H_1^0, H_2^0] = m_1^2|H_1^0|^2 + m_2^2|H_2^0|^2 - m_3^2\left(H_1^0 H_2^0 + H_1^{0^*} H_2^{0^*}\right)$$

$$+\frac{g_1^2 + g_2^2}{8}\left(|H_2^0|^2 - |H_1^0|^2\right)^2$$

Using the minimization conditions $\partial V/\partial H_i^0|_{<H_i^0>=v_i} = 0$

$$\tan^2\beta = \frac{\mathrm{v}_2^2}{\mathrm{v}_1^2} = \frac{m_1^2 + M_Z^2/2}{m_2^2 + M_Z^2/2} \qquad \sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}$$

Expanding $V[H^0_i]$ in terms of $H_i^0 = \mathrm{v}_i + (H_i^0 + iA_i)/\sqrt{2} \quad i = 1, 2$

One can obtained the scalar and pseudoscalar mass matrices ==>

Tree Level Mass Predictions

$$V[A_i] \to (A_1 \quad A_2) \begin{bmatrix} m_1^2 + \frac{M_Z^2}{2}\cos 2\beta & m_3^2 \\ m_3^2 & m_1^2 - \frac{M_Z^2}{2}\cos 2\beta \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

Using the minimization conditions

$$\Rightarrow \det M_A^2 = 0 \qquad\qquad \mathrm{Tr}[\mathrm{M_A^2}] = \mathrm{m_1^2} + \mathrm{m_2^2}$$

CP-odd neutral Higgs Sector:

One Goldstone boson with zero mass (eaten by Z boson)

One physical state: $A = \cos\beta A_2 + \sin\beta A_1$

$$m_A^2 = m_1^2 + m_2^2 = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2$$

Tree Level Mass Predictions

CP-even neutral Higgs Sector:

$$V[H_i^0] \to \begin{pmatrix} H_1^0 & H_2^0 \end{pmatrix} \begin{bmatrix} M_Z^2 \cos^2\beta + m_A^2 \sin^2\beta & -(M_Z^2 + m_A^2)\cos\beta\sin\beta \\ -(M_Z^2 + m_A^2)\cos\beta\sin\beta & M_Z^2 \sin^2\beta + m_A^2 \cos^2\beta \end{bmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}$$

Two physical states:

$$h = \cos\alpha H_2^0 - \sin\alpha H_1^0$$
$$H = \sin\alpha H_2^0 + \cos\alpha H_1^0$$

$$m_{h,H}^2 = \frac{m_A^2 + M_Z^2}{2} \pm \frac{\sqrt{(m_A^2 + M_Z^2)^2 - 4M_Z^2 m_A^2 \cos^2 2\beta}}{2}$$

Charged Higgs Sector: (Similar to CP-odd Higgs sector)

One Goldstone boson with zero mass $\Rightarrow \det M_{H_\pm}^2 = 0$

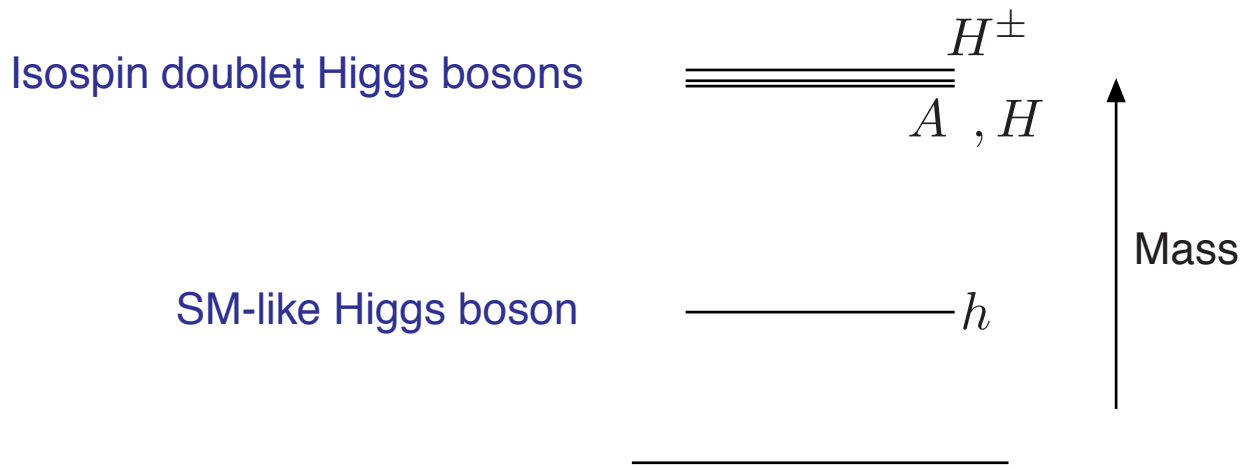
One physical state: $H^\pm = \cos\beta H_2^\pm + \sin\beta H_1^\pm$

