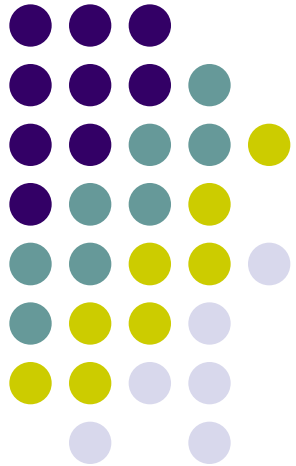
Special Relativity I



Aftermath of M-M Experiment

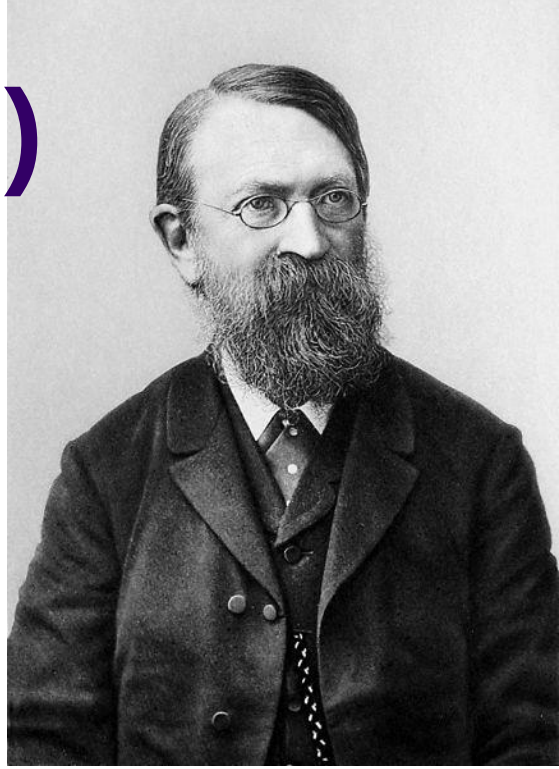
After the Michelson-Morley experiment, the scientists were left with two equally unpleasant alternatives:

- either Maxwell's equations were incorrect (but they explained all the electromagnetic phenomena so well!...)
- or Galilean principle was wrong/incomplete (yet Newtonian mechanics works so well everywhere, from orbits of planets to everyday experience on Earth!...)

Various explanations were immediately proposed.

- George FitzGerald suggested that all objects moving through the ether were physically contracted depending on their speed: the faster they were moving, the larger was this contraction. Objects moving with the speed of light were contracted to zero length (!) – still called “FitzGerald contraction”.
- Others proposed “ether drag”: all moving bodies dragged ether around them along with them (but then it could not be frictionless).

Ernst Mach (1838-1916)



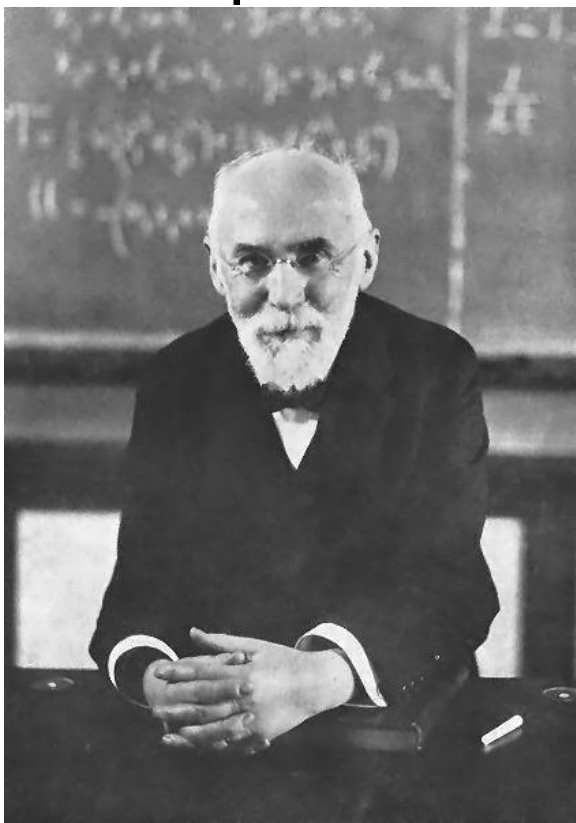
- Both explanations were unsatisfactory. Austrian physicist and philosopher Ernst Mach offered a different explanation:

  The Michelson-Morley experiment was designed to detect ether. No ether was detected, there-fore, there was no ether at all.

- This explanation fully follow the rules of science. However, it was difficult to accept since there was no alternative theory (and since Mach was a weirdo)!

Hendrik Lorentz (1823-1928)

Based on the FitzGerald hypothesis, Lorentz discovered a *coordinate transformation*, i.e. a way of relating two different reference frames, which kept the speed of light, and thus Maxwell's equations invariant. This transformation has been called Lorentz transformation ever since.

