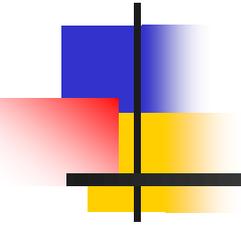
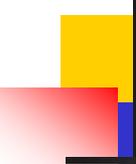


Big Bang



- 
- After he developed his GR, Einstein attempted to see what his theory predicts for the universe as a whole. In the curved space-time there is a possibility to construct a universe with positive curvature, which will be finite but without an edge. Apparently, Einstein did not like an idea of an infinite space-time.
 - His model had only one problem: since gravity is an attractive force, such a universe had either to expand or to contract, but it could not stay still. But astronomers of the time knew that the universe (i.e. Milky Way) was not expanding or contracting.
 - Thus, Einstein decided that his equations were wrong. Later he called this his "greatest blunder".



Cosmological Constant Is Born

- He decided to correct his equations by adding a new force that was not known before. This force should be repulsive, it should compensate the gravitational attraction. He called it the ***cosmological constant***, because the energy that was producing this force was constant everywhere.
- Thus, he was able to construct a stationary model of the universe. This model had only one problem: it was unstable.



What is outside the Universe?

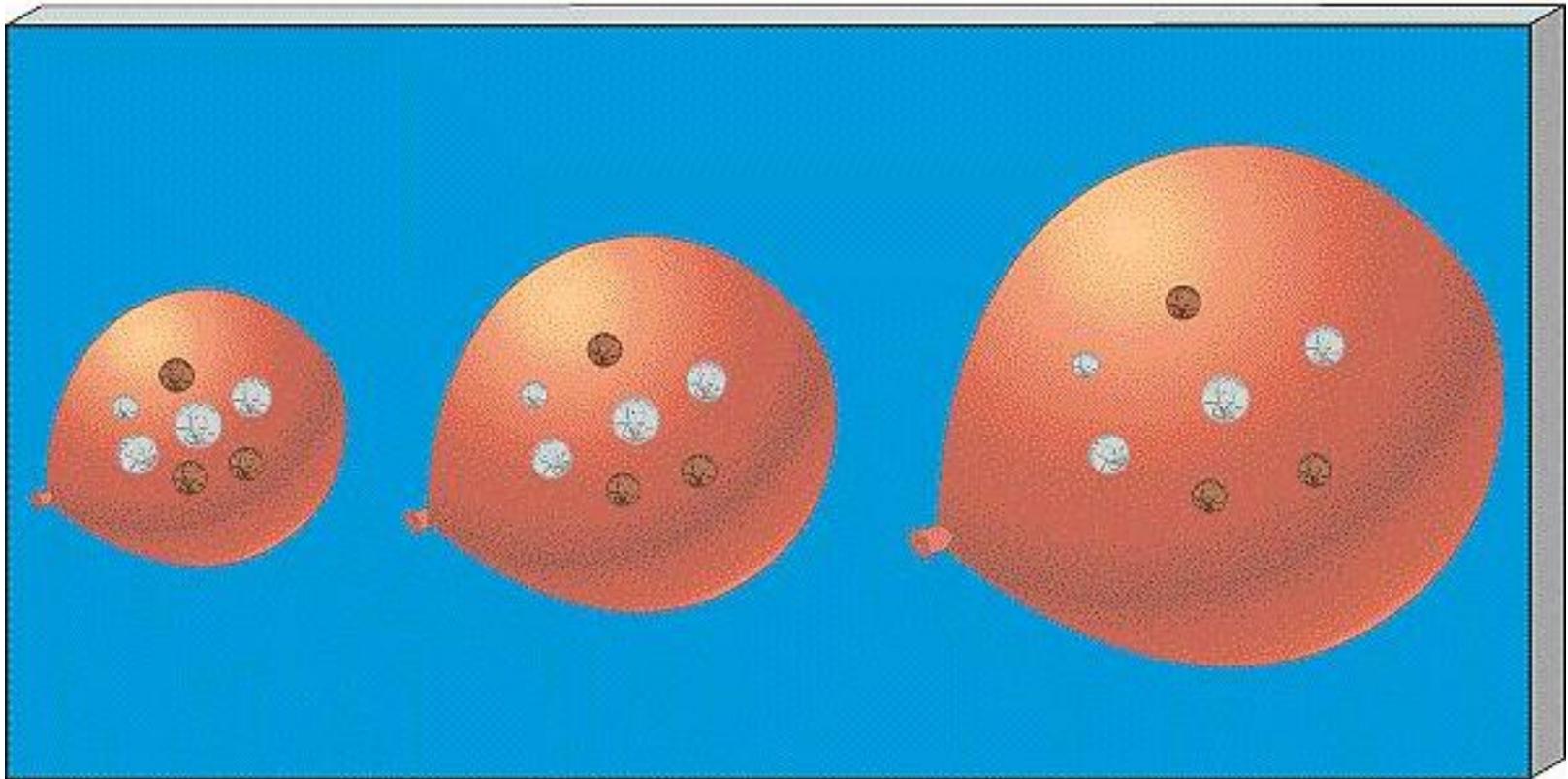
- A** Nothing
- B** Empty space
- C** This question is scientifically meaningless

