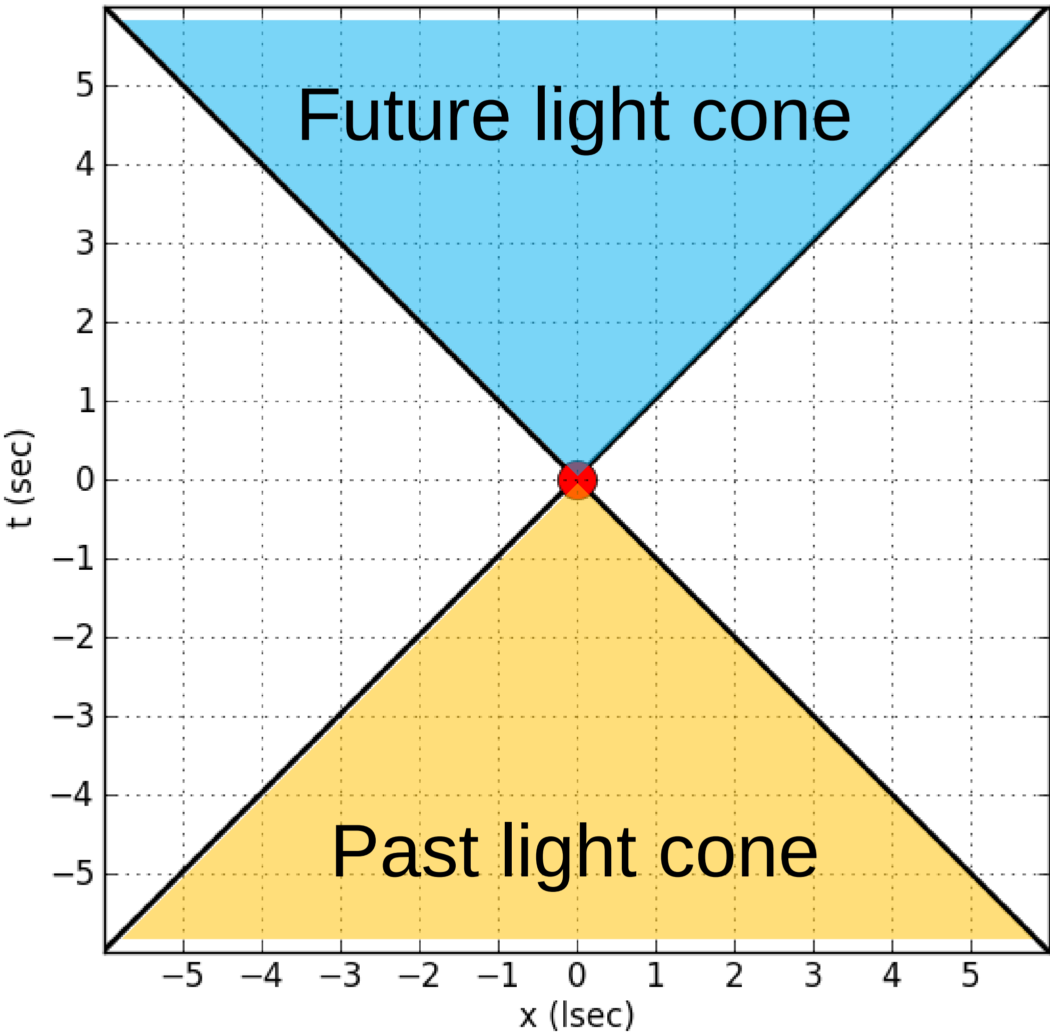
Special Relativity II

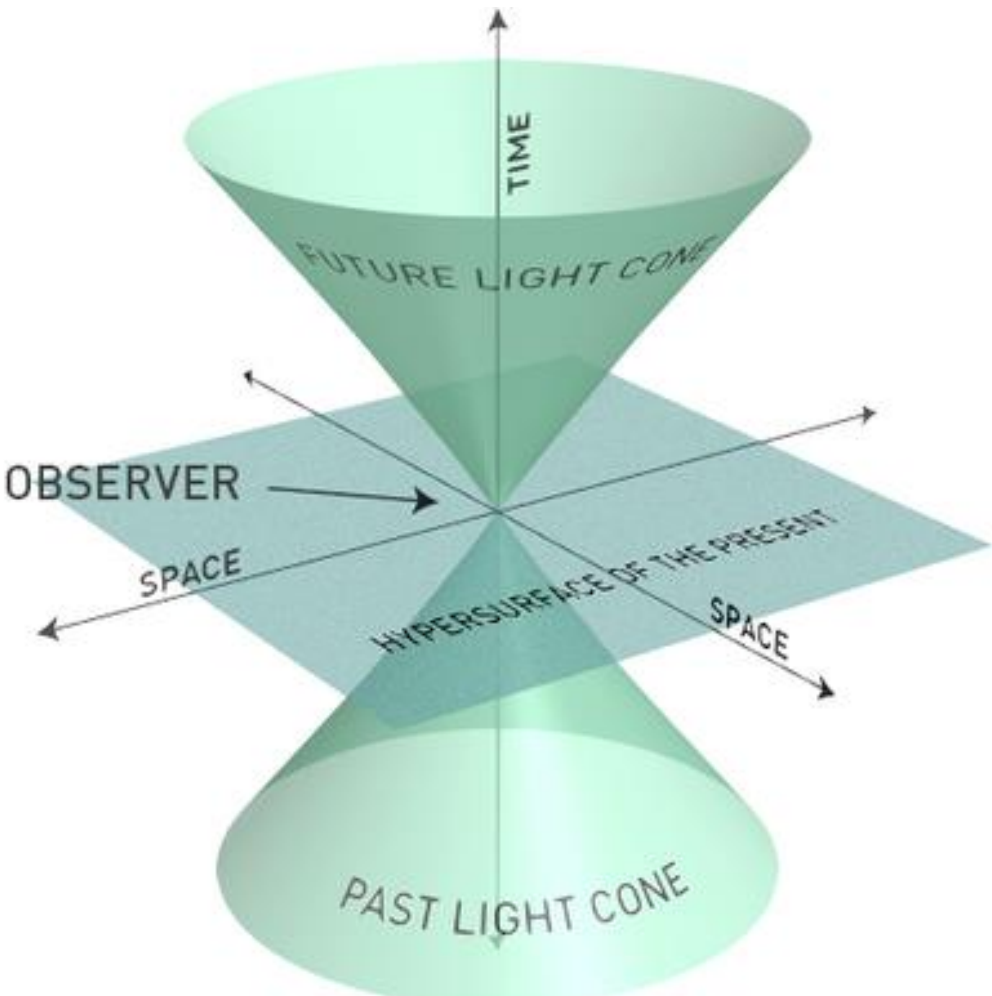
Future And Past Light Cones



Future light cone: all the events you can influence.

Past light cone: all the events that could have influenced you.

Why Cones?



Future light cone: all the events you can influence.

Past light cone: all the events that could have influenced you.

Question

How to observe a galaxy 10 billion years ago?

A. Find a galaxy 10 billion light years away.
B. Jump in a time machine and go 10 billion years back.
C. Travel to a parallel universe where time flows slower.
D. This is absolutely impossible.

