1 General Calculations for Angular Distribution

$$
\begin{aligned}
\frac{d\sigma}{d\Omega_Z d\Omega_l} &= \sum_{\lambda_{l^+}\lambda_{l^-}\lambda_\gamma} \Big| \sum_{J_{Z\gamma}\lambda_Z\lambda_q\lambda_{\bar{q}}} [2J_{Z\gamma}+1] T^{J_{Z\gamma}}_{\lambda_Z,\lambda_\gamma,\lambda_q,\lambda_{\bar{q}}} d^{J_{Z\gamma}}_{\lambda_{Z\gamma}\lambda_{q\bar{q}}}(\cos\theta_Z) e^{i(\lambda_{q\bar{q}}-\lambda_{Z\gamma})\phi_Z} \\
&\quad \otimes A_{\lambda_Z,\lambda_{l^+},\lambda_{l^-}} d^1_{\lambda_Z\lambda_{l^+l^-}}(\cos\theta_l) e^{-i(\lambda_Z-\lambda_{l^+l^-})\phi_l} \Big|^2 \\
&= \sum_{\lambda_{l^+}\lambda_{l^-}\lambda_\gamma} |e^{i\lambda_{l^+l^-}\phi_l} e^{i\lambda_\gamma} \sum_{J_{Z\gamma}\lambda_Z\lambda_q\lambda_{\bar{q}}} [2J_{Z\gamma}+1] T^{J_{Z\gamma}}_{\lambda_Z,\lambda_\gamma,\lambda_q,\lambda_{\bar{q}}} d^{J_{Z\gamma}}_{\lambda_{Z\gamma}\lambda_{q\bar{q}}}(\cos\theta_Z) e^{i(\lambda_{q\bar{q}}-\lambda_Z)\phi_Z} \\
&\quad \otimes A_{\lambda_Z,\lambda_{l^+},\lambda_{l^-}} d^1_{\lambda_Z\lambda_{l^+l^-}}(\cos\theta_l) e^{-i\lambda_Z\phi_l} \Big|^2 \\
&= \sum_{\lambda_{l^+}\lambda_{l^-}\lambda_\gamma} \Big| \sum_{J_{Z\gamma}\lambda_Z\lambda_q\lambda_{\bar{q}}} [2J_{Z\gamma}+1] T^{J_{Z\gamma}}_{\lambda_Z,\lambda_\gamma,\lambda_q,\lambda_{\bar{q}}} d^{J_{Z\gamma}}_{\lambda_{Z\gamma}\lambda_{q\bar{q}}}(\cos\theta_Z) e^{i(\lambda_{q\bar{q}}-\lambda_Z)\phi_Z} \\
&\quad \otimes A_{\lambda_Z,\lambda_{l^+},\lambda_{l^-}} d^1_{\lambda_Z\lambda_{l^+l^-}}(\cos\theta_l) e^{-i\lambda_Z\phi_l} \Big|^2
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\frac{d\sigma}{d\Omega_Z d\Omega_l} = \sum_{\lambda_{l^+l^-}\lambda_\gamma} \Big| \sum_{J_{Z\gamma}\lambda_Z\lambda_{q\bar{q}}} & [2J_{Z\gamma}+1] T^{J_{Z\gamma}}_{\lambda_Z,\lambda_\gamma,(\lambda_{q\bar{q}})} d^{J_{Z\gamma}}_{\lambda_{Z\gamma}\lambda_{q\bar{q}}}(\cos\theta_Z) \\
& \otimes A_{\lambda_Z,(\lambda_{l^+l^-})} d^1_{\lambda_Z\lambda_{l^+l^-}}(\cos\theta_l) e^{i[\lambda_{q\bar{q}}\phi_Z-\lambda_Z(\phi_Z+\phi_l)]} \Big|^2
\end{aligned}
} \tag{1}
$$

