

# Vector bosons and direct photons

## *Lecture 1*

CTEQ  
school  
2013



John Campbell, Fermilab

# Introduction

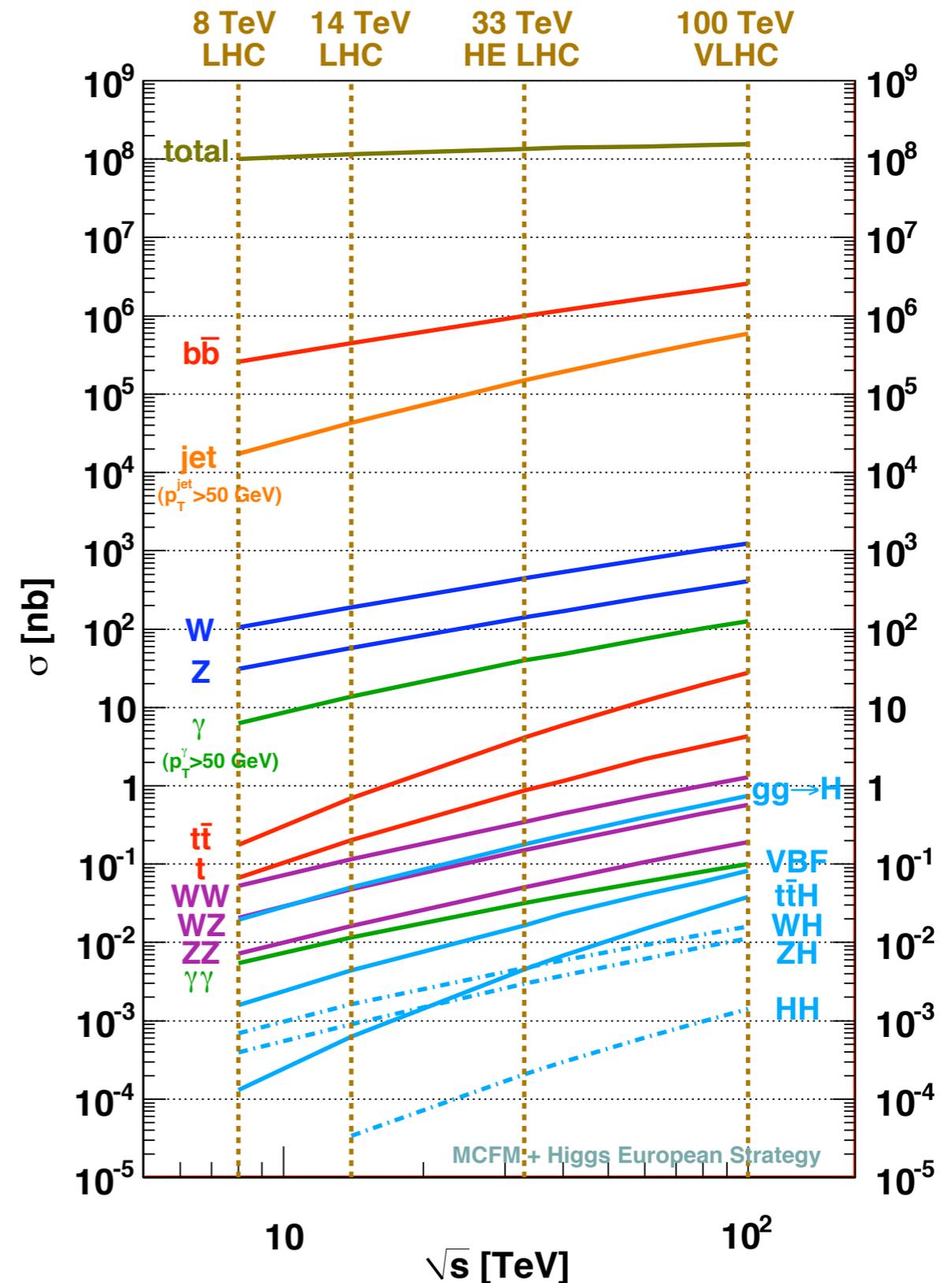
- I am a theorist interested in hadron-collider phenomenology.
- Main interest: higher order corrections in QCD.
- Author of next-to-leading order Monte Carlo code **MCFM**.
  
- Two lectures, today and tomorrow.
- For questions or comments:
  - discussion sessions tonight and tomorrow night;
  - or, email: [johnmc@fnal.gov](mailto:johnmc@fnal.gov)
  
- Some material taken from “QCD for Collider Physics” by Ellis, Stirling, Webber  
- excellent resource for further details on many subjects covered here.

# Outline of lectures

- Overview of vector boson basics.
- Underlying theory of W,Z production.
- Discussion of the direct photon process.
- Di-photon production.
  
- The importance of multi-boson production.
- Review of selected di-boson phenomenology.
- Beyond inclusive di-boson measurements.

# Setting the scene

- Cross sections for producing W, Z bosons and photons are huge.
  - radiating additional jets (approx. factor of  $\alpha_s$ ) still leaves large cross sections.
  - multiple boson production still significant versus BSM rates.
- **Experimentally** important:
  - clean final states good for calibration (leptons, photons).
  - leptons, missing energy (+jets) crucial backgrounds.
- **Theoretically** important:
  - expect well-understood cross sections, test of new calculations.



# Electroweak Feynman rules

- Electroweak interaction Lagrangian:

$$\begin{aligned} \mathcal{L} = & \sum_f \bar{\psi}_f \left( i\not{\partial} - m_f - g_W \frac{m_f H}{2M_W} \right) \psi_f \\ & - \frac{g_W}{2\sqrt{2}} \sum_f \bar{\psi}_f (\gamma^\mu (1 - \gamma_5) T^+ W_\mu^+ + \gamma^\mu (1 + \gamma_5) T^- W_\mu^-) \psi_f \\ & - e \sum_f Q_f \bar{\psi}_f A \psi_f - \frac{g_W}{2 \cos \theta_W} \sum_f \bar{\psi}_f \gamma^\mu (V_f - A_f \gamma_5) \psi_f Z_\mu \end{aligned}$$

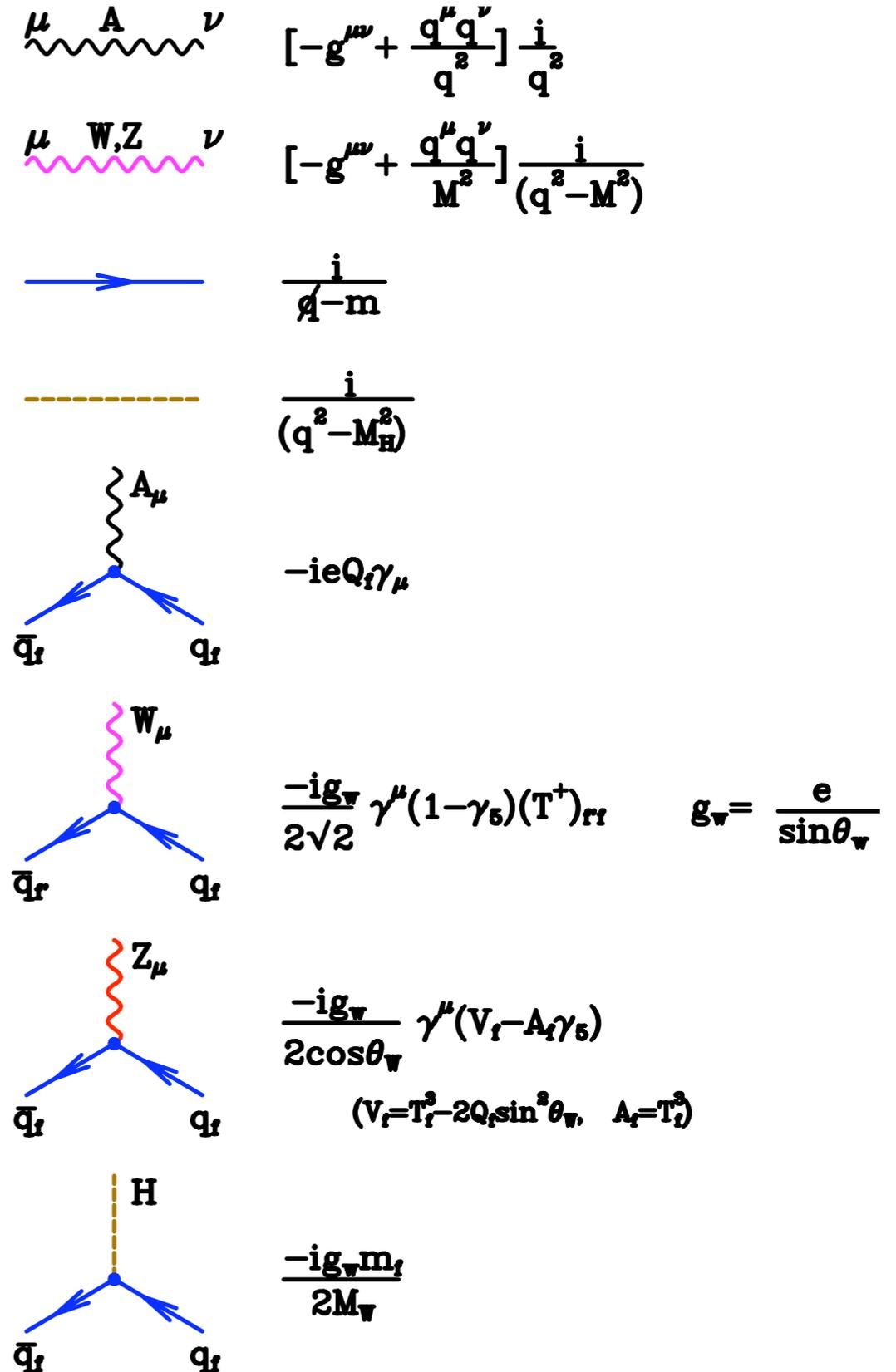
- W boson couples to left-handed fermions
- Z boson couples to both, different strengths:

$$(V_f - A_f \gamma_5) = \frac{V_f + A_f}{2} (1 - \gamma_5) + \frac{V_f - A_f}{2} (1 + \gamma_5)$$

- vector and axial couplings in terms of weak isospin  $T_f^3 = \pm 1/2$

$$\begin{aligned} V_f &= T_f^3 - 2Q_f \sin^2 \theta_W \\ A_f &= T_f^3 \end{aligned}$$

$$\begin{aligned} V_u &\approx 0.2, \quad V_d \approx -0.35, \quad V_\nu = \frac{1}{2}, \quad V_e \approx -0.04 \\ A_u &= \frac{1}{2}, \quad A_d = -\frac{1}{2}, \quad A_\nu = \frac{1}{2}, \quad A_e = -\frac{1}{2} \end{aligned}$$



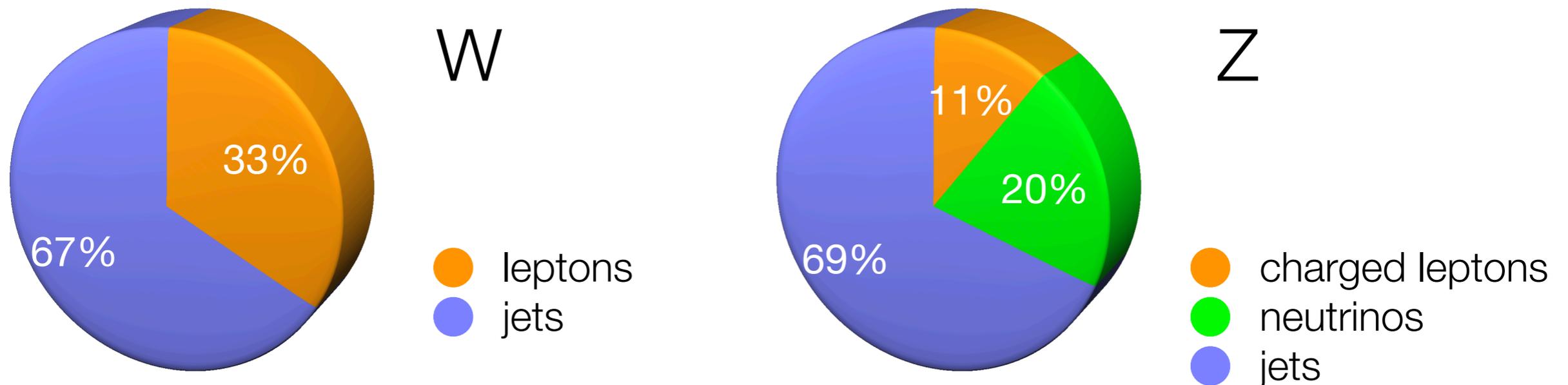
# W and Z decays

- Partial decay widths at leading order:

$$\Gamma(W^+ \rightarrow f \bar{f}') = C \frac{G_F M_W^3}{6\sqrt{2}\pi} \quad \Gamma(Z \rightarrow f \bar{f}) = C \frac{G_F M_Z^3}{6\sqrt{2}\pi} (V_f^2 + A_f^2)$$

C=1 leptons / C=3 quarks

- W decays:** 3 charged leptons (C=1), 2 open generations of quarks (C=3)  
 $\Rightarrow \text{Br}(W^+ \rightarrow e^+ \nu) = 1/9.$
- Z decays:**  $V_e \approx 0, |A_e| = |A_\nu| = |V_\nu| \Rightarrow \text{Br}(Z \rightarrow \nu \bar{\nu}) \approx 2 \times \text{Br}(Z \rightarrow e^+ e^-)$



- Large fraction of decays into difficult-to-measure modes.

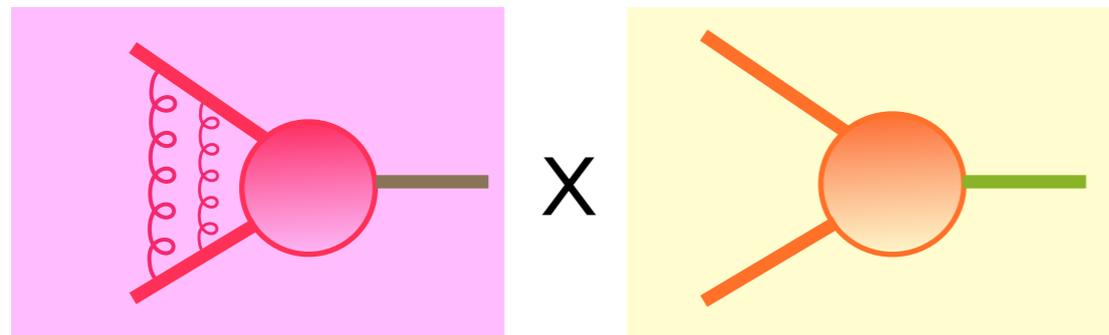
# W and Z production

- You have already seen a sketch of the NLO corrections to these processes and discussed some of the simple phenomenology such as rapidity distns.
  - you will also get more later on (NLO and matching, pdf fits)
- Instead, I will focus on different aspects of W and Z production and the underlying theory.
- Historically, these processes have provided an essential role in extending the perturbative description to higher orders, beyond NLO QCD.
  - they are the simplest non-trivial calculations, containing only a single scale
  - an electroweak final state, so QCD corrections only occur in production
- Of course, improving the accuracy of the predictions important in its own right
  - very large cross sections for basic physics objects
  - improved extractions of fundamental quantities, e.g.  $M_W$ , pdfs
- First up: going from NLO to NNLO QCD.

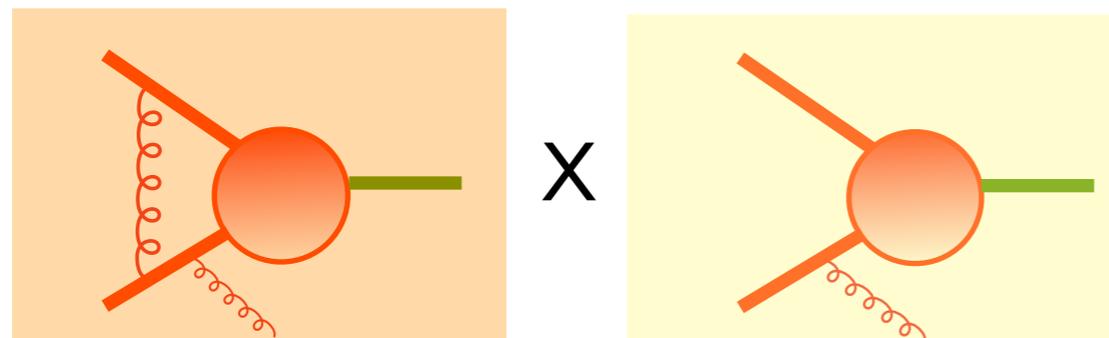
# 2→1 processes at NNLO

(out of my purview, but same story for  $gg \rightarrow H$ )

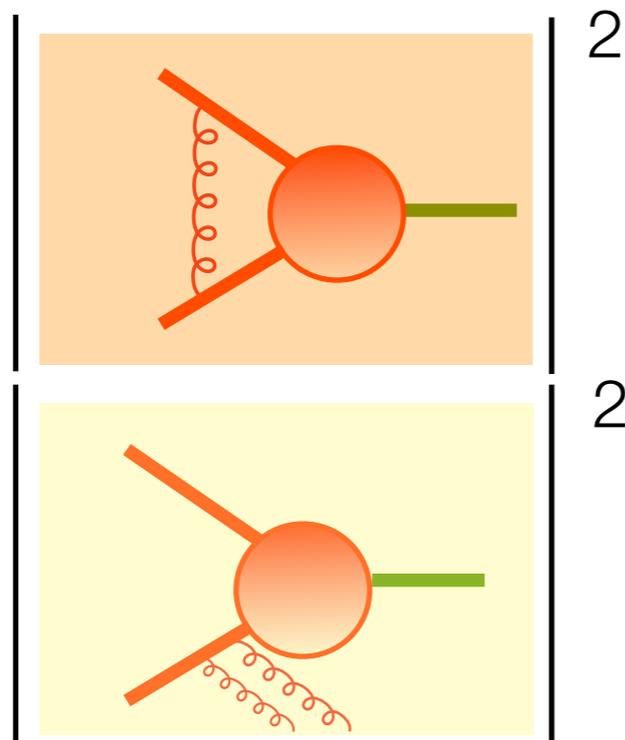
- Much more than just virtual/real combination at NLO - many ways of making  $g_s^4$ .



needs 2-loop amplitude, for many years the unknown ingredient

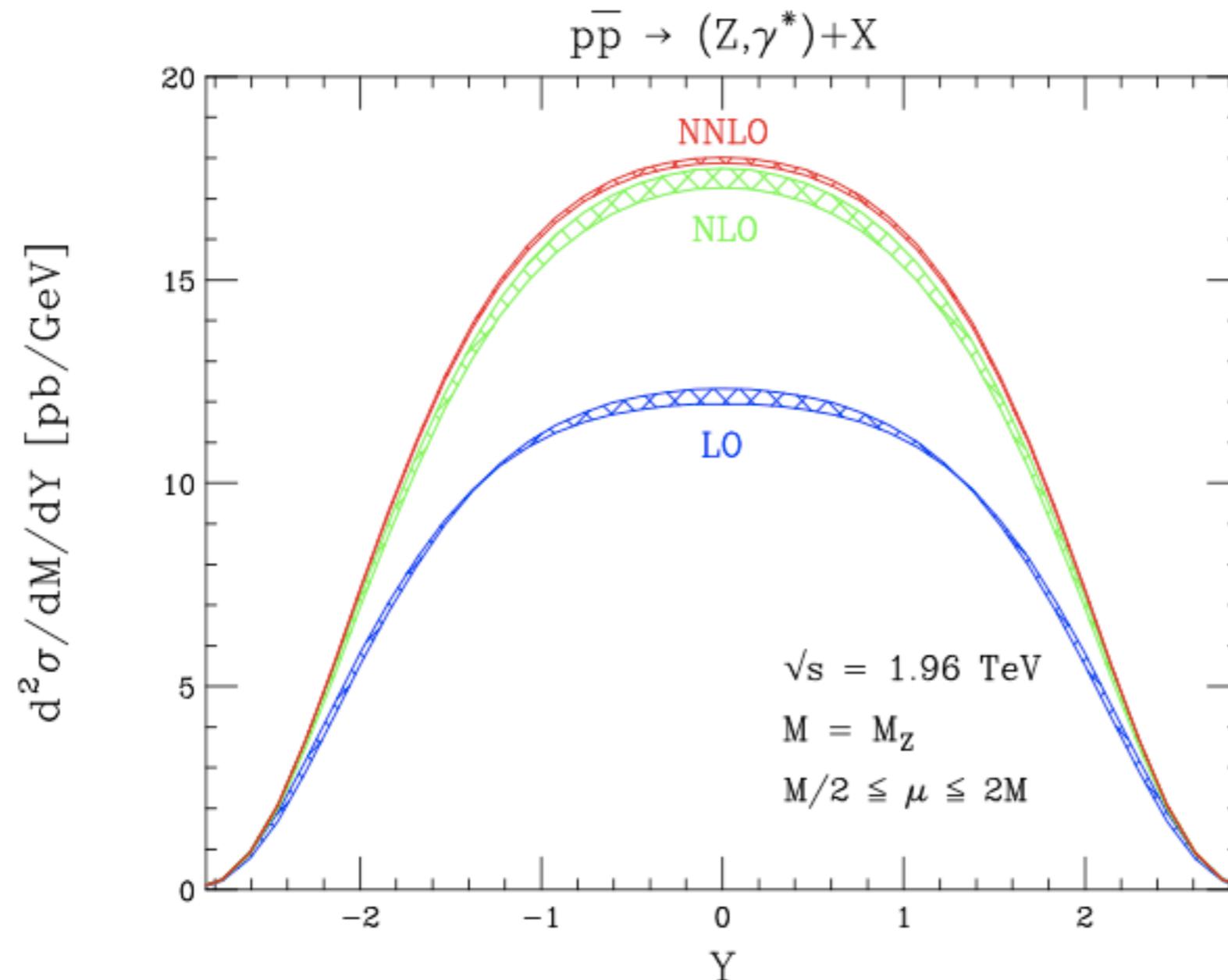


known one-loop corrections, but more soft and collinear divergences



on the surface looks the easiest; in fact, untangling structure of singularities the key to the calculation

# NNLO result



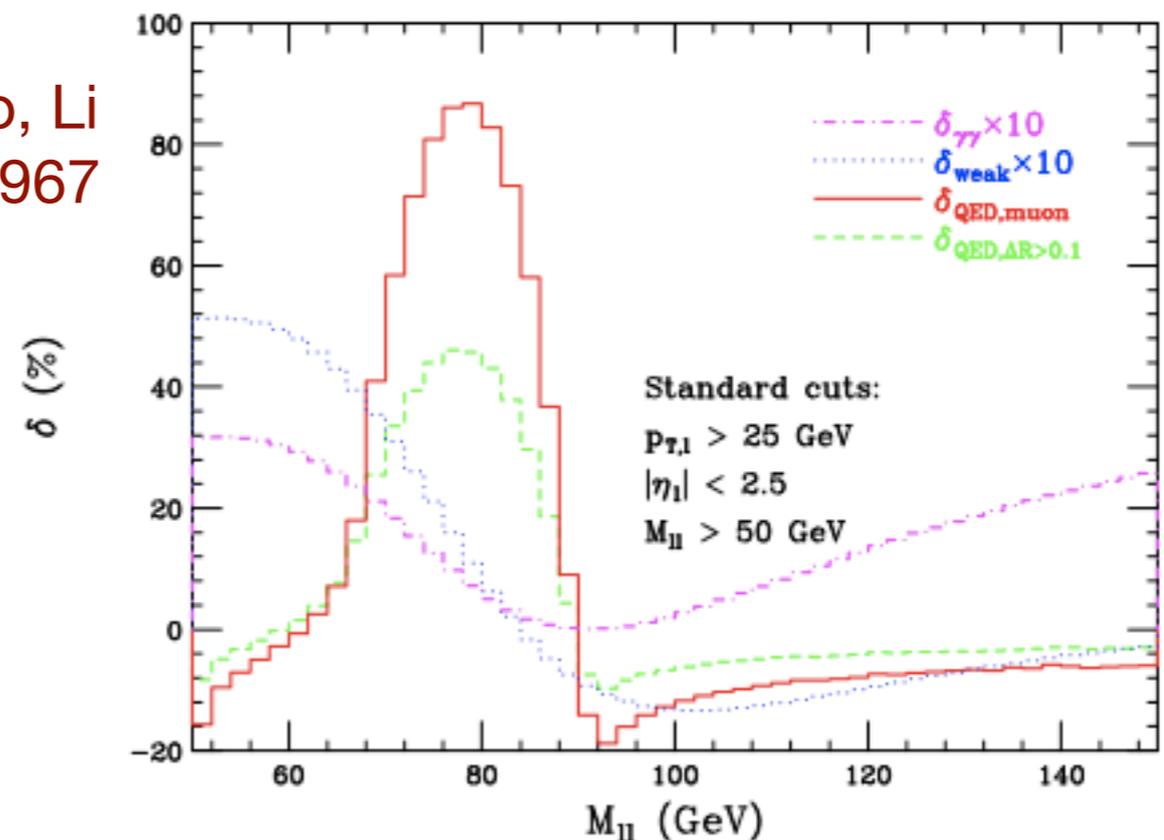
Anastasiou, Dixon,  
Melnikov, Petriello (2004)

- Very large correction from LO to NLO does not repeat from NLO to NNLO  
→ stabilisation of the expansion.
- NNLO outside NLO error estimate, now has error of a few percent.

# Beyond NNLO QCD

- Numerically, expect NNLO QCD ( $\alpha_s^2$ ) to be at the same order as **NLO QED and electroweak effects** ( $\alpha$ ).
  - virtual loops of photons, W, Z bosons;
  - real radiation of photons;
  - since W and Z bosons are massive and are explicitly reconstructed in the detector (and put in different event samples) no need to add their effects.
- Especially important near the Z peak.
- Sensitivity to the definition of the lepton (“bare” or “dressed”, recombined with photon).
- Should include photon-induced contributions  $\gamma\gamma \rightarrow \ell + \ell$ .
- Open issue: how to combine QCD and QED contributions.

Petriello, Li  
1208.5967

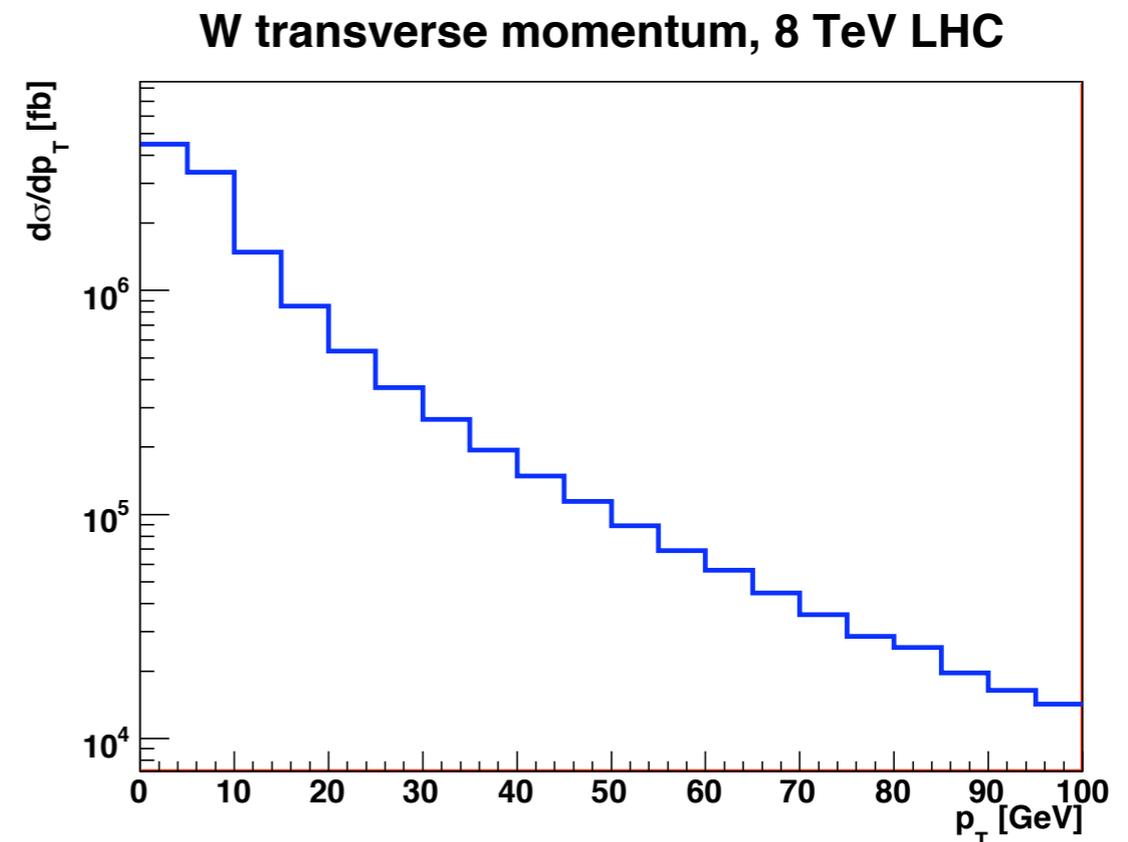


# A different approach

- Extending the perturbative description order-by-order is one tactic.
- However there are limitations present at each order that can be better-handled with a different approach.
  - associated with the extra radiation present at higher orders.
- Start by looking at the **transverse momentum distribution of the W** as given by a NLO calculation of the total rate.

1) All of the genuine NLO corrections, from the 1-loop virtual diagrams, enter at  $p_T=0$ . In fact they are large and negative  $\rightarrow$  get any answer you want in the first bin, depending on the bin width.

2) prediction for any  $p_T>0$  is really just a leading-order one, from real radiation diagrams.



