Newton and his Laws

The Law of Gravity

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- At the time of Newton it was perfectly understood that there existed a force called "gravity" that made all objects fall to the ground.
- Newton conjectured that the same force was responsible for the Moon orbiting the Earth and the planets orbiting the Sun.

- If that was indeed the case, the acceleration acting on the Moon should be about 3600 weaker than the acceleration of objects to fall to the ground.

- Since the distance to the Moon was about 60 times the size of the Earth, the force of gravity had to obey the *inverse square law*:

$$\frac{1}{3600} = \frac{1}{60^2}$$

- Using the inverse square law for the gravitational force, Newton was able to derive all three Kepler's law of planetary motions.

- Using additional arguments, Newton finally arrived at the formula that gives the force of gravity between two objects with masses M_1 and M_2:

$$F_g = G\frac{M_1 M_2}{R^2}$$

where R is the distance between two objects, and G is a ***fundamental constant***, i.e. a number that is the same at all times and everywhere in the universe.

If one of the objects is much larger than the other (as, for example, the case of the Sun and a planet), then the mass of the larger object is usually denoted by *M*, and the mass of the smaller object is denoted by *m*:

$$F_g = G\frac{mM}{R^2}$$

Measuring *G*

- Newton's gravitational constant ***G*** is by far the ***worst*** known gravitational constant:

$$G = (6.6726 \pm 0.0003)\times 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$$

- The reason for that is that gravity can be measured very precisely, but it always comes as ***GM***, but it is *very* hard to measure masses of various objects accurately. We do know $\boldsymbol{GM_\odot}$ for the Sun better:

$$GM_\odot = 1.32712440018(8)\times 10^{20}\,\frac{\mathrm{m}^3}{\mathrm{s}^2}$$

- Now we can understand why all the objects fall to the ground with the same acceleration (and, thus, in the same time, if they fall from the same height).
- From Newton's Second Law:

$$mg = F_g = G\frac{mM}{R^2}$$

- *m* can be cancelled on both side, so it disappears.

$$g = G\frac{M}{R^2}$$

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There is no *m* any more in this equation, which means that *g* is independent on the mass of a falling object. At the surface of the Earth

$$g = 9.8\,\mathrm{m/s}^2$$

Oops! Do we have a problem?

- In the equation:

$$g = G\frac{M}{R^2}$$

G is constant (does not change no matter what), *M* is the mass of the Earth (does not change no matter what), but *R* is the distance to the center of the Earth, and it can change.

- On Skydeck (Sears tower) we are 412 meters further from the center of the Earth, and we therefore should weight less on Skydeck than on the ground.

Question: True or false?

Gravitational vs Inertial Mass

Recall: all objects fall to the ground with the same acceleration, because the gravity force is proportional to the mass:

$$\cancel{m}g = G\frac{\cancel{m}M}{R^2} \quad \Rightarrow \quad g = G\frac{M}{R^2}$$

But who said that two little m(s) are the same?

- ***Inertial mass*** is a measure of inertia, it enters the Second Law of Newton:

$$F = m_{\mathrm{in}} a$$

- ***Gravitational mass*** is a measure of how a body reacts to the force of gravity:

$$F = G m_{\mathrm{gr}} \frac{M}{R^2}$$

There is no a priori reason why these two should be the same!

(Weak) Equivalence Principle

- Equality of inertial and gravitational masses is called a ***(weak) equivalence principle***: inertial and gravitational masses are ***equivalent.***

$$m_{\rm in} = m_{\rm gr}$$

- Equivalence principle has been verified experimentally.

Tests of Equivalence Principle

1590, Galileo Galilei:	1 part in 50
1686, Isaac Newton:	1 part in 1,000
1832, Friedrich Bessel:	1 part in 50,000
1908, Baron von Eotvos:	1 part in 100 million
1930, J. Renner:	1 part in 1 billion
1964, Dicke et al:	1 part in 100 billion
1972, Braginsky, Panov:	1 part in 1 trillion
2008, Adelberger et al:	1 part in 30 trillion
2013, Galileo:	1 part in 100 quadrillion

Coordinates

- Science is based upon observations. We can observe space and time by measuring them.
- Any spatial position can be characterized by three numbers - **coordinates**. They are usually denoted by letters x, y, and z. Time is represented by the letter t.
- Thus, any point in space at every instant in time can be fully described by four numbers: (x,y,z,t).

