1 Basics

Many scientific and industrial processes are so sensitive that it is necessary to omit the disturbing influence of air. For example a particle accelerator could never reach energies of several GeV if the particles would be decelerated by colliding with molecules of the air. Therefore vacuum technology is a basic tool in modern science and engineering.

The experiments described in this report were designed to study some basic concepts of vacuum technology: different methods of pressure measurement, the characteristics of a rotary slide valve vacuum pump and the influence of tubes and pipes on the overall efficiency of the system.

We are not going to give a detailed explanation of the experimental assembly and the theoretical backgrounds (please refer to the description of the experiment or to appropriate literature for these), but will be concentrating on the discussion of our results.

2 Pressure Measurement

At very low pressures it is difficult to use conventional U-Tube manometers because they would be too inaccurate. Two possible alternatives are the McLeod manometer which performs its measurements on a compressed sample of the gas and the Pirani manometer, which measures the heat conductivity of the gas, which is proportional to its pressure in the orders of magnitude that are of interest here. Therefore, a thin tungsten wire is brought into the recipient. Its temperature (which is proportional to its electrical resistance) is kept constant by varying the current flux through it. The electrical power that is needed for this is a measure for the heat conductivity of the gas.

2.1 Calibration of a Pirani Manometer

The disadvantage of a Pirani manometer is the fact that it needs to be calibrated before use. In our case the calibration was done with a McLeod manometer and an ordinary U-Tube manometer. The calibration curves are shown in figure (1). The red curve shows the electrical current through the Pirani, the black one visualizes the electrical power calculated from the current measurements. Note the bilogarithmic scale.
Figure 1: The calibration curve of the Pirani manometer

From around $10^{-1}$ mbar to 1 mbar the electrical power and (therefore the measured heat conductivity) are approximately proportional to the pressure. In this range a straight line can be fitted to the data points which simplifies the conversion from electrical currents to pressure a lot in the following experiments.

For higher pressures the heat conductivity does not depend on the gas pressure so much any more. This is in good agreement with the kinetic gas theory, which states that heat conductivity is independant of pressure as long as the mean free path $\lambda$ of the gas particles is some orders of magnitude smaller than the dimensions of the gas volume, while it is proportional to pressure if $\lambda$ gets greater than these dimensions.

For very low pressures, the calibration curves in figure (1) get flat again. This is due to the non-vanishing heat conductivity of the Manometer itself: In this area the heat flux through the metal wires of the Pirani becomes relevant.

Therefore, a straight line approximation is valid only for pressure between $10^{-1}$ and 1 mbar respectively currents between 6 mA and 15 mA. For higher and lower currents the conversion needs to be done manually.

### 2.2 Discussion of Errors and Inaccuracies

In figure (1), error bars are given for both pressure and electrical power. As you can see, the pressure measurement was quiet accurate in the low pressure area, where the McLeod could be used to perform good measurements.

For high pressures (above 10 mbar) however, the U-Tube manometer had to be used instead of the McLeod which is why the measurements are much more inaccurate in this area (errors of up to $\pm 5$ mbar).
The accuracy of the current measurement is quiet good because a digital multimeter was used for it.

What is astonishing however is the sharp bend in the curve which appears between 1 mbar and 10 mbar. The reason for this seems to be that the digital multimeter automatically switched to a different measurement mode here because the currents grew too big. This change of the multimeters inner resistor affects our calibration curve here. However it cannot be treated as an error because this kind of non-linearities is why the manometer is calibrated before use.

3 The Pumping speed of the Rotary Slide Valve Pump

3.1 Experimental setup and results

In order to measure the pumping speed of our vacuum pump, a piston probing unit was evacuated at a constant pressure which was regulated at the pump inlet. A simple clock was used to measure the overall pumping time at several remaining volumes (100 ml to 10 ml) This measurement was performed three times to get enough information for a statistical analysis.

