Homework # 3

Out Fri, Oct 17. Due Fri, Oct 24

1. (15 points) Verify the accuracy of the freeze-out approximation for the non-relativistic particle: consider an equation for the annihilation reaction of some particle X:

$$\frac{dn_{\rm X}}{dt} = -3Hn_{\rm X} + \langle v\sigma\rangle \left(n^2_{\rm X,eq} - n^2_{\rm X}\right) \tag{1}$$

In order to get a definite answer, assume that the statistical weight of X is $g = 2$, its mass is 30GeV, and that

$$a(t) = \left(\frac{t}{2.4\times 10^{20}{\rm s}}\right)^{1/2}.$$

Adopt the annihilation rate

$$\langle v\sigma\rangle = 3\times 10^{-26}\frac{{\rm cm}^3}{\rm s}\left(\frac{T}{1{\rm GeV}}\right)^{\alpha}.$$

Solve the full equation (1) for $\alpha = 0$, 1, and 4 and find $\bar{n}_\infty$ as the limit of the comoving number density $\bar{n}_X(t)$ of particle X when t goes to infinity, and compare these values with the values you obtain using a freeze-out approximation.

3. (10 points) Compute the temperature of all three neutrino species assuming that τ neutrino decouples when the photon temperature is between 106 and 135 MeV, but both electron and muon neutrino decouple as in the standard theory (at temperatures between 0.5 and 106 MeV).