Frame of Reference

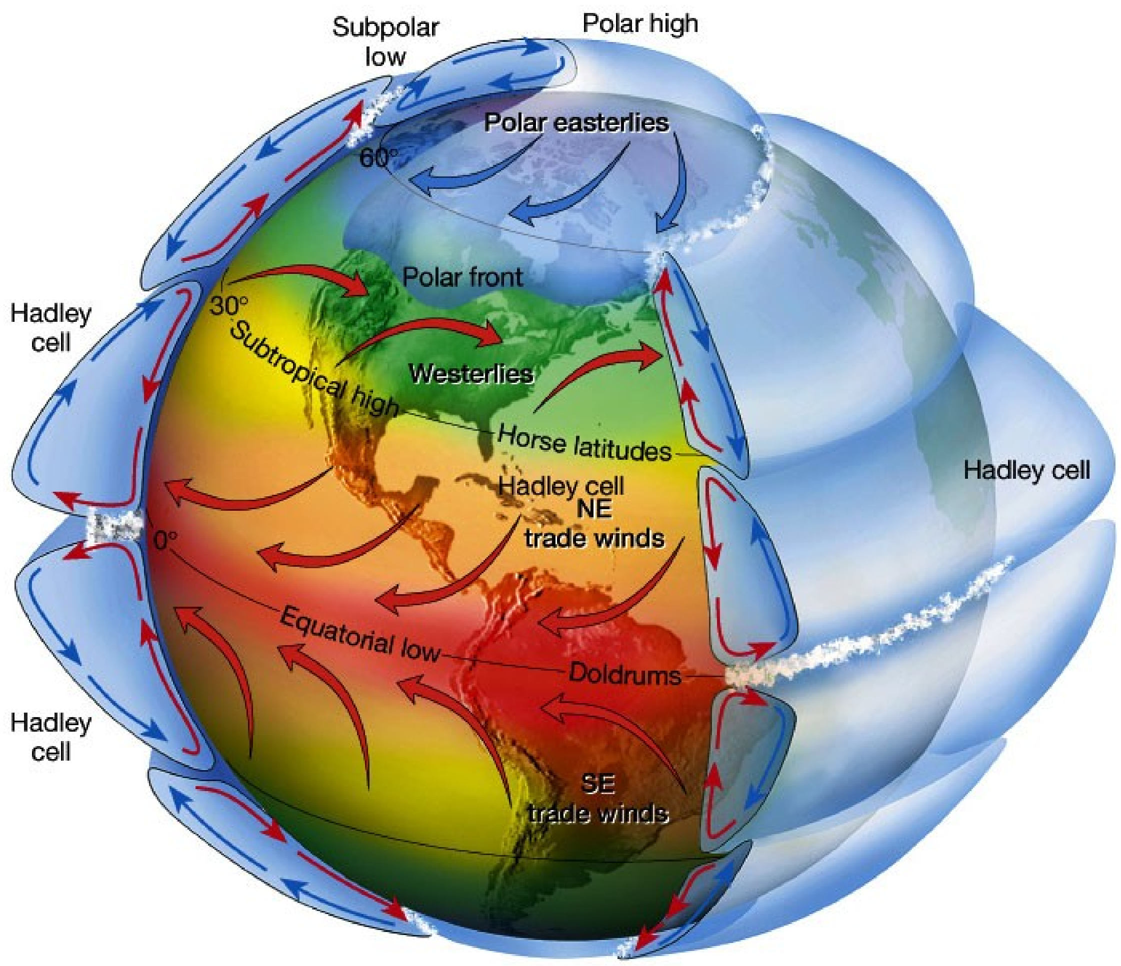
- Different observers may have different sets of coordinates (x,y,z,t). A set of coordinates specific to a particular observer is called a **frame of reference**.
- Not all frames of reference are equal. There is a special subset of all possible frames of reference called **inertial reference frames**. They are associated with observers that move freely, with no external force acting on them.

Inertial Forces

- In the non-inertial frame of reference there appear ***"fictitious" forces*** such as ***centrifugal*** and ***Coriolis*** forces. These forces are called **inertial forces**.