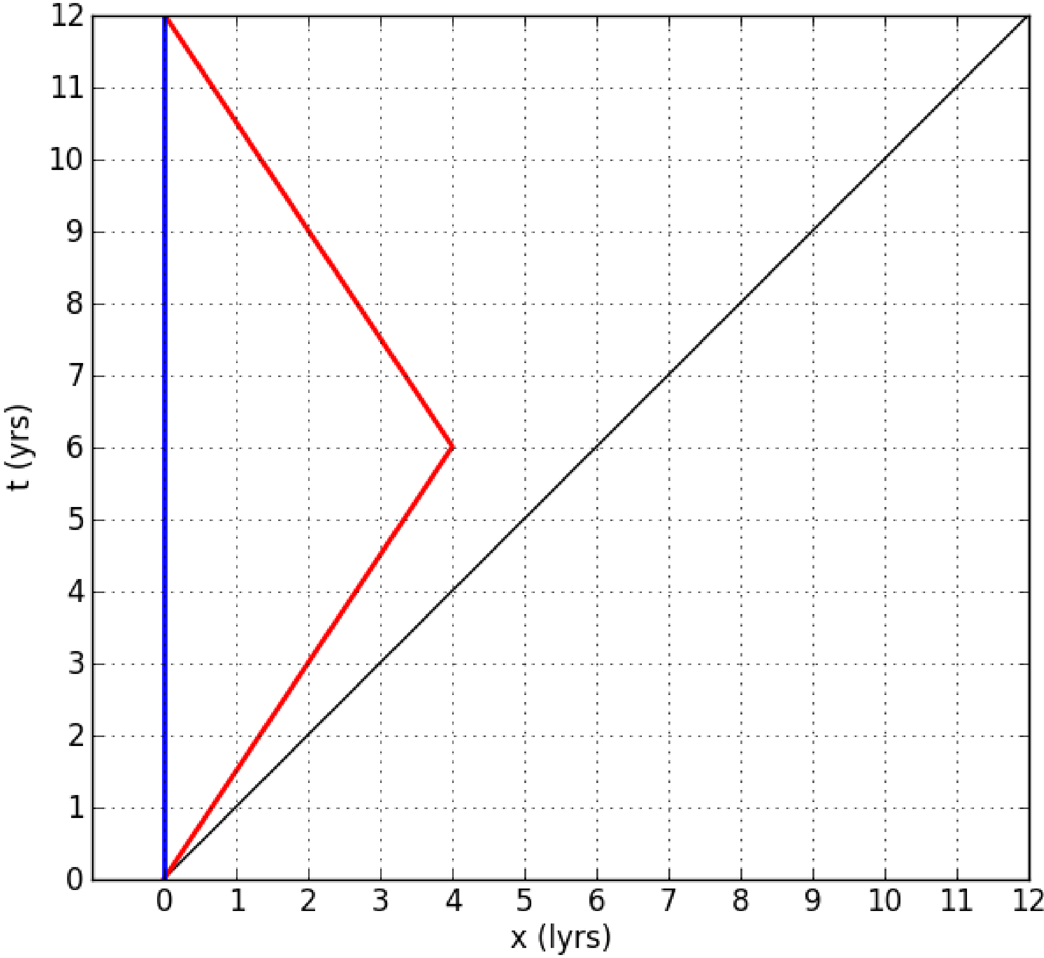
Twin Paradox

Andy and Betty are twins. Betty rides in a spaceship at 2/3 the speed of light to Alpha Centauri and back. Andy stays on Earth and waits for her return. Who will be older when they meet?

What if some time after Betty had left, Andy jumps into a spaceship and flies after her even faster, and eventually catches up with her. Who will be older when they meet?

Twin Paradox

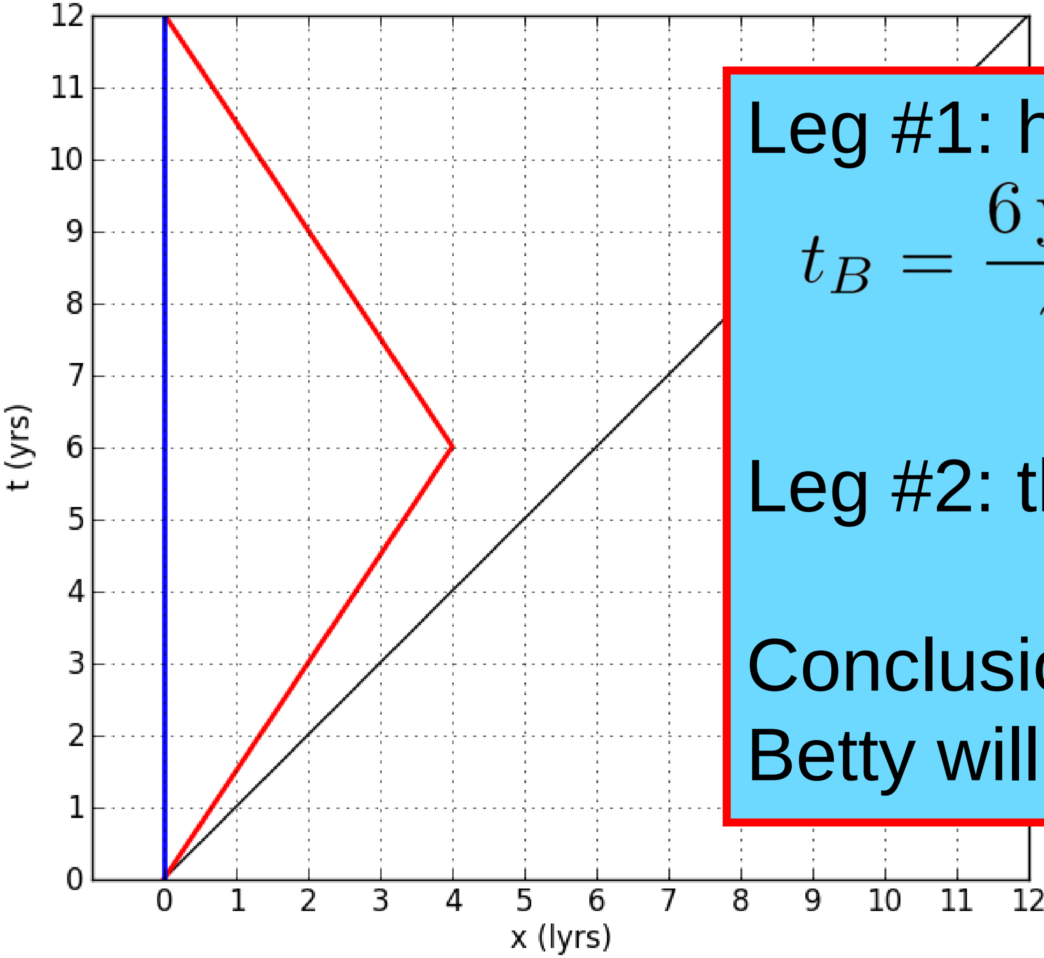
It takes Betty 12 Earth years to get back, hence Andy will be 12 years older. What about Betty?