Sinse $\lambda_{l^+}, \lambda_{l^-}, \lambda_q$ and $\lambda_{\bar{q}}$ separately are only distinguishing between T and A amlitudes and do not matter for the rest of the summation, I make following notation:

$$
\begin{gathered}
T^{J_{Z\gamma}}_{\lambda_Z,\lambda_\gamma,(\lambda_{q\bar{q}})} = \sum_{\lambda_q\lambda_{\bar{q}}}^{\lambda_q-\lambda_{\bar{q}}=\lambda_{q\bar{q}}} T^{J_{Z\gamma}}_{\lambda_Z,\lambda_\gamma,\lambda_q,\lambda_{\bar{q}}} \\
A_{\lambda_Z,(\lambda_{l^+l^-})} = \sum_{\lambda_{l^+}\lambda_{l^-}}^{\lambda_{l^+}-\lambda_{l^-}=\lambda_{l^+l^-}} A_{\lambda_Z,\lambda_{l^+},\lambda_{l^-}} \\
\lambda_{q\bar{q}} = \lambda_q - \lambda_{\bar{q}}; \quad \lambda_{l^+l^-} = \lambda_{l^+} - \lambda_{l^-} \\
\lambda_{l^+l_-} \equiv \lambda_{ll}; \quad \lambda_{q\bar{q}} \equiv \lambda_{qq}
\end{gathered}
$$

For squaring following formulas are used:

If a_i and α_i are real:

$$
|\sum_i a_i e^{i\alpha_i\phi_i}|^2 = \sum_i \sum_{j\geqslant i} (2-\delta_{ij}) a_i a_j \cos(\alpha_i\phi_i - \alpha_j\phi_j)
$$

in general, helicity amplitudes are complex numbers, so, if $z_i = a_i + ib_i$ and

let's also put some x_i real functions in general calculation (which represent the wigner d functions in our distribution):

$$
\begin{aligned}
|\sum_i z_i x_i e^{i\alpha_i\phi_i}|^2 =& |\sum_i a_i x_i e^{i\alpha_i\phi_i} + i\sum_i b_i x_i e^{i\alpha_i\phi_i}|^2 \\
=& |\sum_i a_i x_i e^{i\alpha_i\phi_i}|^2 + |\sum_i x_i b_i e^{i\alpha_i\phi_i}|^2 \\
=& \sum_i \sum_{j\geqslant i} (2-\delta_{ij}) a_i a_j x_i x_j \cos(\alpha_i\phi_i - \alpha_j\phi_j) \\
&+ \sum_i \sum_{j\geqslant i} (2-\delta_{ij}) b_i b_j x_i x_j \cos(\alpha_i\phi_i - \alpha_j\phi_j) \\
=& \sum_i \sum_{j\geqslant i} (2-\delta_{ij}) x_i x_j (a_i a_j + b_i b_j) \cos(\alpha_i\phi_i - \alpha_j\phi_j)
\end{aligned}
$$

Which results in

$$\frac{n(n+1)}{2}$$

members of sumation.

2 $Z\gamma \to ll\gamma$ angular distribution for $J_{Z\gamma} = 0$

$$\frac{d\sigma}{d\Omega_Z d\Omega_l} = a\cos^2\theta + b\cos\theta + c$$

where:

$a = T_1^2[(\frac{1}{2}A_{\frac{1}{2},-\frac{1}{2},1})^2 + (\frac{1}{2}A_{-\frac{1}{2},\frac{1}{2},1})^2 - (A_{-\frac{1}{2},-\frac{1}{2},1} + A_{\frac{1}{2},\frac{1}{2},1})^2] + T_2^2[(\frac{1}{2}A_{\frac{1}{2},-\frac{1}{2},-1})^2 + (\frac{1}{2}A_{-\frac{1}{2},\frac{1}{2},-1})^2 - (A_{-\frac{1}{2},-\frac{1}{2},-1} + A_{\frac{1}{2},\frac{1}{2},-1})^2]$

$b = \frac{1}{2}[T_1^2[(A_{-\frac{1}{2},\frac{1}{2},1})^2 + (A_{\frac{1}{2},-\frac{1}{2},1})^2] + T_2^2[(A_{-\frac{1}{2},\frac{1}{2},-1})^2 + (A_{\frac{1}{2},-\frac{1}{2},-1})^2]]$

$c = T_1^2[(\frac{1}{2}A_{\frac{1}{2},-\frac{1}{2},1})^2 + (\frac{1}{2}A_{-\frac{1}{2},\frac{1}{2},1})^2 + (A_{-\frac{1}{2},-\frac{1}{2},1} + A_{\frac{1}{2},\frac{1}{2},1})^2] + T_2^2[(\frac{1}{2}A_{\frac{1}{2},-\frac{1}{2},-1})^2 + (\frac{1}{2}A_{-\frac{1}{2},\frac{1}{2},-1})^2 + (A_{-\frac{1}{2},-\frac{1}{2},-1} + A_{\frac{1}{2},\frac{1}{2},-1})^2]$