This measurement led to an average volume throughput of \( \frac{\text{d}V}{\text{d}t} = 3.96 \cdot 10^{-7} \text{ m}^3/\text{h} \)

The pumping speed \( S \) can now be calculated by using the equation

\[
p_0 \frac{\text{d}V}{\text{d}t} = S \cdot p
\]

where \( p_0 \) is the atmospheric pressure of around 1000 mbar \( \pm 20 \text{ mbar} \) against which the pump has to work, and \( p \) is the pressure at the inlet of the pump, which was adjusted to 0.4 mbar \( \pm 0.025 \text{ mbar} \) (corresponding to a current flow of 8 mA in the Pirani manometer) in our experiment. The error of \( p \) is due to the inaccurate graphical conversion of the Pirani current into pressure by using the calibration curve recorded in the first experiment.

The pumping speed calculated with the above formula is

\[
S = 3.57 \pm 0.24 \text{ m}^3/\text{h}
\]

This value is in very good agreement with the manufacturers specification of 3.7 \text{ m}^3/\text{h} . It is a little smaller, however, because the conditions (tube sizes etc.) were probably not optimal in our experiments..

3.2 Statistical error analysis

Some explications must be given on the error of \( S \) which is composed of the errors of the constants \( p_0 \) and \( p \) (see above) and the statistical standard deviation of the average \( \frac{\text{d}V}{\text{d}t} \). This is the result of applying the student function to the standard deviation of the measurement results. Its numerical value is \( \Delta \frac{\text{d}V}{\text{d}t} = 5.150 \cdot 10^{-9} \text{ m}^3/\text{h} \). By applying
the Gaussian error propagation function

\[
\frac{\Delta S}{S} = \sqrt{\left( \frac{\Delta p_0}{p_0} \right)^2 + \left( \frac{\Delta dV/dt}{dV/dt} \right)^2 + \left( \frac{\Delta p}{p} \right)^2}
\]  

(2)

one obtains \( \frac{\Delta S}{S} = 6.7\% \) which is equivalent to the absolute value given above: \( \Delta S = 0.24 \text{ m}\text{³}/\text{h} \).

4 Effective pumping speed

4.1 Experimental setup and graphical analysis

The final part of our experiments consisted of a study of the effective pumping speed, a quantity which depends on the efficiency of the pump, which was discussed in the previous section, and on the conductance of tubes and pipes. The experimental setup consisted of a brass recipient (\( V = 3 \text{ l} \)) which was connected to the pump over differently sized tubes: One with a diameter of 25 mm which is appropriate for our pump and two capillaries with diameter of 2 mm respectively 3 mm.

During the evacuation process pressure was measured at fixed time intervals with the Pirani manometer. The results are shown in figure (2). Note the semilogarithmic scale. For the 25 mm tube the measurement was performed three times. As the results are reproducable very well, only one curve is shown in the diagram. As one might expect, the evacuation takes significantly longer for the thin capillaries than for the large tube.

This experiment demonstrates that tube size is essential to the efficiency of a vacuum system. This means one does not only need a powerful pump to create good vacua but also a system of tubes and pipes with very low resistances to the gas flow.

4.2 Mathematical analysis

From the definition of \( S \)

\[
V \frac{dp}{dt} = -Sp
\]

(3)

one can derive a formula for the pressure at a given time \( p(t) \):

\[
p(t) = p_0 \exp \left( -\frac{S}{V} t \right)
\]

(4)

\[
S_{eff} = -V \cdot \frac{\Delta \ln p}{\Delta t}
\]

(5)

Although figure (2) shows that this formula is only valid for low pressures (ideally, the curves should be straight lines because of the semilogarithmic scale), the gradient of the curve should be equal to the exponent \(-S/V \cdot t\) in equation (4).
Consider for example the curve for the 25 mm tube at a pressure of 0.7 mbar. Its gradient at this point is

$$\frac{\Delta \ln p}{\Delta t} = \frac{\ln p_1 - \ln p_2}{t_1 - t_2} = 0.3352$$

For $p_1$, $p_2$, $t_1$, $t_2$ the values from two neighbouring measurements were used. Applying equation 5 gives

$$S_{eff} = 3.62 \frac{m^3}{h}$$

Performing similar calculations for the 2 mm capillary gives the following results:

$$S_{eff}(5\text{mbar}) = 0.791 \frac{m^3}{h}$$
$$S_{eff}(0.3\text{mbar}) = 0.057 \frac{m^3}{h}$$

These values are quite inaccurate because of the inaccurate determination of $\Delta \ln p/\Delta t$ from only two neighbouring measurements. If the curves in figure (2) were straight lines, one could calculate the gradient with a regression algorithm which would be much more accurate.