A. Betty will be younger.
B. Betty will be older.
C. Betty will be the same age as Andy.

Twin Paradox

It takes Betty 12 Earth years to get back, hence Andy will be 12 years older. What about Betty?



Leg #1: her proper time

$$t_B = \frac{6\,\mathrm{yrs}}{\gamma} = \frac{6\,\mathrm{yrs}}{1.34} = 4.5\,\mathrm{yrs}$$

Leg #2: the same

Conclusion:
Betty will be only 9 yrs older!

The Paradox of the Twin Paradox

The actual paradox of the twin paradox is that motion is relative:

- In Andy's reference frame, Andy is at rest and Betty is moving.
- In Betty's reference frame, Andy is moving and Betty is at rest.

Hence, there cannot be any difference between them!!!

The Paradox of the Twin Paradox

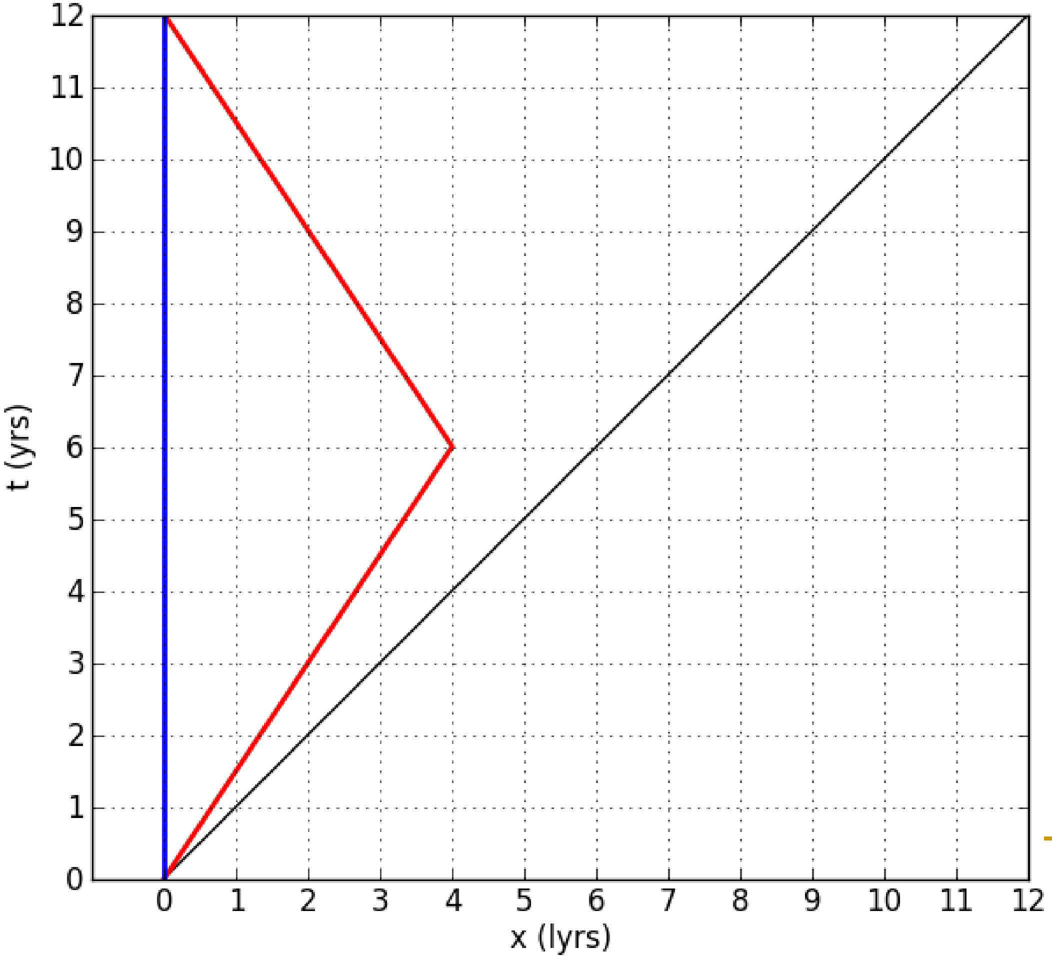
What is going on?