$$\mathrm{Tr}[\mathrm{M}_{\mathrm{H}^\pm}^2] = \mathrm{m}_\mathrm{A}^2 + \mathrm{m}_\mathrm{W}^2 = \mathrm{m}_{\mathrm{H}^\pm}^2$$

The decoupling limit for the Higgs bosons

If $m_A \gg m_Z$, then: $m_h^2 = M_Z^2 \cos^2 2\beta$

- h has the same couplings as would a Standard Model Higgs boson of the same mass
- $\alpha \approx \beta - \pi/2$
- A, H, $H^\pm$ form an isospin doublet, and are much heavier than h



Higgs Spectrum

- The two Higgs doublets carry eight real scalar degrees of freedom.
- Three of them are the charged and CP-odd Goldstone bosons that are absorved in the longitudinal components of the W and the Z.
- Five Higgs bosons remain: Two CP-even, one CP-odd, neutral bosons, and a charged Higgs boson (two degrees of freedom).
- Generically, the electroweak breaking sector (Goldstones and real Higgs) is contained in the combination of doublets

$$\Phi = \cos\beta H_1 + \sin\beta i\tau_2 H_2^*,$$

$$\text{such that} < \phi > = \text{v}$$

while the orthogonal combination contains the other Higgs bosons

Couplings to gauge bosons and fermions

(normalized to SM values)

$$\mathrm{hZZ,\ hWW,\ ZHA,\ WH^{\pm}H} \longrightarrow \sin(\beta-\alpha)$$
$$\mathrm{HZZ,\ HWW,\ ZhA,\ WH^{\pm}h} \longrightarrow \cos(\beta-\alpha)$$

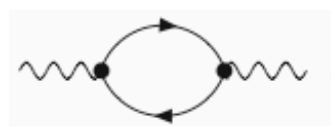
$$\mathrm{(h,H,A)}\ u\bar{u} \longrightarrow \cos\alpha/\sin\beta,\ \ \sin\alpha/\sin\beta,\ \ 1/\tan\beta$$
$$\mathrm{(h,H,A)}\ d\bar{d}/l^+l^- \longrightarrow -\sin\alpha/\cos\beta,\ \ \cos\alpha/\cos\beta,\ \ \tan\beta$$

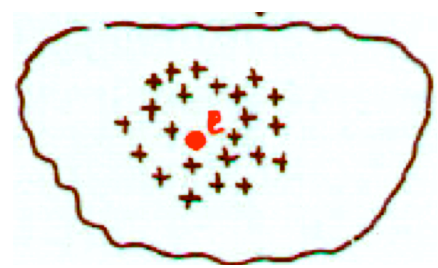
Quantum Corrections $\longleftrightarrow$ Evolution of Higgs quartic couplings below the SUSY breaking scale

Parameters that one measures at low energies (large distances) are not the fundamental ones, and they are modified by quantum corrections

Example: consider a charge in the vacuum, it polarizes the vacuum by inducing production of virtual particles/antiparticles that screen the charge

Same with couplings and masses: RG equations allow to relate fundamental parameters to those at low energies





Large $\tan\beta$ Limit

Given that $m_t = h_t v_2$ and $m_b = h_b v_1$

$$\downarrow$$

v_2 large keeps h_t perturbative

- If one makes h_b large, of the order of h_t, $\tan\beta$ is about 50
- For this limit to happen $m_3^2 \simeq 0$.
- Then, the doublet H_2 contains the Goldstone modes and the "physical" SM-like Higgs boson, while H_1 contains a scalar, a pseudoscalar and a charged Higgs boson.
- Physical Higgs mass ($m_2^2 = -M_Z^2/2$)

$$m_h^2 = 2\lambda_2 v^2 = M_Z^2$$

Radiative Corrections to the MSSM Higgs Masses

Important corrections due to incomplete cancellation of particles and sparticle effects. Mainly top & stop loops and sbottom loops for tanb > 10

- The RG evolution of λ_2 is given by

$$\frac{d\lambda_2}{dt} \simeq -\frac{3}{8\pi^2}\left[\lambda_2^2 + \lambda_2 h_t^2 - h_t^4\right]$$

with $t = log(M_{SUSY}^2/Q^2)$.