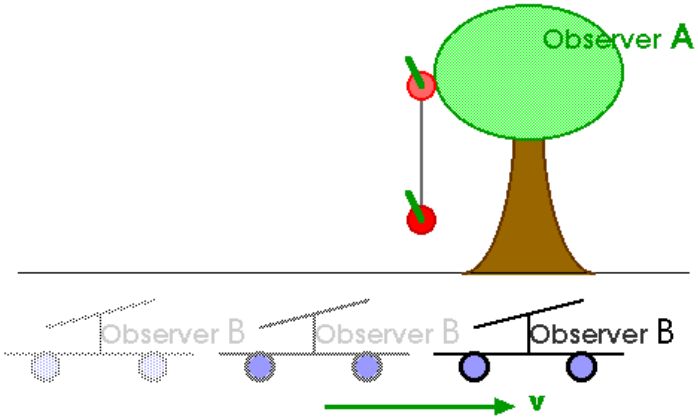


- *Maxwell's equations are invariant under the Lorentz transformation.*
- *Newton's equations are invariant under the Galilean transformation.*

Coordinate Transformations

Galilean transformation:

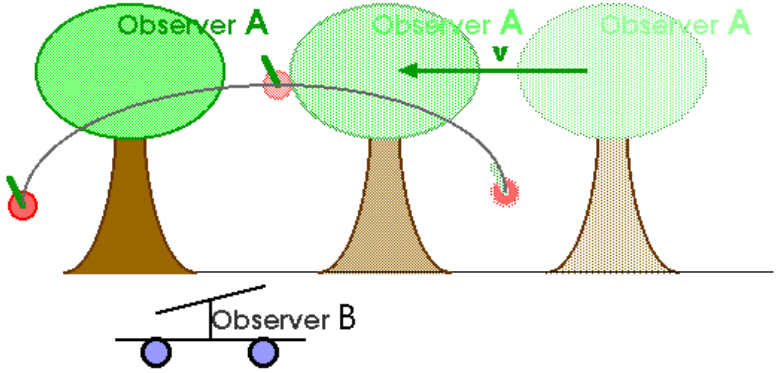
$$\begin{aligned} x' &= x + vt \\ t' &= t \end{aligned}$$



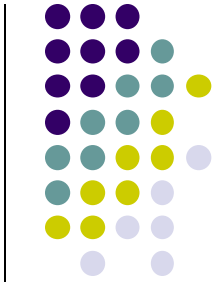
Lorentz transformation:

$$\begin{aligned} x' &= \gamma(x + vt) \\ t' &= \gamma(t + \frac{vx}{c^2}) \end{aligned}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



Lorentz Transormation



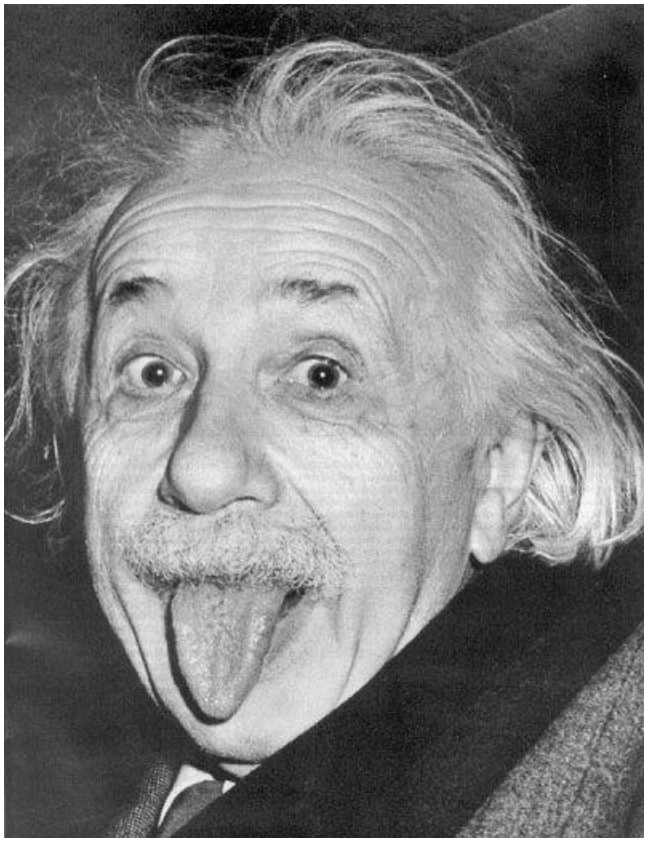
For speeds much less than the speed of light, both transformations are identical.

However, for speeds close to the speed of light, Lorentz transformation predicted weird things: lengths should contract and time intervals should increase (time dilation). This seemed so radical, that few people were ready to accept this.

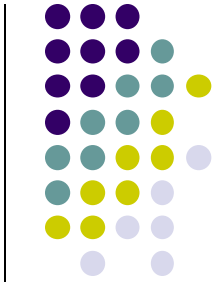
Then there came a patent examiner from Bern...