A. Einstein's theory is wrong, there is no relativity.
B. Nick's calculation is wrong.
C. Andy and Betty are not twins.
D. Even if they are twins, they are not equal in some other respect.
E. Women live longer, they must be aging slower.

Twin Paradox Resolved

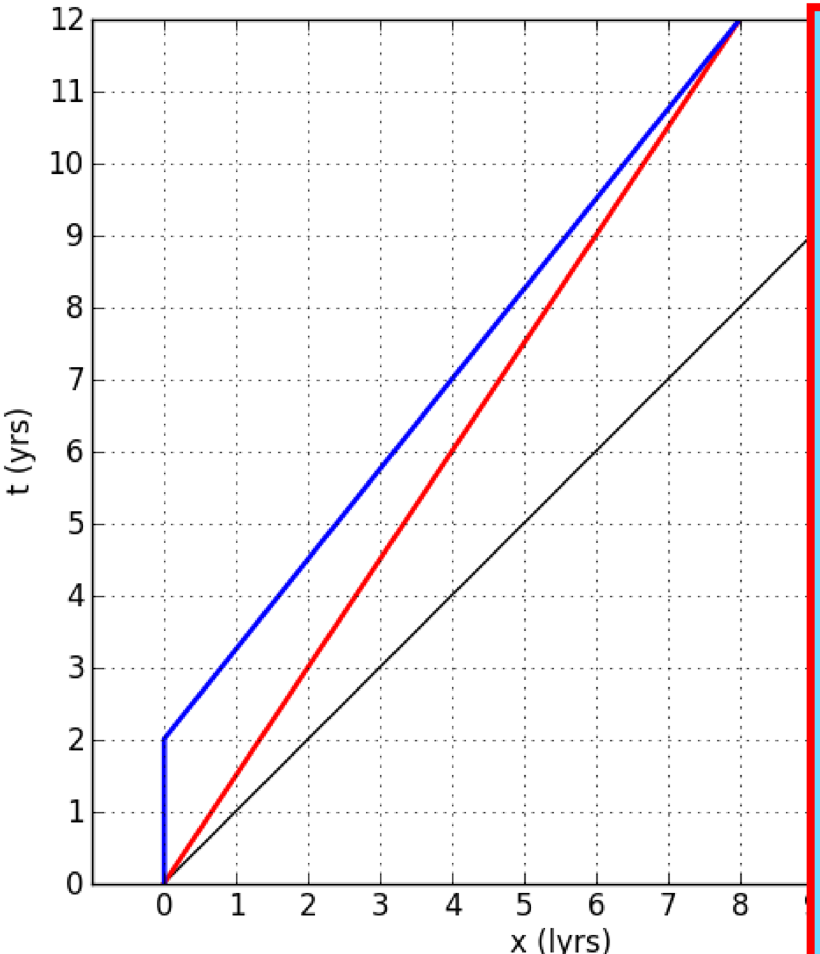
Betty is ***not*** an inertial observer. In SR (as we have just saw) time really flows slower in non-inertial reference frames.