- These forces are fictitious in a sense that there is no physical interaction responsible for these forces. However, a person in a non-inertial frame of reference will feel them quite real!

Coriolis Force



Centrifugal Force

- The Second Law of Newton looks very different in inertial and non-inertial reference frames!

- In the ***inertial*** frame of reference:

$$F_{\mathrm{gr}} = m a_{\mathrm{cp}}$$

- In the non-inertial frame of reference:

$$F_{\mathrm{gr}} = F_{\mathrm{cf}}$$

Kepler's Laws Made Easy

- First Law:

$$m\frac{d^2\vec{r}}{dt^2} = -G\frac{mM}{r^2}\vec{e}_r$$

  - Solution of this equation is the ellipse. There is no deep physics there, just math.
- Second Law:

$$\frac{V^2}{R} = \frac{GM}{R^2}$$

  - When ***R*** is smaller, ***V*** is larger – planets move faster when they are closer to the Sun.

Kepler's Third Law

Period for a circular orbit: $P = {2\pi R}/{V}$

Plug this into the last equation from the previous slide, and we get:

$$P^2 = \frac{4\pi^2}{GM} R^3$$

If we measure ***P*** in years and ***R*** in AU, then

$$GM_\odot = 4\pi^2$$

- What is important is that the relationship between the size of the system (in this case *R*) and a measure of how fast objects are moving (in this case the period *P*) depends on the total mass of the system.
- Thus, if we know the size of a gravitating system, and how fast objects inside this system are moving, we can apply an appropriately modified form of the Kepler's law to measure the total mass of the system.

This is one of only two direct ways to measure masses of astronomical objects!

Strong Equivalence Principle

- A weak equivalence principle implies that all objects move the same way in an inertial and in a freely falling reference frame.
- Albert Einstein formulated a ***strong equivalence principle***:
  - ***All laws of nature are the same in inertial and freely falling reference frames.***
- Einstein's theory of relativity follows from there, but that is another story…

Energy

- Thomas Young (1773 – 1829) first coined the term "energy". He also disproved Newton's corpuscular theory of light.
- At the time of Newton, the concept of "energy" existed. Leibnitz called it "vis viva".
- Both, Leibnitz and Newton understood the process of energy conversion – for example, the kinetic energy of motion gets transformed into heat by friction.

- *There is a fact, or if you wish, a law, governing natural phenomena that are known to date. There is no known exception to this law; it is exact, so far we know. The law is called* ***conservation of energy****; it states that there is a certain quantity, which we call energy, that does not change in manifold changes which nature undergoes. That is a most abstract idea, because it is a mathematical principle; it says that there is a numerical quantity, which does not change when something happens. It is not a description of a mechanism, or anything concrete; it is just a strange fact that we can calculate some number, and when we finish watching nature go through her tricks and calculate the number again, it is the same.*

——The Feynman Lectures on Physics

Many Faces of Energy

- Energy comes in many faces:
  - ***Kinetic***
  - ***Thermal***
  - ***Gravitational***
  - ***Electromagnetic***
  - ***Rest energy***
  - ***Nuclear***
  - ***Atomic***
  - ***Chemical***

Conservation of Energy

- The conservation of energy implies that the sum of **all** kinds of energy of a closed (i.e. not interacting with something else) system is always conserved (as long as the system exists).
- Any particular kind of energy does not have to be conserved.

There are no exceptions to this law!!!
Never ever!!!
Nowhere!!!

Binding Energy

- Energy has a *sign* – it can be positive or negative. Negative energy is also called **binding energy**.
- If an object has binding energy, some other energy needs to be expended to disperse or destroy that object.
- Gravitational energy is always negative (= binding); nuclear, atomic, or chemical energy can be positive or negative.

Gravitational Energy

- Gravitational energy is always binding – gravity always pulls things together.

$$-G\frac{M^2}{R}$$

- If an objects gets more massive or smaller, then it binding energy gets more negative (often it is said – mathematically incorrectly – that its "*binding energy increases*").
- That results in **production** of some other energy.

Escape Velocity

- Conversely, to take a part of a gravitating object away requires an **expenditure** of energy.
- To send a spaceship off the Earth (an asteroid off the solar system, a star off a galaxy, …), the expended energy should be converted into the kinetic energy of motion.
- The speed that corresponds to that energy is called the **escape velocity**.

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}}$$