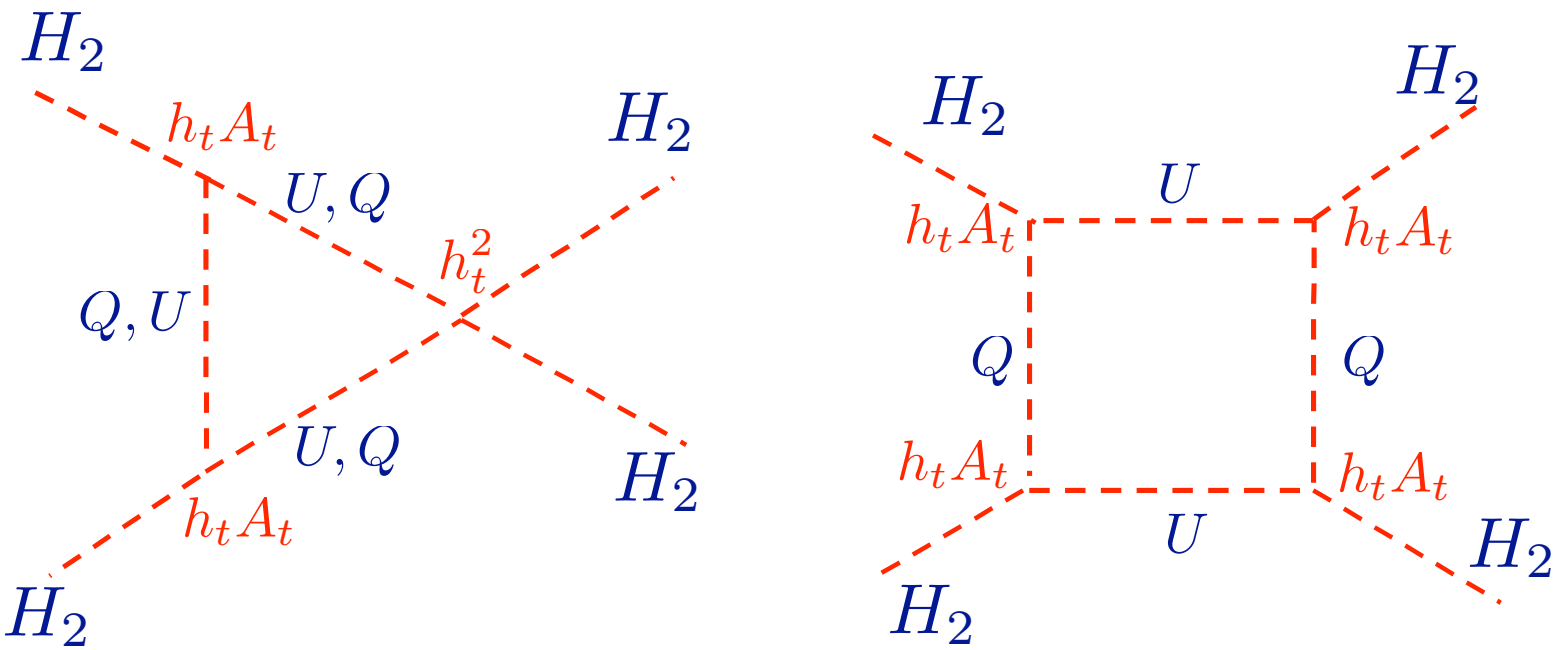
- For large values of $\tan\beta = v_2/v_1$, the Higgs H_2 is the only one associated with electroweak symmetry breaking.
- The Higgs boson mass is approximately given by $m_h^2 = 2\lambda_2 v^2$

$$m_h^2 \simeq M_Z^2 + \frac{3m_t^4}{4\pi^2 v^2}\left[\log\left(\frac{M_{SUSY}^2}{m_t^2}\right) + \frac{X_t^2}{M_{SUSY}^2}\left(1 - \frac{X_t^2}{12M_{SUSY}^2}\right)\right]$$

$\underline{M_{SUSY}^2}$ $\rightarrow$ averaged stop squared mass $\underline{X_t = A_t - \mu/\tan\beta}$ $\rightarrow$ stop mixing parameter

