Combination of Single Top Quark Production Results from CDF

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Abstract

Recently, the CDF experiment at the Fermilab Tevatron has used complementary methods to make multiple measurements of the singly produced top quark cross section. All analyses use the same dataset with more than 2 fb^{-1} of CDF data and event selection based on *W+2* or *W+3* jet events with at least one b-tagged jet. However, due to differences in analysis techniques these results are not fully correlated and a combination provides improved experimental precision. This poster outlines the procedures used to combine the single-top results from these CDF measurements. Two independent methods are used to combine the results: a super-discriminant using a neuro-evolution technique to optimize the neural network based on expected sensitivity, and a method based on the technique of the best linear unbiased estimate. The coombination results in a measurement of the single-top production cross section and also the CKM matrix element V_{TB}.

Introduction

The error on the background prediction for single top searches is larger than the single top signal prediction! In an attempt to seperate signal from background three advanced techniques were used in the CDF search:

- Likelihood Function (LF)
- Matrix Element (ME)
- Neural Network (NN)

In each analysis the multivariate discriminant was used to build template distributions for signal and background and and the data was fit to extract the signal component.

Although the analyses use the same event selection they rely on different observables and correlations between these observables to discriminate between signal and background. These differences mean that a combination should provide additional sensitivity to single top quark production.

Two Combinations

Two very different strategies were developed to combine the CDF single top analyses:

- NEAT : The event-by-event discriminants of the individual analyses are used to build a new super-discriminant via a neural network. The neural network weights and topolgy are optimized using a technique known as neuro-evolution of augmenting topologies (NEAT). This combination is basically an entire new analysis.
- BLUE : The measured cross sections of the individual analyses are combined by taking a weighted average while taking correlations between the analyses into account. The method of best linear unbiased estimator (BLUE) minimizes a chi-squared while taking correlations and systematic errors into account.

BLUE Methodology

The combination method based on BLUE uses a simple X^2 method:

$$\chi^2(\mu) = \sum_i^N \left(\frac{m_i - \mu}{\sigma_i}\right)^2$$

Which can be expressed in matrix form:

$$\chi^2(\mu) = \delta^T(\mu)\cdot \mathbf{S}^{-1}\cdot\delta(\mu)$$

where:

$$\text{(column vector) } \delta_i = m_i - \mu$$
$$\text{(covariance matrix) } \mathbf{S}_{(i,i)} = \sigma_i^2$$

Multiple uncertainties and correlations between uncertainties of each measurement can be incorporated:

$$\mathbf{S} = \sum_e^E \mathbf{S}_e \qquad \mathbf{H} \equiv \mathbf{S}^{-1}$$

where:

$$\text{sum}_H \equiv \sum_{i=1}^N\sum_{j=1}^N \mathbf{H}_{i,j}$$

$$\mathbf{S}_{e(i,j)} = \sigma_{ei}\cdot\sigma_{ej}\cdot\rho_{e(i,j)}$$
$$\sigma_{ei} \equiv e\text{th error on the } i\text{th measurement}$$
$$\rho_{e(i,j)} \equiv \text{correlation of the } e\text{th error between } i\text{th and } j\text{th measurement}$$

A weight can be calculated for each measurement: $w_i \equiv \sum_{j=1}^N \frac{\mathbf{H}_{i,j}}{\text{sum}_H}$

With this information BLUE tells us:

$$\mu_{\text{best}} = \sum_{i=1}^N w_i\cdot m_i, \text{ and}$$
$$\sigma_\mu = \frac{1}{\sqrt{\text{sum}_H}}$$

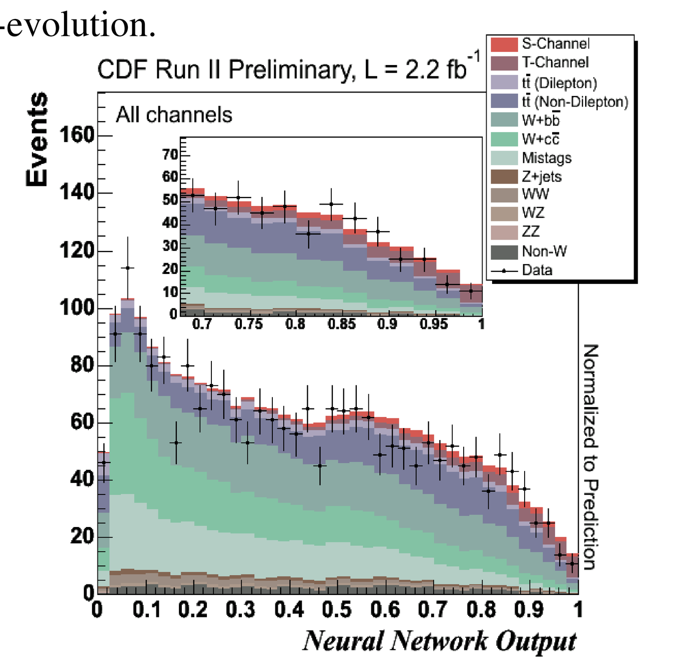
The beauty and power of BLUE is that a complicated minimization is replaced by a more simple matrix diagonalization!