# One approach

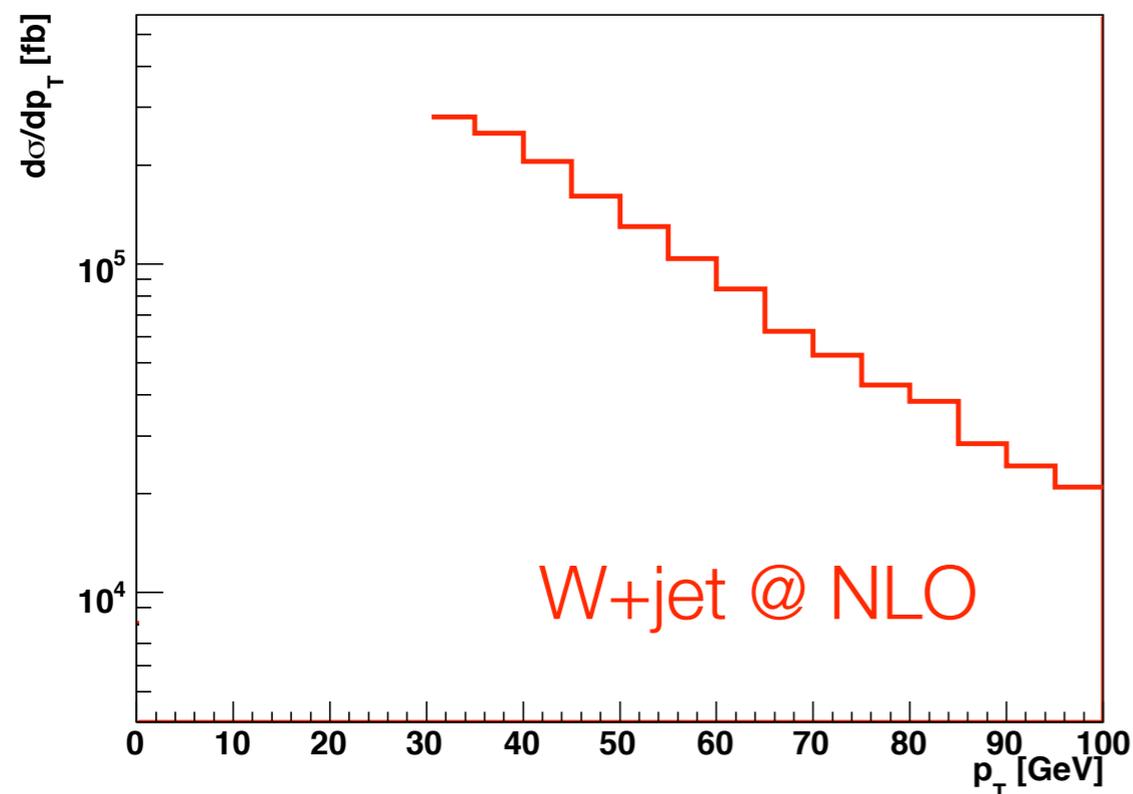
- Easiest to tackle the second problem first: improving prediction at high  $p_T$ .
- Recognize that at large  $p_T$  we can just compute the corrections to the process,

$$pp \rightarrow W + \text{jet}$$

with the jet providing a non-zero recoil even at LO.

- The calculation requires the definition of a jet, specifying a minimum transverse momentum;  $W$   $p_T$  with NLO accuracy above this cut (25 GeV here).

W transverse momentum, 8 TeV LHC



# One approach

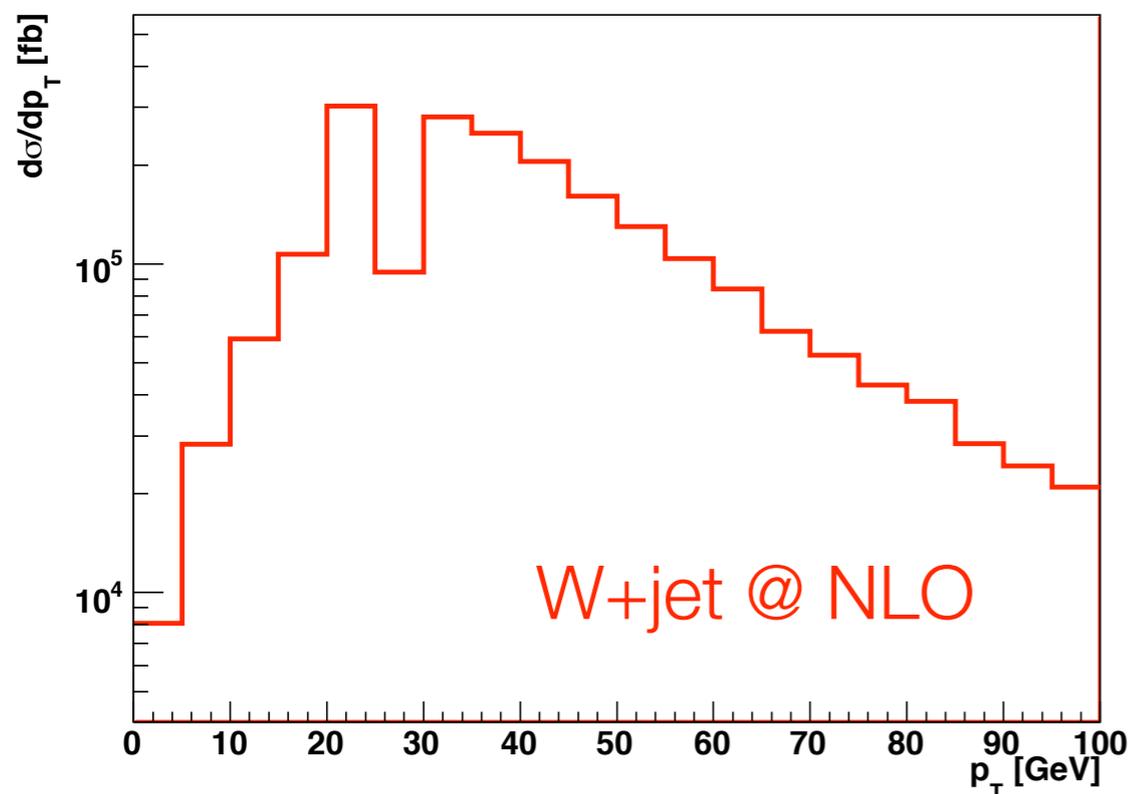
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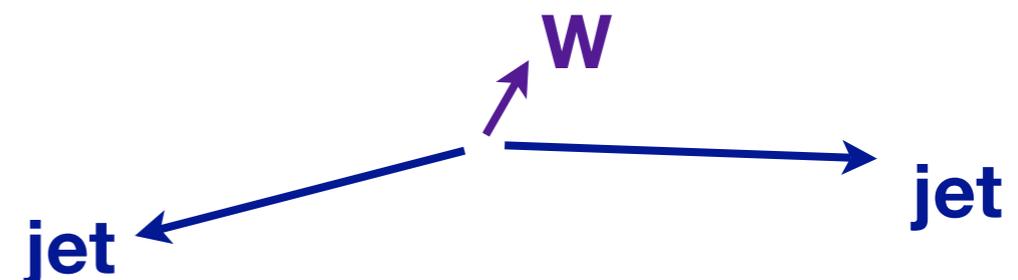
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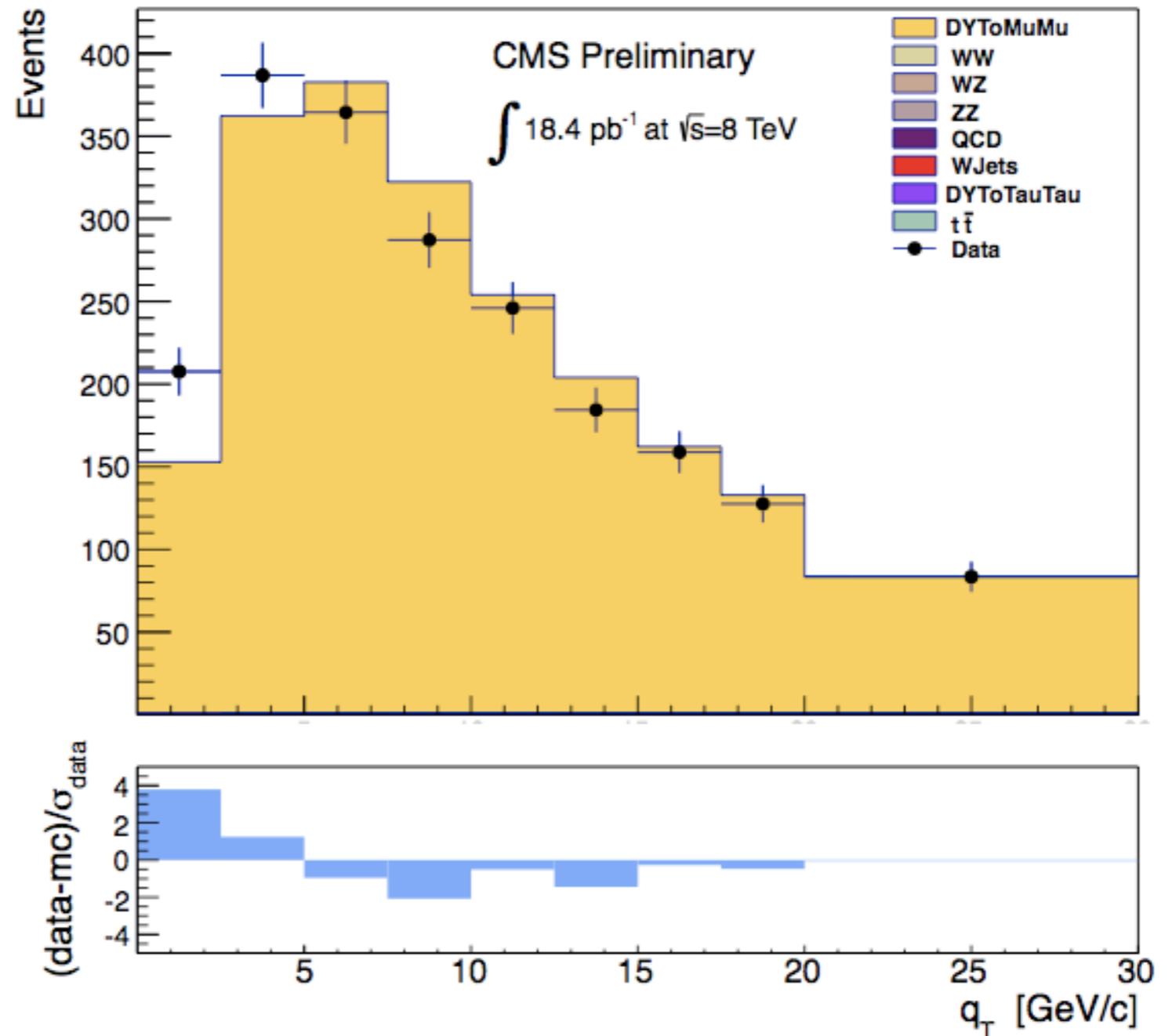
- Below the jet cut,  $W$   $p_T$  is populated by configurations with two almost-balancing jets  $\rightarrow$  LO only.



- Prediction unreliable adjacent to jet cut.

# What now?

- At high  $p_T$  the prediction does not depend on the cut
  - how small can we take it?
  - how can the behaviour at small  $p_T$  be fixed?
- In particular, how can a perturbative description produce turn-over like the one seen in data?
- Solution: need to account for all possible recoils against multiple partons in a systematic fashion.
  - identify relevant terms in the cross section and include effects to all orders → **resummation**.

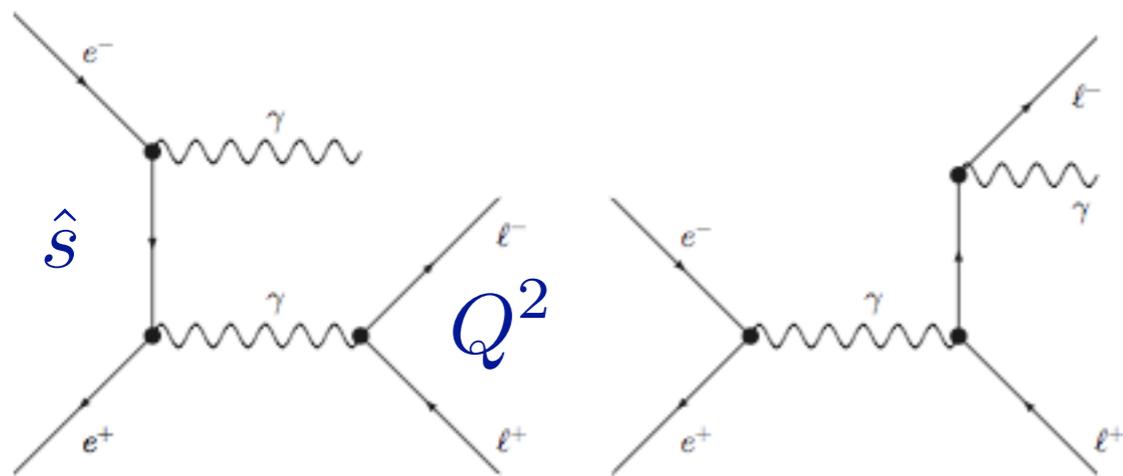


# Introduction to resummation

- To have a feeling for how resummation works, simplest to back off to a lepton collider: **trade quarks for electrons and gluons for photons.**

Parisi and Petronzio (1979)

- Look at the form of the cross section at small transverse momenta; consider virtual photon only (no Z).



Form of doubly-differential cross section at small  $Q_T$ :

$$\frac{d\hat{\sigma}_R}{dQ_{\perp}^2 dQ^2} = \frac{4\alpha^2}{3\hat{s}} \frac{\alpha}{\pi Q_{\perp}^2 Q^2} \frac{\hat{s}^2 + Q^4}{\hat{s} - Q^2} = \hat{\sigma}_0 \frac{\alpha}{\pi Q_{\perp}^2 Q^2} \frac{\hat{s}^2 + Q^4}{\hat{s} - Q^2}$$

$$(Q = m_{\ell^+\ell^-}, \hat{s} = m_{e^+e^-})$$

- Integrate over  $Q^2$  close to partonic threshold:  $s(1 - \delta) \leq Q^2 \leq s - Q_T^2$

$$\frac{d\hat{\sigma}_R}{dQ_{\perp}^2} = \hat{\sigma}_0 \frac{\alpha}{\pi} \frac{1}{Q_{\perp}^2} \left[ \log \frac{s}{Q_{\perp}^2} + \mathcal{O}(1) \right] \longrightarrow d\hat{\sigma}_R \approx \hat{\sigma}_0 \frac{\alpha}{\pi} \frac{dQ_{\perp}^2}{Q_{\perp}^2} \log \frac{s}{Q_{\perp}^2}$$

- Further, integrate out  $Q_T$  up to given  $p_T$ :  $\int d\hat{\sigma}_R = \hat{\sigma}_0 \frac{\alpha}{\pi} \int_0^{p_{\perp}^2} \frac{dQ_{\perp}^2}{Q_{\perp}^2} \log \frac{\hat{s}}{Q_{\perp}^2}$

# Sketch of DLLA

- As it stands we cannot analyse the behaviour as  $Q_T \rightarrow 0$ : problem caused by the usual collinear divergence.
- But we know that in a NLO calculation this divergence is cancelled by the virtual loop contribution at exactly  $Q_T=0$ 
  - result is then finite, giving an  $(\alpha/\pi)$  correction to  $\sigma_0$
- Dropping this term since it is not logarithm-enhanced, we thus have:

$$\hat{\sigma}_0 = \int (d\hat{\sigma}_R + d\hat{\sigma}_V) = \int_0^{p_\perp^2} (d\hat{\sigma}_R + d\hat{\sigma}_V) + \hat{\sigma}_0 \frac{\alpha}{\pi} \int_{p_\perp^2}^{\hat{s}} \frac{dQ_\perp^2}{Q_\perp^2} \log \frac{\hat{s}}{Q_\perp^2}$$

full result with correction dropped
what we wanted on previous slide (+ $\sigma_V$ )
an integral we can do!

- Rearrange and do the integral:

$$\int_0^{p_\perp^2} (d\hat{\sigma}_R + d\hat{\sigma}_V) = \sigma_0 \left( 1 - \frac{\alpha}{\pi} \int_{p_\perp^2}^{\hat{s}} \frac{dQ_\perp^2}{Q_\perp^2} \log \frac{\hat{s}}{Q_\perp^2} \right)$$

$$= \sigma_0 \left( 1 - \frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_\perp^2} \right)$$

This is the **double leading-log approximation (DLLA)** - all single log and constant terms have been dropped.

# Multiple emission

- Single photon contribution to cross section:  $\sigma_0 \left(1 + \epsilon^{(1)}\right) \equiv \sigma_0 \left(1 - \frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_{\perp}^2}\right)$
- Factorization in the soft limit leads to an (approximate) simple form for  $n$ -photon contribution:

$$\epsilon^{(n)} = \frac{1}{n!} \left( -\frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_{\perp}^2} \right)^n$$

- At this point straightforward to account for multiple photon emissions:

$$\Sigma(p_{\perp}^2) \equiv \sigma_0 \sum_{n=0}^{\infty} \epsilon^{(n)} = \sigma_0 \sum_{n=0}^{\infty} \frac{1}{n!} \left( -\frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_{\perp}^2} \right)^n = \sigma_0 \exp \left( -\frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_{\perp}^2} \right)$$

**Sudakov form factor:** no emissions harder than  $p_{\text{T}}$

- Recover differential distribution by taking derivative:

$$\frac{d\Sigma(p_{\perp}^2)}{dp_{\perp}^2} = \sigma_0 \frac{\alpha}{\pi} \left( \frac{1}{p_{\perp}^2} \log \frac{\hat{s}}{p_{\perp}^2} \right) \exp \left( -\frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_{\perp}^2} \right) \quad \begin{array}{l} \text{finite as } p_{\text{T}} \rightarrow 0 \\ \text{(tends to zero)} \end{array}$$

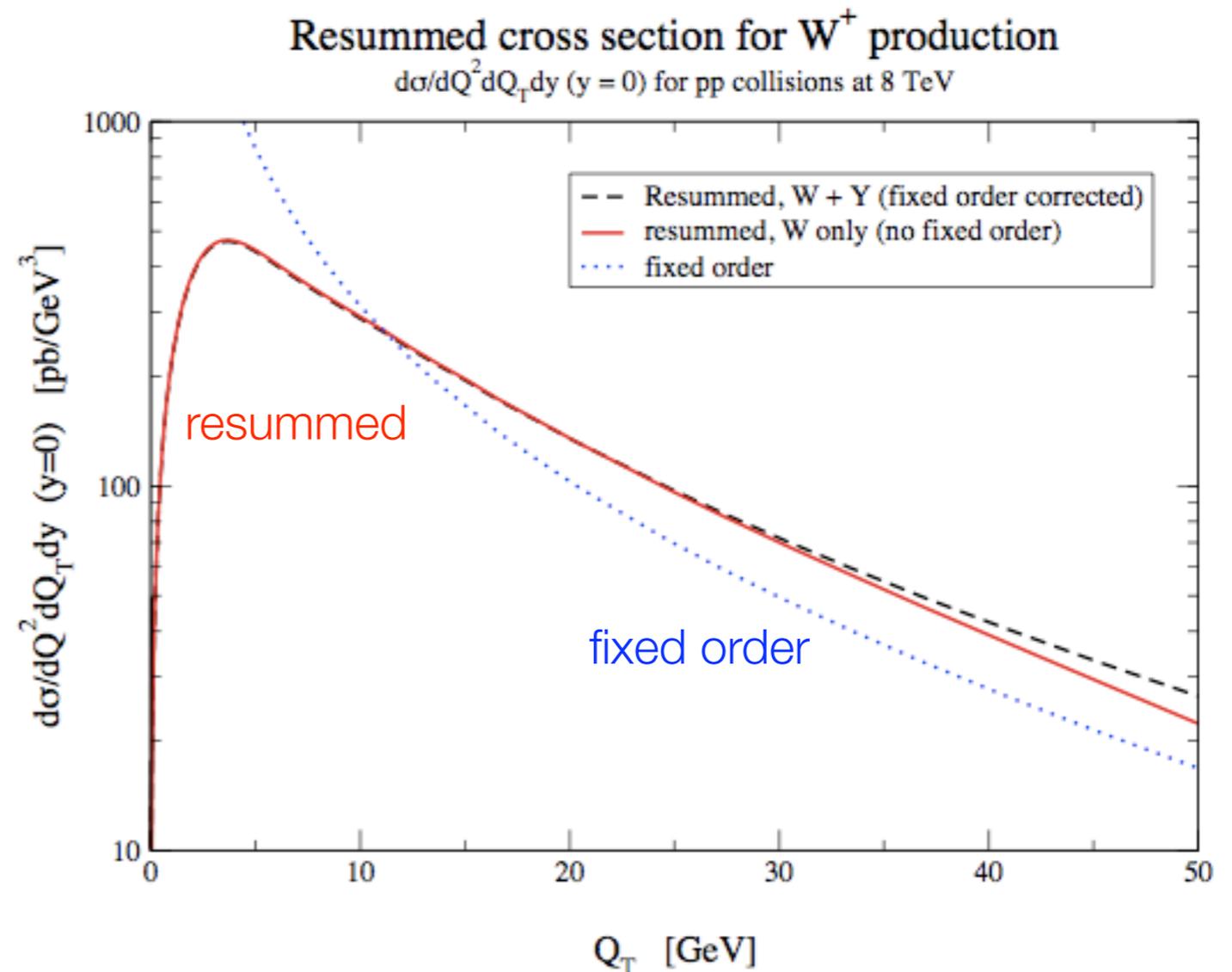
# Comments

- This is far from the end of the story:
  - although there is no divergence, the cross section is now too suppressed.
  - the behaviour is modified by sub-leading logarithms.
  - the treatment of multiple emission is also over-simplified, since emissions are all considered independent with no accounting for mom. conservation.
  - a proper treatment of this is beyond the scope of these lectures, but involves Fourier-transforming from momentum to **impact parameter** space.
- **Recipe to get back to QCD:**
  - remember effect of colour, so additional factor of  $C_F$
  - go from e.m. to strong coupling, remembering dependence on scale

$$\longrightarrow \exp\left(-\frac{\alpha_s(p_\perp^2)C_F}{2\pi} \log^2 \frac{\hat{s}}{p_\perp^2}\right) \quad (\text{additional complications for very small } p_\perp \text{ due to Landau pole})$$

# Resummation in action

- More complicated than presented here:
  - accounts for momentum conservation
  - matching onto fixed order form at high  $p_T$
  - “Collins-Soper-Sterman” resummation formalism, as implemented in **RESBOS** code.

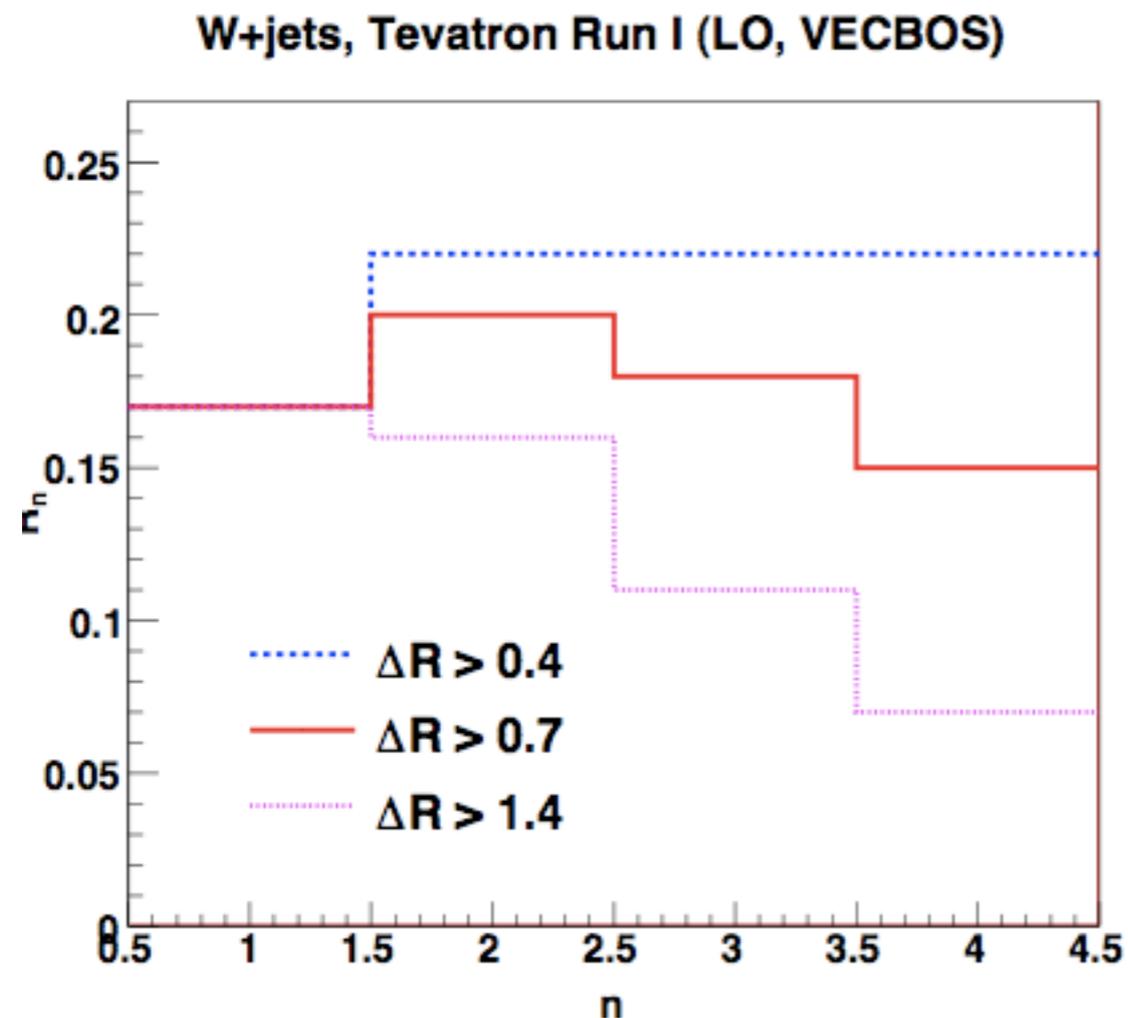


# W,Z + jets

- Go back to the higher order corrections we were considering before and now **categorise by the number of jets** in the final state, i.e. consider  $W+n$  jet,  $Z+n$  jet production.
- Motivation from both sides again:
  - final states with leptons, missing transverse momentum, jets
    - basic experimental signatures of New Physics, e.g. “MET+jets” SUSY
    - backgrounds to top production ( $W$ +jets) and Higgs studies
  - need to be understood to good precision
- At the forefront of developing theoretical tools on the “multiplicity frontier”
  - computation of amplitudes involving many jets, NLO corrections
  - systematic improvement of parton shower predictions - matching, merging and the inclusion of higher-order corrections

# LO predictions

- Instrumental in developing (very efficient) recursion relations for computing helicity amplitudes.
- **Berends-Giele** recursion implemented in VECBOS (1990).
  - first calculation of W+4 jets, leading background to top production.



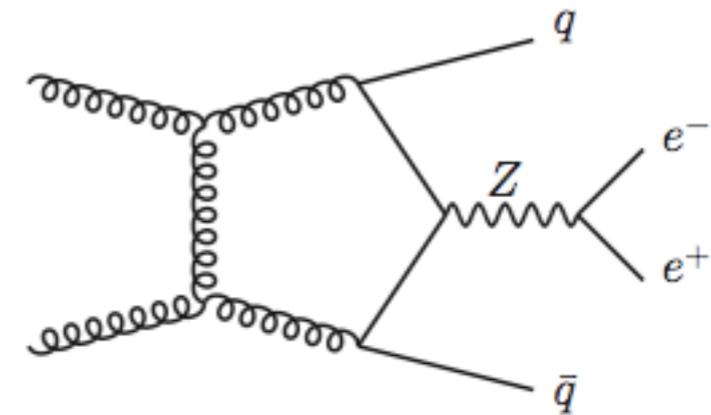
Similar recursive techniques now used in ALPGEN, SHERPA, Madgraph

Useful observation about the scaling of the cross section with additional jets:

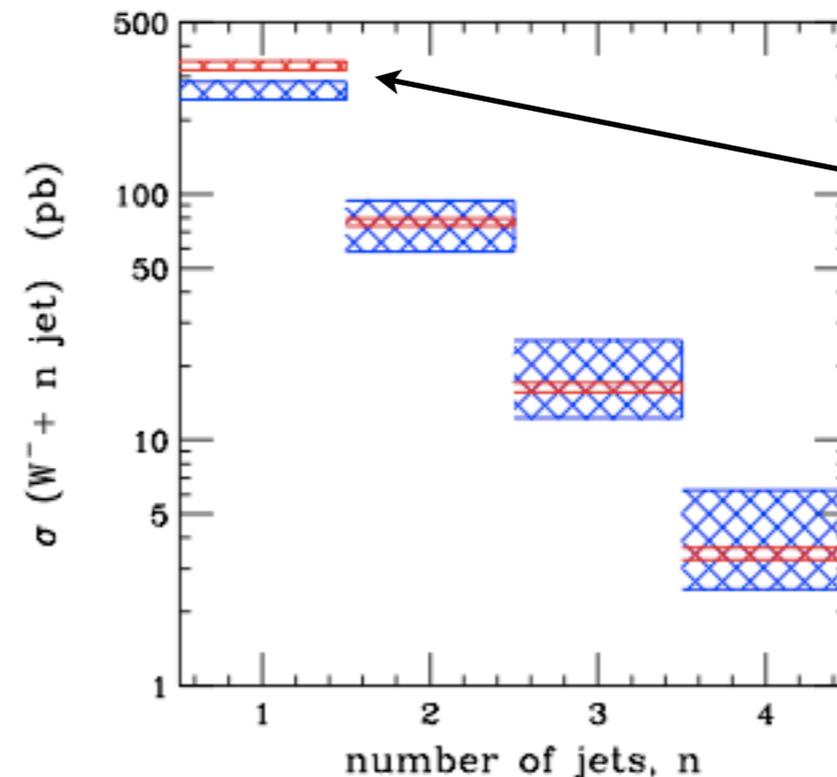
$$R_n = \frac{\sigma(W + n \text{ jets})}{\sigma(W + (n - 1) \text{ jets})}$$

# W/Z + jets at NLO

- Moving to jets (plural) requires evaluation of “pentagon” and higher-point loop integrals.
- V+2 jet case could be handled with usual technology but more than that required new methods.
- This inspired the rise of analytic and numerical **on-shell unitarity** techniques that form the basis of the loop calculations inside the latest theoretical tools
  - e.g. **BlackHat**, **GoSam** and **aMC@NLO**. (c.f. F. Krauss lectures)



- In the arena of V+jets, BlackHat+SHERPA provides predictions for for up to 5 additional jets.
- Scale-dependence of cross-sections reduced from **LO** to **NLO**.