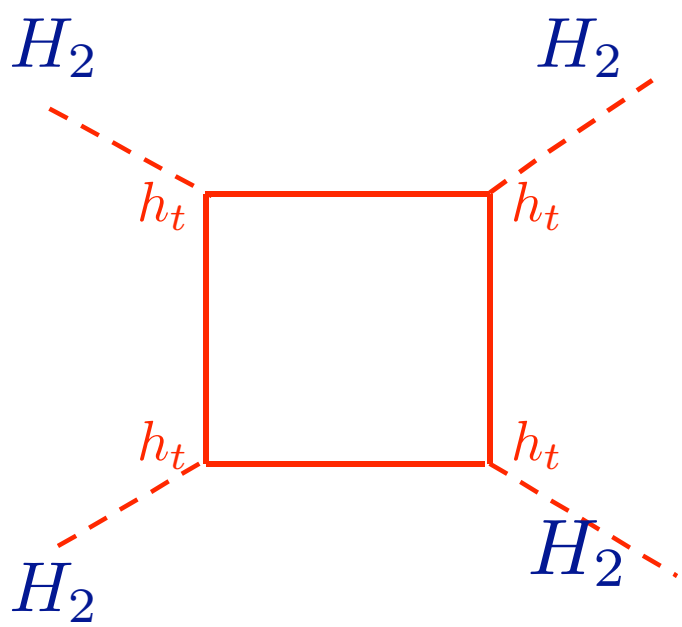
Log terms from RG evolution and A_t terms from threshold effects at M_{SUSY}

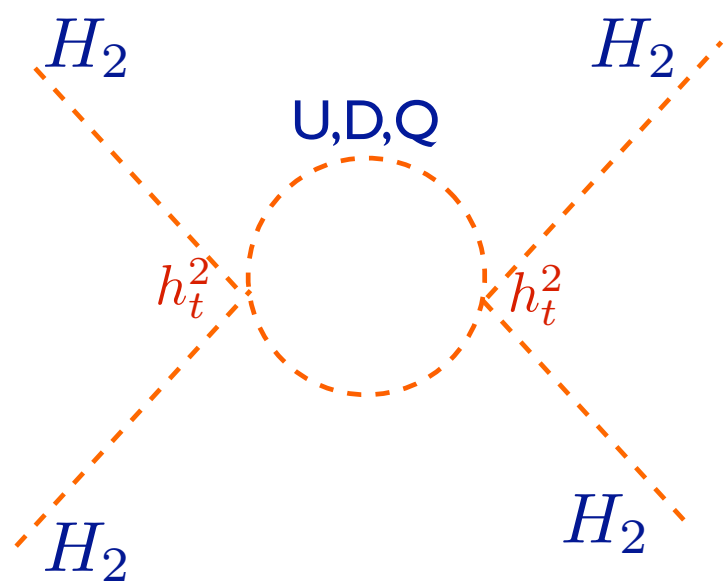
Diagrams Contributing to the Quartic Coupling



These diagrams provide the finite threshold corrections after decoupling of the top quark superpartners.

Top & Stop Contributions to the Higgs Quartic Coupling





These diagrams provide the dominant logarithmic contribution below the stop quark mass scale. If both masses were equal the log will vanish

Stop Mass Matrix

- The stop, and other squarks, acquire masses that are controlled by the supersymmetry breaking parameters.
- Once the Higgs acquires a v.e.v., the mass matrix is

$$M_{\tilde{t}}^2 = \begin{bmatrix} m_Q^2 + m_t^2 & m_t(A_t - \mu^*/\tan\beta) \\ m_t(A_t^* - \mu/\tan\beta) & m_U^2 + m_t^2 \end{bmatrix}$$

- In general, the existence of A_t and μ denote couplings of the stops to the Higgs bosons, that induce finite corrections to the quartic couplings.

