Homework # 1

Out Fri, Oct 3. Due Fri, Oct 10

As we will discuss later in the course, current observational data suggest that our universe is just entering the period of accelerated expansion powered by the so-called "cosmological constant". During this regime the universe expands exponentially. The realistic model that describes a transition from a matter-dominated expansion ($a(t) \propto t^{2/3}$) to the accelerated expansion ($a(t) \propto \exp(H_0 t)$) is not solvable analytically, but we can design an analytically solvable model that has all the features of the real one.

So, assume that the scale factor evolves with time as

$$a(t) = \left(\frac{t}{t_*}\right)^{2/3} e^{(H_* t)^{1/3}},$$

where t_* and H_* are two free parameters. Clearly, for small t we have a matter-dominated regime, and for large t we have an exponential expansion (this expansion is somewhat slower than the true expansion due to the cosmological constant, but it is still *very* fast).

1. (5 points) This model (just like a real universe) has a rare feature that it has two cosmological horizons. Using the equation that describes the propagation of light in the Friedmann universe,

$$\int_{t_e}^{t_0} \frac{dt}{a(t)} = \frac{x_e}{c} \ ,$$

derive the past horizon - the distance the light travels since the Big Bang ($t_e = 0$) to a given moment t_0 - and the future horizon - the distance the light travels since a given moment t_e until the infinite time ($t_0 = \infty$). You only need to use comoving sizes for the horizons throughout this homework.

2. (2 points) What is the value for the future horizon if the expansion of the universe always decelerates ($H_* = 0$)?

3. (5 points) Derive cosmological redshift z to a distant galaxy as a function of the comoving distance x_e to that galaxy. Plot $z(x_e)$ for $H_* t_0 = 10^{-2}, 10^{-1}, 1, 10, 100$.

4. (3 points) It is often said that during decelerated expansion the horizon "moves outward" and during accelerated expansion the horizon "moves inward and galaxies disappear from view". Is it true?

5. (10 points) Consider now a galaxy at a fixed cosmic redshift $z = z_f$. Plot the comoving distance x_e to such a galaxy as a function of t_0 for $z_f = 1, 10^2, 10^{10}$, making sure that you span values of t_0 well below and well above $1/H_*$ (you will not be able to derive $x_e(z_f)$ analytically, but you are only asked to *plot* it). Now, how about "disappearing from view" galaxies?