1 Angular distribution of simple decay process

Let's consider the decay process $a \to 1 + 2$, where a has a total angular momentum J (which would be it's spin in rest frame, in which we choose to work) and spin projection M. In the rest frame we can describe a with the state vector $|J, M\rangle$. We can describe final state by the following state vector $|\theta, \phi, \lambda_1, \lambda_2\rangle$ as well as with $|j, m, \lambda_1, \lambda_2\rangle$, where j is the total angular momentum of the final state particles and m is the projection of the total angular momentum of those. Using above and exploiting the conservation of angular momentum we have:

$$\begin{aligned} A &= \sum_{j,m} \langle \theta, \phi, \lambda_1, \lambda_2 | j, m, \lambda_1, \lambda_2 \rangle \langle j, m, \lambda_1, \lambda_2 | U | JM \rangle \\ &= \sum_{j,m} \langle \theta, \phi, \lambda_1, \lambda_2 | j, m, \lambda_1, \lambda_2 \rangle \, \delta_{mM} \delta_{jJ} A_{\lambda_1 \lambda_2} \\ &= \sum_{j,m} \langle \theta, \phi, \lambda_1, \lambda_2 | J, M, \lambda_1, \lambda_2 \rangle \, A_{\lambda_1 \lambda_2} \end{aligned} \tag{1}$$

In equation (1) we used:

$$\langle j, m, \lambda_1, \lambda_2 | U | JM \rangle = \delta_{mM} \delta_{jJ} A_{\lambda_1 \lambda_2} \tag{2}$$

If this is right, then even here we see, that we have $A_{\lambda_1 \lambda_2}$ parameter without M dependence. This is then shown in equation (B.1.4) in the Richman's Helicity Formalism Write-up.

This point is again mentioned in the equation (B.5.13), where the argument for A not having M (which is λ_Z) subscript is that $\langle \lambda_1 \lambda_2 | U | M \rangle$ must be rotationally invariant.

Let's for the moment assume that above mentioned is right ($A_{\lambda_1\lambda_2}$ does not have M in it) and try to get the full angular distribution for this simple process when decaying particle is Z gauge boson and product particles are two leptons (electrons and muons (no taus), which are light enough to assume that they only have opposite helicities as the result of the decay). So:

$$\frac{d\sigma}{d\Omega_\ell} = \frac{3}{4\pi}\sum_{\lambda_{\ell\ell}} |A_{\ell\ell}|^2 |\sum_{\lambda_Z} d^1_{\lambda_Z\lambda_{\ell\ell}} e^{-i\lambda_Z\phi}|^2 \tag{3}$$

Before I gon into opening these sums let me include here little arithmetics:

$$\begin{aligned}
|\sum_k a_k z_k|^2 &= |\sum_k (a_k(x_k+iy_k))|^2 = |\sum_k a_k x_k + i\sum_k a_k y_k|^2 = \\
&= |\sum_k a_k x_k|^2 + |\sum_k a_k x_k|^2; \\
(\sum_k a_k)^2 &= \sum_i \sum_{j\geq i} (2-\delta_{ij}) a_i a_j
\end{aligned}$$

So if $z_k = e^{i\phi_k} = \cos\phi_k + i\sin\phi_k$ we get:

$$\begin{aligned}
|\sum_k a_k e^{i\phi_k}|^2 &= |\sum_k (a_k(\cos\phi_k + i\sin\phi_k))|^2 = |\sum_k a_k\cos\phi_k + i\sum_k a_k \sin\phi_k|^2 = \\
&= \sum_i \sum_{j\geq i} (2-\delta_{ij}) a_i a_j \cos(\phi_i - \phi_j)
\end{aligned}$$