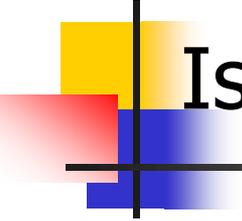


Expansion of the Universe

- Expansion of the Universe really means that the distance between any two inertial observers increases with time. You can think about it as if space is being created in between the objects. Non-inertial observers do not have to participate in the expansion of the Universe.

Expanding Universe Analogy



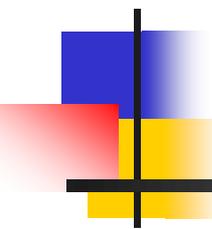


Is this room expanding with the Universe?

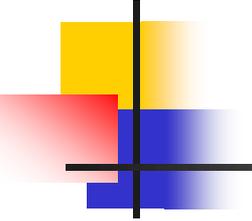
A Yes

B No

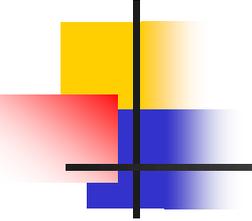
C The question is scientifically meaningless



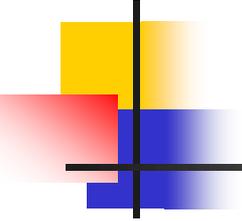
Describing the Universe

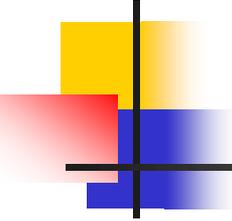
- 
- Einstein assumed that the universe was isotropic and homogeneous. Now we know it is.
-

- Since the universe is the same everywhere, the curvature of space can only be either
 - positive,
 - negative, or
 - the space can be flat.
- Respectively, such a universe is called either
 - ***closed***,
 - ***open***, or
 - ***flat***.

- 
- In GR the universe cannot be stationary - it should either expand or contract.
-

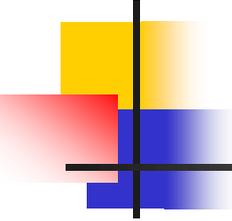
- A physical quantity that describes this contraction or expansion is called a ***scale factor***. The scale factor describes how a distance between two freely moving points changes relative to the distance at the current moment. The scale factor is usually denoted as a , or more rarely as R (our book uses R).
- The scale factor today is 1.

- 
-
- It is correct to think about the expanding universe as if the space itself was expanding or contracting.
 - Inertial observers would participate in this expansion or contraction, but non-inertial do not have to.



Cosmological Redshift

- For a free photon (or a group of photons which is a bundle of light rays), the wavelengths will increase, the frequency will decrease, and we will observe the Doppler redshift, which is called the ***cosmological redshift***. It is normally denoted as z .
- Thus, $z=1$ corresponds to the moment when the universe was twice "smaller" than the current one.



If the universe is infinite, then at $z=1$ its size was $\text{infinity}/2 = \text{infinity}$. Is there a paradox?