The conductances of the capillary can be calculated theoretically as well. At 5 mbar the gas flow is viscous, so the capillary’s conductance is given by the formula

$$L = \frac{\pi d^4}{128 \eta l} \frac{p_1 - p_2}{2}$$  \hspace{1cm} (6)
where \( d \) is the diameter of the tube, \( \eta \) is the viscosity of air, \( l \) is the length of the tube (9.5 cm in our case) and \( p_1 \) and \( p_2 \) are the pressures at both ends of the tube. Here the pressure in the recipient is \( p_1 = 5 \) mbar; for \( p_2 \) we took the lowest pressure that is achievable with our pump: \( p_2 = 0.001 \) mbar. This results in a conductivity of \( L_{\text{viscous}} = 0.204 \, \text{m}^3/\text{h} \). Finally, there is the following theorem about series of conductances:

\[
\frac{1}{S_{\text{eff}}} = \frac{1}{S} + \frac{1}{L}
\]  

(7)

Here, \( S \) is the pumping speed of the vacuum pump. Using the value given by the manufacturer for \( S (3.7 \, \text{m}^3/\text{h}) \) we obtained the resulting effective pumping speed:

\[
S_{\text{eff}}(5\text{mbar}) = 0.194 \, \text{m}^3/\text{h}
\]

For very low pressures, equation (6) is no more valid and has to be replaced by

\[
L = \sqrt{\frac{\pi k T}{18 m_a}} \cdot \frac{d^3}{l}
\]  

(8)

where \( k \) is the Boltzmann constant, \( T \) is the absolute gas temperature and \( m_a \) is the molecular mass of the gas particles. For our 2 mm capillary, one obtains:

\[
L_{\text{molecular}} = 0.037 \, \text{m}^3/\text{h} \approx S_{\text{eff}}
\]

Here, \( S_{\text{eff}} \) is approximated by \( L_{\text{molecular}} \) because the resistance of the capillary is much greater than that of the pump.

The calculated values for \( S_{\text{eff}} \) differ greatly from the experimental results. This might be due to inaccuracies in the given capillary diameter which will affect the results greatly because \( d \) is raised to the fourth respectively third power in equations (6) and (8). Additionally there is a great inaccuracy in the experimental values because they were derived from the gradient of the curves in figure (2) (see above).

Finally, for \( S_{\text{eff}}(5\text{mbar}) \) the efficiency of the pump is possibly greater than we assumed in the theoretical calculations, because the difference between the gas pressure and the atmospheric pressure (against which the pump has to work) is not so great.

5 Minimum pressure

At the end of our experiments we wanted to know the minimum gas pressure that was achievable with our vacuum pump. With a configuration similar to the one we used to measure the effective pumping speed for the 25 mm tube we achieved a current of only 3.6 mA at the Pirani manometer. Although our calibration curve does not go that far down, one can extrapolate that gas pressure at this point should have been around 0.0005 mbar.

It was interesting to observe that, as soon as the pump was switched off, the Pirani current immediately went up to about 4 mA, which corresponds to a gas pressure of 0.001 mbar. This is because of the inevitable leakings in the system.
6 Questions

6.1 Definition of the ideal gas

A gas is called “ideal” if forces between its particles and the volume of the molecules are both neglectable. In vacuum experiments this approximation is valid because the density of the gas is very small.

6.2 Explanation of heat conductivity

The best explanation for the heat conductivity of a gas is given by the kinetic theory: Gas particles collide with the molecules of the hot reservoir and gain kinetic energy. This energy is propagated through the gas volume by collisions of gas particles. Finally accelerated particles will collide with the cold reservoir and give energy to it.