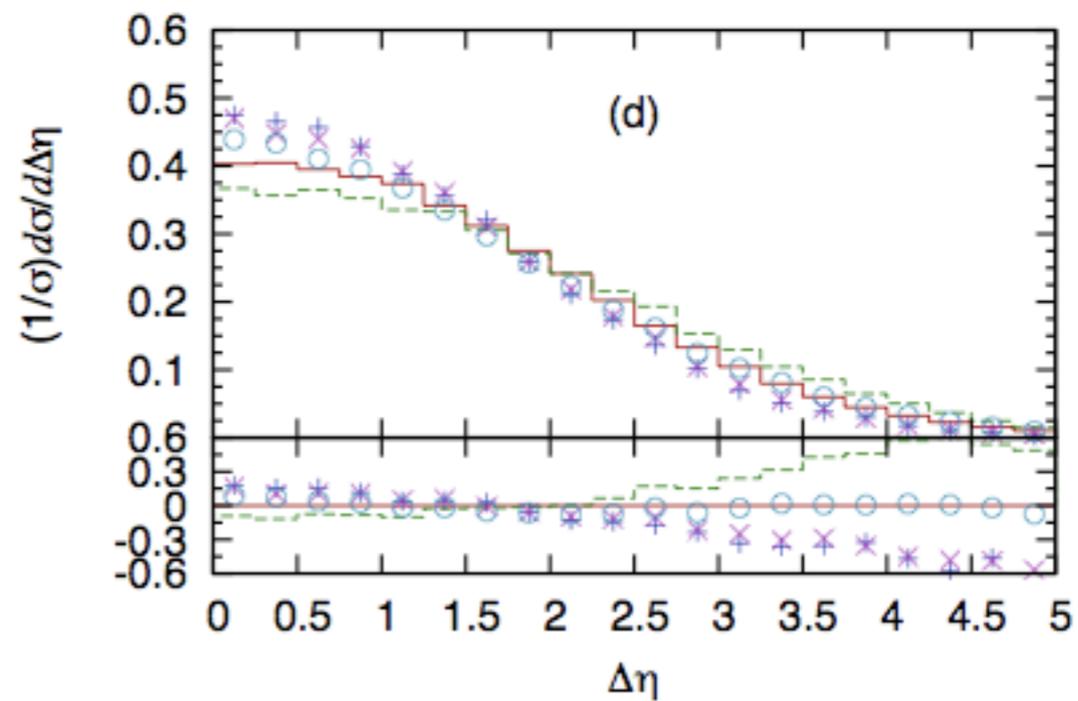
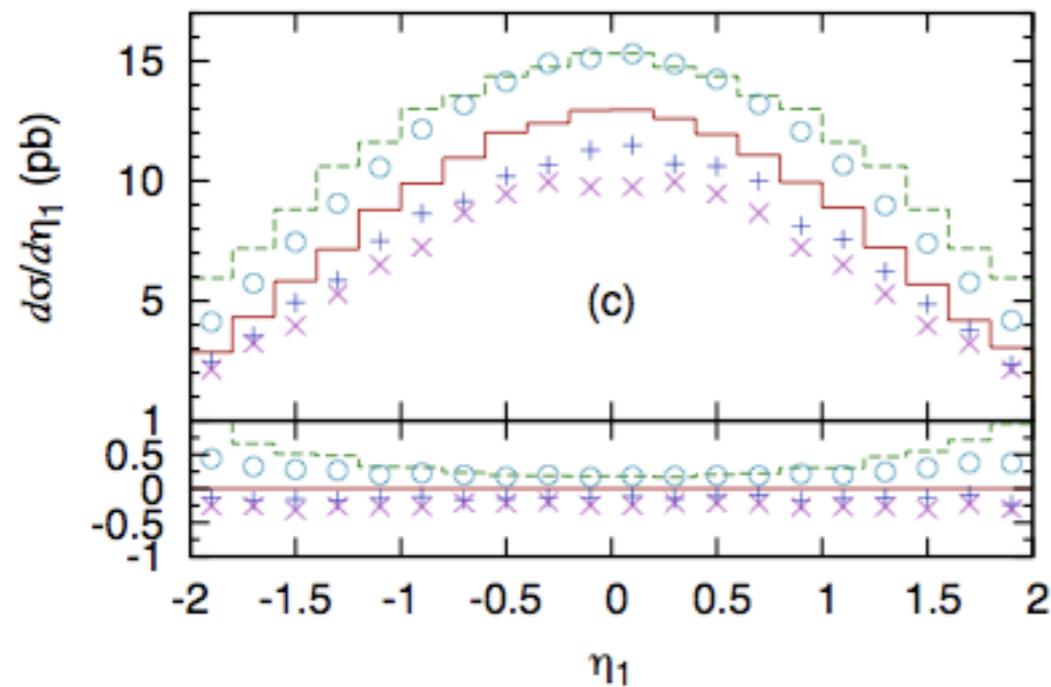
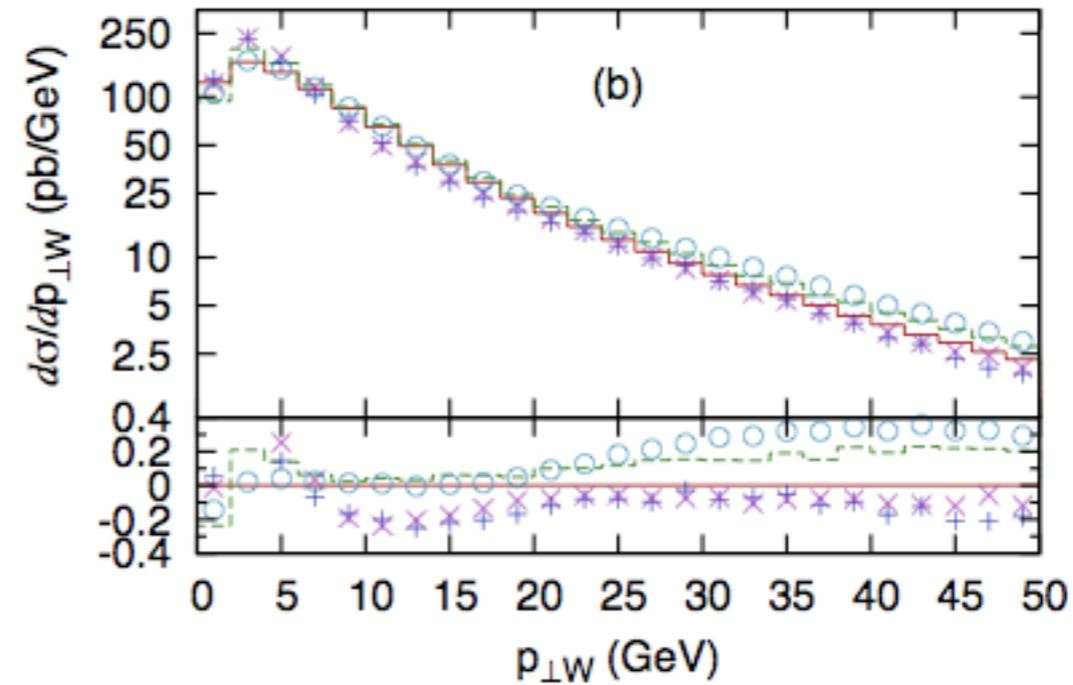
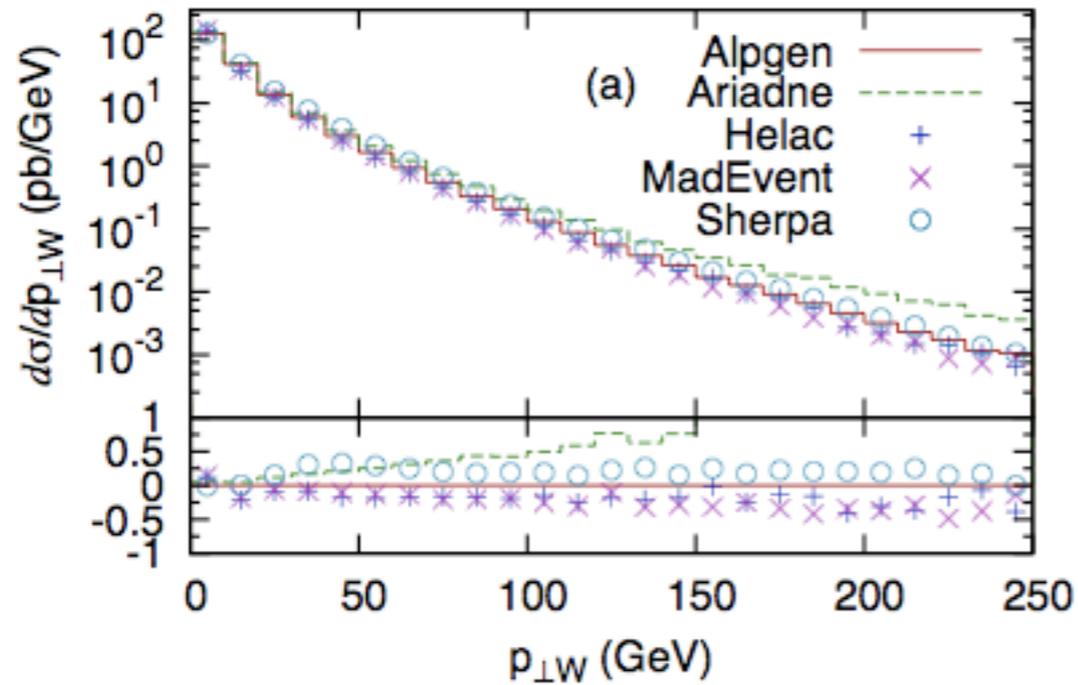


note unusual behaviour for V+1 jet; caused by inclusion of the effect of incident gluons for the first time at NLO

# Improved parton showers

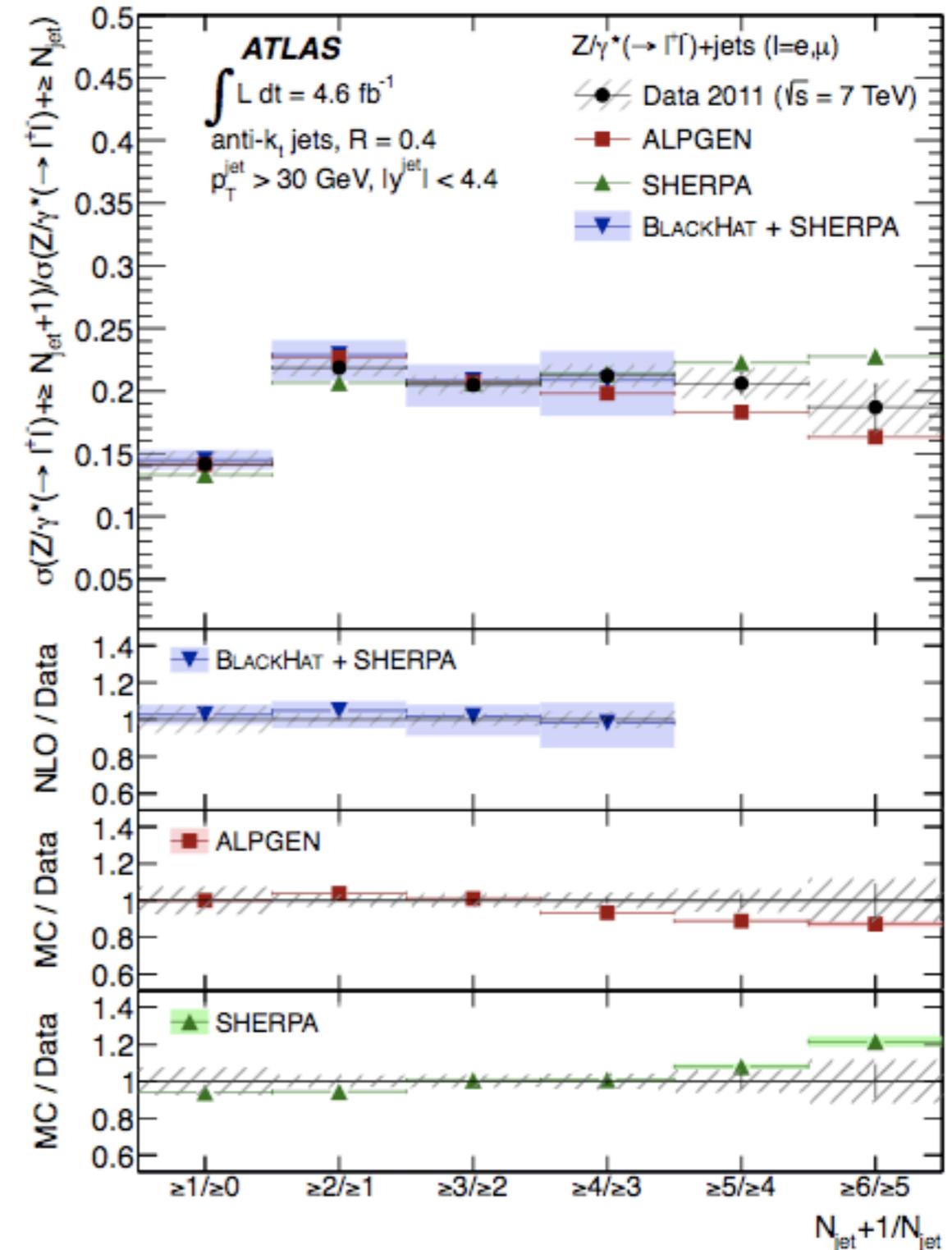
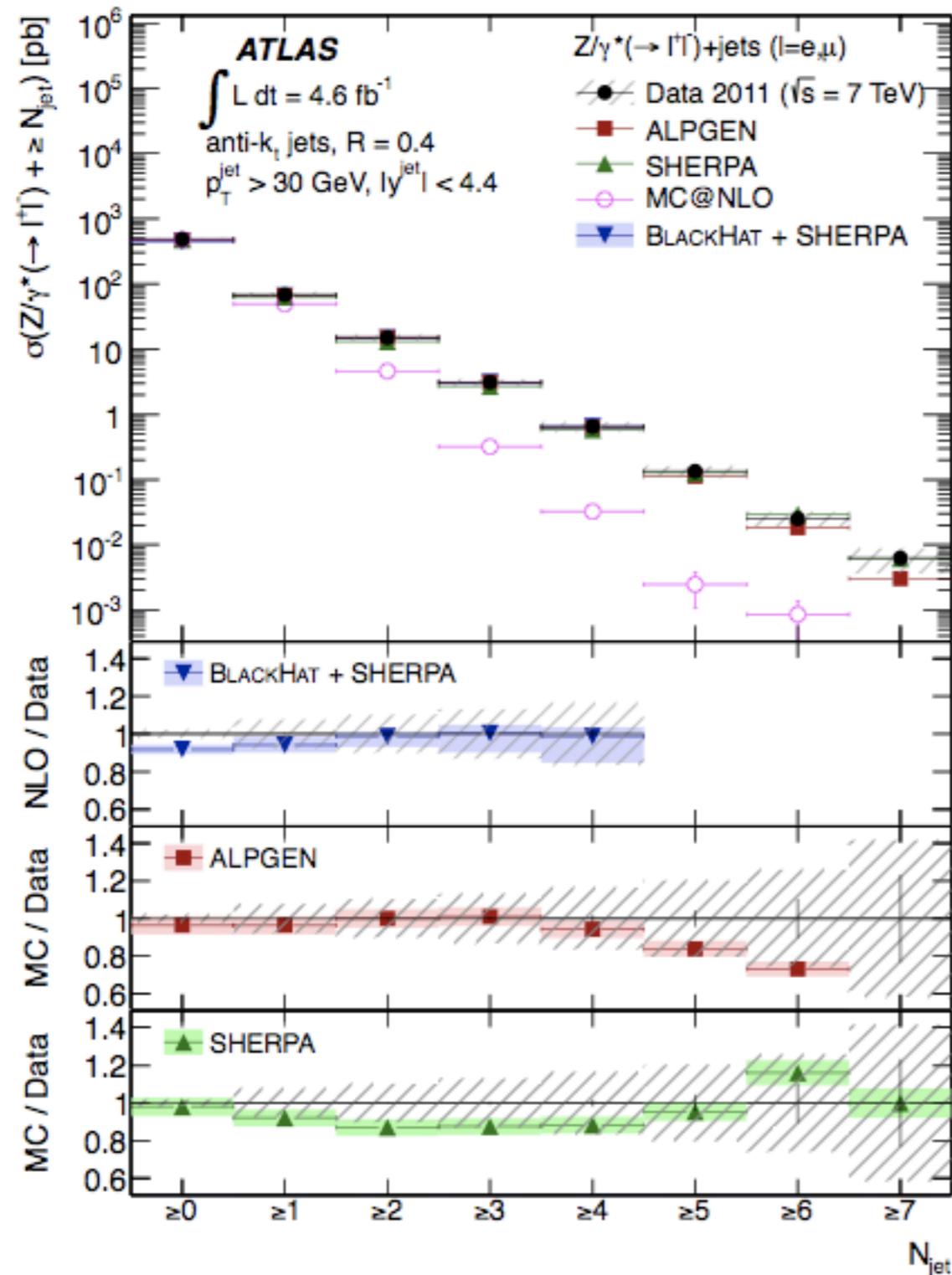
- Test bed for merging schemes at LO.

Alwall et al (2007)



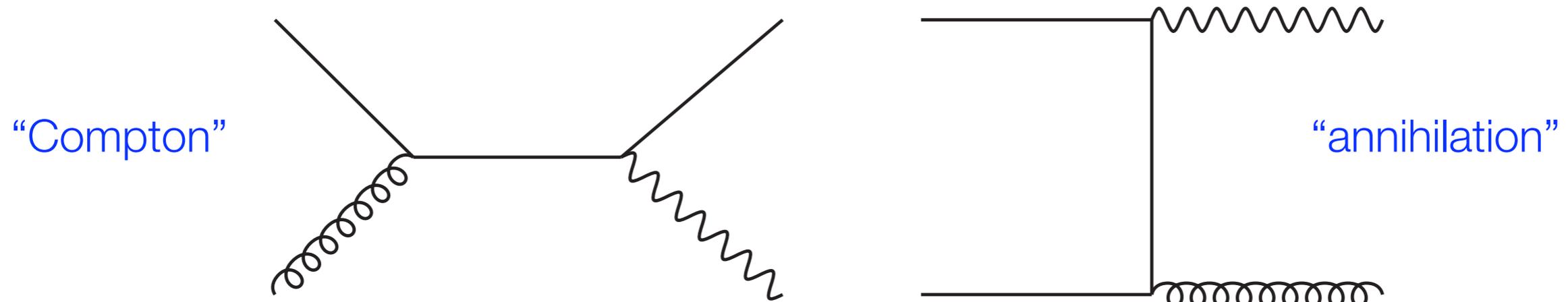
# Recent comparison with data

CERN-PH-EP-2013-023



# Direct photon production

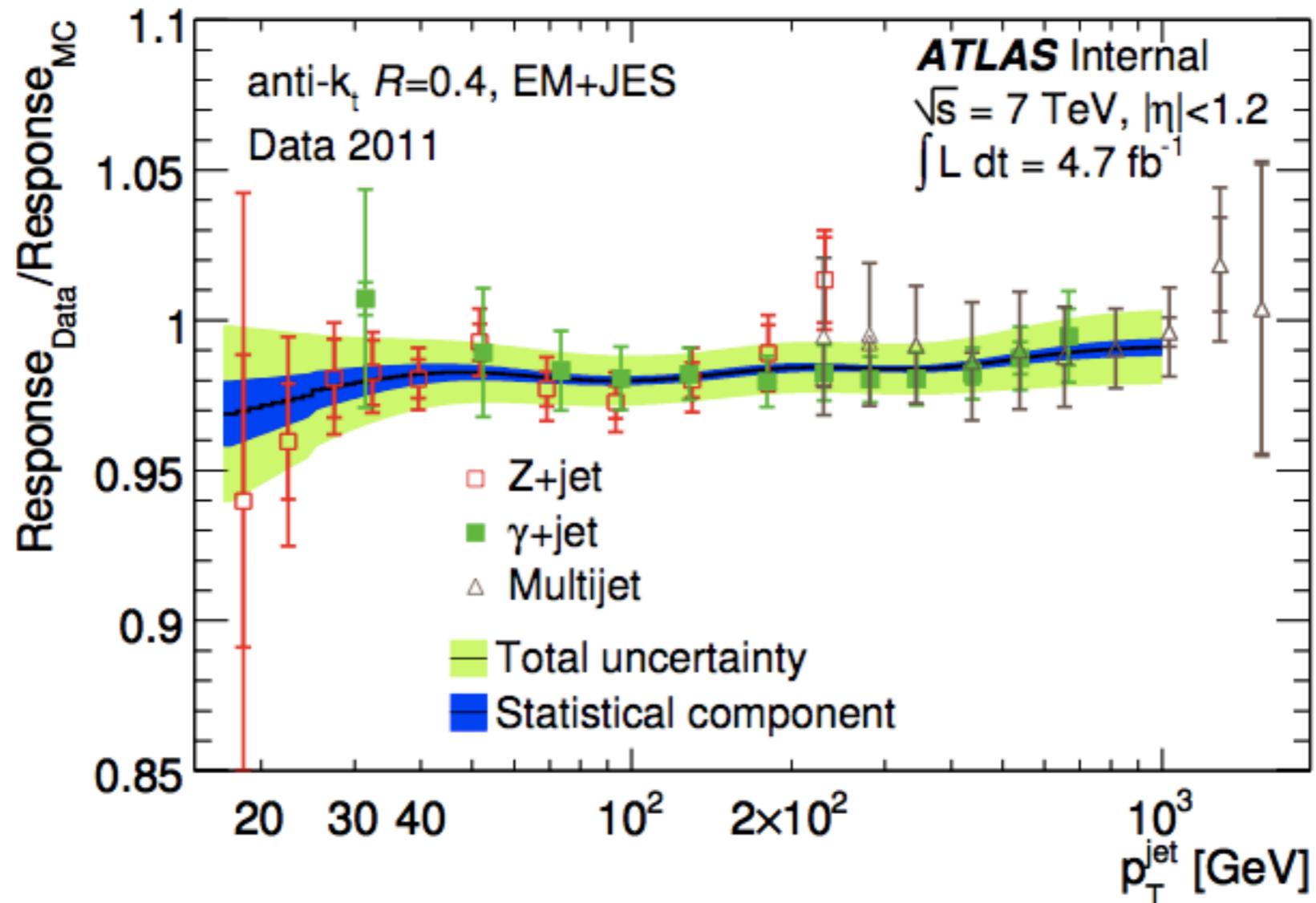
- Unlike  $W$  and  $Z$  production,  $2 \rightarrow 2$  process involving a jet, proceeding through quark- and gluon-initiated channels.



- Leading order kinematics:  $p_T(\text{photon}) = p_T(\text{jet})$ 
  - significance for calibrating detector performance -- well-measured photon probes response of hadronic calorimeters
  - used to measure jet energy scale (JES) and its uncertainty

# JES determination

- Recent ATLAS JES study: [ATLAS-CONF-2013-004](#)



- Photon+jet most important for  $100 < p_T(\text{jet}) < 600$  GeV.

# Amplitudes for photons and jets

- From theoretical point of view, much in common with jet production.
- For example, consider a helicity amplitude for the 2-jet (i.e. 4-parton) process:

$$0 \rightarrow \bar{q}^+(p_1) + q^-(p_2) + g^-(p_3) + g^+(p_4)$$

(this is a **MHV amplitude** - two partons of one helicity, remainder opposite)

- At leading order, amplitude has two color-ordered contributions

$$\mathcal{M}(\bar{q}_1^+, q_2^-, g_3^-, g_4^+) = ig^2 \left[ (T^{a_3} T^{a_4})_{i_2 i_1} M(\bar{q}_1^+, q_2^-, g_3^-, g_4^+) + (T^{a_4} T^{a_3})_{i_2 i_1} M(\bar{q}_1^+, q_2^-, g_4^+, g_3^-) \right]$$

that can be written as simple expressions in terms of **spinor products**:

$$M(\bar{q}_1^+, q_2^-, g_3^-, g_4^+) = \frac{\langle 13 \rangle \langle 23 \rangle^3}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle},$$

$$\langle ij \rangle = \langle i- | j+ \rangle = \bar{u}_-(p_i) u_+(p_j)$$

$$[ij] = \langle i+ | j- \rangle = \bar{u}_+(p_i) u_-(p_j)$$

$$M(\bar{q}_1^+, q_2^-, g_4^+, g_3^-) = -\frac{\langle 13 \rangle \langle 23 \rangle^3}{\langle 12 \rangle \langle 24 \rangle \langle 34 \rangle \langle 31 \rangle}$$

$$\langle ij \rangle \sim \sqrt{s_{ij}} \quad (\text{up to a phase})$$

- Denominators signal soft and collinear divergences
  - recognise color ordering from  $\langle 23 \rangle \langle 41 \rangle$  and  $\langle 24 \rangle \langle 31 \rangle$
  - $\langle 34 \rangle$  is remnant of triple-gluon vertex propagator

# Direct photon amplitude

- **Simple prescription to obtain photon amplitudes:**
  - replace corresponding color matrix in decomposition with identity matrix
  - change overall coupling

$$\begin{aligned} \mathcal{M}(\bar{q}_1^+, q_2^-, \gamma_3^-, g_4^+) &= ieQ_q g (T^{a_4})_{i_2 i_1} [M(\bar{q}_1^+, q_2^-, g_3^-, g_4^+) + M(\bar{q}_1^+, q_2^-, g_4^+, g_3^-)] \\ &\equiv ieQ_q g (T^{a_4})_{i_2 i_1} M(\bar{q}_1^+, q_2^-, \gamma_3^-, g_4^+) , \end{aligned}$$

- Performing combination analytically is useful:

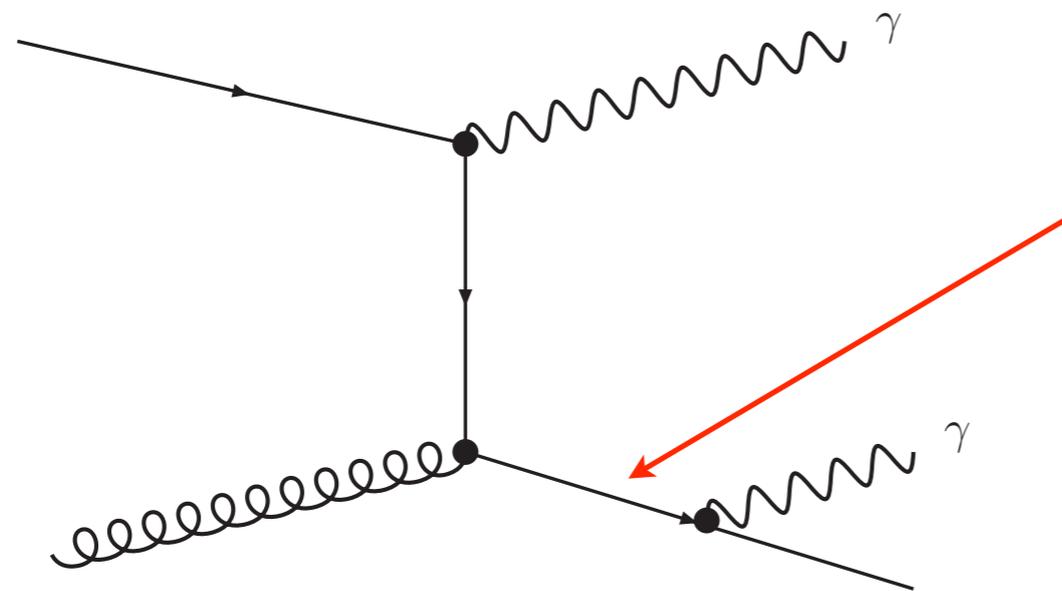
$$\begin{aligned} M(\bar{q}_1^+, q_2^-, \gamma_3^-, g_4^+) &= \frac{\langle 13 \rangle \langle 23 \rangle^3}{\langle 12 \rangle \langle 23 \rangle \langle 24 \rangle \langle 34 \rangle \langle 31 \rangle \langle 41 \rangle} (\langle 24 \rangle \langle 31 \rangle - \langle 23 \rangle \langle 41 \rangle) \\ &= \frac{\langle 13 \rangle \langle 23 \rangle^3}{\langle 23 \rangle \langle 24 \rangle \langle 31 \rangle \langle 41 \rangle} , \end{aligned}$$

Schouten identity:  
 $\langle ab \rangle \langle cd \rangle = \langle ac \rangle \langle bd \rangle + \langle ad \rangle \langle bc \rangle$

- As it must, remnant of triple-gluon propagator cancels.
- Form of amplitude identical for 2-photon process.
- Very useful for recycling complicated amplitudes with more jets.