NEAT Methodology

The NEAT combination method takes the discriminating variable from each analysis and combines them into one super-analysis. The new super-discriminant is built using a neural network which is optimized for expected sensitivity via neuro-evolution.

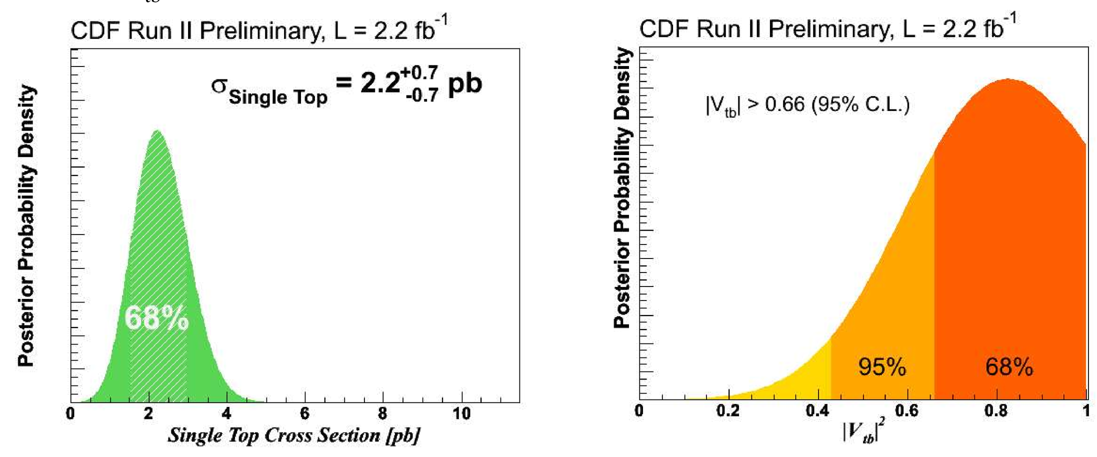
A separate discriminant is optimized for each of the following channels:

- 2-jet, 1-tag
- 2-jet, 2-tag
- 3-jet, 1-tag
- 3-jet, 2-tag



NEAT Results : Cross Section

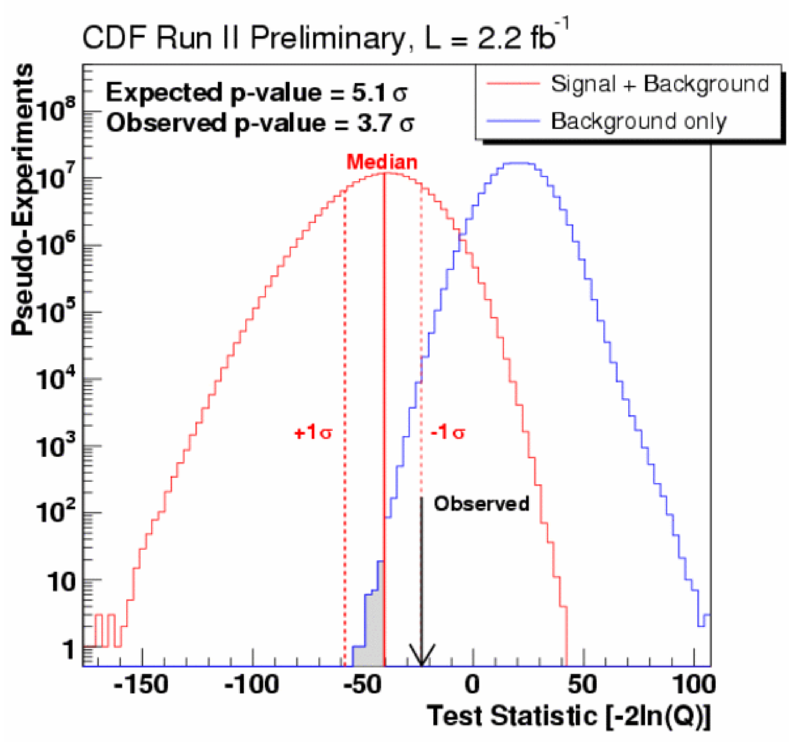
Signal and background templates are built from the NEAT output distributions. The data is fit to these templates using a binned likelihood technique to extract the single top cross sections and the CKM matrix element $|V_{tb}|$.



