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# The GSI anomaly

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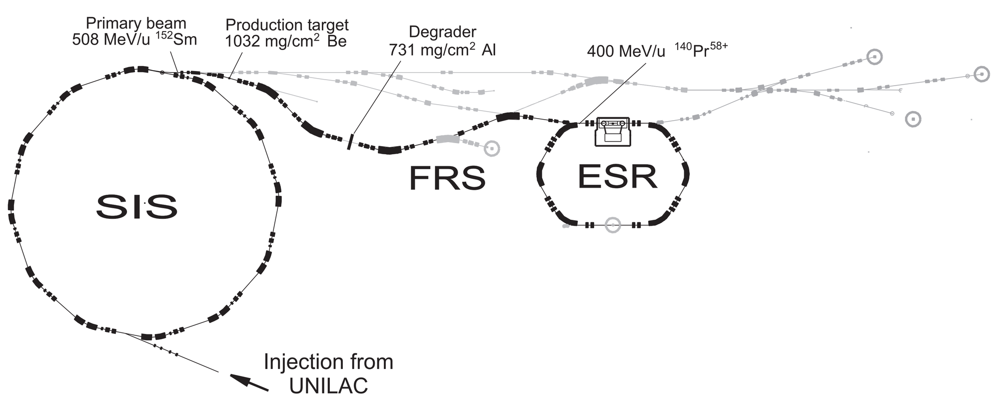
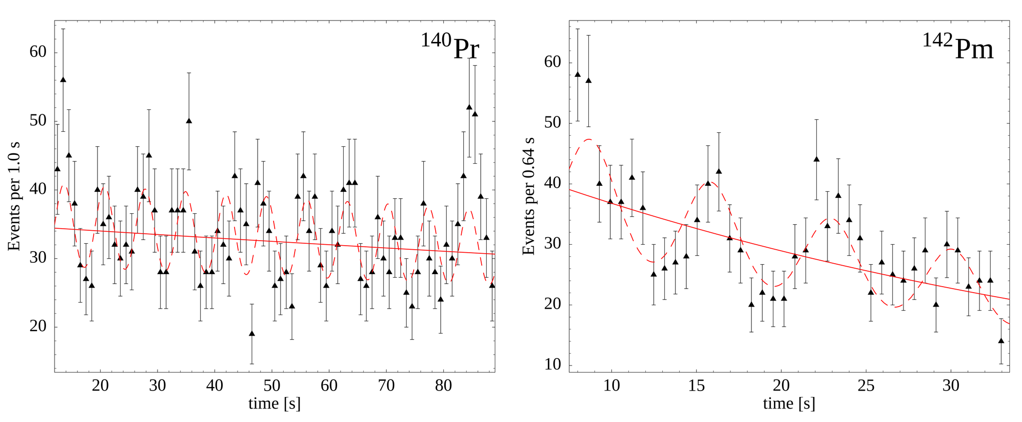
## Abstract

Several controversial attempts have been made to explain oscillating decay rates of heavy ions, found at GSI Darmstadt. Here, we give a critical overview of the existing literature on this anomaly, and show that the effect cannot originate from the neutrino mass splitting. It could, however, be explained by hypothetical internal excitations of the mother ions ($\sim 10^{-15}$ eV). A new run, which is scheduled for the near future, will hopefully help to clarify the situation.

## Orbital electron capture decays of hydrogen-like heavy ions




The accelerator facility at GSI Darmstadt can produce monoisotopic beams of highly ionized heavy atoms and store them for extended periods of time in the Experimental Storage Ring (ESR). There, an experiment has been performed in which electron capture decays of hydrogen-like ${}^{140}_{59}\mathrm{Pr}^{58+}$ and ${}^{142}_{61}\mathrm{Pm}^{60+}$ ions have been studied using time resolved Schottky mass spectrometry [1]. This technique is able to detect the change of an ion's revolution frequency which occurs when it decays from the mother state (${}^{140}_{59}\mathrm{Pr}^{58+}$ resp. ${}^{142}_{61}\mathrm{Pm}^{60+}$) into the daughter state (${}^{140}_{58}\mathrm{Ce}^{58+}+\nu_e$ resp. ${}^{142}_{60}\mathrm{Nd}^{60+}+\nu_e$). If the number of stored ions is sufficiently small ($\lesssim 3$), Schottky mass spectroscopy thus allows for a measurement of the individual decay times.
After many measurements, the following distributions of the decay times have been obtained:




The distributions show the expected exponential behavior, but contain a superimposed oscillation with a period of $T \sim 7$ s, which is so far unexplained.
Statistical fluctuations are excluded as the origin of the anomaly at the 99% confidence level [1]. Most systematical errors can be excluded as well because time-resolved Schottky mass spectroscopy implies a quasi-continuous monitoring of the stored ions. Therefore, effects like instabilities of the beam orbit or a time-modulation of the detection efficiency would be detectable even before the ions decay.

## Overview of existing literature

There exist several preprints discussing the GSI anomaly, most of them attempting to relate it to the neutrino mass splitting.

- Ivanov, Reda, and Kienle [2] first calculate the amplitudes for the decays ${}^{140}_{59}\mathrm{Pr}^{58+} \to {}^{140}_{58}\mathrm{Ce}^{58+} + \nu_1$, ${}^{140}_{59}\mathrm{Pr}^{58+} \to {}^{140}_{58}\mathrm{Ce}^{58+} + \nu_2$, and ${}^{140}_{59}\mathrm{Pr}^{58+} \to {}^{140}_{58}\mathrm{Ce}^{58+} + \nu_3$ separately, and then sum them coherently. According to their computation, each amplitude receives a different phase factor, so that oscillatory interference terms arise in the decay rate. To match the observed oscillation period $T \sim 7$ s, a value of $\Delta m^2_{21} \sim 2.22(3) \cdot 10^{-4}$ eV$^2$ is required for the solar mass squared difference, in conflict with KamLAND results.
  Ref. [2] suffers from an incorrect treatment of the detection process. Since the neutrino is not detected in the experiment, the contributions from decays into different neutrino mass eigenstates must be summed incoherently rather than coherently. Below, we will give a careful treatment of the detection process, which will show why this is the case. Similar arguments have been given by Giunti [3, 4] and by Burkhardt et al. [5]. (Note that Ivanov et al. have replied to Giunti's comment [6].) Moreover, Giunti has shown that the decay rate computed by Ivanov et al. does *not* reduce to the Standard Model result if the neutrino masses are set to zero [4].

## Overview of existing literature (contd.)

- In a more recent paper [7], Kleinert and Kienle propose an explanation of the GSI anomaly in terms of a "neutrino-pulsating vacuum". The authors interpret the $\nu_e$ emission in EC decays as the absorption of a negative energy $\bar{\nu}_e$ from the Dirac sea forming the vacuum. They assume these negative energy anti-neutrinos to undergo oscillations, and thus come to the conclusion that the rate of EC decay should oscillate as well. However, they neglect the fact that the Dirac sea contains equal numbers of all three anti-neutrino flavors, so that, due to unitarity, its $\bar{\nu}_e$ charge remains constant in time.
- Ivanov, Kryshen, Pitschmann, and Kienle have discussed the possibility that the difference between the value of $\Delta m^2_{21}$ calculated in [2] and the value measured in KamLAND could be explained by loop-induced Coulomb interactions of neutrinos in the vicinity of a nucleus [8]. They even claim that their argument can be used to obtain a value for the sum of neutrino masses, $\sum_j m_j \simeq 0.66$ eV. Of course, this claim rests on the assumption that the results of [2] are correct.
- Faber [9] describes the mother ion as a wave packet, while the daughter ion and the neutrino are treated as plane waves. Using disputable kinematical arguments, Faber comes to the conclusion that at least one of the external states has to be off-shell. Moreover, he treats the detection process in the same way as the authors of [2], erroneously taking the coherent sum over the amplitudes for decays into different neutrino mass eigenstates. Using this incorrect approach, Faber deduces a value of $\Delta m^2_{21} \sim 8 \cdot 10^{-5}$ eV$^2$.
- Another explanation attempt for the GSI anomaly is due to Lipkin [10, 11]. As has been correctly pointed out by Peshkin [12], Lipkin's treatment of the detection process suffers from the same misconception as that in [2, 9], namely that the sum over neutrino mass eigenstates is taken to be coherent rather than incoherent.
- In two papers [3, 4], Giunti has pointed out that the GSI anomaly cannot be explained by the neutrino mass splitting. He shows that the contrary statements in [2, 9] arise due to a projection of the decay amplitude onto the wrong final state. Giunti exemplifies his result with an analogy to the double slit experiment. Finally, he discusses the possibility to explain the GSI anomaly by quantum beats of the mother state. However, the origin of the required energy splitting $\Delta E \sim 10^{-16}$ eV of this state remains to be found.