# Photons in perturbative QCD

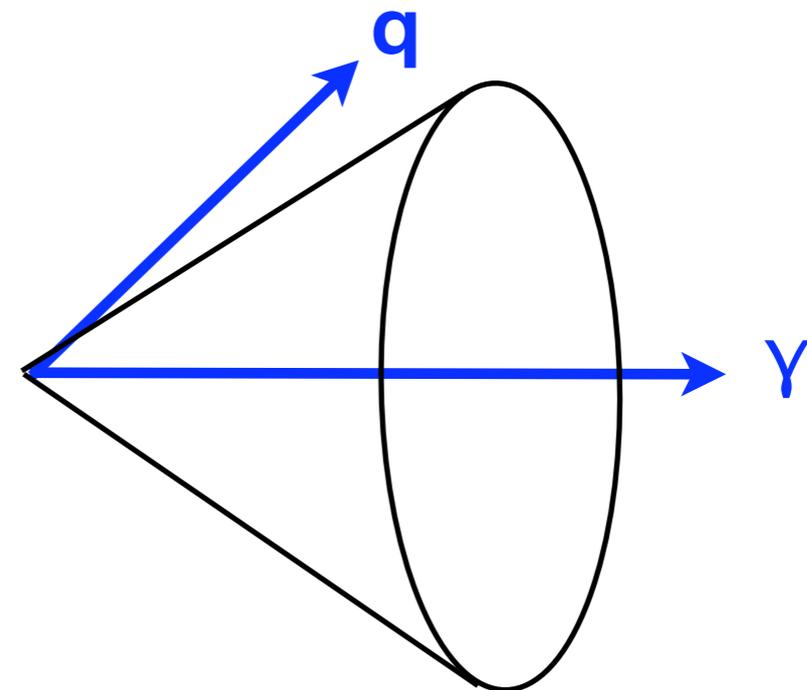
- In the presence of QCD radiation (i.e. beyond LO) cross sections with photons develop additional singularities.



singular propagator  
when quark and  
photon are collinear

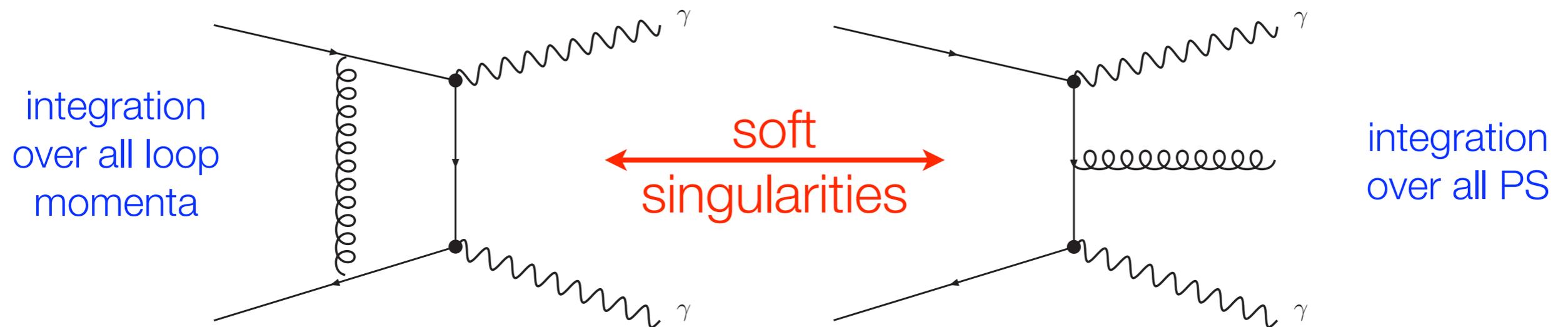
- Naive solution: remove collinear configurations with a cut.

no radiation in  
“isolation cone”



# Cone problems

- Removing quark-photon singularities in this way would be acceptable (but only up to NLO, no all-orders definition like this).
- However, a physically meaningful prediction would also require the same cut on gluons.
- Enforcing such a cut would prohibit the emission of soft gluons inside the cone and be infrared-unsafe: cancellation of virtual/real singularities not complete.



# Theorist solution

- **Frixione (1998)**: allow soft partons, but remove collinear configurations.
- Enforced by a cut of the form:

$$\sum_{R_{j\gamma} \in R_0} E_T(\text{had}) < \epsilon_h p_T^\gamma \left( \frac{1 - \cos R_{j\gamma}}{1 - \cos R_0} \right)$$

- Parton required to be softer as it gets closer to photon.
- No contribution exactly at the collinear singularity.
- This is simple to apply to a theoretical calculation and results in a well-defined cross section.
  - with such a cut, higher order calculations with photons no more difficult than corresponding QCD ones
- Cannot be (exactly) implemented experimentally due to finite detector resolution.
  - must tweak parameters of the cut ( $\epsilon_h$ ,  $R_0$ ) for good agreement with experimental data (ideally, universally)

# Conventional approach

- Usually, isolation cone allows a small amount of hadronic energy inside.

$$\sum_{\in R_0} E_T(\text{had}) < \epsilon_h p_T^\gamma \quad \text{or} \quad \sum_{\in R_0} E_T(\text{had}) < E_T^{\text{max}}$$

- Okay for QCD infrared-safety, but collinear quark-photon singularity again exposed.
- Singularities can be handled by usual higher-order machinery, e.g. dipole subtraction, and exposed:

$$-\frac{1}{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{M_F^2} \right) \frac{\alpha}{2\pi} e_q^2 P_{\gamma q}(z)$$

- Just like initial-state collinear singularities are absorbed into pdfs, these can be defined away.

# Photon fragmentation

- The analogous quantity to pdfs is the **photon fragmentation function**: defined for each flavour of parton.

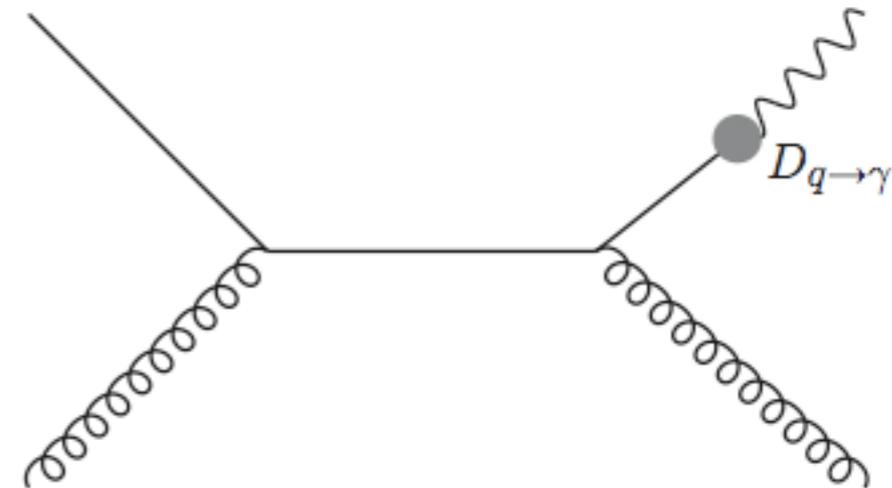
- Inclusion of fragmentation function introduces an additional scale to the problem: **fragmentation scale,  $M_F$** .

- Just like pdfs: non-perturbative input required, but perturbative evolution. Defined order-by-order in pQCD.

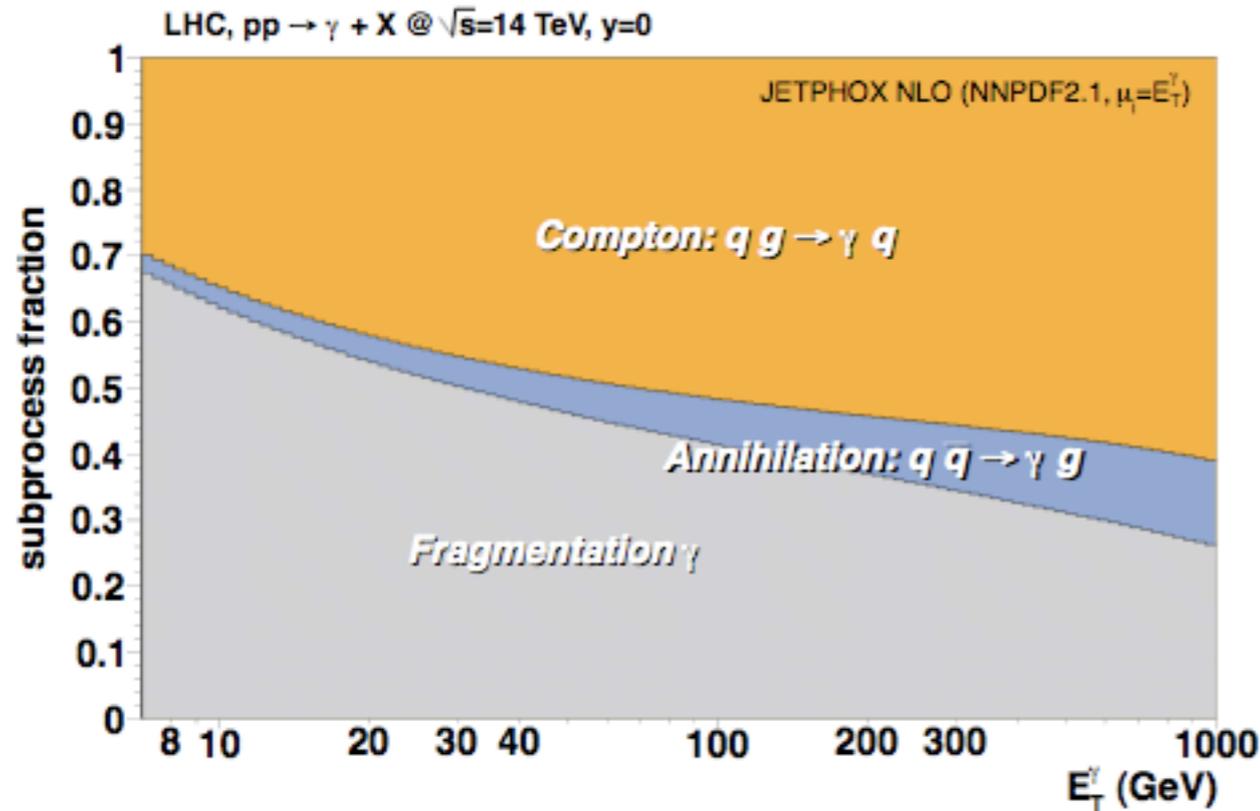
- Using conventional isolation, cross-section now has two components:

$$d\sigma = \underbrace{d\sigma_{\gamma+X}(M_F)}_{\text{direct/prompt}} + \sum_i \underbrace{d\sigma_{i+X} \otimes D_{i \rightarrow \gamma}(M_F)}_{\text{fragmentation}}$$

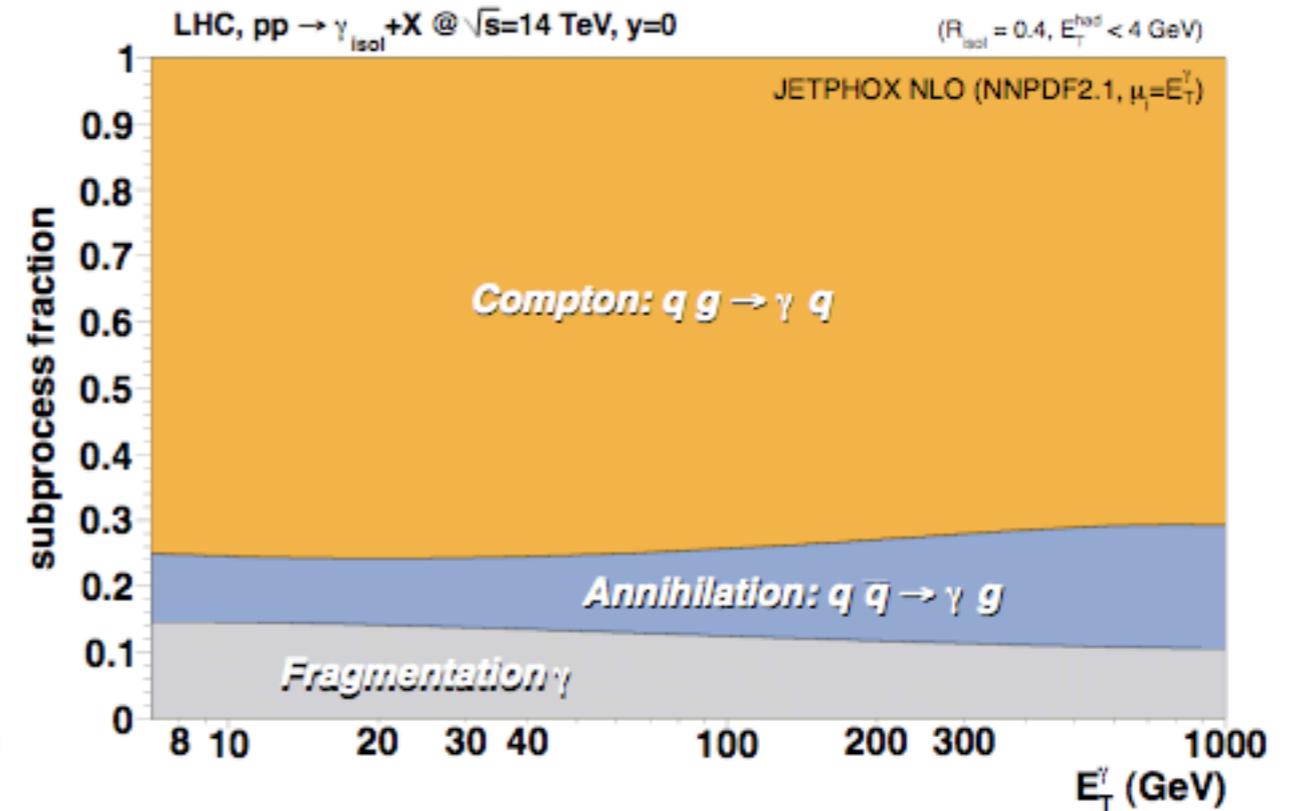
- separation well-defined only for a given  $M_F$ .
- After isolation, the finite remainder from the fragmentation contribution is typically small.



# Size of contributions



inclusive

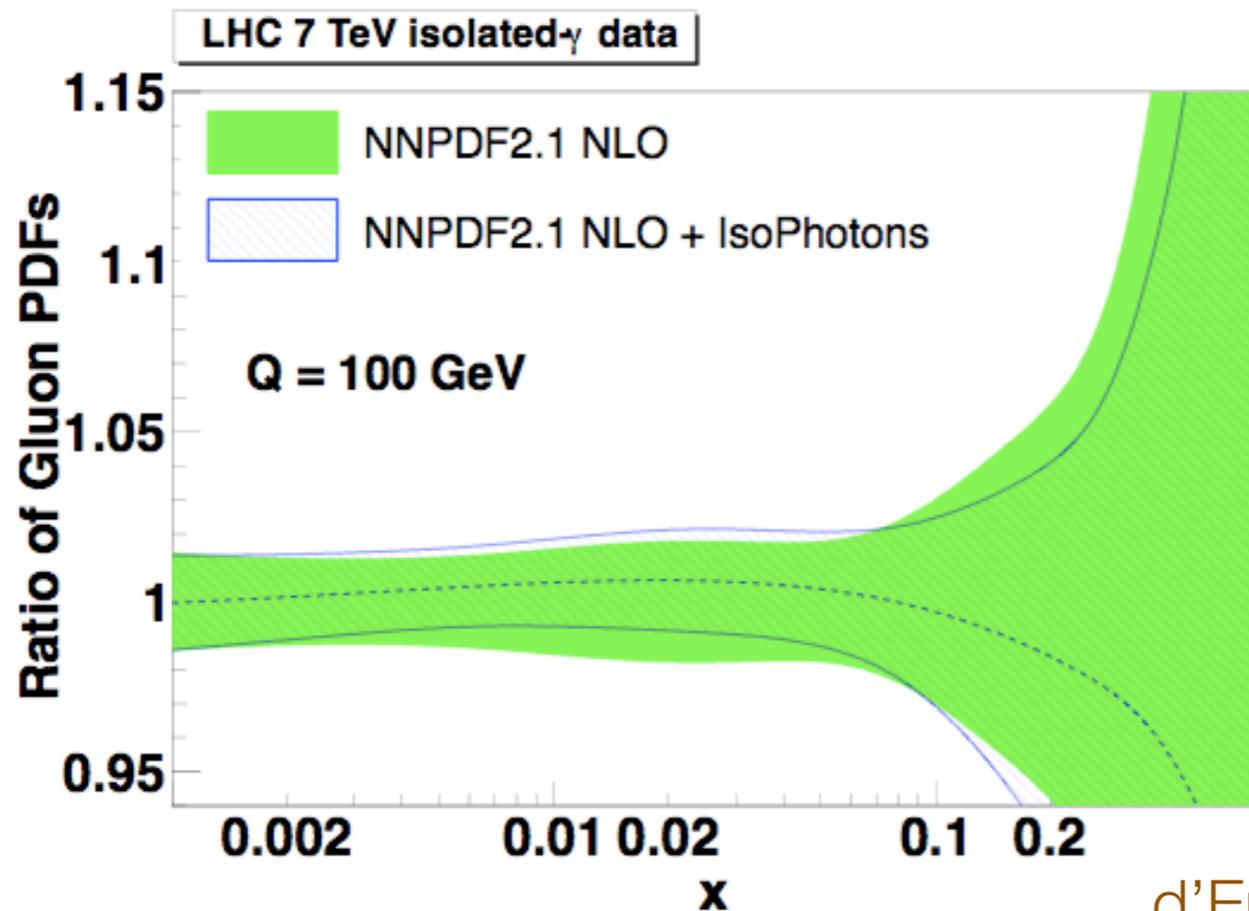


isolated photon

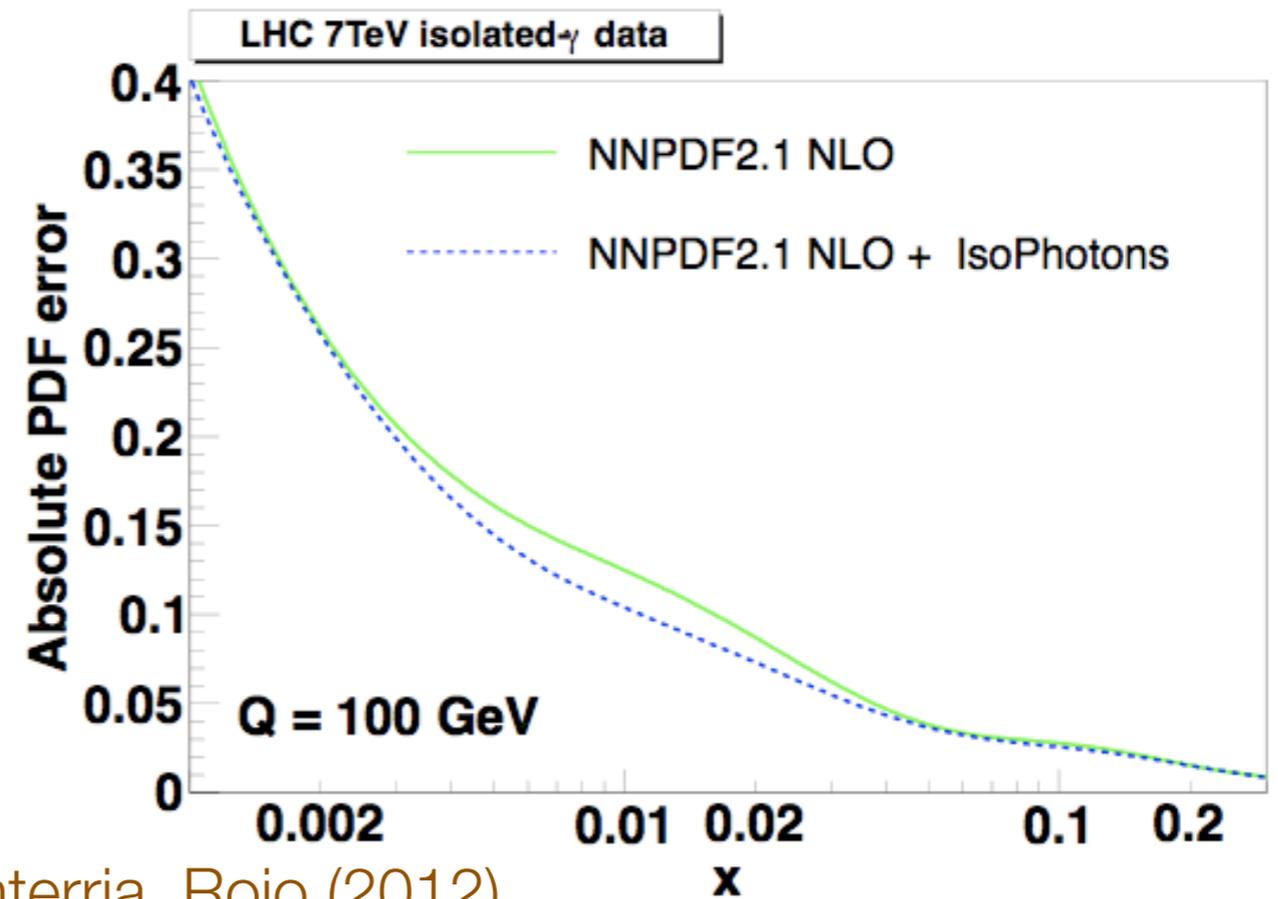
- Direct photon production at NLO in MCFM and **JETPHOX** (shown here).
- In the inclusive case fragmentation contribution is large, even at high  $p_T$ .
- After isolation, both fragmentation and annihilation contributions small.
  - this process is therefore dominated by the Compton mode and thus can potentially provide a useful probe of gluon pdf.

# Pdf improvements from direct photons

- Study using NNPDF with up to 7 TeV LHC data only
  - shows slight improvement in gluon uncertainty
  - potential for improvement with more data, subject to some limitations: only NLO (NNLO becoming the standard), non-perturbative corrections need to be better-understood.

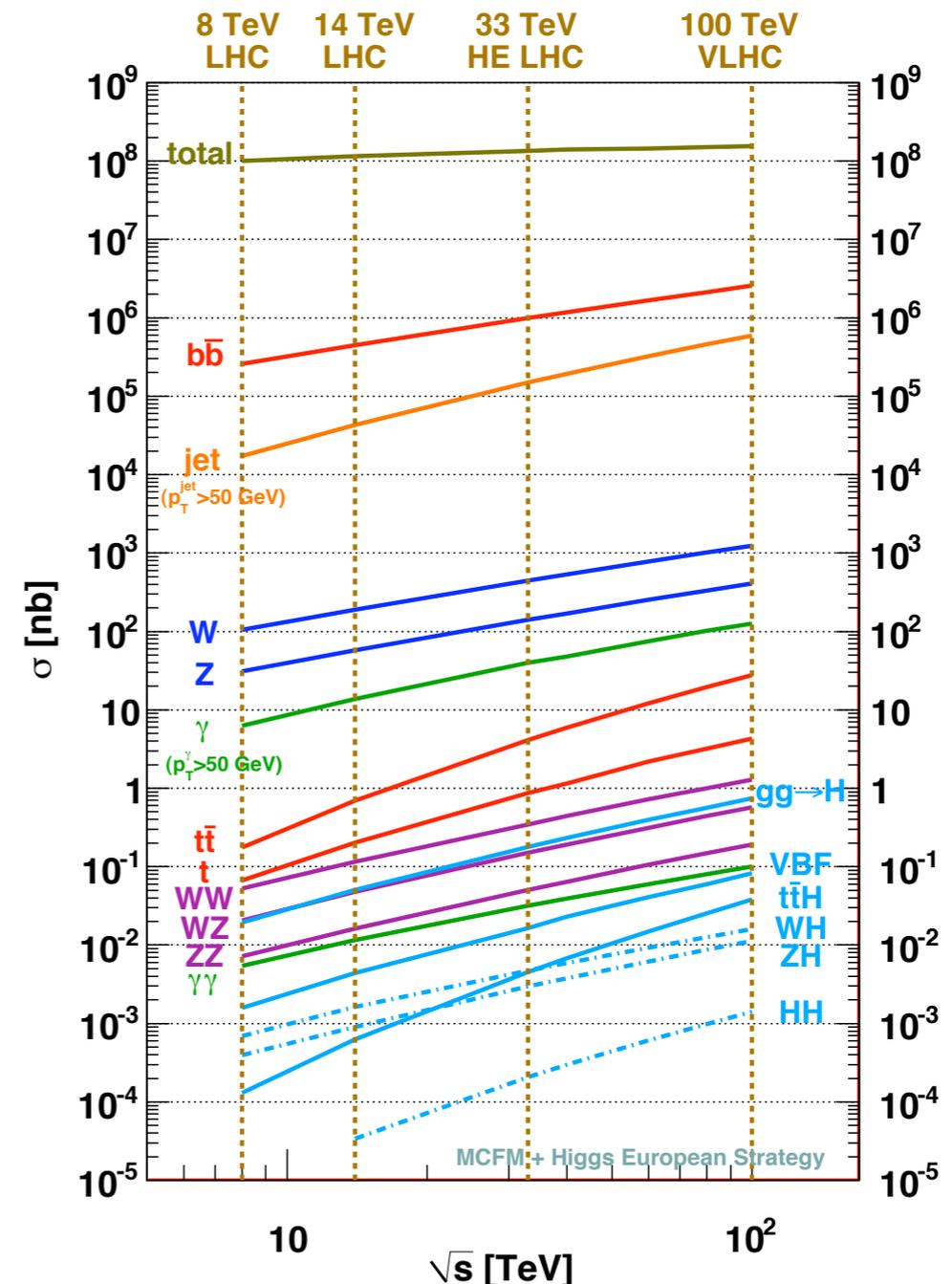


d'Enterria, Rojo (2012)



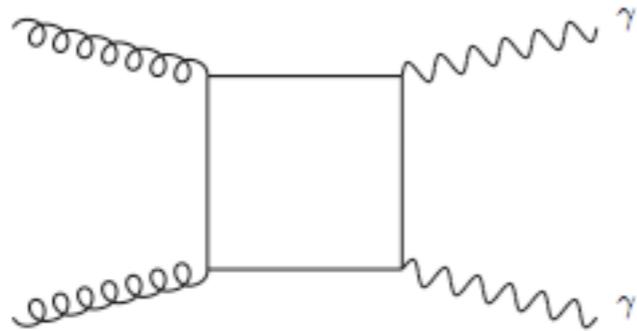
# Diphoton production

- Clear interest as the principal background to the Higgs process,  $gg \rightarrow H \rightarrow \gamma\gamma$ .
  - even though the background is subtracted with a fitting procedure, it would be nice to have some control of this process ab initio.
- Experimentally, significant contamination of this partonic process from the production of jets, or photon+jet, where jets are mis-identified as photons.
  - the cross-sections for these strong processes are so much larger that mis-identification rates as small as  $10^{-4}$  must be handled with care.
- Here, just focus on a few aspects of the basic process:  $q\bar{q} \rightarrow \gamma\gamma$



# Higher order corrections

- NLO corrections included in **DIPHOX** and **MC2FM**.
- A particular class of NNLO contributions is separately gauge-invariant and numerically important at the LHC due to the large gluon flux:



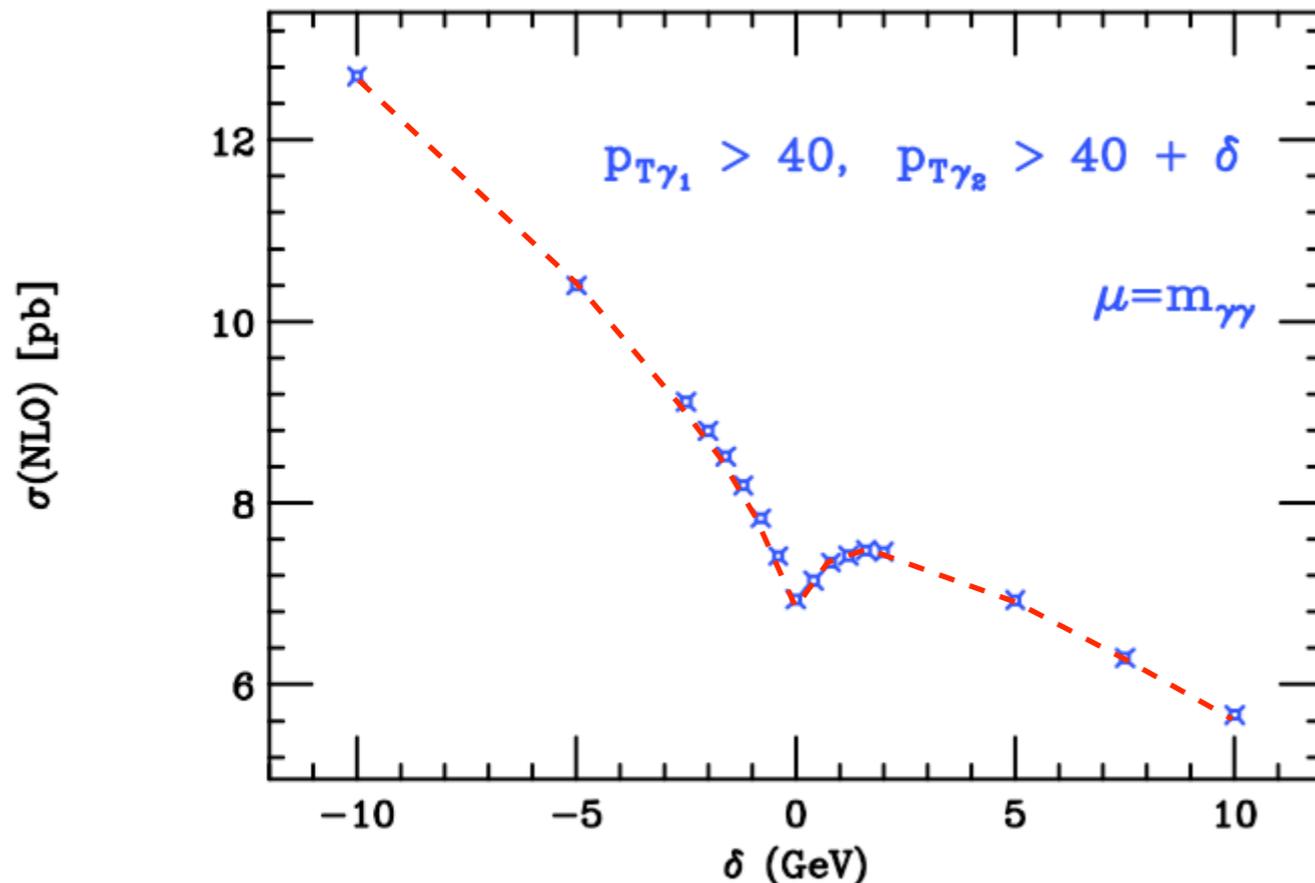
Since there is no tree level  $gg\gamma\gamma$  coupling, **this loop contribution is finite**  
→ can add separately.