Albert Einstein (1879 – 1955)

- Began his career as a patent clerk in 1902.
- In 1905 developed Special Theory of Relativity.
- In 1908 became Assistant Professor at Bern.
- In 1915 developed General Theory of Relativity.
- In 1921 got Nobel Prize.



Relativity Principle

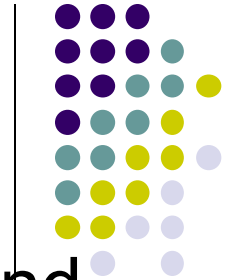


Einstein realized, that these weird things like length contraction and time dilation were not absolute, but *relative*. In other words, they only *appeared*.

Einstein based his theory on the relativity principle:

> *The laws of nature are the same in all inertial frames of reference.*

Special Relativity

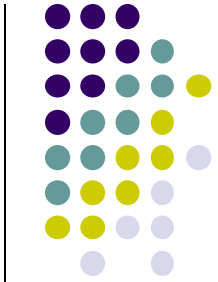


Einstein also believed in Maxwell's equations, and since Maxwell's equations require that the speed of light is the same in all reference frames, he simply stated that:

> *The speed of light in the vacuum is the same in all inertial frames of reference.*

Thus, he accepted Lorentz transformation and "discarded" Galilean transformation. In doing so, he created the **Special Theory of Relativity**, or **SR**.

Question



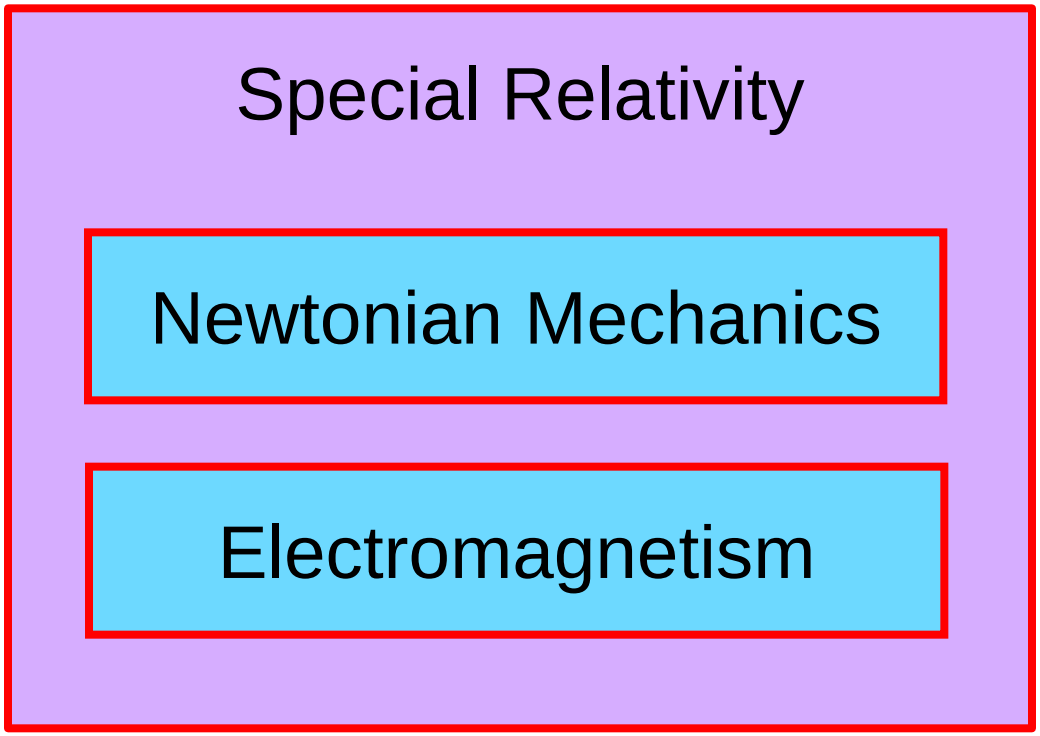
You book says:

> *Einstein had the audacity and courage to abandon Galilean relativity completely, and with it Newtonian mechanics...*

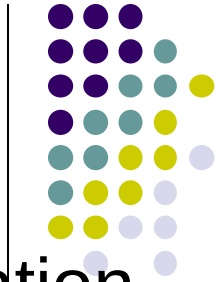
A. True
B. False

Physics Theories

Special Relativity extends Newtonian Mechanics to close-to-c speeds, but it does *not* contradicts it – on the opposite, it includes all of Newtonian Mechanics in its entirety.



γ Factor



- Both, the length contraction, and the time dilation, are described by one quantity, called the boost factor, or simply γ factor, because it is traditionally denoted by a Greek letter γ .

- If an object is moving with respect to a specific reference frame, it appears that all lengths along the direction of motion are contracted γ times, and all clocks on this object are slowed down γ times.