## The detection process

The detection process in the GSI experiment can be well understood in the density matrix formalism. The density matrix of the system is

$$\rho_\psi = |M\rangle\langle M|,$$

where $|M\rangle$ is the time-evolved mother state. The detection of a daughter state $|D\rangle$ is described with the help of the operator

$$\rho_{\mathrm{det}} = \sum_{j=1}^{3} \int d^3p_\nu \, |D;\nu_j,\mathbf{p}_\nu\rangle\langle D;\nu_j,\mathbf{p}_\nu|.$$

Here, the $\nu_j$ stand for the three neutrino mass eigenstates and $\mathbf{p}_\nu$ is the neutrino momentum. Typically, the state $|D\rangle$ will correspond to a wave packet of finite extent in both, momentum and coordinate space. The probability of detecting $|D\rangle$ is

$$\mathcal{P} = \mathrm{tr}\big[\rho_{\mathrm{det}}\rho_\psi\big] = \sum_{j=1}^{3} \int d^3p_\nu \left| \big\langle D;\nu_j,\mathbf{p}_\nu|M\big\rangle \right|^2. \qquad (1)$$

We observe that the sum over neutrino states is incoherent. Therefore, if $\mathcal{P}$ contains oscillatory interference terms, they cannot arise from the neutrino mass splitting, and would also occur in a hypothetical model with only one neutrino flavor. All attempts to explain the GSI anomaly in terms of neutrino mixing are thus foredoomed.
It is however imaginable that different components of the state $|M\rangle$ acquire relative phase differences during their propagation. If several such components could decay into the *same* daughter state $|D;\nu_j,\mathbf{p}_\nu\rangle$, they might induce interference terms in $\mathcal{P}$. To see under which conditions this mechanism could explain the GSI anomaly, we need to compute the matrix element $\big\langle D;\nu_j,\mathbf{p}_\nu|M\big\rangle$. In the following, we will perform this calculation in a wave packet approach.

## Wave packet calculation

Let us first assume the mother and daughter ions to be Gaussian wave packets [13]:

$$|M\rangle \propto \int \frac{d^3p}{\sqrt{2E_{\mathbf{p}}}} \exp\left[-\frac{(\mathbf{p}-\mathbf{p}_{0M})^2}{4\sigma_M^2}\right] e^{iE_{\mathbf{p}}t_M - i\mathbf{p}\mathbf{x}_M}|M,E_{\mathbf{p}},\mathbf{p}\rangle,$$
$$|D\rangle \propto \int \frac{d^3p}{\sqrt{2E_{\mathbf{p}}}} \exp\left[-\frac{(\mathbf{p}-\mathbf{p}_{0D})^2}{4\sigma_D^2}\right] e^{iE_{\mathbf{p}}t_D - i\mathbf{p}\mathbf{x}_D}|D,E_{\mathbf{p}},\mathbf{p}\rangle.$$

