1 Using Parity Conservation to Minimize Number of Helicity Amplitudes

If we have a scattering process:

$$a + b \to c + d$$

during which parity is conserved then:

$$< \lambda_c, \lambda_d | T^J | \lambda_a, \lambda_b > = < \lambda_a, \lambda_b | \Pi T^J \Pi | \lambda_c \lambda_d > = \frac{\eta_c \eta_d}{\eta_a \eta_b} (-1)^{s_c + s_d - s_a - s_b} < -\lambda_c, -\lambda_d | T^J | -\lambda_a, -\lambda_b >$$

where η and s are the parity and spin of the particle. Π is the parity operator.
For the case of the decay:

$$a \to b + c$$

$$< \lambda_b, \lambda_c | U | \lambda_a > = < \lambda_b, \lambda_c | \Pi U \Pi | \lambda_a > = \eta_b \eta_c \eta_a (-1)^{s_b + s_c - s_a} < -\lambda_a, -\lambda_b | U | -\lambda_c, -\lambda_d >$$

In our case we have:

$$q + \bar{q} \to Z + \gamma \to \ell + \bar{\ell} + \gamma$$

with $\eta_Z = -1, \eta_\gamma = -1, \eta_q = 1, \eta_\ell = -1, \eta_{\bar{\ell}} = 1$ and $\eta_{\bar{q}} = -1$

$$\frac{d\sigma}{d\Omega_Z d\Omega_l} = \sum_{\lambda_{\ell\ell} \lambda_\gamma} | \sum_{J_{Z\gamma} \lambda_Z \lambda_{q\bar{q}}} < \lambda_Z, \lambda_\gamma | T^{J_{Z\gamma}} | \lambda_q, \lambda_{\bar{q}} > < \lambda_\ell, \lambda_{\bar{\ell}} | U | \lambda_Z > |^2$$

Although, this process involves a weak interaction, which violates parity conseravtion ($[T^J, \Pi] \neq 0; [U, \Pi] \neq 0$), if we apply parity conservation in this case we will get following relation between helicity amplitudes:

$$T^J_{\lambda_Z, \lambda_\gamma, \lambda_{qq}} = T^J_{-\lambda_Z, -\lambda_\gamma, -\lambda_{qq}}$$
$$A_{\lambda_Z, \lambda_{\ell\ell}} = A_{-\lambda_Z, -\lambda_{\ell\ell}}$$