γ Factor

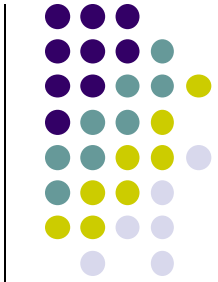
If the speed of an object is much smaller than the speed of light, the γ factor is almost exactly 1, and Newtonian mechanics with Galilean relativity rules.

When the speed of an object approaches the speed of light, the γ factor becomes very large, and then deviations from the Galilean relativity become large too.

γ Factor

v/c	γ
$1/2$	1.15
$2/3$	1.34
$3/4$	1.51
$4/5$	1.67
$5/6$	1.81
0.9	2.3
0.99	7.1
0.999	22.4
0.9999	70.7

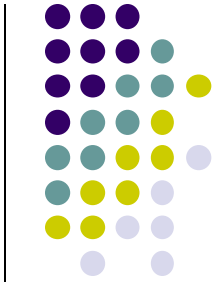
Proper Length and Proper Time



Since the time and the length appear differently to different observers, i.e. they become relative, it is important to have some invariant quantities as well.

- **Proper time** is the time that is measured in the reference frame that is at rest with respect to an object.
- **Proper length** is the length that is measured in the reference frame that is at rest with respect to an object.

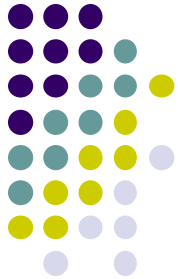
Proper Length and Proper Time



Thus, if you want to measure a proper length of an object, or a proper time interval between two events, you need to be in the reference frame that is at rest with respect to this object or events.

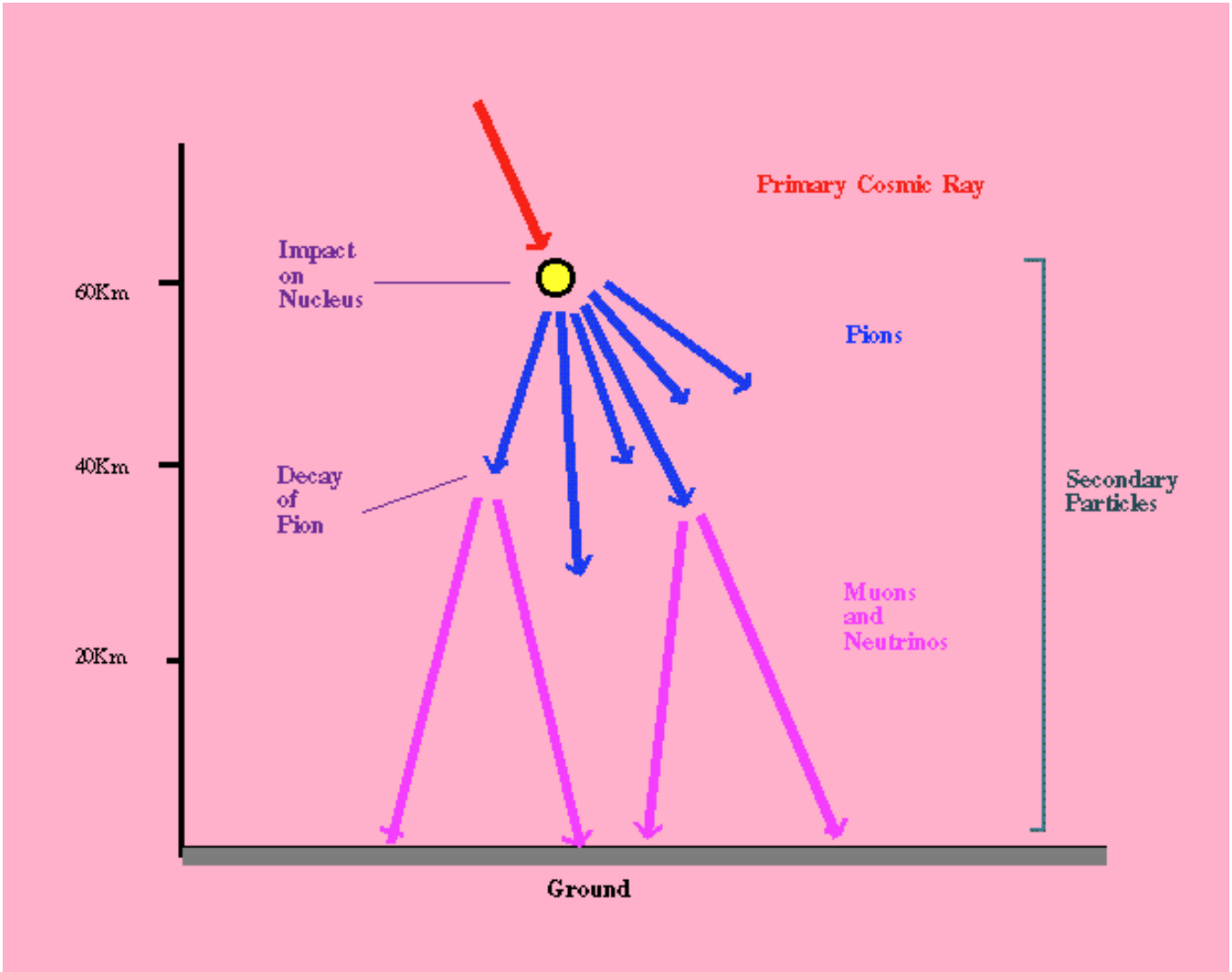
- Proper time interval is always the smallest as measured by various observers.
- Proper length is always the largest.