- A. Yes, there is a paradox, and the modern cosmology cannot address it.
- B. No, there is no paradox, the infinite universe does not expand or contract, since it has nowhere to expand into.
- C. No, there is no paradox, the fact that the universe was twice “smaller” means that all distances between freely moving points were twice smaller, and that's it.



Modeling the Universe



- Einstein was not pleased with his cosmological model, but he could not invent anything better.
- In 1924 Russian physicist Alexander Friedmann found all possible theoretical models for the isotropic and homogeneous universe in GR without a cosmological constant. None of them was stationary. Einstein first claimed that Friedmann made a mistake in his calculations, but later admitted that Friedmann was right.

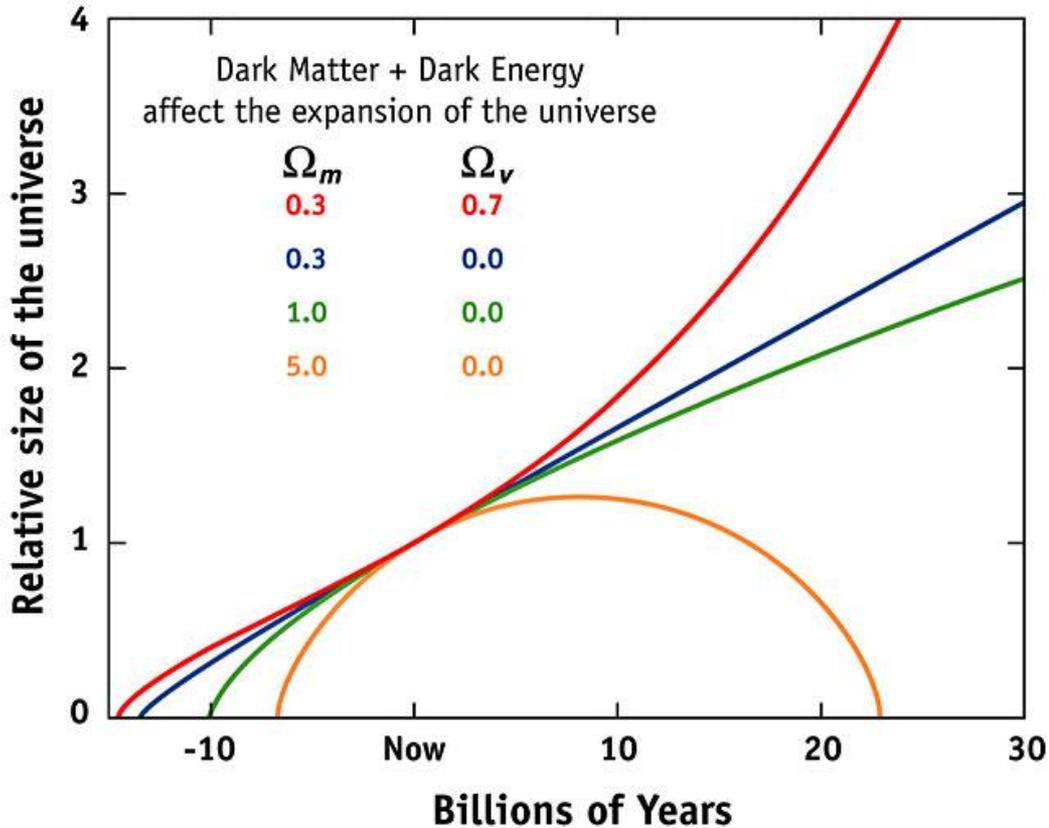


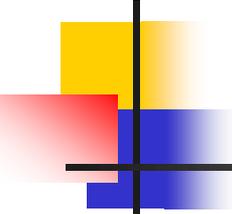
Friedman found out that:

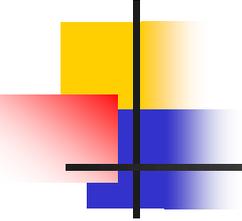
- The universe either expands or contracts.
- All three types of the universe (open, flat, and closed) begin at the state with infinite density and zero size (i.e. a singularity) and expand from there. This process of beginning from the infinite density is now called a ***Big Bang*** (the term itself appeared in 50s).
- The flat and open universes expand forever.