Effect of Quantum Corrections on the Lightest Higgs Mass

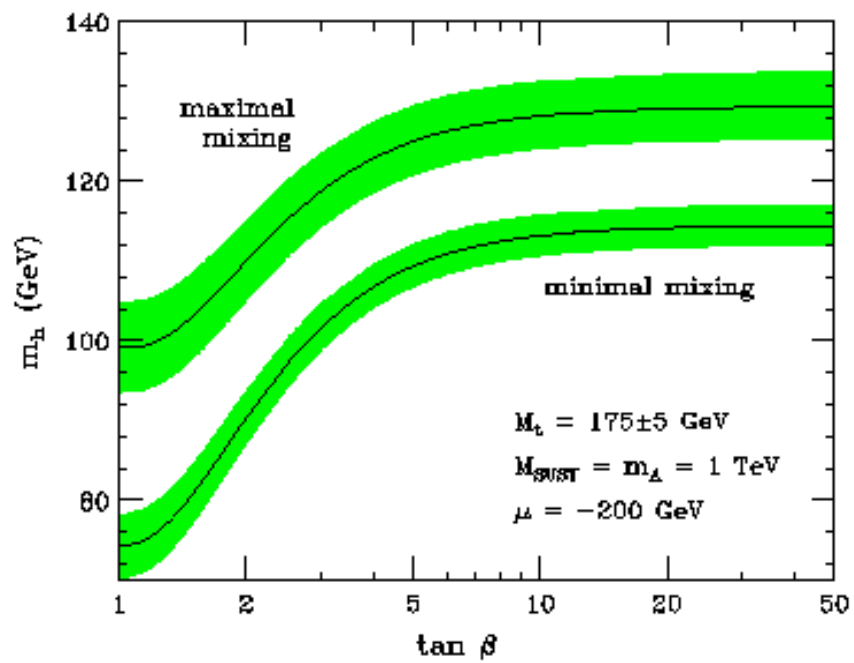
- m_t^4 enhancement
- log sensitivity to stop masses M_S
- depend. on stop mass mixing X_t

After 2 -loop corrections

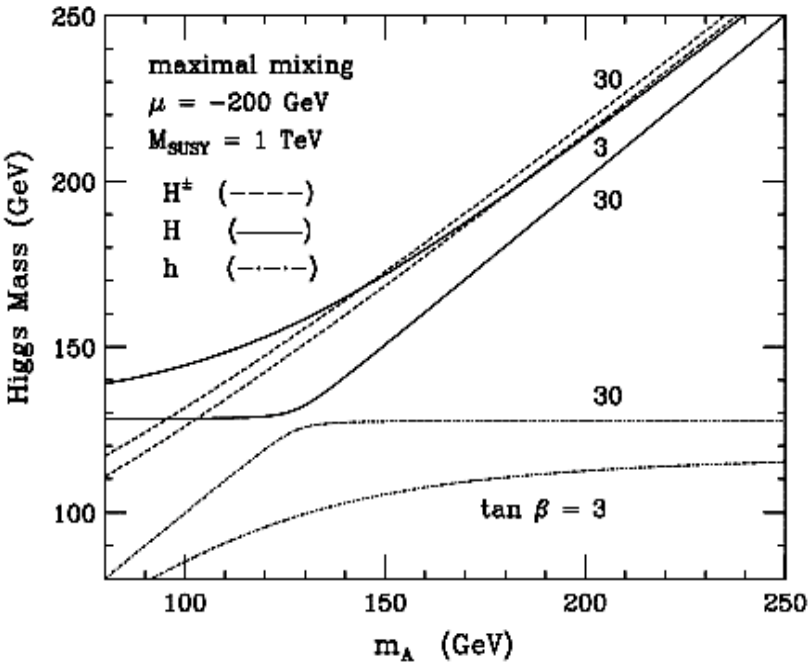
$$m_h \leq 135 \text{ GeV}$$

stringent test of the MSSM

$$M_S = 1 \rightarrow 2 \text{ TeV} \Longrightarrow \Delta m_h \simeq 2 - 5 \text{ GeV}$$
$$\Delta m_t = 1 \text{ GeV} \Longrightarrow \Delta m_h \sim 1 \text{ GeV}$$



MSSM Higgs Masses as a function of M_A



$$m_H^2 \cos^2(\beta-\alpha) + m_h^2 \sin^2(\beta-\alpha) = [m_h^{max}(\tan\beta)]^2$$

- $\cos^2(\beta-\alpha) \to 1$ for large $\tan\beta$, low m_A
  $\Longrightarrow$ H has SM-like couplings to W,Z
- $\sin^2(\beta-\alpha) \to 1$ for large m_A
  $\Longrightarrow$ h has SM-like couplings to W,Z

for large $\tan\beta$:
always one CP-even Higgs with SM-like couplings to W,Z
and mass below $m_h^{max} \leq 135$ GeV