Proper Time Is Real – Atmospheric Muons



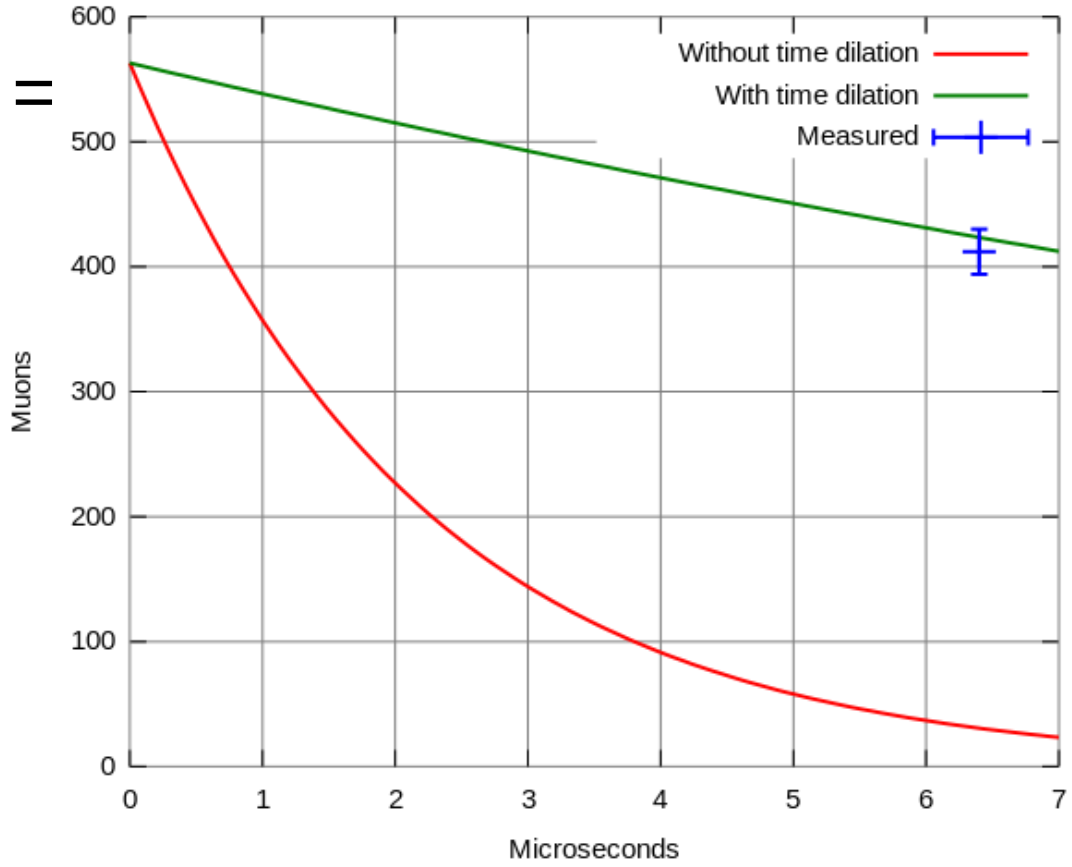
- The Earth is bombarded by energetic particles from space – **cosmic rays**. The most energetic of them would heat a teaspoon of water by 8^{o}C.
- Fortunately, we are protected from them by the atmosphere.

Proper Time Is Real – Atmospheric Muons



Proper Time Is Real – Atmospheric Muons

- Muons decay very quickly – they live only for 2 micro-seconds.
- 2μs * 300,000km/s = 0.6km = 600m



Space-Time

Since both, space distances, and time intervals, become relative, and the property of simultaneity is lost, we should not separate space and time any more, but talk about **space-time**.

Usually, space-time of SR is also called **Minkowski space-time**, due to Hermann Minkowski (we will meet other space-times later).