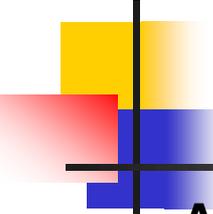
Friedman's Model

EXPANSION OF THE UNIVERSE



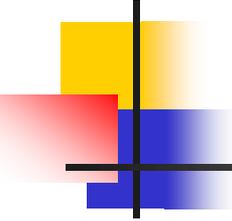
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- The closed universe initially expands, reaches the maximum size, and then contracts to the singularity again (***Big Crunch***). Thus, the closed universe is also “closed in time” (it exists only for a finite time).
 - It takes the closed universe precisely the same time to reach the singularity from a point where it stops expanding as to reach this point from the beginning of the Big Bang.

- 
-
- The curvature of the universe (positive, negative, or zero) is usually labeled by a quantity called k :
 - **closed universe** has $k=+1$ (positive curvature).
 - **flat universe** has $k=0$ (zero curvature, flat).
 - **open universe** has $k=-1$ (negative curvature).



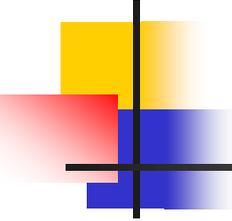
The fate of the universe depends on the amount of matter in the universe:

- A lot of matter: closed universe.
- Just right amount of matter: flat universe.
- Too little matter: open universe.
- Why?
 - A. It is predestined so.
 - B. In a closed universe the matter is more densely packed because there is little room.
 - C. Gravity controls how the universe expands.
 - D. Matter is consumed to make the universe expand.



Matter Density Parameter

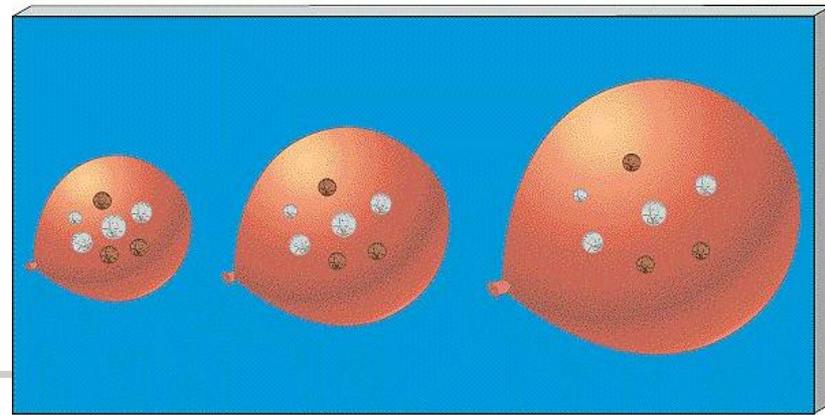
- The amount of matter is quantified by a quantity called *matter density parameter* and is labeled by a Greek letter Omega Ω_M .
- $\Omega_M > 1$: closed universe
- $\Omega_M = 1$: flat universe
- $\Omega_M < 1$: open universe



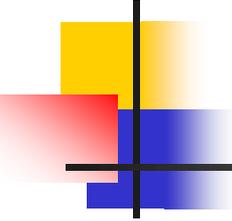
Stick To Your Guns

- The density parameter always stays on the same side of unity: In other word, an open universe is always open, a flat universe is always flat, a closed universe is always closed.
- This is because the current moment, when we are measuring Ω_M , is not a special moment in the history of the universe.

Hubble Law

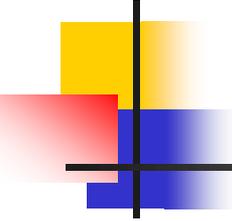


- When the homogeneous and isotropic universe expands, space expands everywhere at the same rate.
- If 3 points A,B,C are equidistant, then C is moving away from A twice faster than B. Hence the relative receding velocity is proportional to the distance: $v = H_0 r$.
- Wait – this is a Hubble Law!!!
- Hence the Hubble Law is a manifestation of the expansion of the universe.