(in fact, finite nature means that one can compute corrections to it, i.e. part of  $N^3LO$ , using just NLO technology)

- Contributes approximately 15-25% of the NLO total, depending on exact choice of photon cuts, scale choice, etc.
- Interesting behaviour of perturbative calculation in the case of photon cuts favoured by the experiment - “staggered”  $p_T$  cuts where second photon not required to be as hard as first
  - useful for purity of the signal or rejection of fake backgrounds

# Staggered photon cuts

- Consider typical cuts of the form:  $p_T^{\gamma_1} > 40 \text{ GeV}$ ,  $p_T^{\gamma_2} > 40 + \delta \text{ GeV}$
- Causes a problem because in the perturbative calculation photons are produced back-to-back with equal  $p_T$ 
  - sensitivity to staggered cut only begins at NLO.



Rather sensitive to value of  $\delta$ , NLO correction becomes very large if cuts are too far apart.

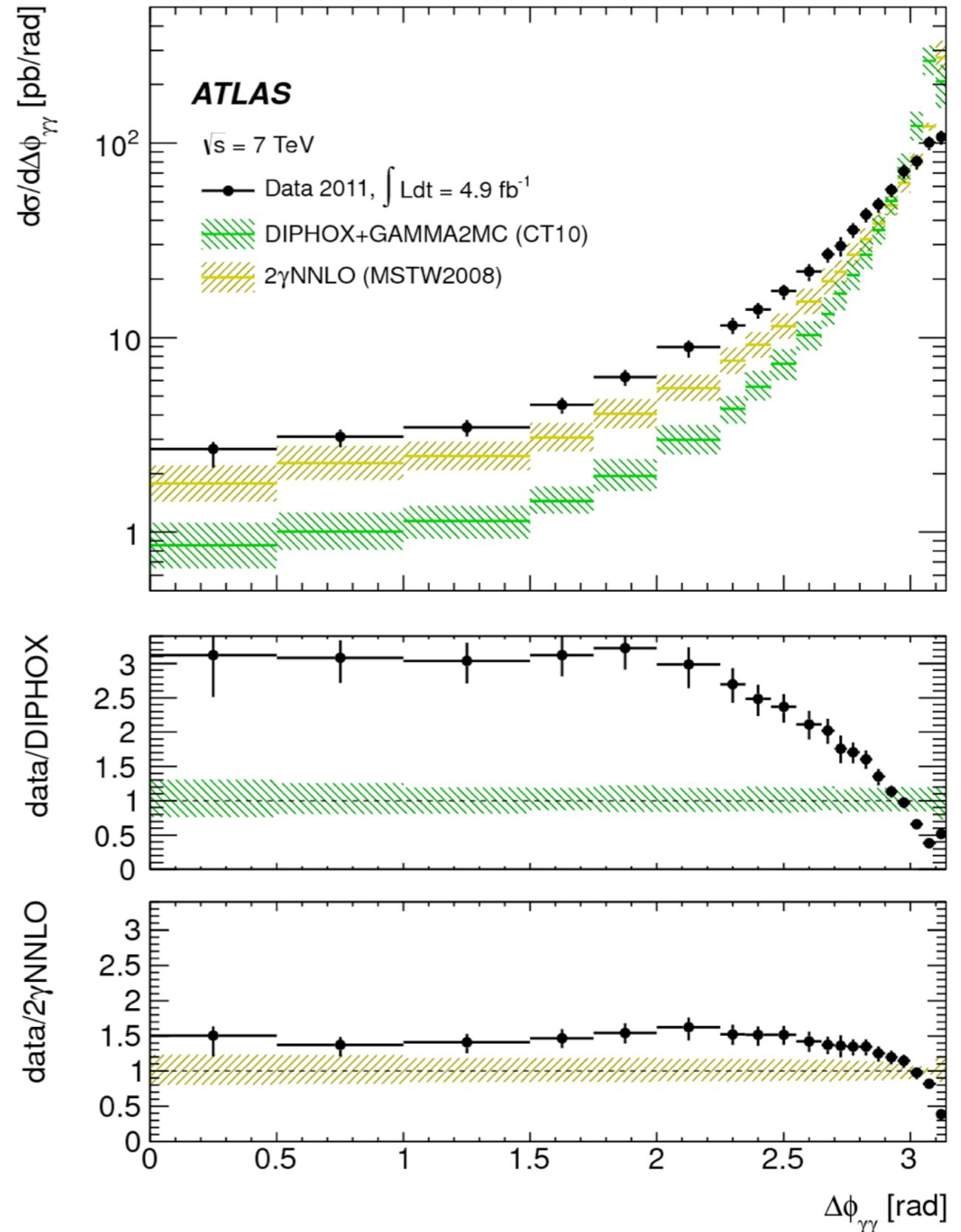
Cusp at  $\delta=0$  due to emission of soft gluons and presence of  $\delta \log \delta$  enhancement (candidate for resummation).

Frixione, Ridolfi (1997)

Lesson: perturbative stability in the threshold region requires “moderate”  $\delta$ .

# NNLO results

- A full NNLO calculation has recently been performed, in the “Frixione” scheme, i.e. no need for fragmentation contributions.  
Catani et al (2012)
- Better description of kinematic regions that are poorly described or inaccessible at NLO.
- Good example: azimuthal angle between photons only non-trivial at NLO in the total cross-section.
- Even better description would require either higher orders or inclusion in parton shower → not yet feasible.



# Summary

- **Overview of vector boson basics.**
  - importance, Feynman rules, decays
- **Underlying theory of W,Z production.**
  - NNLO QCD, NLO EW, DLLA, resummation, W/Z+jets
- **Discussion of the direct photon process.**
  - isolation, fragmentation, sensitivity to pdfs
- **Di-photon production.**
  - subtleties of higher orders in pQCD.

# Vector bosons and direct photons

## *Lecture 2*

CTEQ  
school  
2013



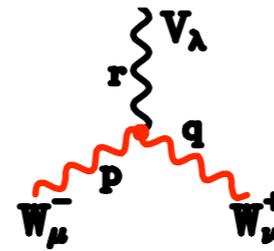
John Campbell, Fermilab

# Outline of lectures

- Overview of vector boson basics.
- Underlying theory of W,Z production.
- Discussion of the direct photon process.
- Di-photon production.
  
- The importance of multi-boson production.
- Review of selected di-boson phenomenology.
- Beyond inclusive di-boson measurements.

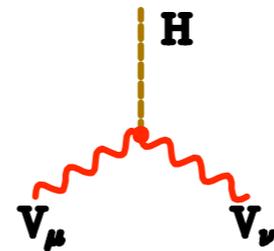
# Weak boson self-interactions

- Now turn to multiple production of vector bosons, with at least one W or Z.
- These have an essentially different character from diphoton production because of **self-interactions**.
- Probes of triple couplings:
  - Higgs production by VBF
  - decays into W and Z pairs
  - **di-boson production**
- Probes of quartic couplings:
  - tri-boson production (and beyond)
  - di-boson production through VBF
- Rich structure predicted by the SM Lagrangian to explicitly test.



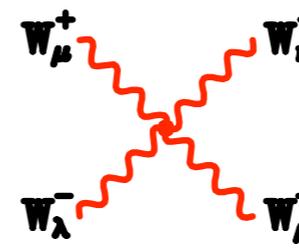
$$+ig_V [(p-q)_\lambda g_{\mu\nu} + (q-r)_\mu g_{\nu\lambda} + (r-p)_\nu g_{\lambda\mu}]$$

(all momenta incoming,  
 $g_\lambda = e, g_\tau = g_V \cos\theta_V$ )

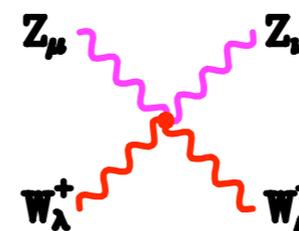


$$+ig_{VH} M_V g_{\mu\nu}$$

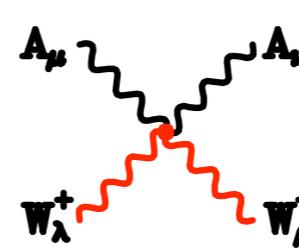
( $g_{W^\pm} = g_V, g_Z = g_V / \cos^2\theta_V$ )



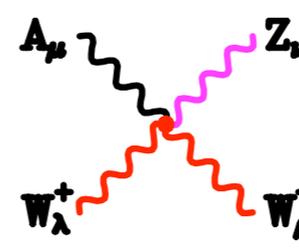
$$+ig_V^2 [2g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}]$$



$$-ig_V^2 \cos^2\theta_V [2g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}]$$



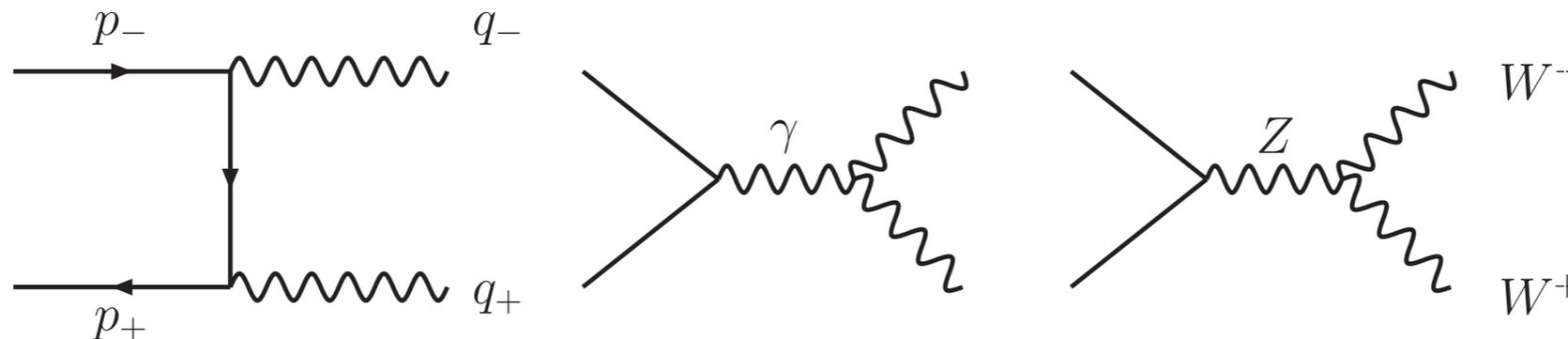
$$-ie^2 [2g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}]$$



$$-ieg_V \cos\theta_V [2g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}]$$

# The special role of self-interactions

- To illuminate the special role self-interactions play, consider the reaction  $e^+e^- \rightarrow W^+W^-$  at a lepton collider.



- Choose frame in which the  $W$  3-momenta are in the  $z$ -direction:
- Polarization vectors of  $W$  ( $\epsilon \cdot q = 0$ ,  $\epsilon^2 = -1$ ):

$$q_{\pm} = (E, 0, 0, \pm q)$$

$$E^2 - q^2 = m_W^2$$

$$\epsilon^{\mu} = (0, 1, 0, 0), \quad \epsilon^{\mu} = (0, 0, 1, 0)$$

transverse (c.f. photon)

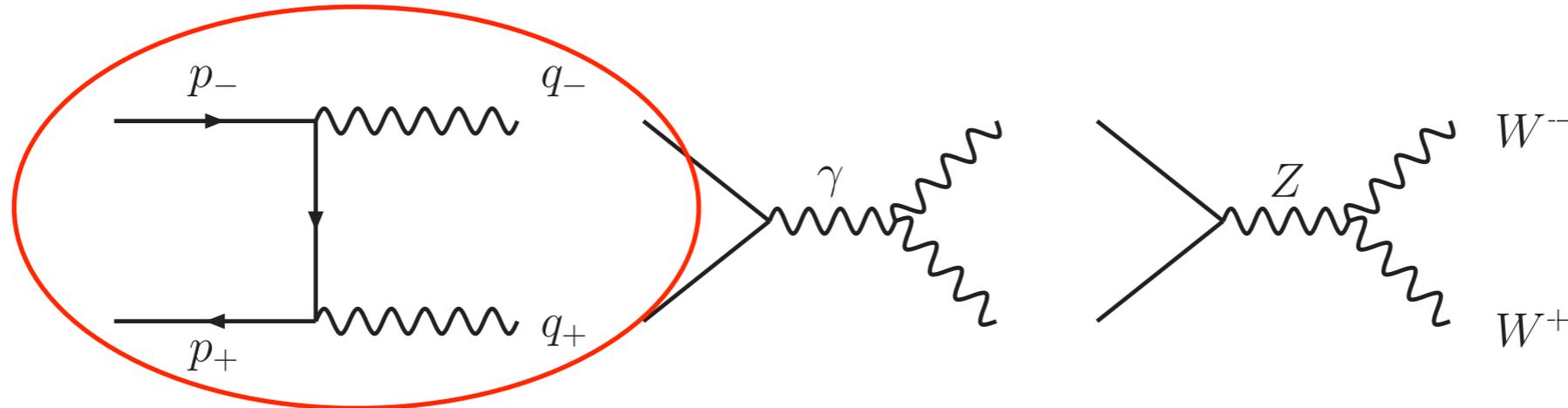
$$\epsilon_{\pm}^{\mu} = \frac{1}{m_W} (q, 0, 0, \pm E)$$

longitudinal (massive bosons)

- Longitudinal mode means diagrams grow as  $E^2 \rightarrow$  focus on this limit.

- in that case can study longitudinal modes by approximating  $\epsilon_{\pm}^{\mu} \rightarrow \frac{1}{m_W} q_{\pm}$

# Longitudinal contribution



- Contribution from first diagram:

$$M = \frac{(-ig_w)^2}{8} \bar{v}(-p_+) \not{q}_+ (1 - \gamma_5) \frac{i}{\not{p}_- - \not{q}_-} \not{q}_- (1 - \gamma_5) u(p_-)$$

- Using longitudinal polarization and keeping only leading term:

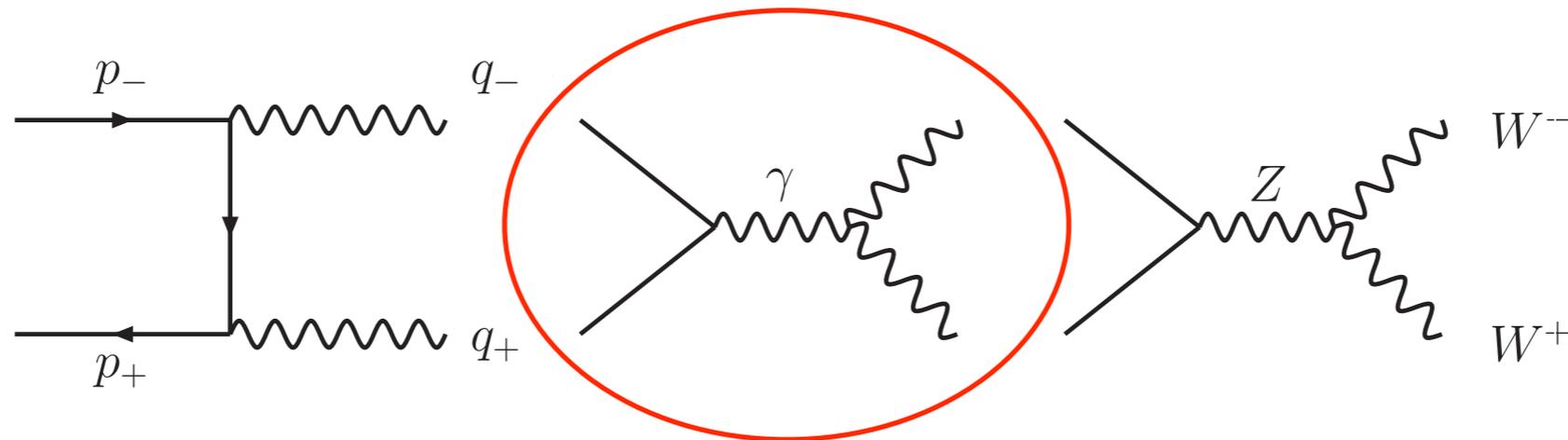
$$M = -i \frac{(-ig_w)^2}{4M_W^2} \bar{v}(-p_+) \not{q}_+ (1 - \gamma_5) u(p_-)$$

(via equation of motion)

- Useful to rewrite using momentum conservation:

$$M = -i \frac{(-ig_w)^2}{8M_W^2} \bar{v}(-p_+) (\not{q}_+ - \not{q}_-) (1 - \gamma_5) u(p_-)$$

# Self-coupling contributions



- Contribution from second diagram:

write in terms of  $g_w$   
using  $e = g_w \sin \theta_w$

$$M = (-ig_w)(ig_w)Q_e \sin^2 \theta_W \bar{v}(-p_+) \gamma^\rho u(p_-) \frac{-ig_{\rho\alpha}}{(q_+ + q_-)^2} \\ \times V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+) \epsilon_\beta(q_-) \epsilon_\delta(q_+).$$

triple-boson vertex

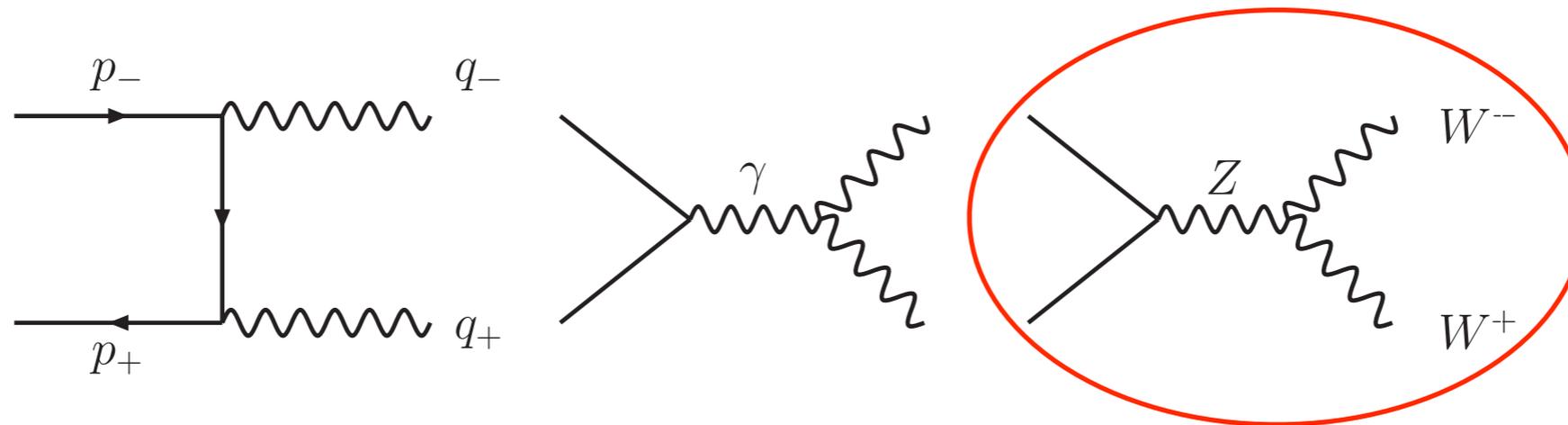
- Triple-boson vertex:  $V^{\alpha\beta\delta}(p, q, r) = g^{\alpha\beta}(p^\delta - q^\delta) + g^{\beta\delta}(q^\alpha - r^\alpha) + g^{\delta\alpha}(r^\beta - p^\beta)$

- Contracted with longitudinal polarizations here:

(discard  $m_W^2$ )

$$V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+) \epsilon_\beta(q_-) \epsilon_\delta(q_+) = -\frac{(q_+ + q_-)^2}{2M_W^2} [q_+^\alpha - q_-^\alpha] + O(1)$$

# Self-coupling contributions



- Similar contribution from third diagram:

vector and axial  
couplings from before

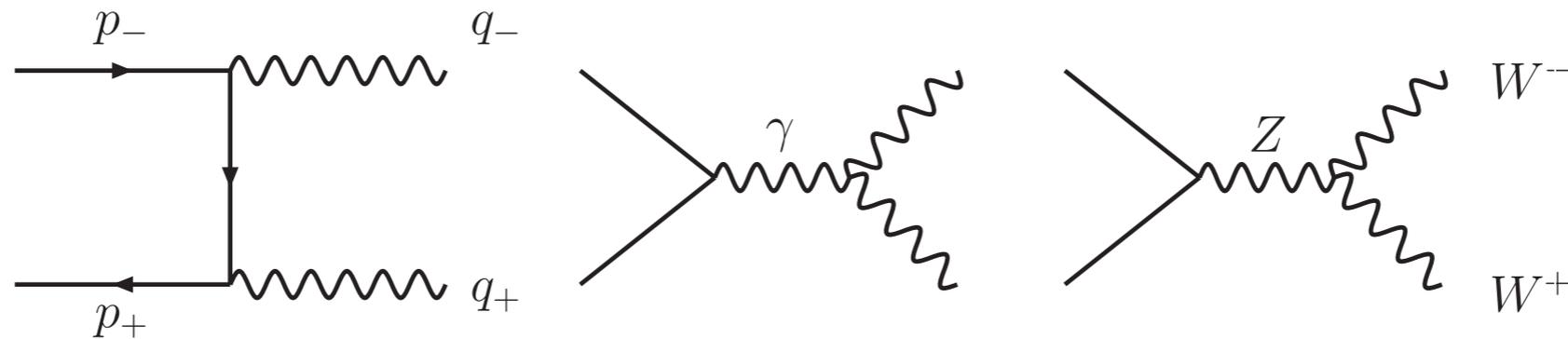
$$M = \frac{(-ig_w)(ig_w)}{2} \bar{v}(-p_+) \gamma^\rho (V_e - A_e \gamma_5) u(p_-) \frac{-ig_{\rho\alpha}}{(q_+ + q_-)^2 - M_Z^2} \times V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+) \epsilon_\beta(q_-) \epsilon_\delta(q_+)$$

- Hence, combining second and third diagrams:

$$M = -i \frac{(-ig_w)^2}{4M_W^2} \bar{v}(-p_+) (\not{q}_+ - \not{q}_-) [2Q_e \sin^2 \theta_W + V_e - A_e \gamma_5] u(p_-)$$

(discarding non-leading terms)

# Total in the high-energy limit



$$M = -i \frac{(-ig_w)^2}{8M_W^2} \bar{v}(-p_+) (\not{q}_+ - \not{q}_-) [1 + 4Q_e \sin^2 \theta_W + 2V_e - (1 + 2A_e)\gamma_5] u(p_-)$$

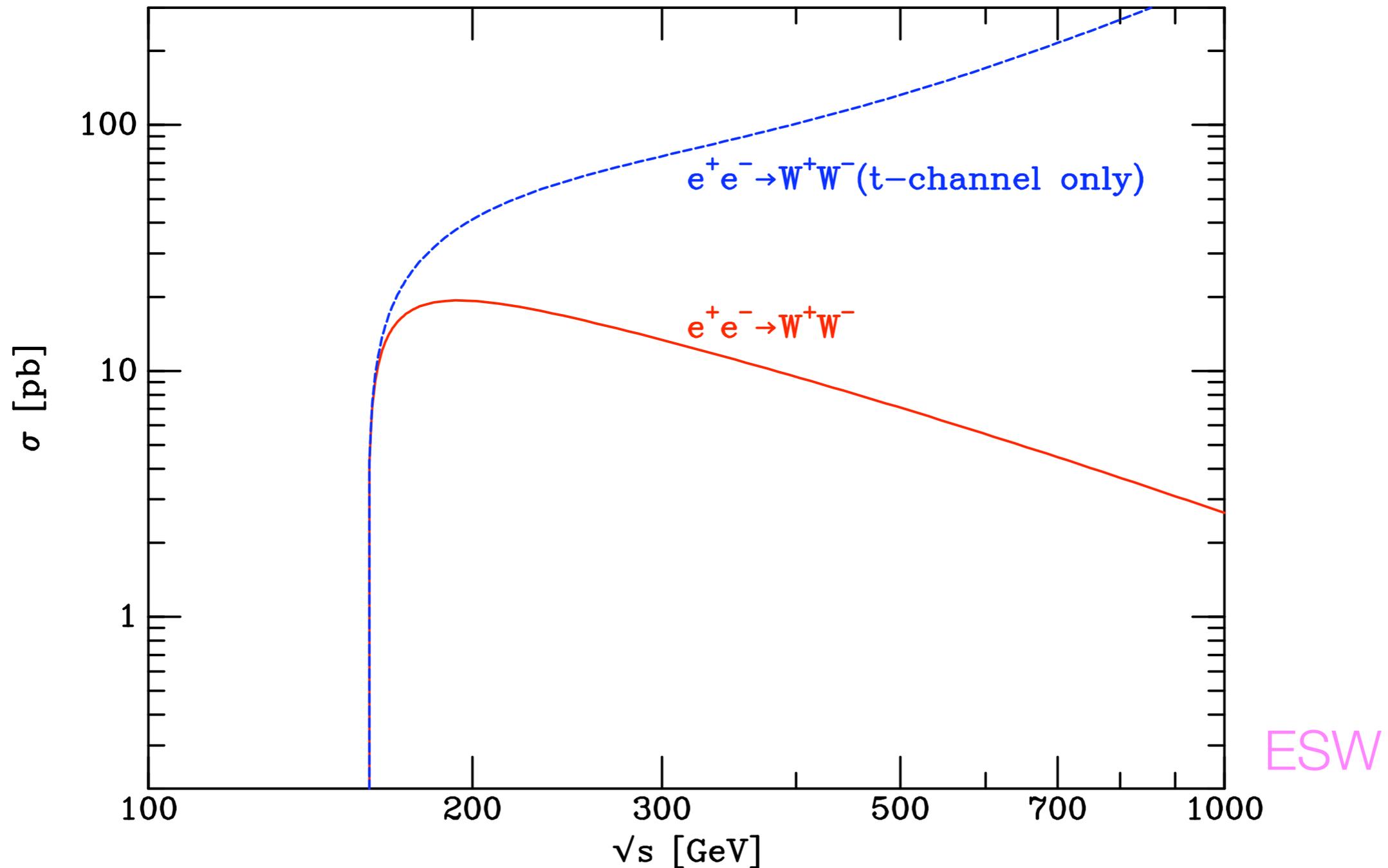
- Recall definitions of Z couplings:  $V_e = -\frac{1}{2} - 2Q_e \sin^2 \theta_W$ ,  $A_e = -\frac{1}{2}$

to see that the **leading high-energy behaviour is cancelled**.