$\downarrow$

if $m_A > m_h^{max}$ $\rightarrow$ $m_h \simeq m_h^{max}$ and $m_H \simeq m_A$

if $m_A < m_h^{max}$ $\rightarrow$ $m_h \simeq m_A$ and $m_H \simeq m_h^{max}$

m_A nearly degenerate with m_h or m_H

Radiative corrections to the Higgs couplings

Important effects through radiative corrections to the CP-even Mass matrix

which defines the mixing angle α

$\rightarrow$ via rad. correc. to $\sin\alpha$ and $\cos\alpha$: govern couplings of Higgs to fermions

$\rightarrow$ via correc. to $\cos(\alpha-\beta)$ and $\sin(\alpha-\beta)$: govern couplings to vector bosons

Important vertex corrections to Higgs-fermion Yukawa couplings through loops of SUSY particles

relevant for large $\tan\beta \geq 20$

can induce important flavor changing neutral and charged current effects

Depending on SUSY Spectrum, radiative corrections to Higgs couplings can change Higgs searches in a very crucial manner

M.C. and Haber Higgs Review: Prog. in Particles and Nuclear Physics 50, 2003

Gaugino/Higgsino Mixing

- Just like the gauge boson mixes with the Goldstone modes of the theory after spontaneous breakdown of the gauge symmetry, gauginos mix with the Higgsinos.
- Mixing comes from the interaction $\sqrt{2} g A_i^* T_a \psi_i \lambda^a$, when one takes $A_i \equiv H_i$, and $\lambda^a \equiv \tilde{W}^a$, $\tilde{B}$, and $\psi_i = \tilde{H}_i$.
- Charged Winos, $\tilde{W}_1 \pm i\tilde{W}_2$, mix with the charged components of the Higgsinos $\tilde{H}_{1,2}$. The mass eigenstates are called charginos $\tilde{\chi}^{\pm}$.
- Neutral Winos and Binos, $\tilde{B}$, $\tilde{W}_3$ mix with the neutral components of the Higgsinos. The mass eigenstates are called neutralinos, $\tilde{\chi}^0$.
- Charginos form two Dirac massive fields. Neutralinos give four massive Majorana states.

Charginos

Consider the chargino Lagrangian in terms of Weyl spinors, in the Wino- Higgsino basis, with $\tilde{W}^{\pm} = \left(\tilde{W}^1 \pm i\tilde{W}^2\right)/\sqrt{2}$

$$\mathcal{L}_{charginos} = -(\tilde{W}^- \tilde{H}_1^-) \begin{pmatrix} M_2 & g_2 v_2 \\ g_2 v_1 & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_2^+ \end{pmatrix} + h.c.$$

M_2 is the soft SUSY breaking gaugino mass term, μ is the Higgsino mass parameter which comes from the superpotential, and the off diagonal terms come from the gaugino-Higgs Higgsino mixing and are always < M_Z

$$\text{Defining: } \psi_j^+ \rightarrow (\tilde{W}^+,\ \tilde{H}_2^+) \text{ and } \psi_j^- \rightarrow (\tilde{W}^-, \tilde{H}_1^-)$$

Chargino Mass Matrix is diagonalized by two matrices V, U such that:

$$M_{\tilde{\chi}^{\pm}}^{Diag} = V^* M_{\tilde{\chi}^{\pm}} U$$

The chargino mass eigenstates are: $\tilde{\chi}_i^+ = U_{ij}\psi_j^+ \qquad \tilde{\chi}_i^- = V_{ij}\psi_j^-$

The chargino eigenstates are two Dirac, charged fermions with masses:

$$m^2_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{2}\Big[|M_2|^2 + |\mu|^2 + 2m_W^2 \mp\sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2}\Big].$$

- If μ is large, the lightest chargino is a Wino, with mass M_2, and its interactions to fermion and sfermions are governed by gauge couplings.
- If M_2 is large, the lightest chargino is a Higgsino, with mass μ, and the interactions are governed by Yukawa couplings.