Time is relative!!!



Twin Paradox II

How about this story?



Betty:

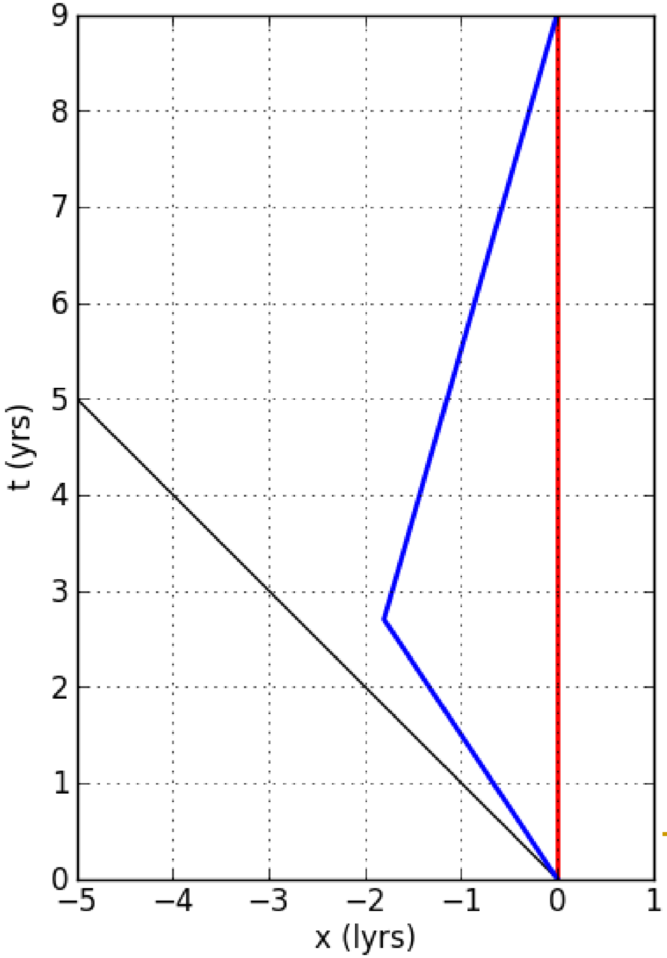
$$t_B = \frac{12\,\text{yrs}}{\gamma} = \frac{12\,\text{yrs}}{1.34} = 9\,\text{yrs}$$

Andy:

$$\begin{aligned} t_A &= 2\,\text{yrs} + \frac{10\,\text{yrs}}{\gamma} \\ &= 2\,\text{yrs} + \frac{10\,\text{yrs}}{1.67} \\ &= 8\,\text{yrs} \end{aligned}$$

Twin Paradox II

Betty's view



Betty:

$$t_B = \frac{9\,\text{yrs}}{\gamma} = \frac{9\,\text{yrs}}{1} = 9\,\text{yrs}$$

Andy:

$$\begin{aligned} t_A &= \frac{2.7\,\text{yrs}}{\gamma(2/3)} + \frac{6.3\,\text{yrs}}{\gamma(2/7)} \\ &= \frac{2.7\,\text{yrs}}{1.34} + \frac{6.3\,\text{yrs}}{1.043} \\ &= 8\,\text{yrs} \end{aligned}$$

Space-Time Diagrams

Important conclusion:

- On space-time diagrams, the proper time is the ***longest*** along the straight line connecting two events.
- The further a line from being straight, the shorter the proper time along that world line.
- This is exactly opposite of our familiar Euclidean space, so be aware.