Important Conclusion

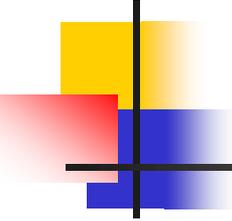
- Cosmological parameters like H_0 and Ω_M change with time. They only have definite values up to a given precision. For example, the 18th decimal place in both those numbers changes every second.
- But since both H_0 and Ω_M change significantly only on a cosmological time-scales, over the periods of billions of years, for any practical purposes these two parameters have definite values.



Cosmological Redshift and Cosmic Time

- If the universe expands with time, it is larger now than it was some time ago, i.e. we can use the cosmological redshift as a measure of cosmic time: $z=1$ does not mean that the universe is twice younger, but there is one-to-one correspondence between the age of the universe and the redshift.
- We will use the redshift as a proxy to cosmic time, whenever it is convenient.

Cosmological Redshift and Cosmic Time

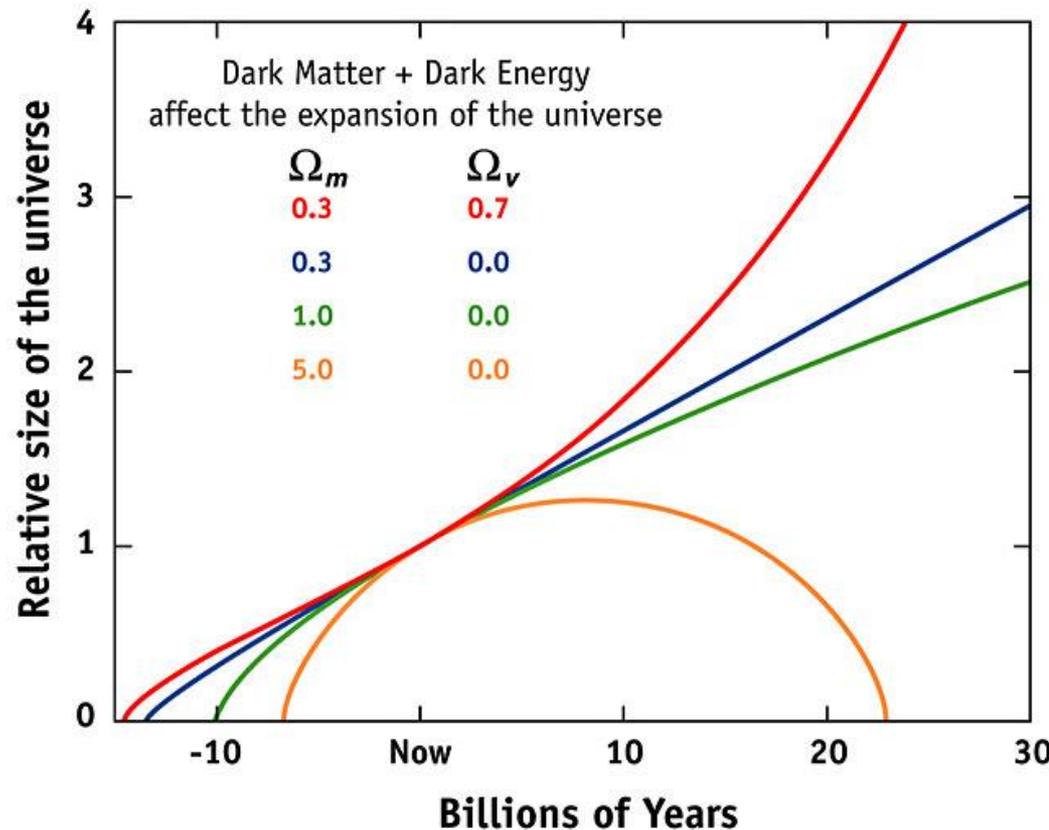


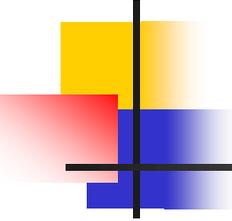
- Redshift really measures the “size” of the universe, i.e. how the space expands. Why can we use it as a measure of time too?
 - A. It is convenient to have a single measure for all quantities.
 - B. We can only see along our past light cone.
 - C. Expanding the whole universe takes time.
 - D. This is just a math trick.