Neutralinos

Consider the neutralino Lagrangian in terms of Weyl spinors, in the Bino-Wino-neutral Higgsino basis, with

$$\mathcal{L}_{neutralinos} = -\frac{1}{2}(\psi^0)^T M_{\tilde{\chi}^0}\psi^0 + h.c.$$

Defining: $\psi_j^0 \rightarrow (i\tilde{B}^0,\ i\tilde{W}^0,\ \tilde{H}_1^0,\ \tilde{H}_2^0)$

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -g_1 v_1/\sqrt{2} & g_1 v_2/\sqrt{2} \\ 0 & M_2 & g_2 v_1/\sqrt{2} & -g_2 v_2/\sqrt{2} \\ -g_1 v_1/\sqrt{2} & g_2 v_1/\sqrt{2} & 0 & -\mu \\ g_1 v_2/\sqrt{2} & -g_2 v_2/\sqrt{2} & -\mu & 0 \end{pmatrix}$$

Neutralino Mass Matrix diagonalized by a matrix Z $\rightarrow M_{\tilde{\chi}^0}^{Diag} = Z^* M_{\tilde{\chi}^0} Z^{-1}$

The neutralino mass eigenstates are: $\tilde{\chi}_i^0 = Z_{ij}\psi_j^0$

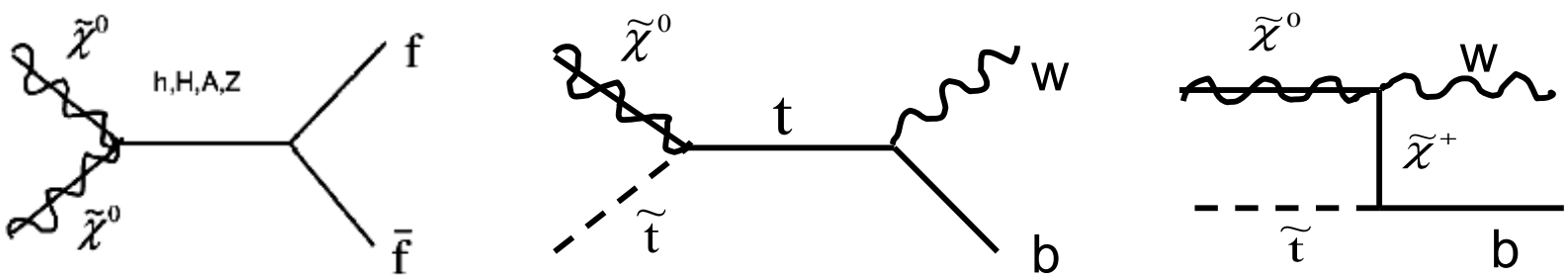
Neutralino Spectrum

- The eigenstates are four Majorana particles.
- If the theory proceeds from a GUT, there is a relation between M_2 and M_1, $M_2 \simeq \alpha_2(M_Z)/\alpha_1(M_Z) M_1 \simeq 2M_1$.
- So, if μ is large, the lightest neutralino is a Bino (superpartner of the hypercharge gauge boson) and its interactions are governed by g_1.
- This tends to be a good dark matter candidate.

$$\Omega_{CDM} \sim 1 \,/\, \int_0^{x_F} \langle \sigma_A \, v \rangle \, dx \qquad x \equiv \frac{M}{T}$$

$0.089 < \Omega_{CDM} h^2 < 0.131$ WMAP at 3 σ

Many processes contribute to the $\tilde{\chi}_1^0 \, \tilde{\chi}_1^0$ annihilation cross section: $\langle \sigma_A \, v \rangle$



Gluinos

The gluino is an $SU(3)_C$ color octet fermion, so it does not have the right quantum numbers to mix with any other state. Therefore, at tree-level, its mass is the same as the corresponding parameter in the soft SUSY-breaking Lagrangian:

$$\boxed{M_{\tilde{g}} = M_3}$$

However, quantum corrections can be quite large, because it is a color octet.

This correction can be of order 5% to 25%, depending on the squark masses!

It tends to **increase** the gluino mass, compared to the tree-level prediction.

Squarks and Leptons

To treat these in complete generality, we would have to take into account arbitrary mixing. So the mass eigenstates would be obtained by diagonalizing:

- a 6×6 (mass)^2 matrix for up-type squarks ($\widetilde{u}_L, \widetilde{c}_L, \widetilde{t}_L, \widetilde{u}_R, \widetilde{c}_R, \widetilde{t}_R$),
- a 6×6 (mass)^2 matrix for down-type squarks ($\widetilde{d}_L, \widetilde{s}_L, \widetilde{b}_L, \widetilde{d}_R, \widetilde{s}_R, \widetilde{b}_R$),
- a 6×6 (mass)^2 matrix for charged sleptons ($\widetilde{e}_L, \widetilde{\mu}_L, \widetilde{\tau}_L, \widetilde{e}_R, \widetilde{\mu}_R, \widetilde{\tau}_R$),
- a 3×3 matrix for sneutrinos ($\widetilde{\nu}_e, \widetilde{\nu}_\mu, \widetilde{\nu}_\tau$)