Relativistic Velocities

- According to Galilean transformation, if a ball is thrown inside a moving train, an observer on the ground would measure its speed as

$$v_{\text{ground}} = v_{\text{train}} + v_{\text{ball}}$$

- What if $v_{\text{train}} = 0.5c$ and $v_{\text{ball}} = 0.6c$?

$$v_{\text{ground}} = 0.5c + 0.6c = 1.1c$$

- Uncle Albert gets very unhappy…

Relativistic Velocities

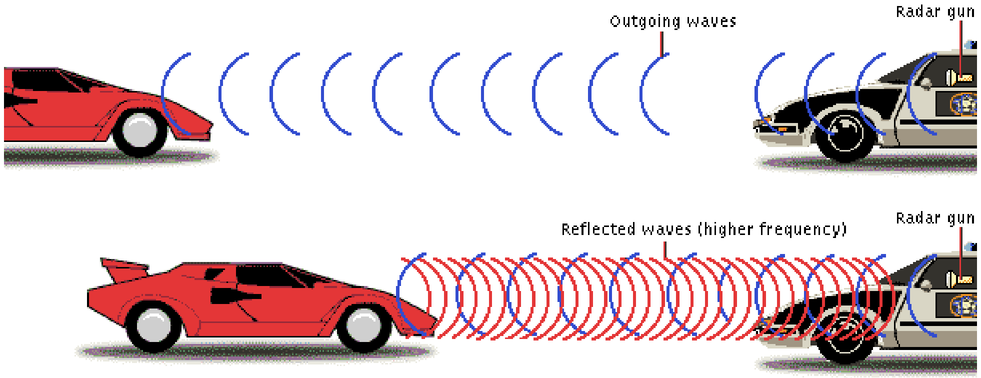
- The Lorentz transformation has a special formula for adding relativistic speeds. If you add two speeds that are less than c, you always get a speed that is less than c.

$$v_{\text{sum}} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{0.5c + 0.6c}{1 + 0.5 * 0.6} = 0.846c$$

Doppler Effect

- Christian Doppler (1803 – 1853) is the Evil Genius of all sensible drivers.
- In 1842 he discovered the Doppler effect: a shift in the frequency (wavelength) of a wave emitted or reflected by a moving source.

60 miles an hour?! But that's impossible!
I've only been driving for 15 minutes!



Doppler Effect II

- Doppler effect is measured by ***redshift:***

$$z = \frac{\lambda_{\mathrm{RECEIVED}} - \lambda_{\mathrm{EMITTED}}}{\lambda_{\mathrm{EMITTED}}}$$

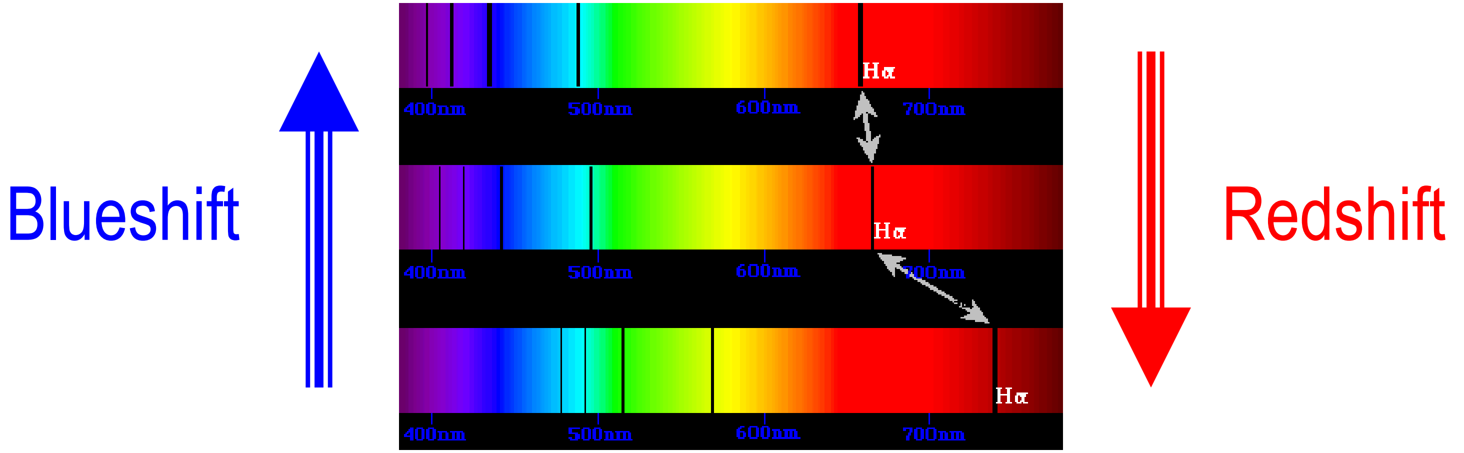
- *Observed* redshift is determined by the ***line-of-sight*** velocity of a source of wave:

$$z = \frac{v}{c}$$

- It tells (almost) nothing about a ***tangential*** (sideways) velocity.