$$|V_{tb}| = 0.88 \pm 0.14\ (\text{exp.}) \pm 0.07\ (\text{theory})$$

NEAT Results : pValue

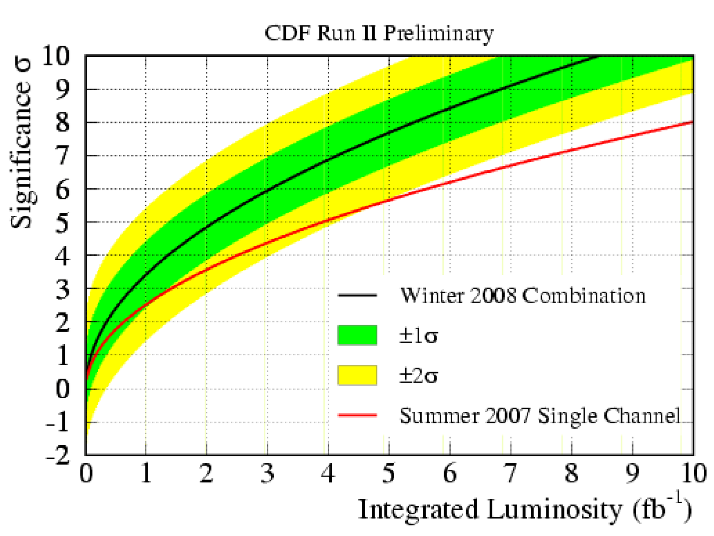
We also compare the data to two hypotheses: H0 assumes that there is no single top production while H1 supposes the Standard Model rate of single top. The likelihood ratio Q = -2ln(p(H1)/p(H0)) is used to perform this comparison. We calculate a p-value for the observed data assuming the null hypothesis (H0) and compare it to our expected p-value, evaluated from ensembles of pseudo-experiments constructed assuming H1 (SM amount of single top).



NEAT : Summary

- NEAT measures a single top cross section of $2.2\ ^{+0.7}_{-0.7}$ pb.
- NEAT calculates |Vtb| = 0.88 ± 0.14 (exp.) ± 0.07 (theory)
- From PE's:
  - NEAT calculates an obvserved sensitivity of 3.7σ
  - NEAT calculates a median expected sensitivity of 5.1σ
  - About a 9% gain in expected sensitivity over any analysis

CDF Sensitivity to Single Top



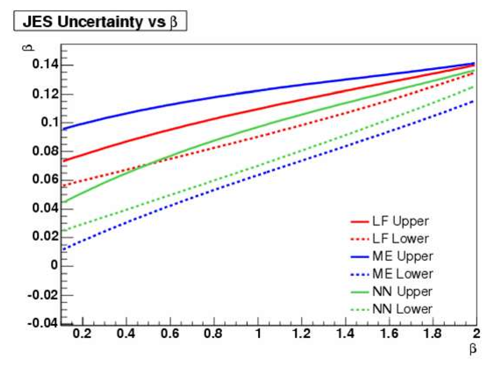
CDF single top analyses performed far better than simple scaling based on additional integrated luminosity!

AIB: Assymetric Iterative BLUE

BLUE needs the measured values, uncertainties, and correlations between each error before the combined result can be calculated. Correlated pseudo-experements are thrown for each analysis and the correlations of resulting cross sections as well as the measured cross sections from each analysis are summarized here:

Analysis	Cross Section (pb)
LF	1.79
ME	2.17
NN	1.97
Theory	2.864

Analysis	LF	ME	NN
LF	100%	59%	74%
ME		100%	61%
NN			100%



In general errors can have dependence on the value measured. Analyses which measure lower will in general have a smaller cross section and this could bias the combination. In order to avoid this bias we run BLUE in an iteratively. Another complication is that errors need not be symmetric. Since BLUE is based on a gaussian approximation it is not able to treat this directly. To handle this we use Assymetric Iterative BLUE (AIB):

AIB us a set of three BLUE combinations. Using average errors (for central value), upper errors, or lower errors.

$$\mathscr{R}_{\text{upper}} = \frac{\sigma_{\text{upper BLUE}}}{\sigma_{\text{upper BLUE}} + \sigma_{\text{lower BLUE}}}$$
$$\sigma_{\text{upper}} = 2\cdot\mathscr{R}_{\text{upper}}\cdot\sigma_{\text{center BLUE}}$$
$$\sigma_{\text{lower}} = 2\cdot(1-\mathscr{R}_{\text{upper}})\cdot\sigma_{\text{center BLUE}}$$
$$\sigma_{\text{center BLUE}} \equiv \frac{\sigma_{\text{upper}} + \sigma_{\text{lower}}}{2}$$

AIB Results

- AIB calculates a single top cross section of $2.1\ ^{+0.7}_{-0.6}$ pb.
- Looking at pseudo-experiments with no signal present:
  - AIB calculates an obvserved sensitivity of 3.7σ
  - AIB calculates a median expected sensitivity of 4.7σ
  - About a 7% gain in expected sensitivity over any analysis
- AIB was also used to calculate consistency of the three analyses.:
  - The combined values had a χ^2 equal to or less than 87% of PE's
  - 14.8% (1.1σ) of PE's with the SM expected single top cross section had a measured cross section of < 2.1 pb.

Conclusions

Two unique combination strategies were successfully used at CDF to combine three independent measurements of single top production at CDF. The three CDF results were combined and shown to be compatible with each other and the standard model though studies using and assymetric and iterative modification to BLUE. The NEAT combination prooved to be the most sensitive by combining the discriminating variables of the individual analyses into a new superanalysis. NEAT measures a single top cross section of $2.2\ ^{+0.7}_{-0.7}$ pb and the CKM matrix element $|V_{tb}|$ = 0.88 ± 0.14 (exp.) ± 0.07 (theory).

A new combination based on 2.7 fb-1 of CDF integrated luminosity is underway.