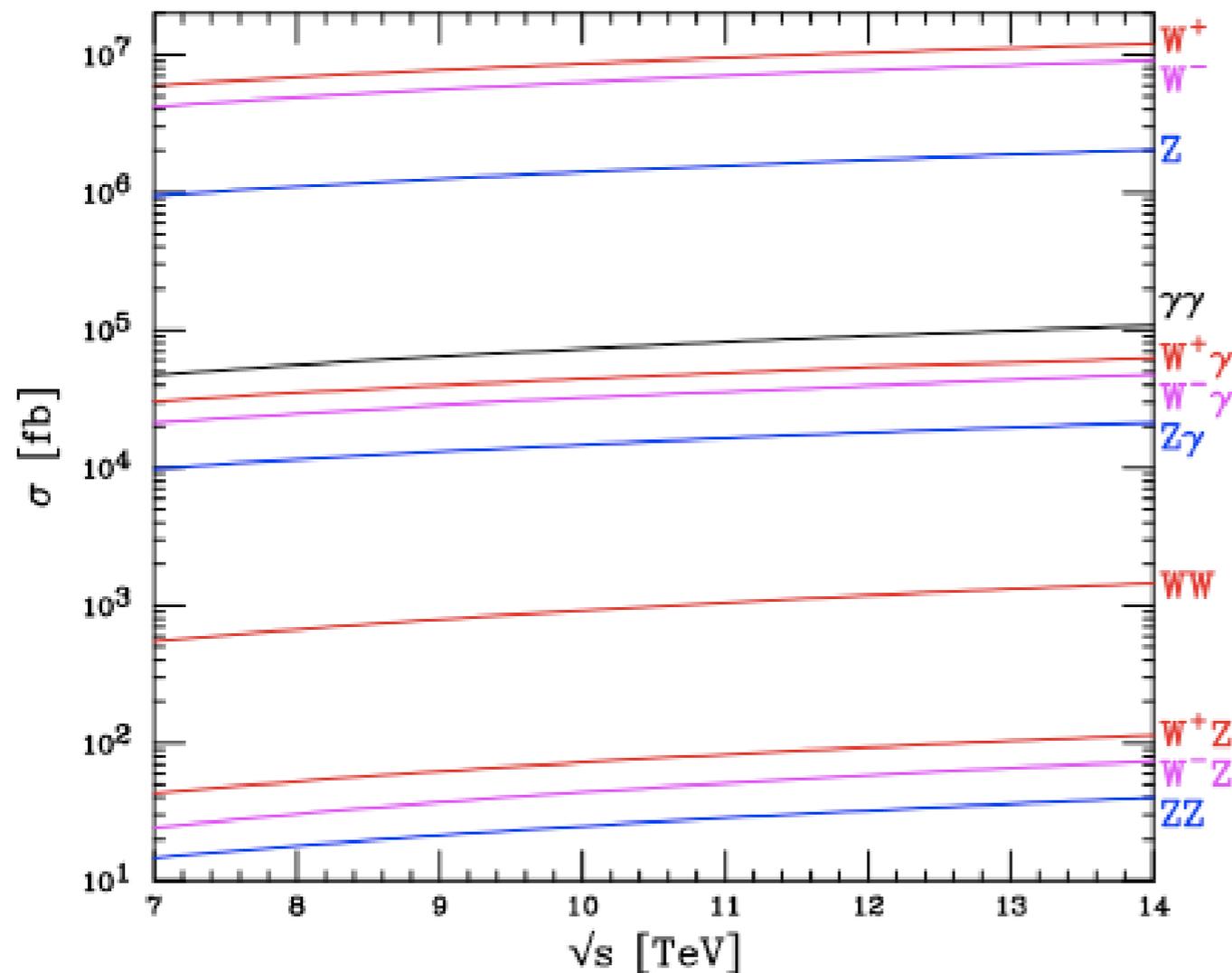
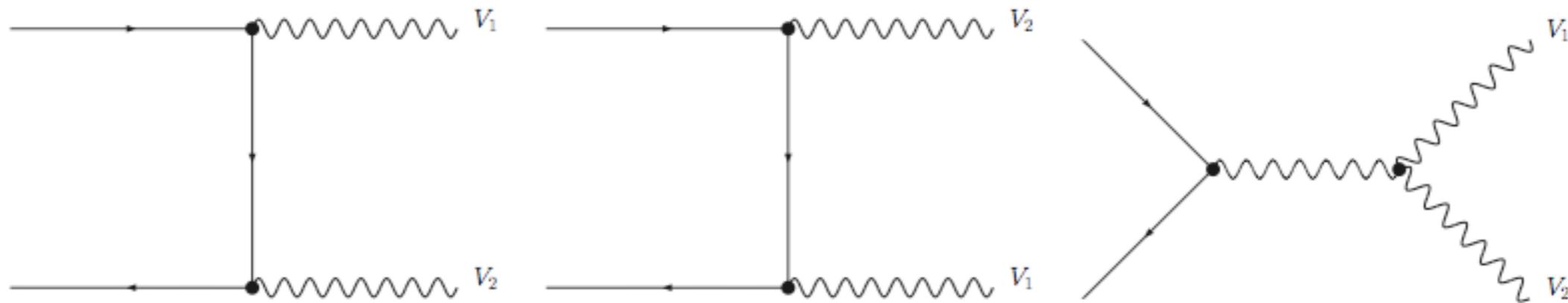
- due to the relationship between the coupling of the W,Z and photon to fermions and the triple-boson couplings
- equivalently**, due to the underlying gauge structure of the weak sector of the Standard Model.
- imperative to test at hadron colliders.

# Strength of high-energy cancellation

- Full result including sub-leading terms.



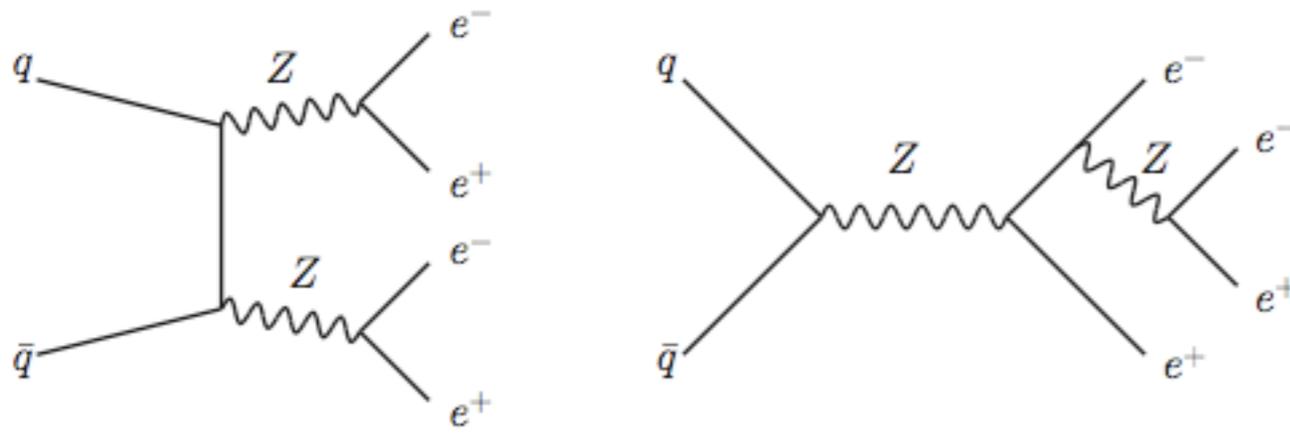
# Diboson production at hadron colliders



- Tri-boson coupling present for all processes except  $Z\gamma$ .
- Processes involving photons strongly-dependent on photon  $p_T$  (and rapidity) cut.
- Further suppression by BRs once decays are included.
- Next-to-leading order corrections known analytically, included in [MCFM](#), [VBFNLO](#).

# Single-resonant diagrams

- Modern calculations of the diboson processes include effects of decays; in that case, EW gauge invariance requires that additional diagrams are included.



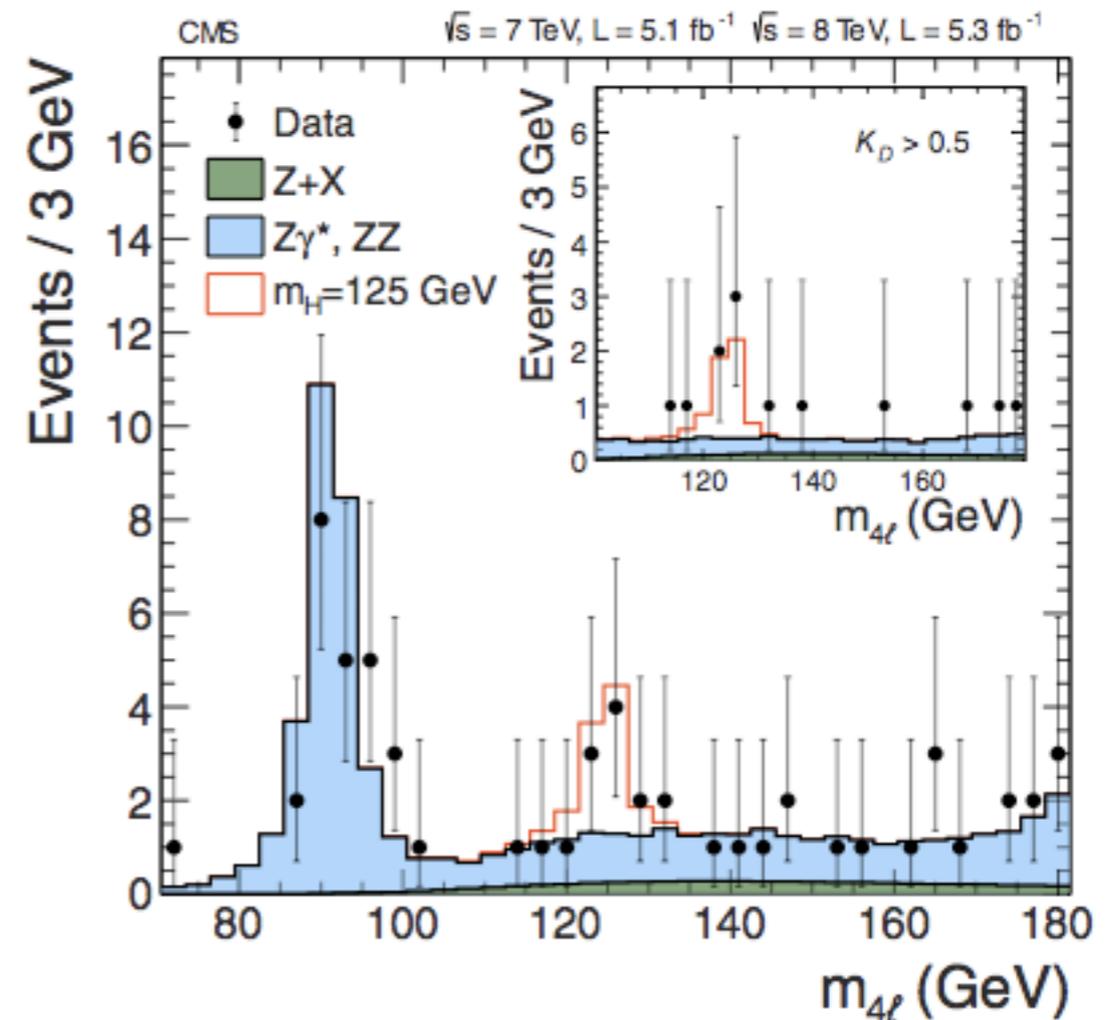
“double”-

“single”-resonant

- Inclusive cross section is dominated by the double-resonant contribution, but other distributions can be sculpted.
- Notably: invariant mass of 4 leptons.
- Useful cross-check of analysis in Higgs search.

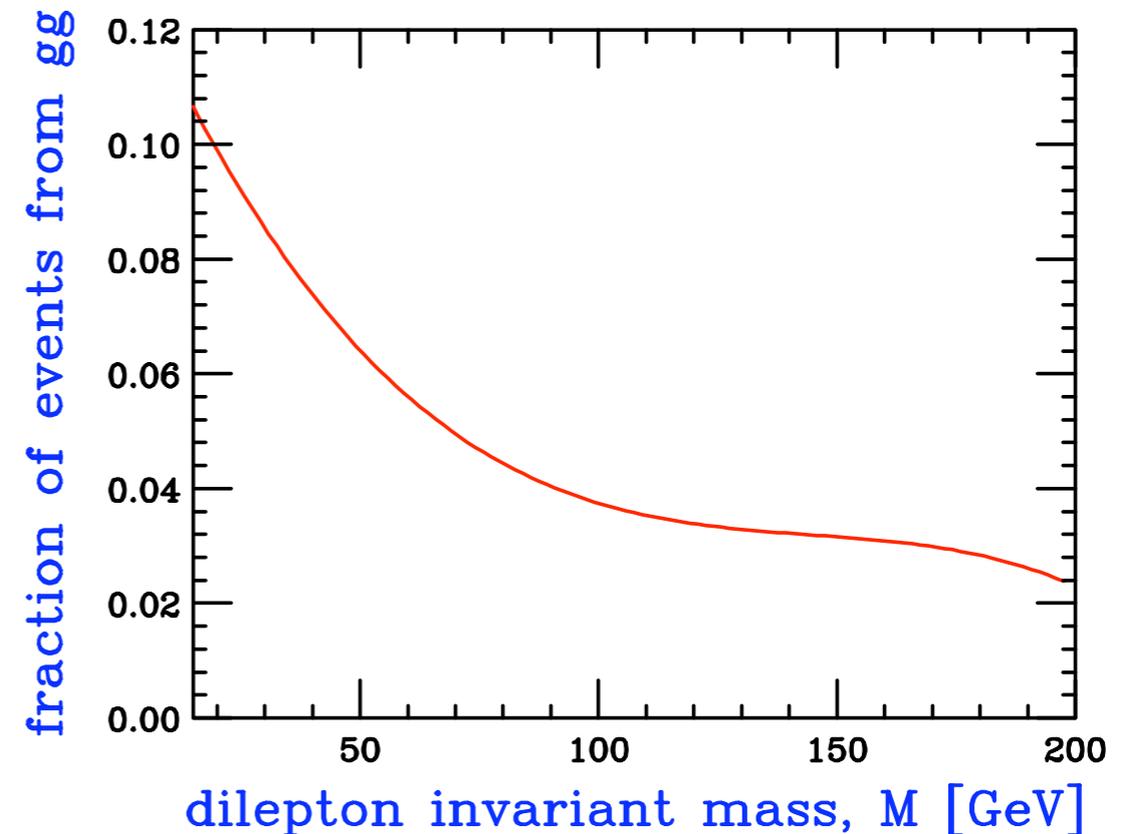
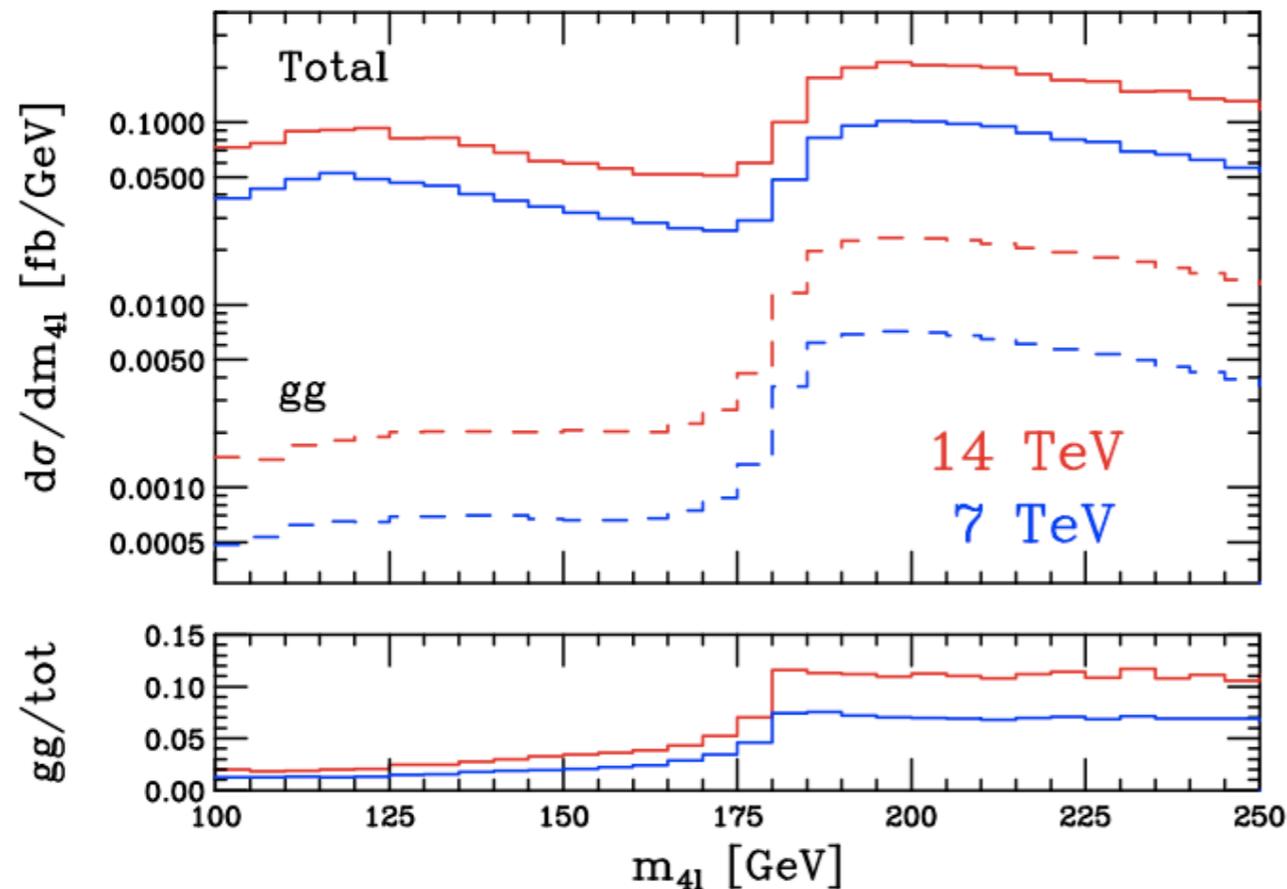
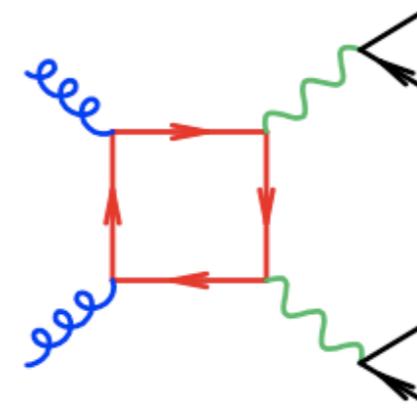
$$qq \rightarrow ZZ \rightarrow e^+e^-e^+e^-$$

CMS-HIG-12-028



# Gluon-induced contributions

- Just like diphoton production, part of NNLO contribution to  $WW$  and  $ZZ$  production is numerically relevant at the LHC.

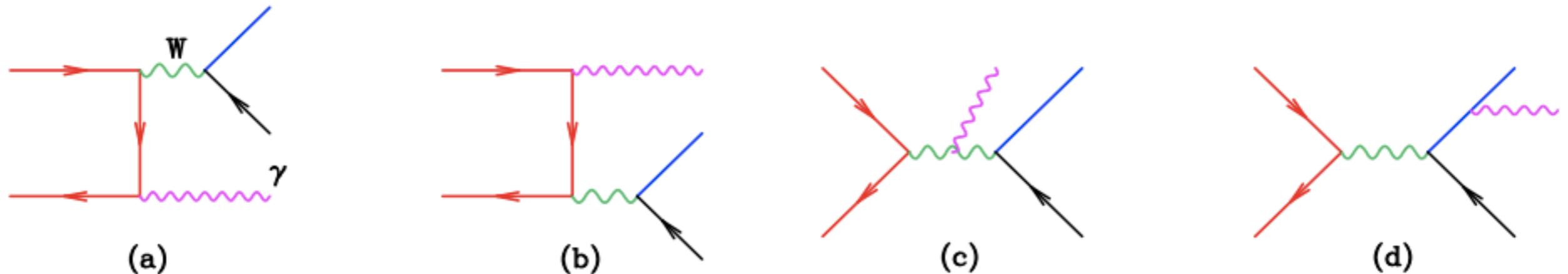


$ZZ$ : small below  $Z$  pair threshold (e.g. H search), but large above

$WW$ : impact of gg contribution enhanced by H analysis cuts such as low dilepton invariant mass

# Photon radiation in decays

- For  $W\gamma$  and  $Z\gamma$  production it is essential to account for the effect of photon radiation from the products of the  $W$  or  $Z$  decay.



- required by EM gauge invariance unless dileptons confined to resonance region  $\rightarrow$  not always easy to enforce experimentally
- effect can be dramatic  $\sigma(e^+\nu\gamma) \neq \sigma(W^+\gamma) \times \text{Br}(W^+ \rightarrow e^+\nu)$

W on-shell  
(no FSR)

W off-shell  
(includes FSR)

Decay	Cuts	$\sigma^{LO}(e^+\nu\gamma)$	$\sigma^{NLO}(e^+\nu\gamma)$
No FSR	Basic $\gamma$	4.88	8.74
	$M_T$ cut		3.78
	Lepton cuts	1.49	2.73
Full	Basic $\gamma$	23.0	30.1
	$M_T$ cut		3.94
	Lepton cuts	1.58	2.85

difference reduced by  
transverse mass cut  
 $M_T(\ell\gamma,\nu) > 90$  GeV  
 $\rightarrow$  no room left to  
radiate in decay

# W+photon amplitude

- Consider the lowest order partonic process (4-momenta in brackets):

$$\bar{u}(p_1) + d(p_2) \rightarrow W^+ + \gamma(p_3)$$

- The helicities of the quarks are fixed by the W coupling but we choose a positive helicity photon. Up to an overall factor amplitude is:

$$Q_u \frac{[23]}{\langle 13 \rangle} + Q_d \frac{[13]}{\langle 23 \rangle}$$

$$Q_u = 2/3 \text{ and } Q_d = -1/3$$

(and we have used  $Q_e = Q_d - Q_u$  to simplify)

- Convert back to more-familiar dot products by extracting overall spinor factor:

$$\frac{[23]}{\langle 13 \rangle} \left( Q_u + Q_d \frac{p_1 \cdot p_3}{p_2 \cdot p_3} \right) \quad (\text{recall, } \langle i j \rangle [j i] = 2p_i \cdot p_j)$$

- Can now evaluate in the partonic c.o.m. Assume the down quark has a positive z component and denote the angle between it and the photon by  $\theta^*$ .
- Amplitude thus proportional to:  $Q_u(1 + \cos \theta^*) + Q_d(1 - \cos \theta^*)$

# Radiation amplitude zero

- Amplitude **vanishes** at the scattering angle given by:

$$\cos \theta^* = \frac{Q_u + Q_d}{Q_d - Q_u} = -\frac{1}{3} \quad (\text{independent of parton energies})$$

- This feature is characteristic of all helicity amplitudes for the emission of photons in multi-boson processes.
- “**Radiation amplitude zero**” (RAZ) the result of interference between diagrams.
- Easy to calculate the corresponding photon rapidity:

$$y_\gamma^* = \frac{1}{2} \log \left( \frac{1 + \cos \theta^*}{1 - \cos \theta^*} \right) \approx -0.35$$

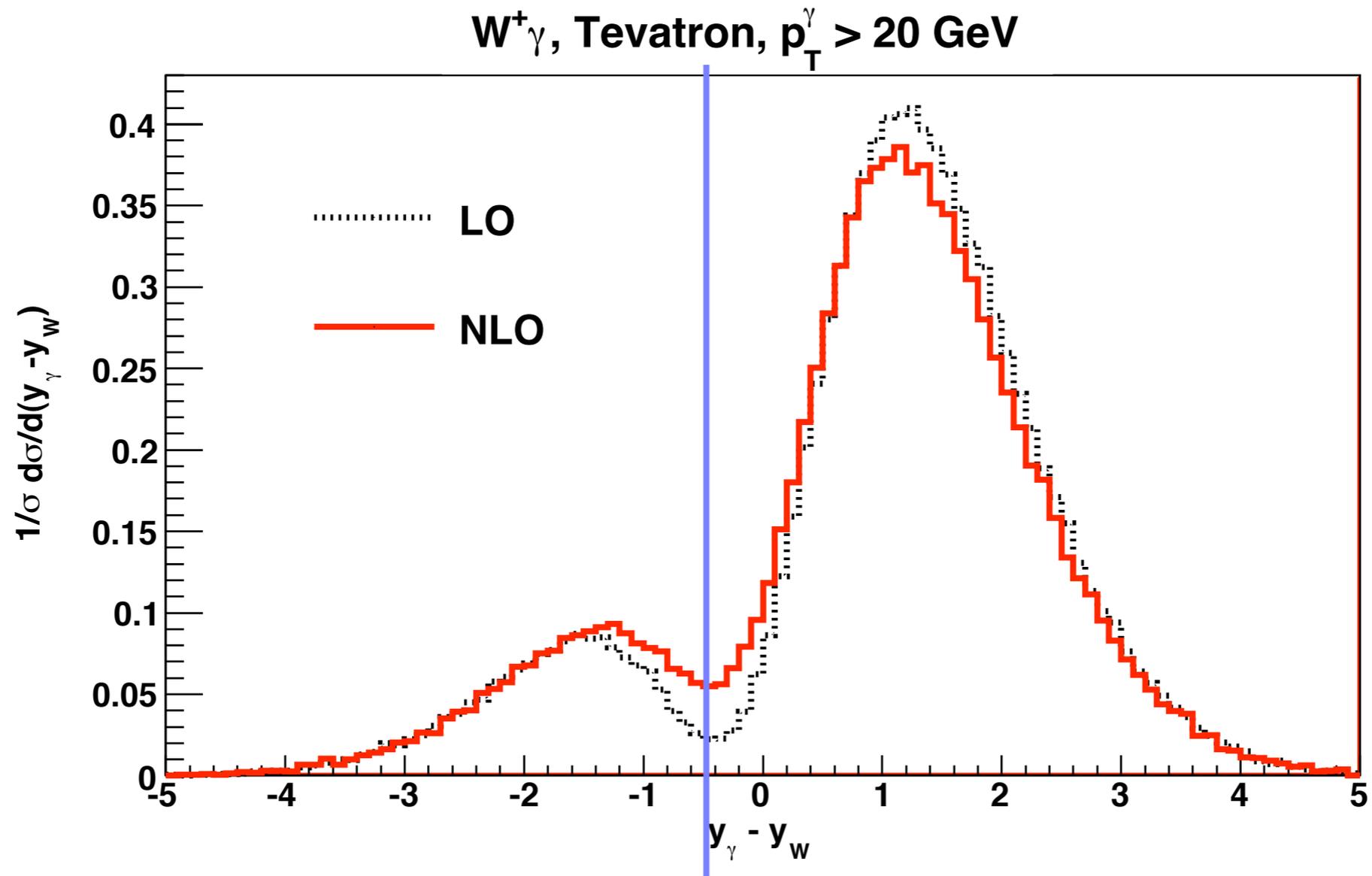
- Rather than reconstructing all objects and trying to boost back to c.o.m, easiest to construct a (boost invariant) rapidity difference:  $\Delta y^* = y_\gamma^* - y_W^*$ .
- For small photon  $p_T$  relative to  $m_W$  the W rapidity in the c.o.m. is given by:

$$y_W^* \approx \frac{1}{2} \log \left( \frac{m_W - p_T^\gamma \cos \theta^*}{m_W + p_T^\gamma \cos \theta^*} \right)$$

# Position of zero

- Expanding for small  $p_T$  gives:  $y_\gamma^* \approx \frac{p_T^{\gamma, \min}}{3m_W}$
- Hence the corresponding zero in the W rapidity distribution is positive, but at a significantly smaller value.
- Rapidity difference, e.g. for typical experimental cuts at 20 GeV:  $\Delta y^* \approx -0.45$  (for the sub-process we looked at).
- **Tevatron**: quark and anti-quark directions coincide with those of protons and anti-protons, to first approximation.
  - prediction for the radiation zero derived above should be reproduced approximately once pdfs are folded in;
  - this pdf dilution means that we do not obtain exact vanishing of the distribution but instead a pronounced dip.
- **LHC**: no well-defined direction for protons, so RAZ should be at  $\Delta y^* = 0$ .