If we assume that soft SUSY breaking parameters are flavor blind:

The first- and second-family squarks and sleptons have negligible Yukawa couplings, so they end up in 7 very nearly degenerate, unmixed pairs $(\widetilde{e}_R, \widetilde{\mu}_R)$, $(\widetilde{\nu}_e, \widetilde{\nu}_\mu)$, $(\widetilde{e}_L, \widetilde{\mu}_L)$, $(\widetilde{u}_R, \widetilde{c}_R)$, $(\widetilde{d}_R, \widetilde{s}_R)$, $(\widetilde{u}_L, \widetilde{c}_L)$, $(\widetilde{d}_L, \widetilde{s}_L)$.

Stop Sector

$\tilde{t}_L, \tilde{t}_R$ Mixing effects relevant due to large Yukawa coupling
lightest mass eigenstate $\tilde{t}_1$ may be the lightest visible sparticle

$$M^2_{\tilde{t}} = \begin{pmatrix} m^2_{Q_3} + m^2_t + D_{\tilde{t}_L} & m_t(A_t - \mu/\tan\beta) \\ m_t(A_t - \mu/\tan\beta) & m^2_{U_3} + m^2_t + D_{\tilde{t}_R} \end{pmatrix} \equiv \begin{pmatrix} m^2_{\tilde{t}_L} & m^2_{\tilde{t}_L R} \\ m^2_{\tilde{t}_L R} & m^2_{\tilde{t}_R} \end{pmatrix}$$

The mass eigenstates are given by

$$\tilde{t}_1 = \cos\theta_{\tilde{t}}\tilde{t}_L + \sin\theta_{\tilde{t}}\tilde{t}_R$$
$$\tilde{t}_2 = -\sin\theta_{\tilde{t}}\tilde{t}_L + \cos\theta_{\tilde{t}}\tilde{t}_R$$

$$\tan 2\theta_{\tilde{t}} = \frac{2m^2_{\tilde{t}_{LR}}}{m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R}}$$

With masses:

$$m^2_{\tilde{t}_{1,2}} = \frac{m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R}}{2} \pm \sqrt{\left(\frac{m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R}}{2}\right)^2 + |m^2_{\tilde{t}^2_{LR}}|^2}$$

Similar for the sbottom sector for large $\tan\beta$

The SUSY Particles of the MSSM

Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	$+1$	$h \quad H \quad A \quad H^{\pm}$	$H_u^0 \quad H_d^0 \quad H_u^+ \quad H_d^-$
squarks	0	-1	$\widetilde{u}_L \quad \widetilde{u}_R \quad \widetilde{d}_L \quad \widetilde{d}_R$ $\widetilde{s}_L \quad \widetilde{s}_R \quad \widetilde{c}_L \quad \widetilde{c}_R$ $\widetilde{t}_1 \quad \widetilde{t}_2 \quad \widetilde{b}_1 \quad \widetilde{b}_2$	" " " " $\widetilde{t}_L \quad \widetilde{t}_R \quad \widetilde{b}_L \quad \widetilde{b}_R$
sleptons	0	-1	$\widetilde{e}_L \quad \widetilde{e}_R \quad \widetilde{\nu}_e$ $\widetilde{\mu}_L \quad \widetilde{\mu}_R \quad \widetilde{\nu}_\mu$ $\widetilde{\tau}_1 \quad \widetilde{\tau}_2 \quad \widetilde{\nu}_\tau$	" " " " $\widetilde{\tau}_L \quad \widetilde{\tau}_R \quad \widetilde{\nu}_\tau$
neutralinos	$1/2$	-1	$\tilde{\chi}_1^0 \quad \tilde{\chi}_2^0 \quad \tilde{\chi}_3^0 \quad \tilde{\chi}_4^0$	$\widetilde{B}^0 \quad \widetilde{W}^0 \quad \widetilde{H}_u^0 \quad \widetilde{H}_d^0$
charginos	$1/2$	-1	$\tilde{\chi}_1^{\pm} \quad \tilde{\chi}_2^{\pm}$	$\widetilde{W}^{\pm} \quad \widetilde{H}_u^+ \quad \widetilde{H}_d^-$
gluino	$1/2$	-1	$\widetilde{g}$	" "