Let's consider $\lambda_{\ell\ell} = -1$ and 1 cases separately just for simplicity:
For $\lambda_{\ell\ell} = -1$

$$\begin{aligned}
\frac{d\sigma}{d\Omega_\ell} &= \frac{3}{4\pi}|A_{-1}|^2 |\sum_{\lambda_Z} d^1_{\lambda_Z -1} e^{-i\lambda_Z\phi}|^2 = \\
&= |A_{-1}|^2[(\sum_{\lambda_Z} d^1_{\lambda_Z -1})^2 + 2\sum_{\lambda_Z}\sum_{\lambda_{Z^*}>\lambda_Z} d^1_{\lambda_Z -1} d^1_{\lambda_{Z^*}-1}\cos(\lambda_Z - \lambda_{Z^*} i)\phi] = \\
&= |A_{-1}|^2[(\sum_{\lambda_Z} d^1_{\lambda_Z -1})^2 + 2[d^1_{-1-1}d^1_{0-1}\cos\phi + d^1_{-1-1}d^1_{1-1}\cos 2\phi + d^1_{0-1}d^1_{1-1}\cos\phi] = \\
&= |A_{-1}|^2[(\sum_{\lambda_Z} d^1_{\lambda_Z -1})^2 + 2[d^1_{11}d^1_{1-1}\cos 2\phi + d^1_{10}\cos\phi(d^1_{1-1} + d^1_{1-1})]
\end{aligned} \tag{4}$$

For $\lambda_{\ell\ell} = 1$

$$\begin{aligned}
\frac{d\sigma}{d\Omega_\ell} &= \frac{3}{4\pi}|A_1|^2|\sum_{\lambda_Z} d^1_{\lambda_Z 1} e^{-i\lambda_Z\phi}|^2 = \\
&= |A_1|^2[(\sum_{\lambda_Z} d^1_{\lambda_Z 1})^2 + 2\sum_{\lambda_Z}\sum_{\lambda_{Z^*}>\lambda_Z} d^1_{\lambda_Z 1} d^1_{\lambda_{Z^*} 1}\cos(\lambda_Z - \lambda_{Z^*} i)\phi] = \\
&= |A_1|^2[(\sum_{\lambda_Z} d^1_{\lambda_Z 1})^2 + 2[d^1_{-11}d^1_{01}\cos\phi + d^1_{-11}d^1_{11}\cos 2\phi + d^1_{01}d^1_{11}\cos\phi] = \\
&= |A_1|^2[(\sum_{\lambda_Z} d^1_{\lambda_Z 1})^2 + 2[d^1_{11}d^1_{1-1}\cos 2\phi - d^1_{10}\cos\phi(d^1_{1-1} + d^1_{1-1})]
\end{aligned} \tag{5}$$

$$\begin{aligned}
d^1_{11} &= \frac{1+\cos\theta}{2} \\
d^1_{10} &= \frac{-\sin\theta}{\sqrt{2}} \\
d^1_{1-1} &= \frac{1-\cos\theta}{2}
\end{aligned} \tag{6}$$

From (6) we get:

$$\begin{aligned}
&(\sum_{\lambda_Z} d^1_{\lambda_Z \pm 1})^2 = 1 \\
&d^1_{11} + d^1_{1-1} = 1 \\
&d^1_{11}d^1_{1-1} = \frac{1-\cos^2\theta}{2} = \frac{\sin^2\theta}{2}
\end{aligned} \tag{7}$$

Using (6) and (7) in (4) and (5) we get:

$$\begin{aligned}
\frac{d\sigma}{d\Omega_\ell} = \frac{3}{4\pi}[|A_{-1}|^2[1 + \frac{\sin^2\theta}{2}\cos 2\phi + \frac{\sin\theta}{\sqrt{2}}\cos\phi] + \\
+ |A_1|^2[1 + \frac{\sin^2\theta}{2}\cos 2\phi - \frac{\sin\theta}{\sqrt{2}}\cos\phi]]
\end{aligned} \tag{8}$$

So as a result (unless I made a mistake in algebra) we have dependence on the azimuthal angle. This in general would not be surprising because we have particle helicity of which cannot be measured, so we sum over its helicities inside the absolute square. But in this case we should also understand that this does not have any physical meaning, because there is not any preferred azimuthal direction to have any kind of distribution other then flat. So this may mean that my initial assumption of not including λ_Z in my parameter was wrong.
Now lets try to include λ_Z in the parameter, redo the calculation and impose

the physical restriction on the parameters such that any azimuthat dependence is alyminated. This is something we usually do in physics.

I will make explicit calculations for this case and send them soon. I though I had them, but I just found a little error.