6.3 Vacuum flask

The heat conductivity of a gas is independant of its pressure only as long as the mean free path is smaller than the dimensions of the gas volume. If one evacuates the envelope of a vaccum flask pressure will be low enough that this condition is no more met. Then heat conductivity becomes proportional to gas pressure.

6.4 Heat conductivities of several materials

The following table lists the heat conductivities of some materials in WK$^{-1}$m$^{-1}$:

<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity (WK$^{-1}$m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>4.01</td>
</tr>
<tr>
<td>Water</td>
<td>0.60</td>
</tr>
<tr>
<td>Air</td>
<td>0.02</td>
</tr>
<tr>
<td>Stone</td>
<td>2.30</td>
</tr>
<tr>
<td>Fat</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Copper is a very good heat conductor. Therefore it should be an excellent material for passive cooling systems as they are used for computer CPUs etc. However it is very expensive, so it has no practical importance as a heat conductor.

Water is not as good a heat conductor as copper because it is a liquid. However, in comparison to other liquids its heat conductivity is relatively large. This is made use of in cooling systems for car engines.

Air has a very poor heat conductivity. This is for example why textile clothes can keep you warm: The air that is contained between them and your body acts as an isolator. However, if air is flowing fast enough, it can be used for cooling systems as well. For example formula 1 car engines are cooled by air.

The heat conductance of stone is quiet good, and it has a high heat capacity. Therefore it has been used for centuries in ovens.

The low heat conductance of fat makes it nature’s first choice for warming living beings.
6.5 Blaise Pascal’s brother-in-law

Blaise Pascal’s brother-in-law lived in Clermont-Ferrand which is 400 m above sea level. As Pascal had no exact measurement devices for the height of a mountain, he might have estimated the height difference between Clermont-Ferrand and the Puy de Dôme as 1000 m. The mercury level in the U-Tube-manometer will fall around 10 cm over this height difference. Pascal will have concluded that mercury is 10,000 times heavier than air.

6.6 Definition of molecular flow

The range of molecular flow is the pressure range in which the mean free path of the gas molecules is as large as or larger than the dimensions of the gas volume. Under these circumstances, many equations of the kinetic theory are no more valid.

6.7 Pumping time for a 1 mm capillary

For a capillary that has a diameter of only 1 mm the effective pumping speed will be very low, so the pressure will fall very slowly. The conductivity of the capillary is therefore given by equation (6), the equation for viscous gas flow. Beginning with equation (3) one can derive:

\[ V \frac{dp}{dt} \approx -Lp = -\frac{\pi d^4}{128 \eta l} \cdot \frac{p}{2} \cdot \rho \]

\[ \frac{1}{p^2} \frac{dp}{dt} = \frac{\kappa}{V} \frac{dt}{t} \]

\[ \frac{1}{p_0} - \frac{1}{p} = \frac{\kappa}{V} t \]

\[ p = \frac{1}{p_0} + \frac{\kappa}{V} t \]

According to this formula, one can expect a pressure of 351 mbar after 10 minutes if a 1 mm capillary is used.

6.8 Mean free path in ultra high vacuums

In ultra high vacuums — that is at pressures around \( 4 \cdot 10^{-11} \) mbar — one can expect a great mean free path of the gas molecules. A numerical value can be derived from the kinetic gas theory, which provides us with the following formula:

\[ \lambda = \frac{1}{\sqrt{32} \cdot \rho \cdot F} \]  

(9)
Here \( \lambda \) is the mean free path and \( F = 1.3 \cdot 10^{-19} \text{m}^2 \) is the cross section of the air molecules. The particle density \( \rho \) in ultra high vacuums is given by

\[
\rho = \frac{3p}{m_ac^2} \tag{10}
\]

As \( 1/2m_ac^2 = 3/2k_BT \) this is equivalent to

\[
\rho = \frac{p}{k_BT} \tag{11}
\]

At a pressure of \( 4 \cdot 10^{-11} \text{mbar} \) and a temperature of 300 K the particle density is \( \rho = 9.657 \cdot 10^{11} \text{m}^{-3} \). Now we can apply equation (9) and obtain the result:

\[
\lambda = 1.41 \cdot 10^6 \text{m}
\]