# Radiation zero at the Tevatron



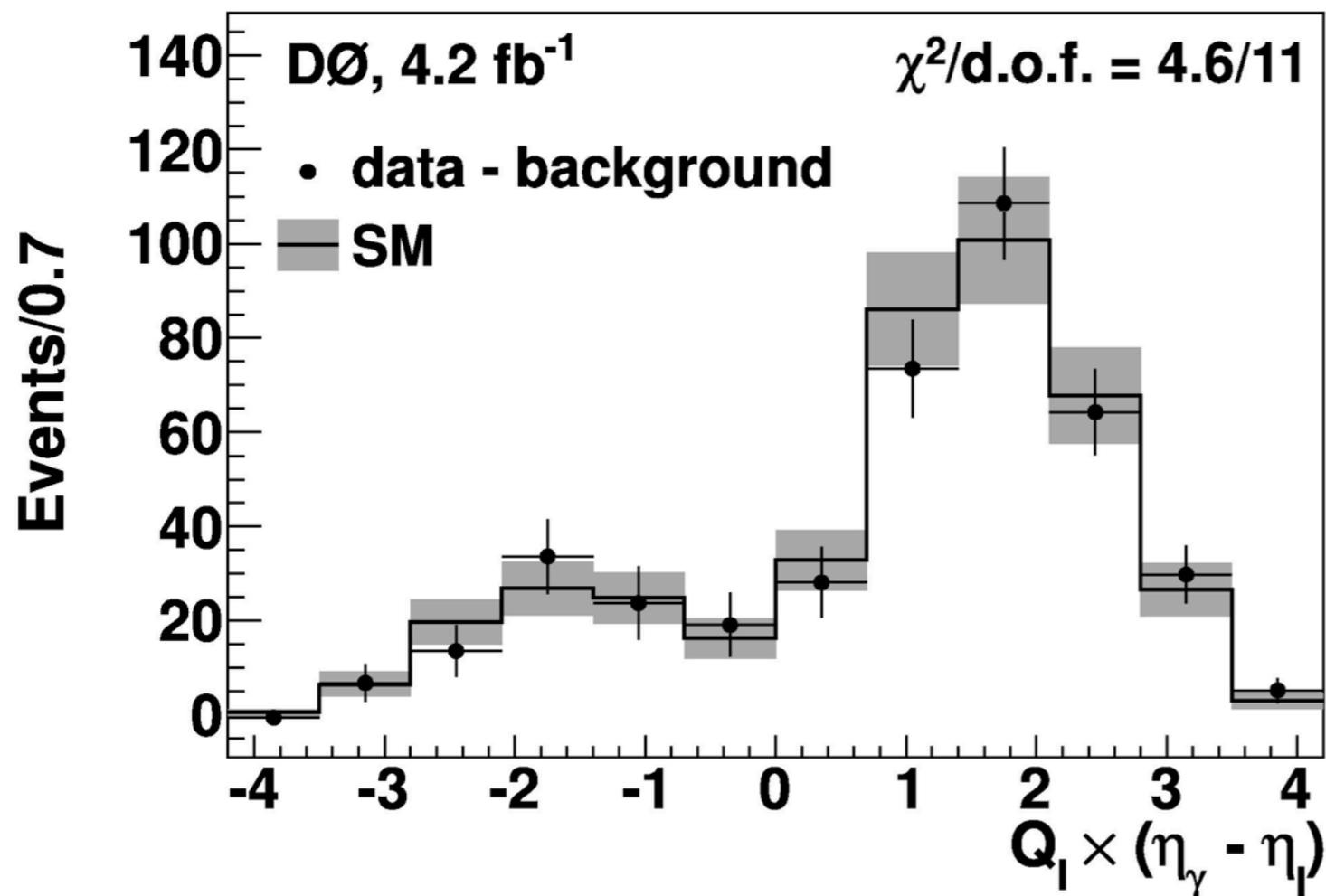
expected position of RAZ

Amplitude zero a feature of the LO amplitude only  
→ partially washed out at higher orders

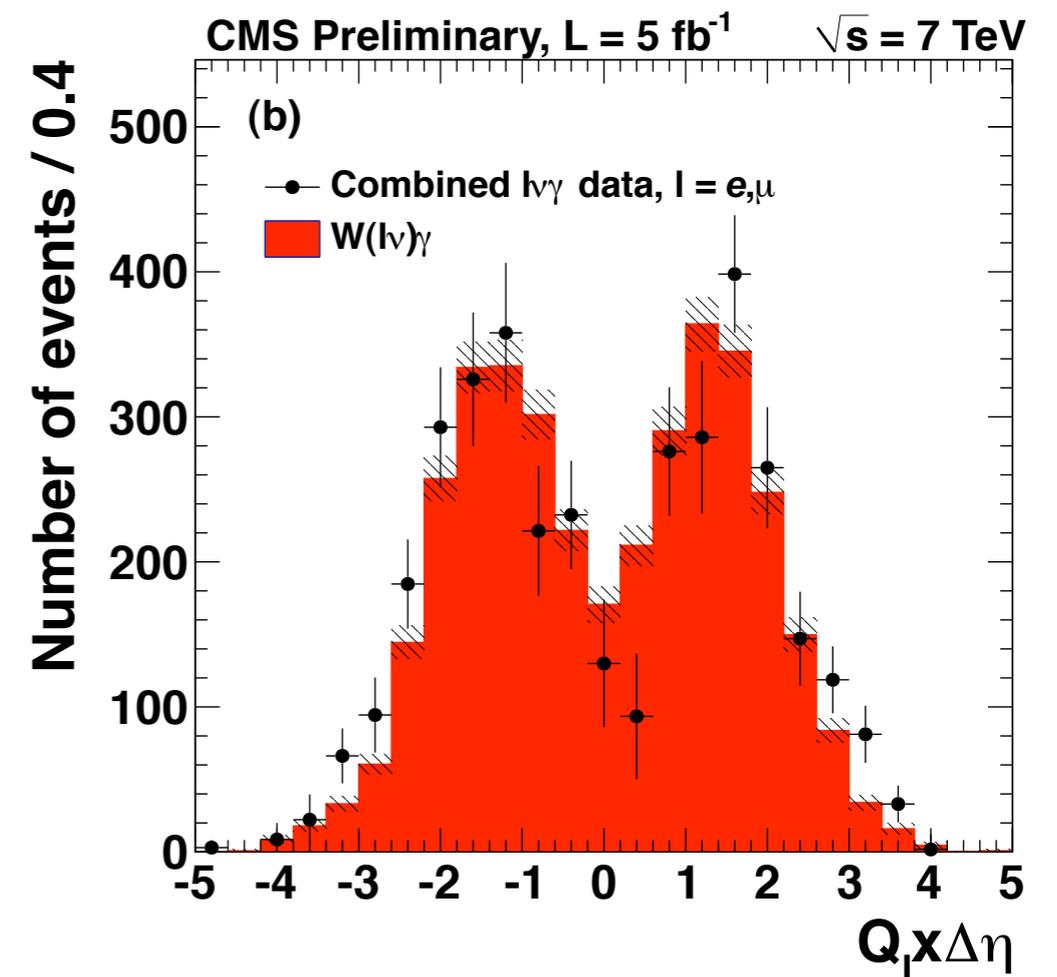
# Experimental evidence for RAZ

- Experimental issues that wash out dip:
  - easiest to use lepton rapidity rather than  $W$  (retains much information)
  - contamination from photon radiation in  $W$  decay

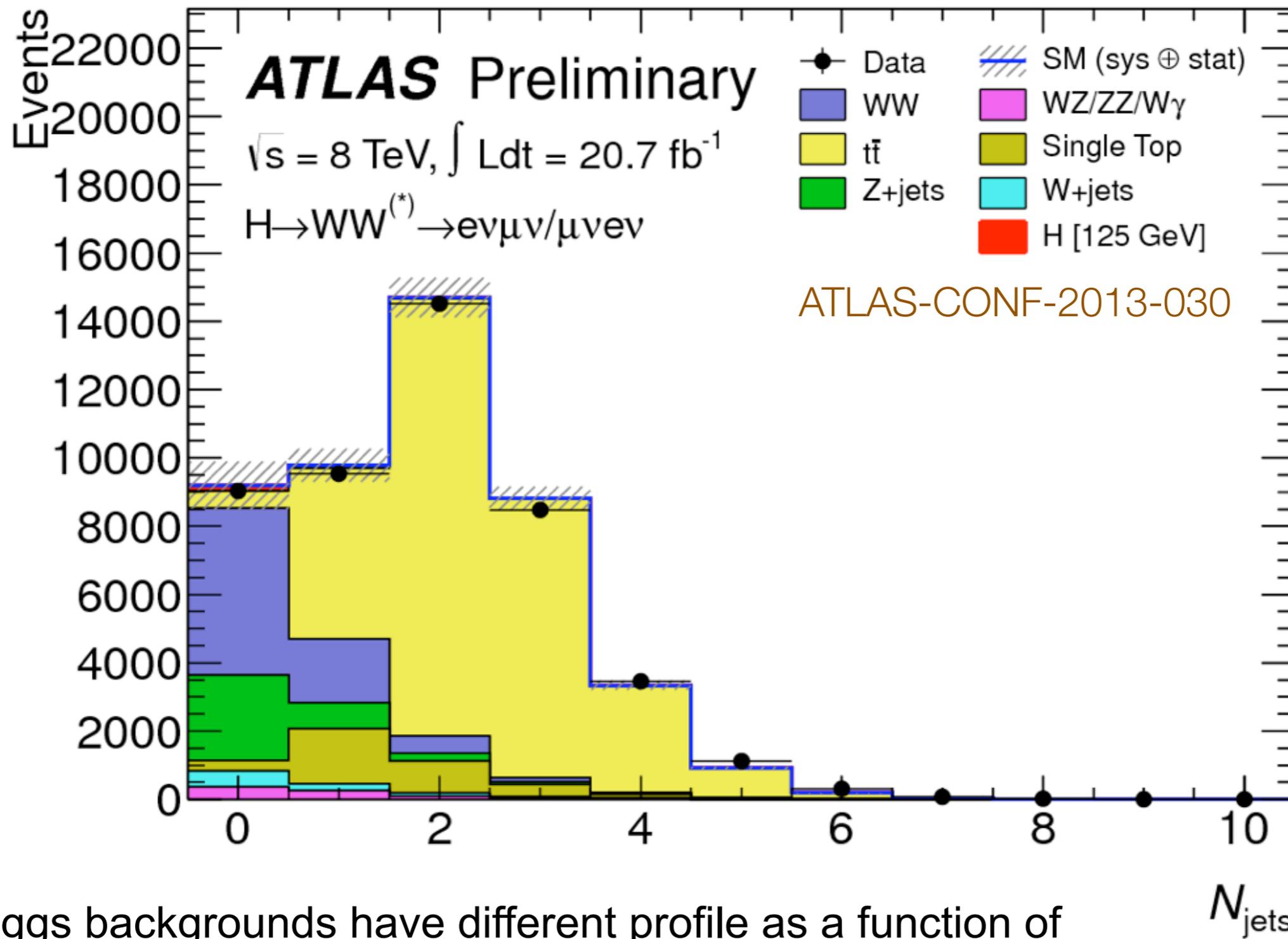
D0, arXiv 1109.4432



CMS, PAS-EWK-11-009



# WW: the importance of jet-binning



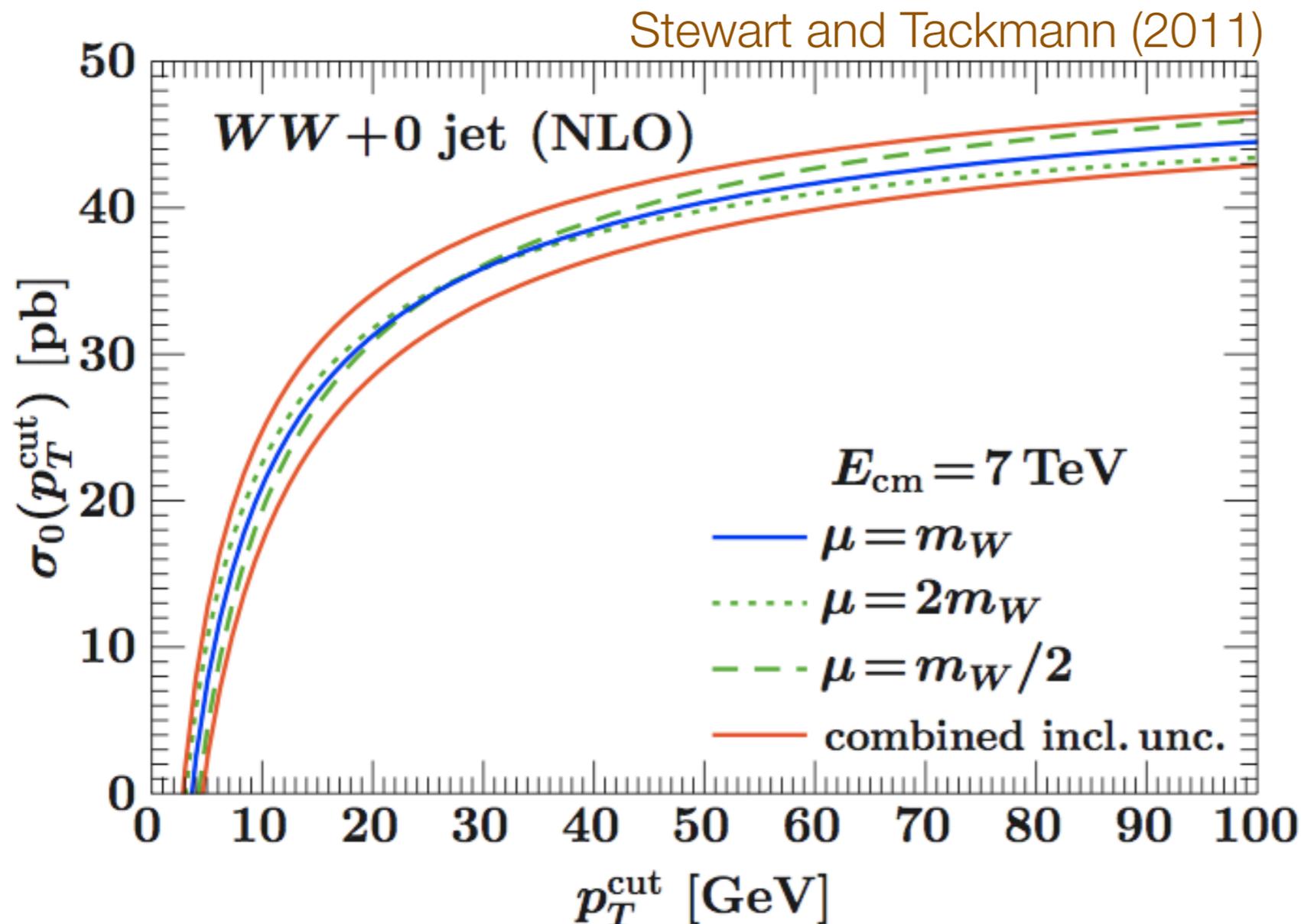
- Higgs backgrounds have different profile as a function of number of jets  $\rightarrow$  important to understand theory the same way.

# Jet vetoes

- Top backgrounds naturally contain jets: at least partly understood via well-known weak interaction.
- In contrast,  $WW$  process only produces jets through QCD.
  - jet-binned cross sections can be subject to large uncertainties.
- The reason is that the veto is explicitly removing part of the real radiation that is responsible for ensuring that infrared divergences cancel
  - the incomplete cancellation that results introduces a logarithm into the perturbative expansion
  - consider an inclusive  $WW$  cross section at NLO; vetoing jets to obtain the 0-jet cross-section is removing a term of order  $\alpha_s$
  - however, the derivation of the Sudakov factor we sketched earlier tells us that we're actually introducing a factor more like  $\alpha_s \log^2[2m_W/p_T^{\text{veto}}]$ ; for typical values of the veto this factor is numerically large  $\sim 3$ .
- we should therefore expect worse perturbative behaviour.

# Vetoed uncertainties

- However, the naive method of scale variation can be too optimistic and result in uncertainties for vetoed cross sections smaller than for the inclusive case.
- The accidentally-small variation can be undone by assuming the scale uncertainties in the 0-jet and 1-jet bins are uncorrelated.



$$\Delta_{0\text{-jet}}^2 = \Delta_{\text{incl.}}^2 + \Delta_{1\text{-jet}}^2$$

New uncertainty much larger across the range of  $p_T$

Some empirical evidence that this may be *too conservative*

Real answer is to resum the logarithms → much work in H case.

# Anomalous triple gauge couplings

- aTGCs usually described in terms of additional interactions in the Lagrangian:

$$\mathcal{L}_{anom} = ig_{WWZ} \left[ \Delta g_1^Z (W_{\mu\nu}^* W^\mu Z^\nu - W_{\mu\nu} W^{*\mu} Z^\nu) + \Delta \kappa^Z W_\mu^* W_\nu Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_{\rho\mu}^* W_\nu^\mu Z^{\nu\rho} \right] + ig_{WW\gamma} \left[ \Delta \kappa^\gamma W_\mu^* W_\nu \gamma^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_{\rho\mu}^* W_\nu^\mu \gamma^{\nu\rho} \right]$$

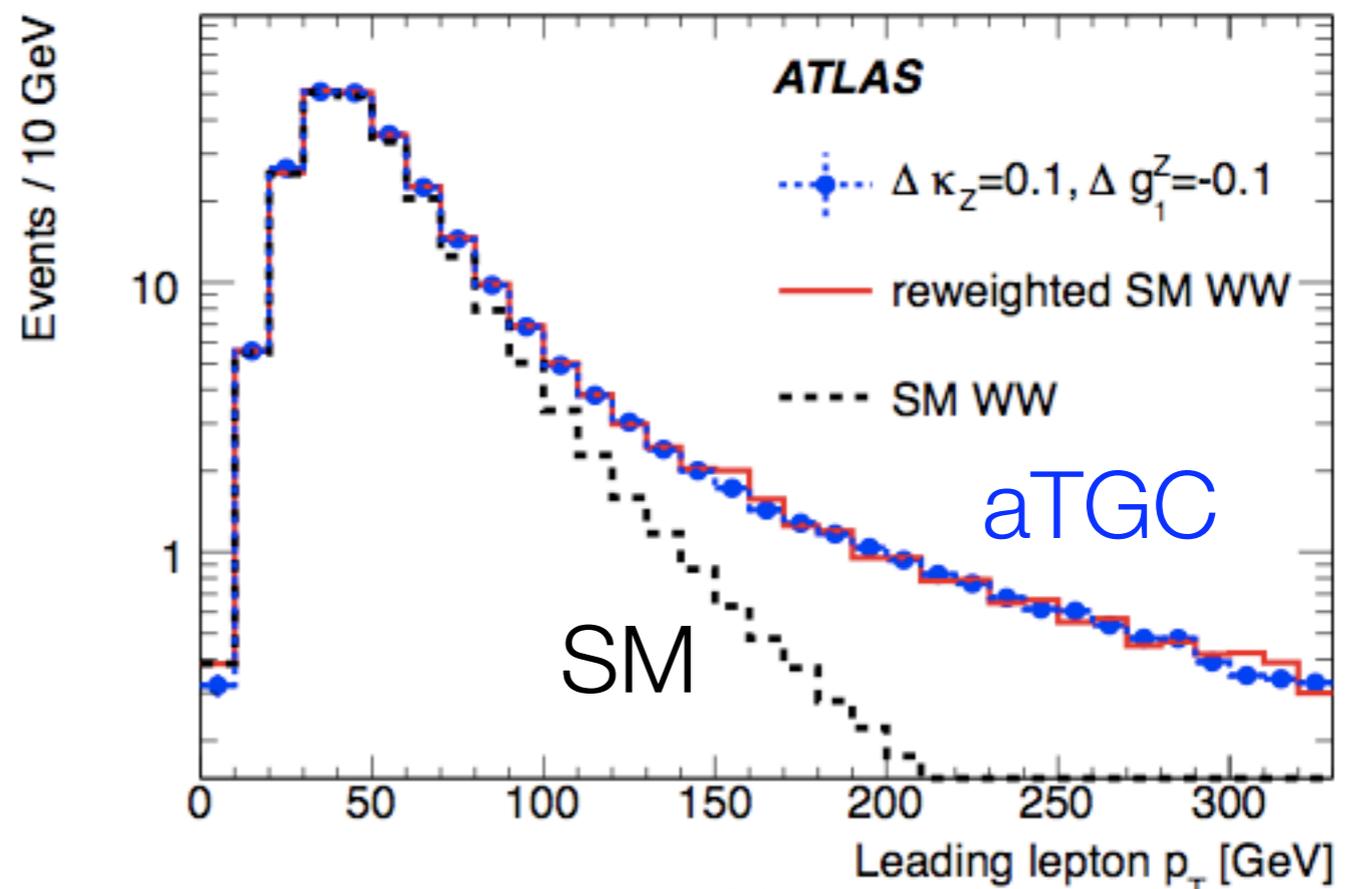
- Most general contribution that separately conserves C and P.

- Operators do not change the predicted cross-section significantly, but instead alter distributions at high  $p_T$ , invariant mass, etc.

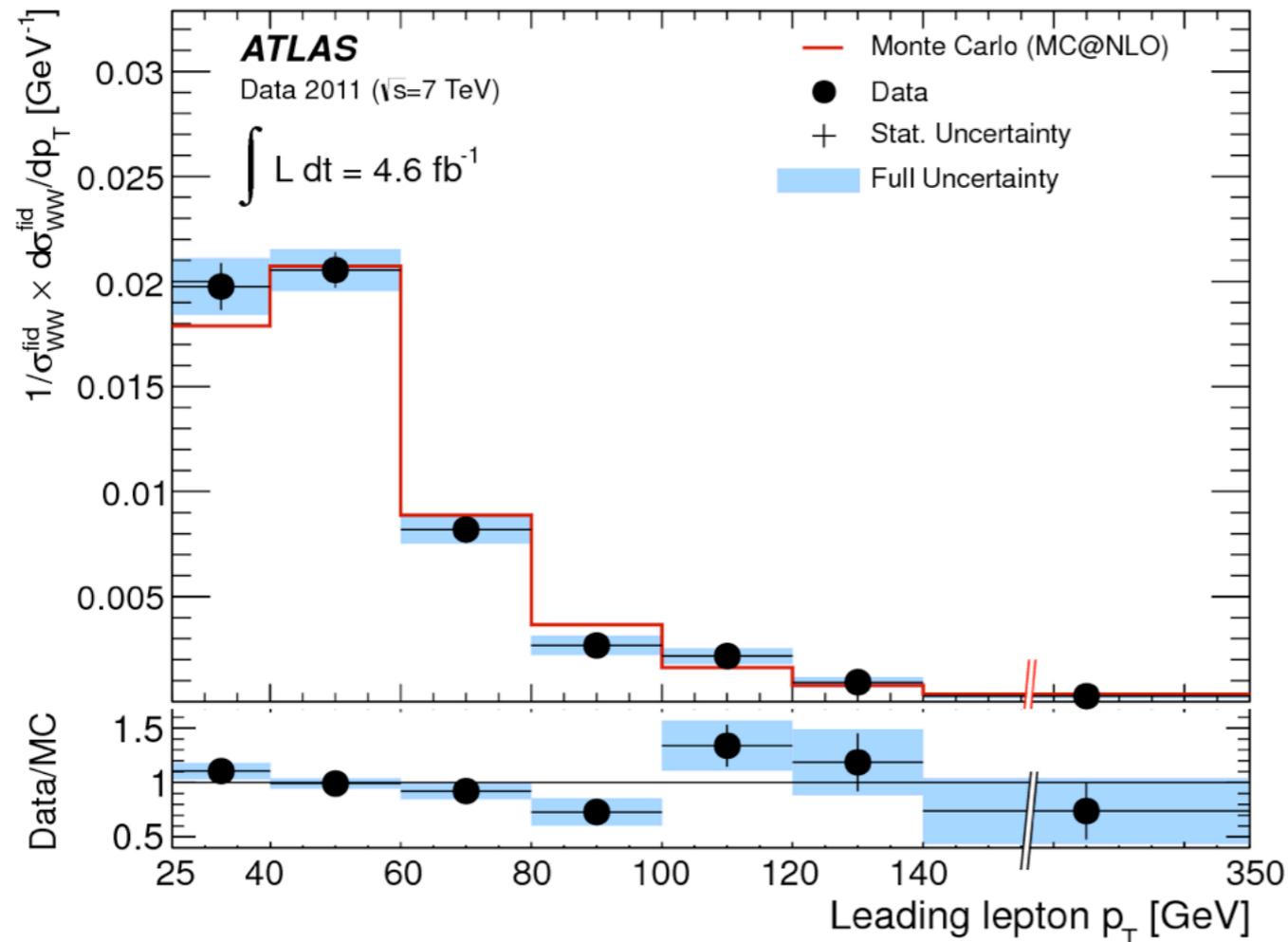
- This plot, for illustration, uses values of parameters outside current exclusion.

- need to look for small deviation in tail.

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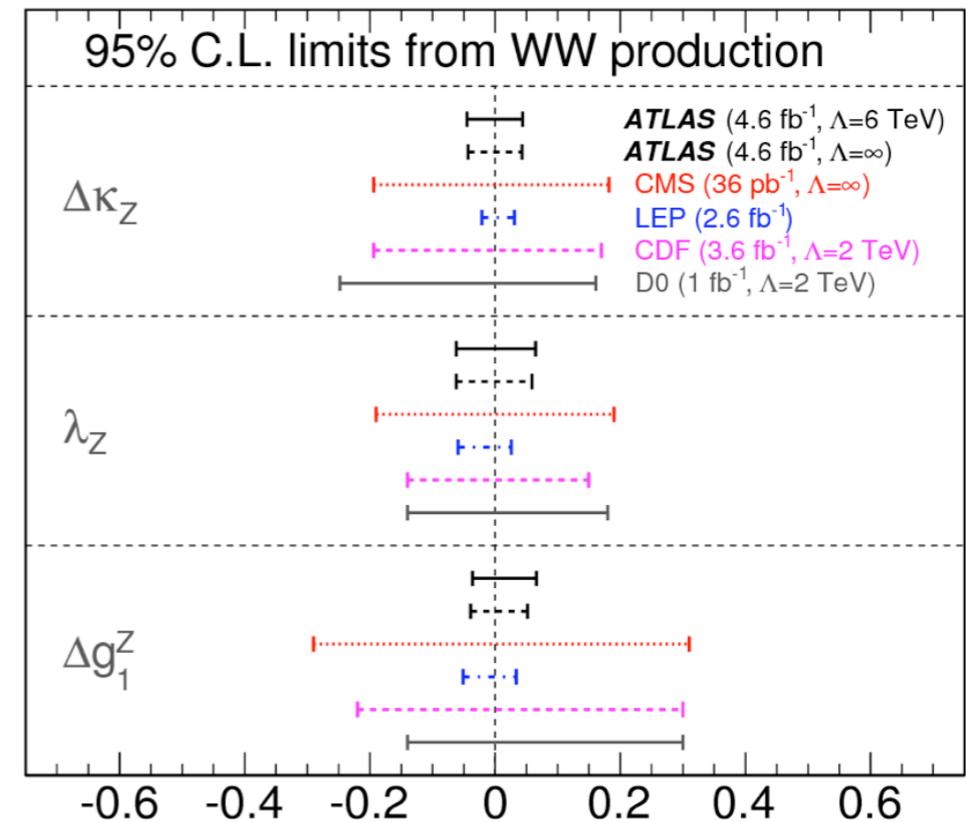


# Example of result



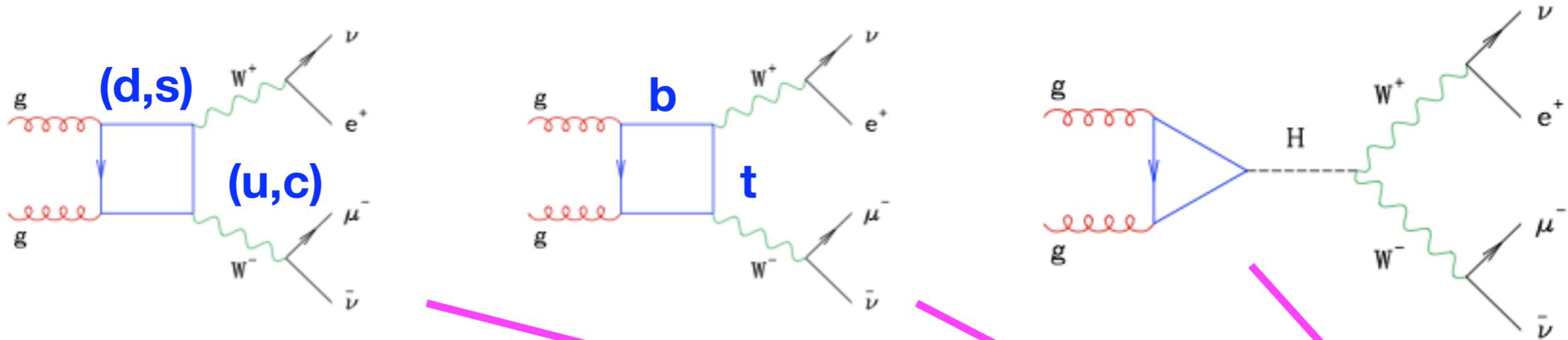
Comparison with CMS,  
Tevatron and LEP

## ATLAS WW analysis



# A bit beyond vector bosons ...

- The gluon-initiated WW contribution has the same external particles as the Higgs production process. Should calculate full amplitude before squaring:



$$\mathcal{A}_{\text{full}} = \delta^{a_1 a_2} \left( \frac{g_w^4 g_s^2}{16\pi^2} \right) \mathcal{P}_W(s_{34}) \mathcal{P}_W(s_{56}) [2 \mathcal{A}_{\text{massless}} + \mathcal{A}_{\text{massive}} + \mathcal{A}_{\text{Higgs}}]$$

- Is the interference important? Need to check in view of importance to extracting couplings.
- How do we define signal and background?
  - at what point is the Higgs boson just another SM contribution?

# Notation

$$\mathcal{A}_{\text{full}} = \delta^{a_1 a_2} \left( \frac{g_w^4 g_s^2}{16\pi^2} \right) \mathcal{P}_W(s_{34}) \mathcal{P}_W(s_{56}) \underbrace{[2 \mathcal{A}_{\text{massless}} + \mathcal{A}_{\text{massive}} + \mathcal{A}_{\text{Higgs}}]}_{\mathcal{A}_{\text{box}}}$$

- Background only:  $\sigma_B \longrightarrow |\mathcal{A}_{\text{box}}|^2$
- Signal only:  $\sigma_H \longrightarrow |\mathcal{A}_{\text{Higgs}}|^2$
- This is the usual approach. To include the effect of interference define:

$$\sigma_i \longrightarrow 2\text{Re}(\mathcal{A}_{\text{Higgs}} \mathcal{A}_{\text{box}}^*)$$

- Cross section in the presence of the Higgs, i.e. including also the interference:

$$\sigma_{H,i} = \sigma_H + \sigma_i$$

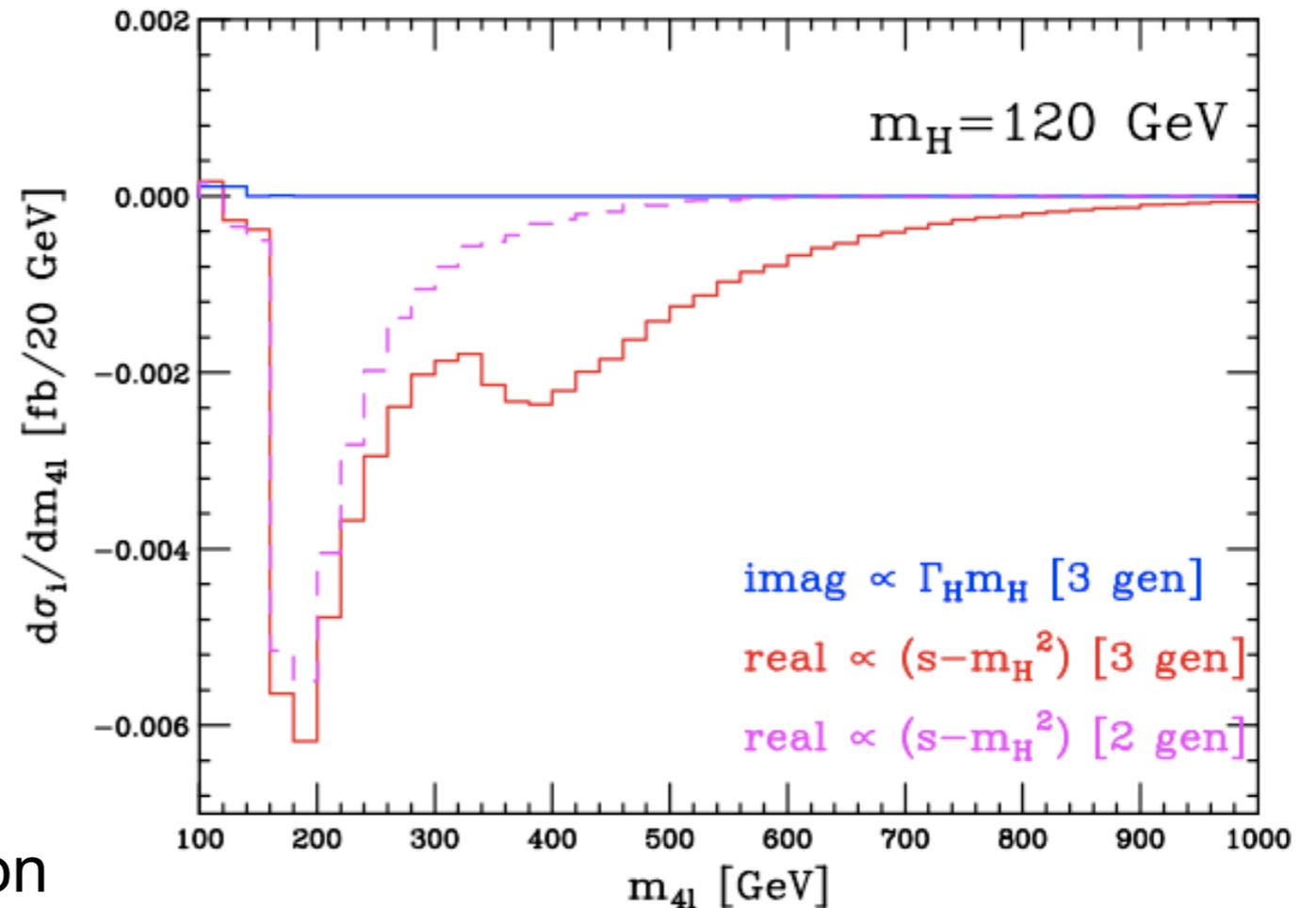
- Can then compare results for  $\sigma_H$  and  $\sigma_{H,i}$ .