Here, $\mathbf{p}_{0M}$ and $\mathbf{p}_{0D}$ are the central momenta of the wave packets, and $\sigma_M$, $\sigma_D$ are the momentum resolutions imposed by the experiment. The complex phase factors fix the spacetime location of the wave packets: At $t = t_M$, the peak of $|M\rangle$ is located at $\mathbf{x}_M$, and at $t = t_D$, the peak of $|D\rangle$ is located at $\mathbf{x}_D$.
The amplitude of the transition from $|M\rangle$ into $|D;\nu_j,\mathbf{p}_\nu\rangle$ is

$$\big\langle D;\nu_j,\mathbf{p}_\nu|M\big\rangle \sim \exp\big[-f(\mathbf{p}_{0M},\mathbf{p}_{0D},\mathbf{x}_M,\mathbf{x}_D,t_M,t_D)\big] \cdot \exp\big[i\phi(\mathbf{p}_{0M},\mathbf{p}_{0D},\mathbf{x}_M,\mathbf{x}_D,t_M,t_D)\big]. \qquad (2)$$

The real factor $\exp[-f(\mathbf{p}_{0M},\mathbf{p}_{0D},\mathbf{x}_M,\mathbf{x}_D,t_M,t_D)]$ enforces sufficient overlap of the wave packets, but is non-oscillatory. The complex phase factor $\exp[i\phi(\mathbf{p}_{0M},\mathbf{p}_{0D},\mathbf{x}_M,\mathbf{x}_D,t_M,t_D)]$ is oscillatory, but it is irrelevant for the modulus of the matrix element appearing in $\mathcal{P}$ (cf. eq. (1)).
Thus, the wave packet formalism, which is the most general way of treating the kinematics of particle physics experiments, shows that pure kinematics cannot explain the GSI anomaly.
We will see below that the situation is different if hypothetical internal degrees of freedom of the mother ion are considered.

## Effect of hypothetical internal excitations

We will now construct a hypothetical situation in which the GSI oscillations *can* be explained by a quantum mechanical interference effect, namely by quantum beats of the mother ion [13]. This possibility has been pointed out previously in [3, 4, 12]. Let us assume that the state of the mother ion is split into several sublevels $|M_\sigma\rangle$ with different masses $m_\sigma$. Moreover, we assume that, for some reason, the production process creates the mother ion in a superposition of these states, i.e.

$$|M\rangle = \sum_\sigma \alpha_\sigma |M_\sigma\rangle\,,$$

with $\sum_\sigma |\alpha_\sigma|^2 = 1$. Then, eq. (2) gets modified into

$$\big\langle D;\nu_j,\mathbf{p}_\nu|M\big\rangle \sim \sum_\sigma \alpha_\sigma \exp\big[-f_\sigma + i\phi_\sigma\big]\,,$$

so that $\mathcal{P}$ contains interference terms of the form $\exp[i(\phi_\rho - \phi_\tau)]$.
Assuming a superposition of only two substates $|M_\sigma\rangle$, working in the frame $\mathbf{p}_{0D} = 0$, assuming $\sigma_M = \sigma_D$, and expanding the phase in $\Delta m^2 = m_2^2 - m_1^2$, we find

$$\mathcal{P} \propto 1 + \sin 2\theta \cos[\Delta m^2 (t_D - t_M)/2\bar{E}_M]\,,$$

where the parameterization $\alpha_1 = \cos\theta$, $\alpha_2 = \sin\theta$ has been used. $\bar{E}_M$ is the average central energy of the mother wave packets.
To explain the GSI anomaly, $\Delta m^2 \sim 2.2 \cdot 10^{-4}$ eV$^2$ is required, or equivalently $|m_2 - m_1| \sim 8.4 \cdot 10^{-16}$ eV.

## References

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