Doppler Effect III

- If the source is receding, the redshift ***z*** is positive (proper ***redshift***).
- If the source is approaching, the redshift ***z*** is negative – ***blueshift***.



Rest energy

- One of the biggest differences between Newtonian Mechanics and SR is the existence of **rest energy**.
- Einstein found that the energy of a moving particle in SR is

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \approx mc^2 + \frac{mv^2}{2} + \ldots$$

Extra term

Newtonian kinetic energy

Rest Energy

- He called that extra term the "rest energy".

$$E_0 = mc^2$$

- Wrong forms (try it and meet your doom).

$$E_0 = m_0 c^2$$

$$E = m_0 c^2$$

$$E = mc^2$$

Rest Energy

- The rest energy is enormous compared to other types of energy. The rest energy of an average person, if fully used, is enough to satisfy the energy needs of the whole country for .

* That's why in "Star Trek" they use matter-antimatter reactors.

Rest Energy

- Total energy $E = \gamma mc^2$.
- Thus, in order to accelerate an object to, say, 0.9c ($\gamma = 2.3$), we need to spend 130% of its rest energy!
- To get a 1 ton spaceship to that speed will take all the energy the US consumes in ½ year.
- Another ½ year will take the ship to 0.96c.
- Another ½ year will take the ship to 0.98c.
- Another ½ year will take the ship to 0.987c.

“Relativistic Mass”

- In SR all formulas for physical quantities differ from Newtonian Mechanics:

$$E \quad \frac{mv^2}{2} \quad \gamma mc^2$$

$$\vec{p} \quad m\vec{v} \quad \gamma m\vec{v}$$

$$\vec{F} \quad m\vec{a} \quad \gamma m\vec{a}_\perp + \gamma^3 m\vec{a}_\parallel$$

“Relativistic Mass”

- Somehow, historically, the momentum equation produced most of irritation.
- To avoid it, bad textbooks introduce a “relativistic mass”:

$$\vec{p} = \gamma m \vec{v} = m_{\mathrm{REL}} \vec{v}$$

$$m_{\mathrm{REL}} = \gamma m$$

- That is a total hokum. The mass of an object is the sum of masses of its atoms and does not depend on how an object moves.

“Relativistic Mass”

- For example, in the relativistic second law of Newton there are 2 masses:

$$\vec{F} = \gamma m \vec{a}_\perp + \gamma^3 m \vec{a}_{||}$$

- “Transverse mass” is $m_T = \gamma m$.
- “Longitudinal mass” is $m_L = \gamma^3 m$.

- *Easy recipe for weight loss: just approach the scale sideways!*

"Relativistic Mass"

- The easiest way to fail this class:

$$m_{\mathrm{REL}} = \gamma m$$

Interstellar Travel

- Ok, let’s imagine we finally found someone up there. How do we go visit? (Or, we reached the stage of colonial expansion, if it happens.)
- Special Relativity (SR) limits the speed of any interstellar travel to below the speed of light (300,000 km/s = 7.2 uph).
- Interstellar distances are humongous:
  - Proxima Centari: 4.22 lyr.
  - Center of the Galaxy: 27,700 lyr.

Center of the Galaxy

- To reach the center of the Milky Way in 10 years, one has to travel on average at 0.999,999,935c.
- To send a 1,000 ton spaceship with such a speed requires all the energy the Sun emits in 1 second.
- Theoretically, this seems possible. But what’s the point? – travelers will incur a 55,000 year time difference!

One Way Ticket

- Essentially, any interstellar travel beyond the very vicinity of the Sun is a one-way ticket – there is no sense of coming back.
- If we don't care about coming back, how far can we go? That depends on how fast we accelerate.
- A convenient acceleration is 1 g (would feel like the gravity on Earth).

One-gee All The Way…

Edge of the Solar system	30AU	11 days
Nearest star	4.22 lyr	2.3 yrs
Center of the MW	27,700 lyr	10.6 yrs
Andromeda galaxy	2.5M lyr	15 yrs
Cosmic horizon	50,000M lyr	25 yrs