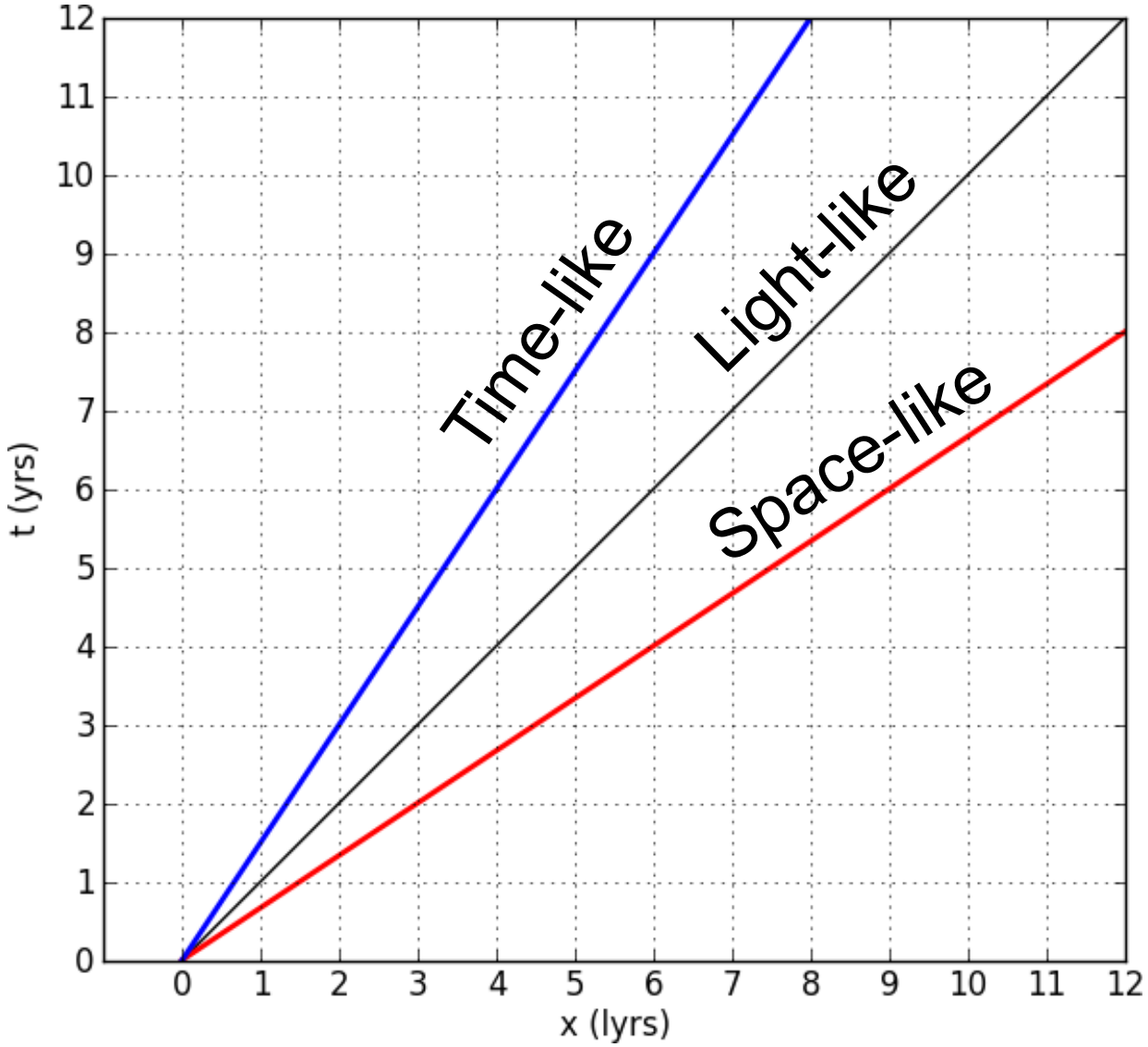
Space-Time Diagrams

From now on we will measure distance in *light-years* (light-second, light-hours, etc). One light-year is the distance light travels in one year.

- 1 light-second = c*1 second = 300,000 km
- 1 light-hour = c*1 hour = 10^9 km
- 1 light-year = c*1 year = 10^{13} km

A key advantage of such units – light now travels at 45 degrees.