$and:$

$T_1 = \sum_{\lambda_q\lambda_{\bar{q}}}^{\lambda_q-\lambda_{\bar{q}}=0} T^0_{\lambda_q,\lambda_{\bar{q}},1,1}$

$T_2 = \sum_{\lambda_q\lambda_{\bar{q}}}^{\lambda_q-\lambda_{\bar{q}}=0} T^0_{\lambda_q,\lambda_{\bar{q}},-1,-1}$

From $\cos\theta_Z$ distribution (not shown today) we see that $J_{Z\gamma} = 0$ is not a case, but it just was a first simple step to consider. Next step is to look at $J_{Z\gamma} = 1$ for both $fitFunc(\theta_Z, \theta_l, \phi_Z - \phi_l)$ and result of analytical calculation fits. I

am working on incorporating of efficiencies and acceptances to get meaningful results.

3 $Z\gamma \to ll\gamma$ angular distribution for $J_{Z\gamma} = 1$

3.1 $\lambda_\gamma = -1$

$(J_{Z\gamma} = 1; \lambda_\gamma = -1) \Rightarrow \lambda_Z = -1, 0$
$A\mid_{\lambda_Z=-1}^{\lambda_{q\bar{q}}=-1} = T^1_{-1,-1,(-1)} d^1_{0,-1}(\cos\theta_Z) e^{i\phi_Z} A_{-1,(\lambda_{l^+l^-})} d^1_{-1\lambda_{l^+l^-}}(\cos\theta_l) e^{i\phi_l}$
$A\mid_{\lambda_Z=-1}^{\lambda_{q\bar{q}}=0} = T^1_{-1,-1,(0)} d^1_{0,0}(\cos\theta_Z) A_{-1,(\lambda_{l^+l^-})} d^1_{-1\lambda_{l^+l^-}}(\cos\theta_l) e^{i\phi_l}$
$A\mid_{\lambda_Z=-1}^{\lambda_{q\bar{q}}=1} = T^1_{-1,-1,(1)} d^1_{0,1}(\cos\theta_Z) e^{-i\phi_Z} A_{-1,(\lambda_{l^+l^-})} d^1_{-1\lambda_{l^+l^-}}(\cos\theta_l)$

$A\mid_{\lambda_Z=0}^{\lambda_{q\bar{q}}=-1} = T^1_{-1,0,(-1)} d^1_{-1,-1}(\cos\theta_Z) A_{0,(\lambda_{l^+l^-})} d^1_{0\lambda_{l^+l^-}}(\cos\theta_l)$
$A\mid_{\lambda_Z=0}^{\lambda_{q\bar{q}}=0} = T^1_{-1,0,(0)} d^1_{-1,0}(\cos\theta_Z) e^{-i\phi_Z} A_{0,(\lambda_{l^+l^-})} d^1_{0\lambda_{l^+l^-}}(\cos\theta_l)$
$A\mid_{\lambda_Z=0}^{\lambda_{q\bar{q}}=1} = T^1_{-1,0,(1)} d^1_{-1,1}(\cos\theta_Z) e^{-i2\phi_Z} A_{0,(\lambda_{l^+l^-})} d^1_{0\lambda_{l^+l^-}}(\cos\theta_l)$

Let's write out only exponent powers:

1.$\phi_Z + \phi_l$
2.ϕ_l
3. $-\phi_Z + \phi_l$
4.
5. $-\phi_Z$
6. $-2\phi_Z$

So the cos's in crossterms after squaring will be:

$1-2 \Rightarrow \cos(\phi_Z)$
$1-3 \Rightarrow \cos(2\phi_Z)$
$1-4 \Rightarrow \cos(\phi_Z + \phi_l)$
$1-5 \Rightarrow \cos(2\phi_Z + \phi_l)$
$1-6 \Rightarrow \cos(3\phi_Z + \phi_l)$
$2-3 \Rightarrow \cos(\phi_Z)$
$2-4 \Rightarrow \cos(\phi_l)$
$2-5 \Rightarrow \cos(\phi_Z + \phi_l)$
$2-6 \Rightarrow \cos(2\phi_Z + \phi_l)$
$3-4 \Rightarrow \cos(-\phi_Z + \phi_l)$
$3-5 \Rightarrow \cos(\phi_l)$
$3-6 \Rightarrow \cos(\phi_Z + \phi_l)$
$4-5 \Rightarrow \cos(\phi_Z)$
$4-6 \Rightarrow \cos(2\phi_Z)$
$5-6 \Rightarrow \cos(\phi_Z)$