The Age of the Universe

- The Hubble constant measures how fast the universe expands right now. It tells nothing about how fast the universe was expanding in the past, or how old the universe is.

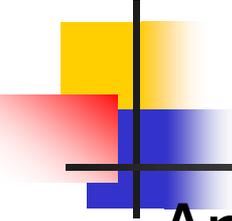
EXPANSION OF THE UNIVERSE





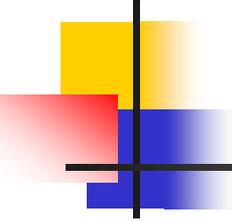
The Age of the Universe

- The age of the universe is inversely proportional to H_0 , but is a complex function of Ω_M . If $H_0 = 70 \text{ km/s/Mpc}$ and $\Omega_M = 1$, then the universe is about 10 billions years old.



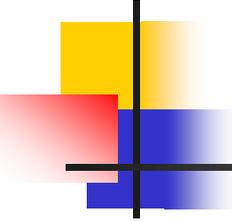
Cosmological Horizon

- An important feature of the Big Bang theory is the ***cosmological horizon***. Since the universe has only a finite age, light travels only a finite distance during the age of the universe. Thus, we cannot see arbitrary far: there are galaxies in the universe that are further away than the "light-age" of the universe, so we cannot see them. The sphere around us with the radius equal to the "light-age" of the universe is called the *cosmological horizon*. It is about 15,000 megaparsecs.



Cosmological Constant

- Without a cosmological constant all cosmological models are *decelerating*, i.e. the expansion of the universe is slowing down (but it does not have to stop completely).
- Cosmological constant acts as a repulsive force, so the expansion of the universe can become *accelerating*.
- The cosmological constant is usually labeled by the Greek letter “lambda” Λ .



Cosmological Constant

- Recall the symbolic form of Einstein equations:

$$\mathbf{Geometry = Energy}$$

- With the cosmological constant this becomes:

$$\mathbf{Geometry + \Lambda = Energy}$$

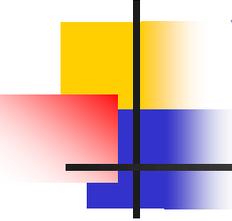
- One can do a math trick:

$$\mathbf{Geometry = Energy - \Lambda}$$

- Is there a problem in doing this?

A. Yes

B. No



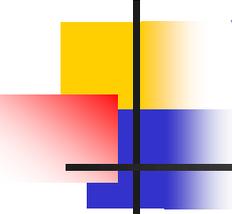
Cosmological Constant and “Dark Energy”

- Math does not care about the cause and effect – but we do! We cannot just move Λ to the right side, but we can hypothesize existence of new form of energy that has repulsive gravity:

$$\text{Geometry} = \text{Normal-Energy} \\ + \text{Dark-Energy}$$

- Physicists generally dislike the cosmological constant and prefer to talk about the Dark Energy.

Cosmological Constant and “Dark Energy”

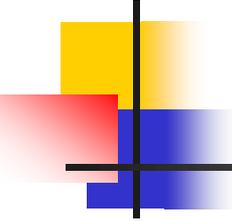


- Hence, there are two alternative ways to make the expansion of the universe accelerate:

Geometry + Λ = Normal-Energy

- or

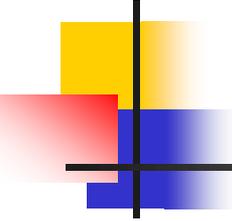
**Geometry = Normal-Energy
+ Dark-Energy**



Flat Universe

- When we talk about the dark energy, we quantify its abundance with its own density parameter, Ω_Λ (sometime also labeled as Ω_V).
- In the presence of dark energy the condition for the universe to be flat changes:

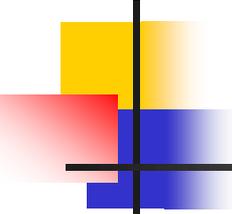
$$\Omega_{\text{TOT}} = \Omega_M + \Omega_\Lambda = 1$$



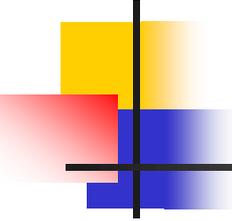
Is Dark Energy Even Possible?