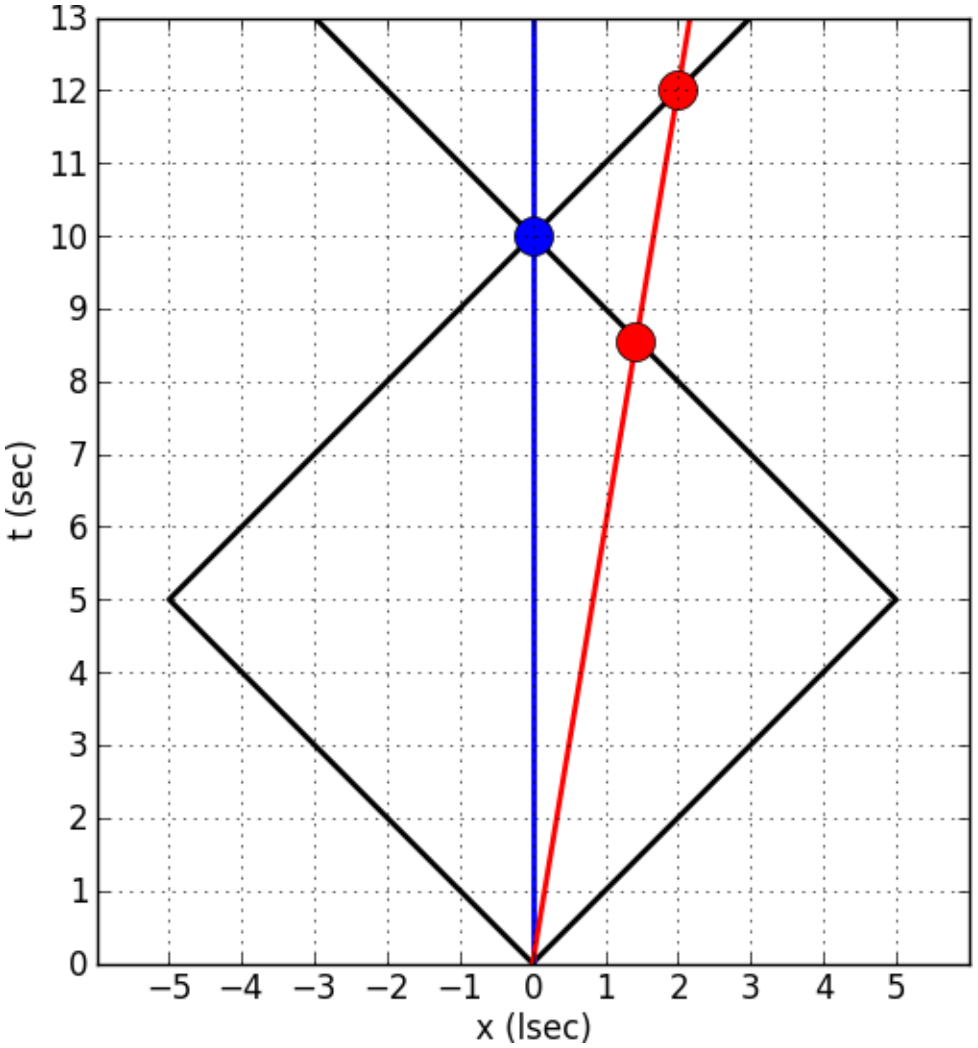
Space-Time Diagrams



- Material objects travel ***only*** along time-like world lines.

- Light (and other massless particles) travel along light-like world lines.

Why Special Relativity Is Called “Relativity”

