1

The distribution function:

$$\frac{d\sigma}{d\Omega_\ell d\Omega_Z} \equiv W(\Omega) = \sum_n^{N_p} A_n \omega_n(\Omega) \tag{1}$$

The likelihood function:

$$L = \prod_{n=1}^{N_D} P(\Omega_n) \tag{2}$$

and from here we construct $-\ln L$

$$\begin{aligned}
\mathscr{L} = -\ln L &= -\ln \prod_{n=1}^{N_D} \frac{W(\Omega_n)\epsilon(\Omega_n)}{\int W(\Omega)\epsilon(\Omega)d\Omega} \\
&= -\sum_{n=1}^{N_D} \ln\left(W(\Omega_n)\epsilon(\Omega_n)\right) + \sum_{n=1}^{N_D} \ln\left(\int W(\Omega)\epsilon(\Omega)d\Omega\right) \\
&= -\sum_{n=1}^{N_D} \ln\left(\epsilon(\Omega_n)\sum_{n'}^{N_p} A_{n'}\omega_{n'}(\Omega_n)\right) + N_D \ln\left(\sum_{n'}^{N_p} A_{n'} \int \omega_{n'}(\Omega)\epsilon(\Omega)d\Omega\right) \\
&= \underbrace{-\sum_{n=1}^{N_D} \ln \epsilon(\Omega_n)}_{1} \underbrace{-\sum_{n=1}^{N_D} \ln\left(\sum_{n'}^{N_p} A_{n'}\omega_{n'}(\Omega_n)\right)}_{2} + N_D \ln\left(\sum_{n'}^{N_p} A_{n'} \underbrace{\int \omega_{n'}(\Omega)\epsilon(\Omega)d\Omega}_{3}\right)
\end{aligned} \tag{3}$$

Part 1 in equation 4 doesn't depend on the A_i parameters thus is irrelevant for the minimization and can be dropped. So the final expression for the likelihood function is:

$$\boxed{\mathscr{L} = -\ln L = -\sum_{n=1}^{N_D} \ln\left(\sum_{n'}^{N_p} A_{n'}\omega_{n'}(\Omega_n)\right) + N_D \ln \sum_{n'}^{N_p} A_{n'}\epsilon_{n'}} \tag{4}$$

We can also construct extended likelihood function

$$EL = L \times \frac{\left[\mathcal{L}\int W(\Omega)\epsilon(\Omega)d\Omega\right]^{N_D}}{N_D!} e^{-\mathcal{L}\int W(\Omega)\epsilon\Omega d\Omega} \tag{5}$$

so from this we get following $-\ln EL$

$$
\begin{aligned}
\mathscr{E}\mathscr{L} = &- \ln(EL) = -\sum_{n=1}^{N_D} \ln\left(\sum_{n'}^{N_p} A_{n'}\omega_{n'}(\Omega_n)\right) + N_D \ln \sum_{n'}^{N_p} A_{n'}\epsilon_{n'} + \\
&+ \ln(N_D!) - N_D \ln\left(\mathcal{L}\int W(\Omega)\epsilon(\Omega)d\Omega\right) + \mathcal{L}\int W(\Omega)\epsilon(\Omega)d\Omega = \\
&= -\sum_{n=1}^{N_D} \ln\left(\sum_{n'}^{N_p} A_{n'}\omega_{n'}(\Omega_n)\right) + N_D \ln \sum_{n'}^{N_p} A_{n'}\epsilon_{n'} - \\
&- N_D \ln \mathcal{L} \sum_{n'}^{N_p} A_{n'}\epsilon_{n'} + \mathcal{L}\sum_{n'}^{N_p} A_{n'}\epsilon_{n'} = \\
&= -\sum_{n=1}^{N_D} \ln\left(\sum_{n'}^{N_p} A_{n'}\omega_{n'}(\Omega_n)\right) + \cancel{N_D \ln \sum_{n'}^{N_p} A_{n'}\epsilon_{n'}} - \\
&- N_D \ln \mathcal{L} - \cancel{N_D \ln \sum_{n'}^{N_p} A_{n'}\epsilon_{n'}} + \mathcal{L}\sum_{n'}^{N_p} A_{n'}\epsilon_{n'}
\end{aligned}
\tag{6}
$$

Thus

$$
\boxed{\mathscr{E}\mathscr{L} = -\sum_{n=1}^{N_D} \ln\left(\sum_{n'}^{N_p} A_{n'}\omega_{n'}(\Omega_n)\right) + \mathcal{L}\sum_{n'}^{N_p} A_{n'}\epsilon_{n'}}
\tag{7}
$$