- When Einstein introduced his cosmological constant, he did it on purely adhoc grounds. He did not think about it as some “dark energy”.
- However, particle physicists since 60s have been actually discovered particles that may behave as Dark Energy!
- None of the known ones are abundant enough to make up enough of Dark Energy, but at least we know such stuff can exist.

The Ultimate Fate of the Universe

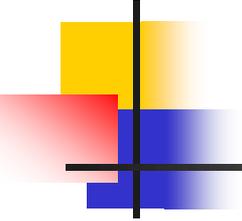


- The universe is expanding now. In the future it has only two possibilities:
- it expands forever if $\Omega_{\text{TOT}} \leq 1$.
- it stops expanding and re-collapses if $\Omega_{\text{TOT}} > 1$.

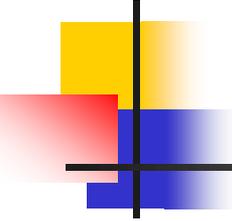


If the universe re-collapses:

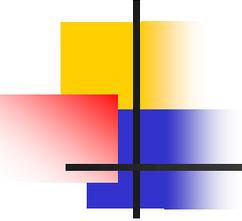
- The universe is still expanding, so it will not start recollapsing for at least another 100 billion years.
- By the time the universe stops expanding and begins recollapsing, all of the stars will be dead.
- About 100 billion years after it begins recollapsing, most of galaxies will collapse into huge black holes.
- The universe will continue recollapsing until it becomes so small that those black holes begin to touch each other.

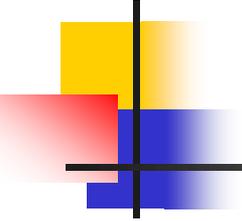
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- The black holes will be merging with each other as crazy. Perhaps, they will do it everywhere more or less uniformly, then the universe will collapse in a Big Crunch.
 - Perhaps, one of the black holes will become much larger than the others. Such a universe is not homogeneous and isotropic any more, and only God knows what will happen then...

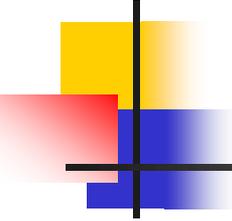
If the universe expands forever:



- In about another 10 billion years galaxies will use up their gas and new stars will stop forming.
- After about 100 billion years last stars will die. The universe will be filled with stellar "cinder": white dwarfs, neutron stars, and black holes.
- In another 1000 billion years most of the stellar remnants in galaxies will collapse to a central black hole, leaving a few stellar remnants to orbit it, and throwing a few more into the empty intergalactic space.

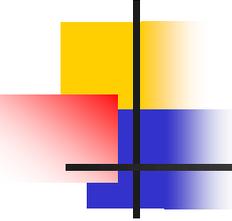
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- In about 10^{40} years protons will decay, leaving the ``lepton desert''.
 - In about 10^{110} years all black holes will evaporate, and galaxies will dissolve.
 - After that the universe will be filled with very slowly moving electrons, neutrinos, very low energy photons, and a few unattached blobs of electrons and neutrinos, all that is left of stars.

- 
-
- No matter what the fate of the universe is, the perfect cosmological principle is wrong, the universe will not be like it is now.
 - Your book says: *“If we pass this way but once, we must make the most of it.”*



Olber's Paradox

- Newton's universe was infinite in space and time and uniformly filled with stars. Kepler was the first to realize that in such a universe the sky should be ablaze: during infinite time stars emit infinite amount of light!
- Several scientists discussed this paradox, which for some reason is now called ***Olbers' paradox***, even if Heinrich Olbers was not the first to mention it.



In the expanding universe:

- Stars existed only for a finite time since they formed after the Big Bang.
- Light from remote stars is redshifted and becomes invisible.
- Any one of those two features is enough to explain the darkness of the night sky. Thus, in the Big Bang theory the Olbers' paradox is resolved with a large safe margin!