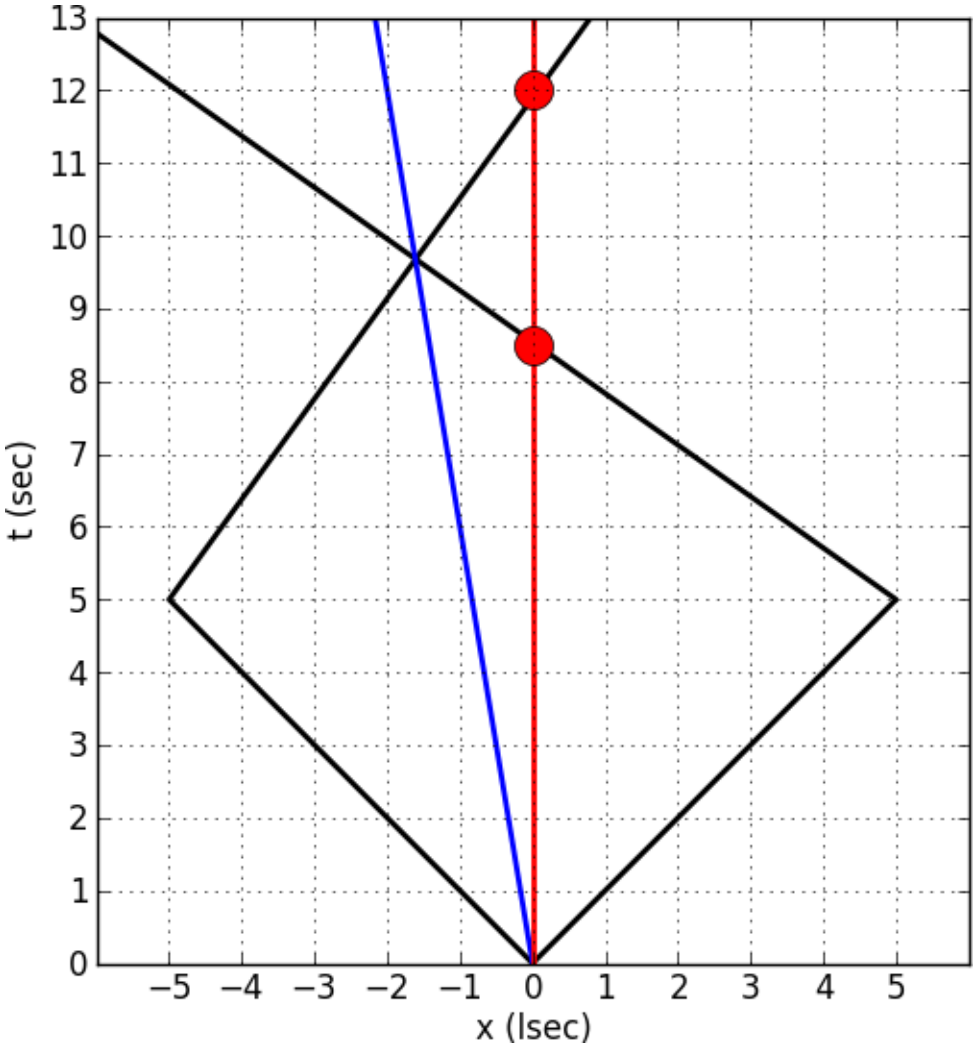


Blue sees two events as simultaneous.

Red sees them as happening at different times.

Not surprising, Red travels towards one of them.

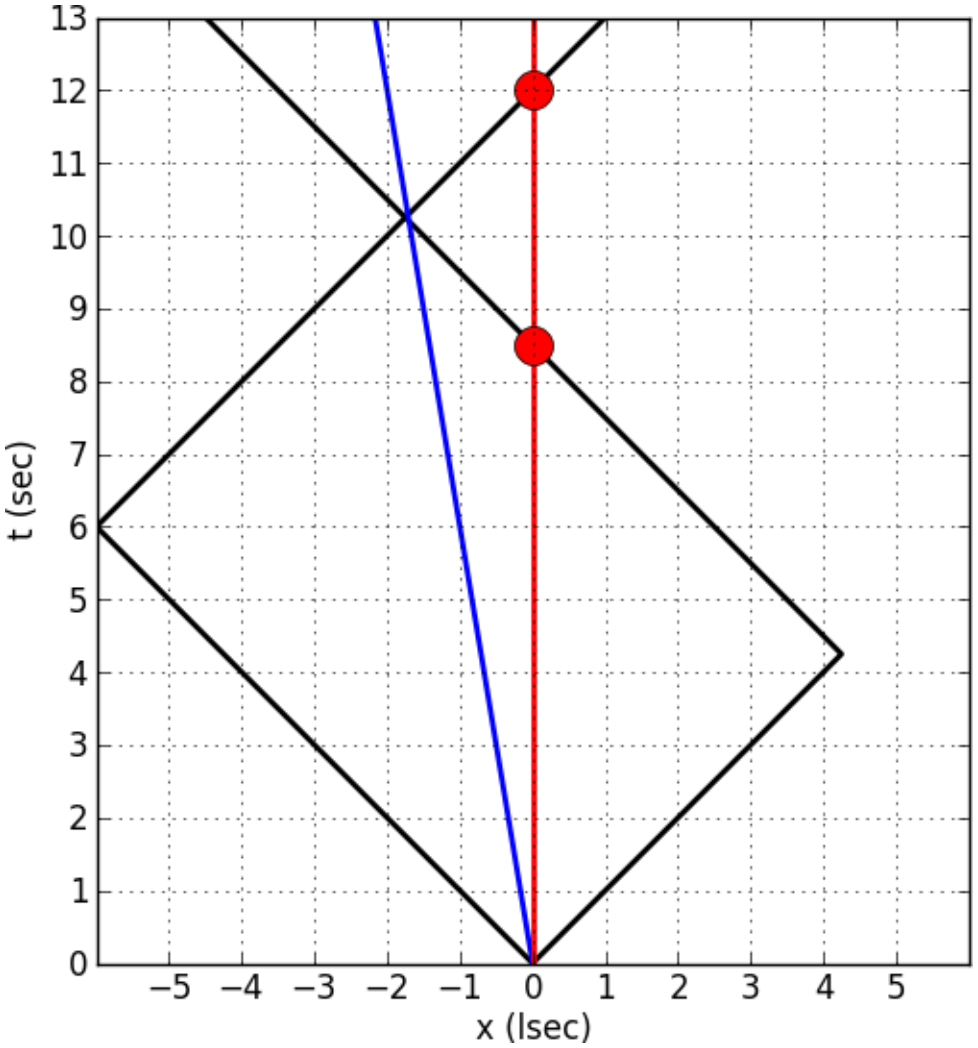
Why Special Relativity Is Called “Relativity”



Red reference frame.

Is there anything wrong in this picture?

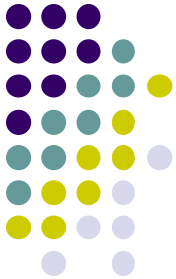
Why Special Relativity Is Called “Relativity”



Red reference frame.

Red sees the two events as **not** simultaneous!

Why Special Relativity Is Called “Relativity”



In Special Relativity not all observers agree on whether a pair of events are simultaneous or not.

Different observers may see two causally-disconnected events as happening in a different order.

Causally connected events always happen in the same order.