# Analyzing the interference

- Separate interference by **Re** and **Im** parts of propagator:

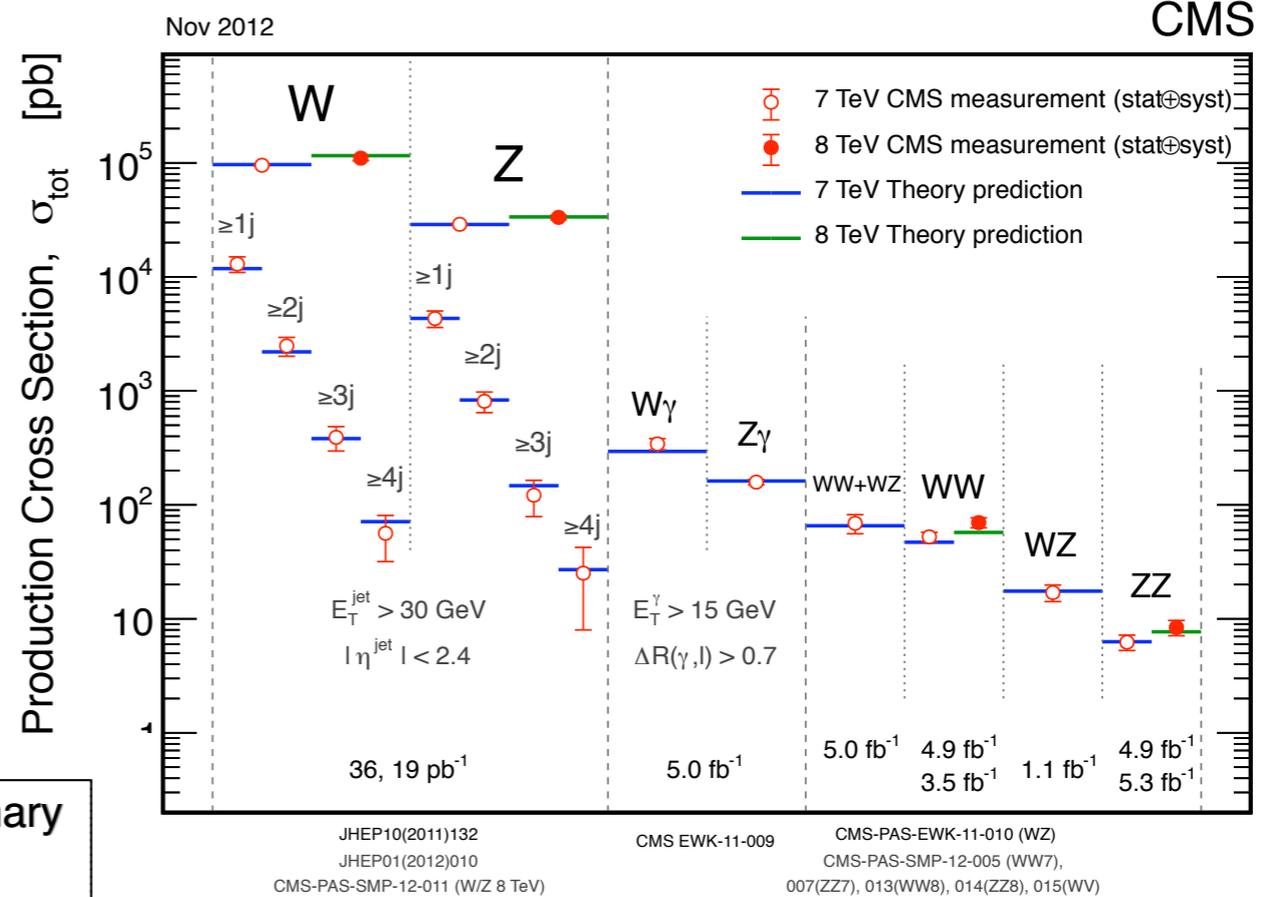
$$\delta\sigma_i = \frac{(\hat{s} - m_H^2)}{(\hat{s} - m_H^2)^2 + m_H^2 \Gamma_H^2} \Re \left\{ 2\tilde{\mathcal{A}}_{\text{Higgs}} \mathcal{A}_{\text{box}}^* \right\} + \frac{m_H \Gamma_H}{(\hat{s} - m_H^2)^2 + m_H^2 \Gamma_H^2} \Im \left\{ 2\tilde{\mathcal{A}}_{\text{Higgs}} \mathcal{A}_{\text{box}}^* \right\}$$

- For our light Higgs the second term is negligible.
- If the full  $s$ -dependence of the first term can be represented by factor from the propagator, it should vanish on integration (odd about the Higgs mass).
  - but  $s$ -dependence is more complicated because the box diagrams favour large invariant masses (W pairs).
- Long destructive tail required by unitarity; integrated contribution significant, (negative) 10-15%.

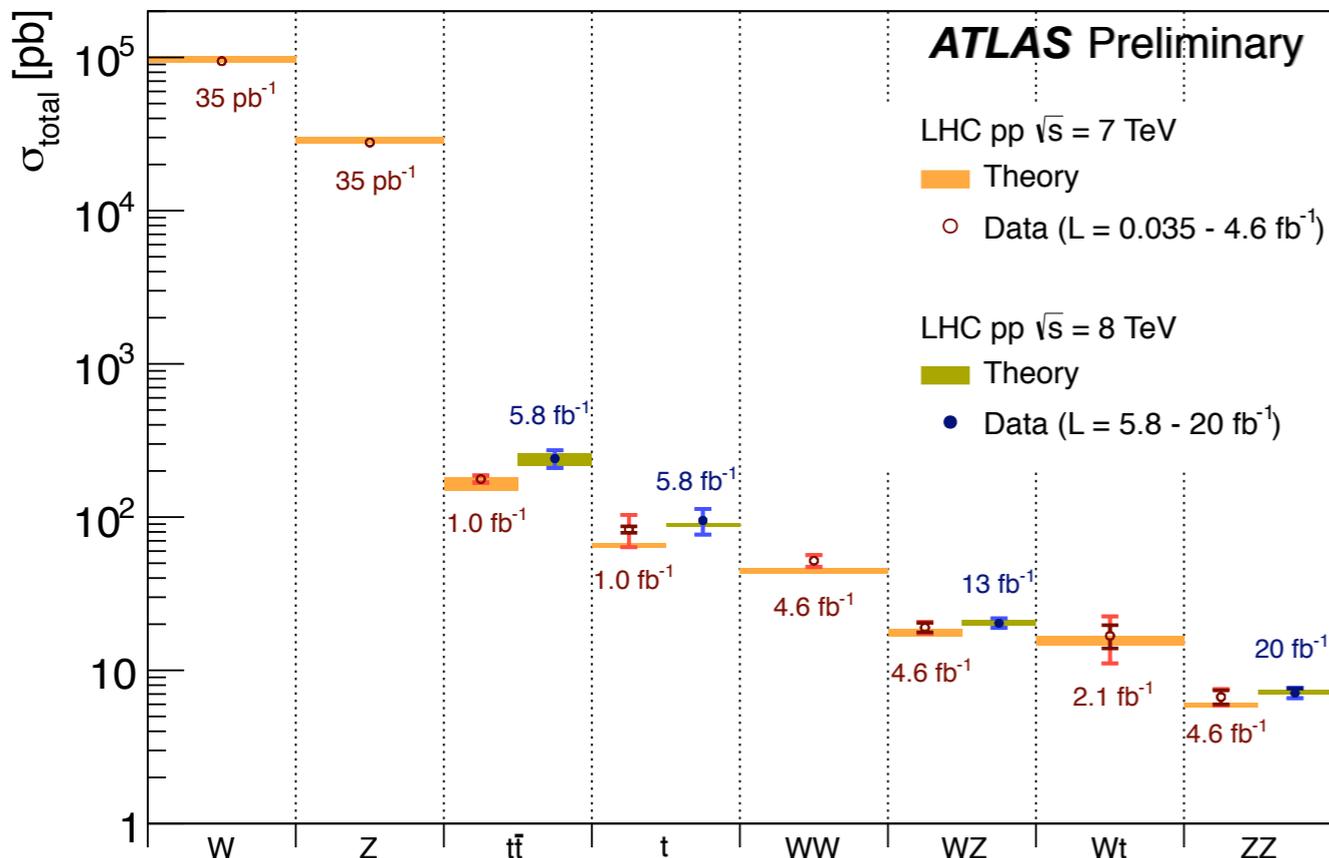


# Vector bosons: experimental summary

Good consistency with expectations of NNLO (W/Z) and NLO (dibosons) for all processes in both experiments.



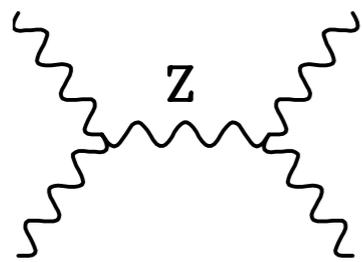
*Slight exception:* WW has a small error and looks high throughout.



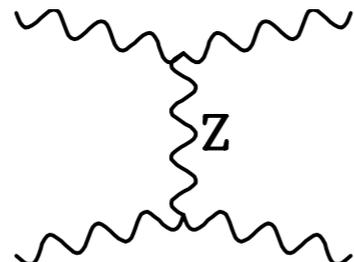
# Vector boson scattering

- One way of probing the electroweak sector further is through **vector boson scattering**.
- Simplest to consider the amplitudes not a hadron collider but in the pure scattering process:

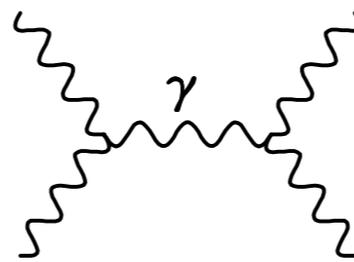
$$W^+(p_+) + W^-(p_-) \rightarrow W^+(q_+) + W^-(q_-)$$



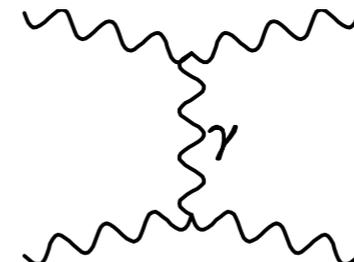
(a)



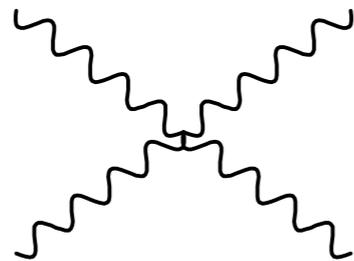
(b)



(c)



(d)



(e)

Five diagrams all involving self-couplings of the vector bosons

# High-energy limit (again)

- Once again consider the high-energy behaviour, concentrating on leading behaviour given by the scattering of longitudinal W bosons.
- Incoming W's along the z-axis:

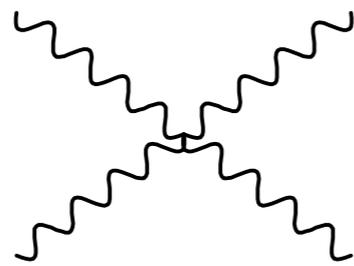
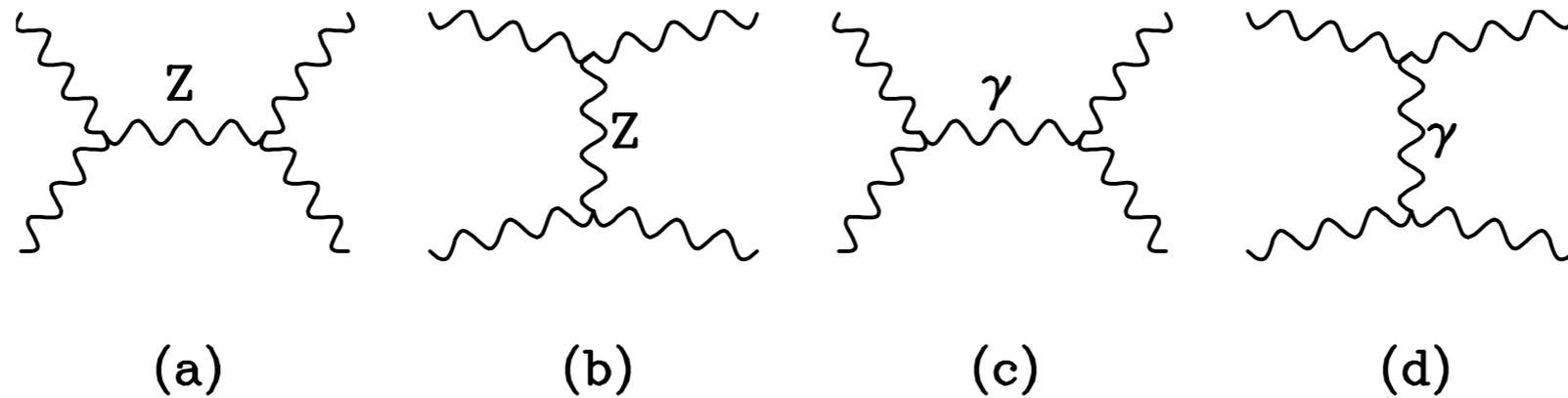
$$\begin{aligned}p_{\pm} &= (E, 0, 0, \pm p), \\q_{\pm} &= (E, 0, \pm p \sin \theta, \pm p \cos \theta)\end{aligned}$$

and longitudinal polarizations a slight generalization of previous form:

$$\begin{aligned}\varepsilon_L(p_{\pm}) &= \left( \frac{p}{M_W}, 0, 0, \pm \frac{E}{M_W} \right), \\ \varepsilon_L(q_{\pm}) &= \left( \frac{p}{M_W}, 0, \pm \frac{E}{M_W} \sin \theta, \pm \frac{E}{M_W} \cos \theta \right)\end{aligned}$$

- Use these to calculate the form of the diagrams in the high-energy limit, i.e. dropping terms without factor of  $p^2/m_W^2$ .

# Result

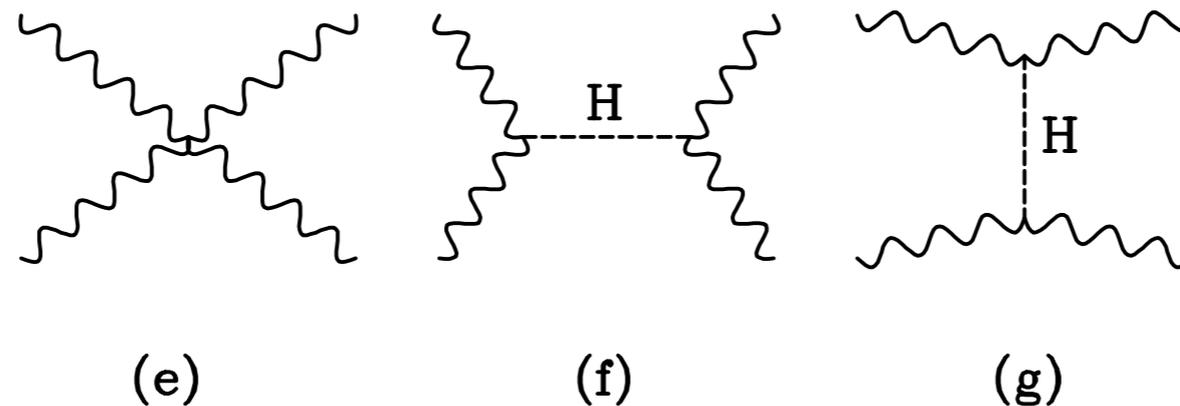
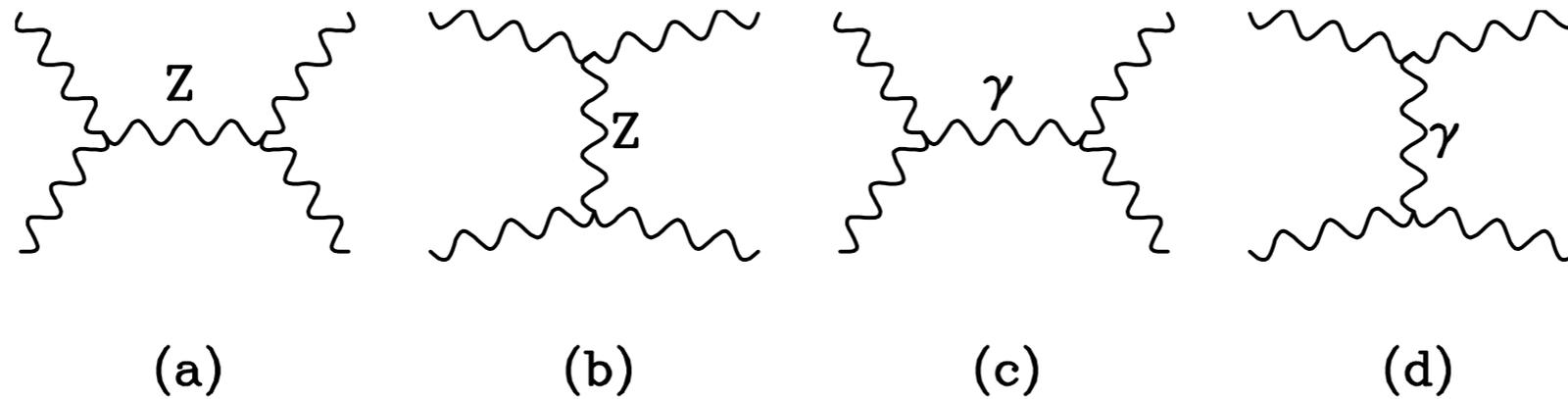


Leading term ( $p^4$ ) cancels,  
but  $p^2$  remains:

$$T^e = g_w^2 \left\{ \frac{p^4}{M_W^4} \left[ -3 + 6 \cos \theta + \cos^2 \theta \right] + \frac{p^2}{M_W^2} \left[ -4 + 6 \cos \theta + 2 \cos^2 \theta \right] \right\}$$

$$T^{a-d} = g_w^2 \left\{ \frac{p^4}{M_W^4} \left[ 3 - 6 \cos \theta - \cos^2 \theta \right] + \frac{p^2}{M_W^2} \left[ \frac{9}{2} - \frac{11}{2} \cos \theta - 2 \cos^2 \theta \right] \right\}$$

# Result (continued)



Cancellation  
now complete

$$T^{f-g} = g_w^2 \left\{ \frac{p^2}{M_W^2} \left[ -\frac{1}{2} - \frac{1}{2} \cos \theta \right] - \frac{M_H^2}{4M_W^2} \left[ \frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right] \right\}$$

$$T^e = g_w^2 \left\{ \frac{p^4}{M_W^4} \left[ -3 + 6 \cos \theta + \cos^2 \theta \right] + \frac{p^2}{M_W^2} \left[ -4 + 6 \cos \theta + 2 \cos^2 \theta \right] \right\}$$

$$T^{a-d} = g_w^2 \left\{ \frac{p^4}{M_W^4} \left[ 3 - 6 \cos \theta - \cos^2 \theta \right] + \frac{p^2}{M_W^2} \left[ \frac{9}{2} - \frac{11}{2} \cos \theta - 2 \cos^2 \theta \right] \right\}$$

# Discussion

- As a result,  $WW$  scattering amplitude does not diverge at high-energy.
  - however, it may still be too large for perturbative unitarity to hold

- Can examine using a partial-wave analysis.

Lee, Quigg and Thacker (1977)

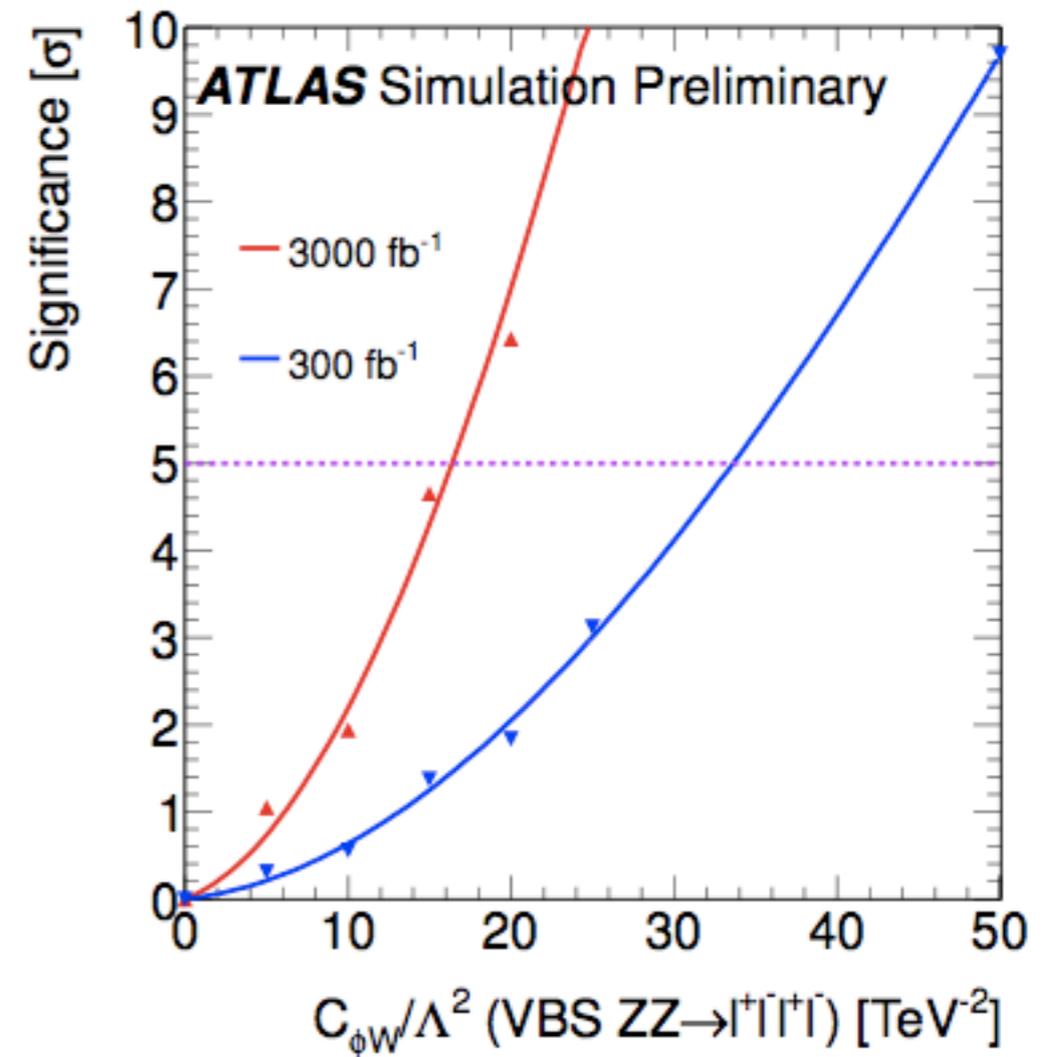
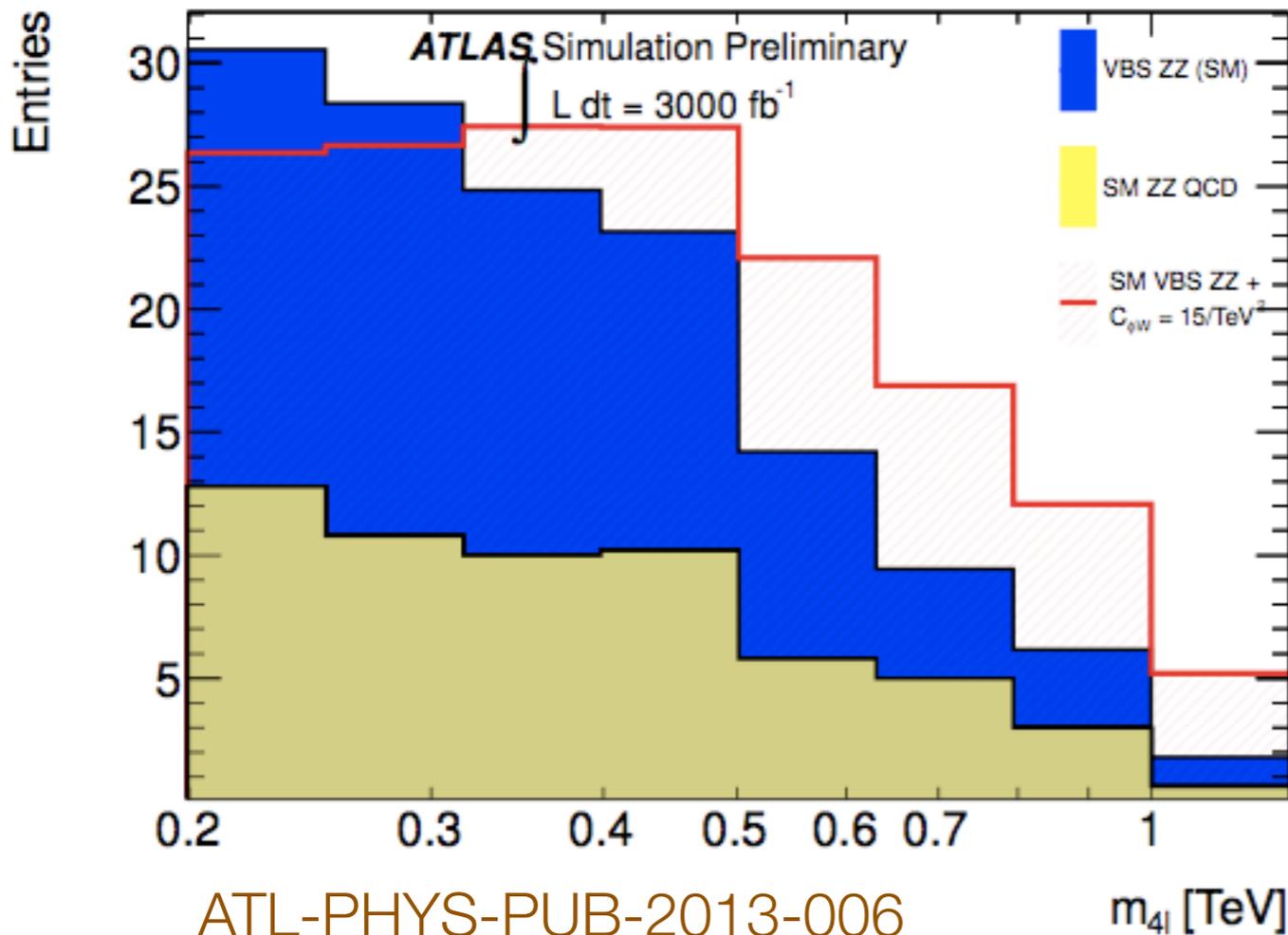
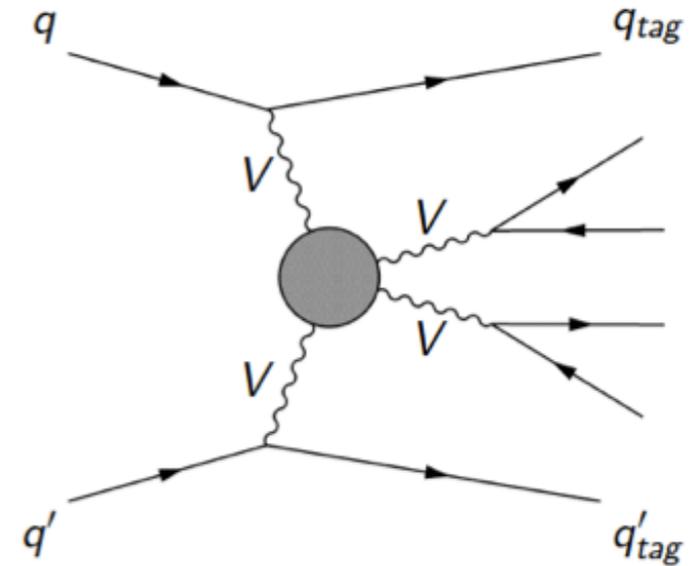
- Looking at all channels of vector boson scattering and requiring unitarity results in a constraint on the Higgs boson mass:

$$M_H < \left( \frac{8\sqrt{2}\pi}{3G_F} \right)^{\frac{1}{2}} \approx 1 \text{ TeV}$$

- Observation of a Higgs boson violating this bound would have meant strong interactions of  $W, Z$  bosons that could not be described perturbatively.
- Even with a light Higgs, it is possible that it is not entirely responsible for the unitarization at high energies
  - essential to probe vector boson scattering to look for anomalous couplings/ hints of new particles.

# Recent study

- Like vector-boson fusion: induce scattering in association with two forward jets.
- Sensitivity to operators not probed in diboson production ( $C_{\phi W}$  here).
- $\sigma_{SM} \sim 0.5$  pb (w/o decays), need v.high luminosity.



# Tri-boson production

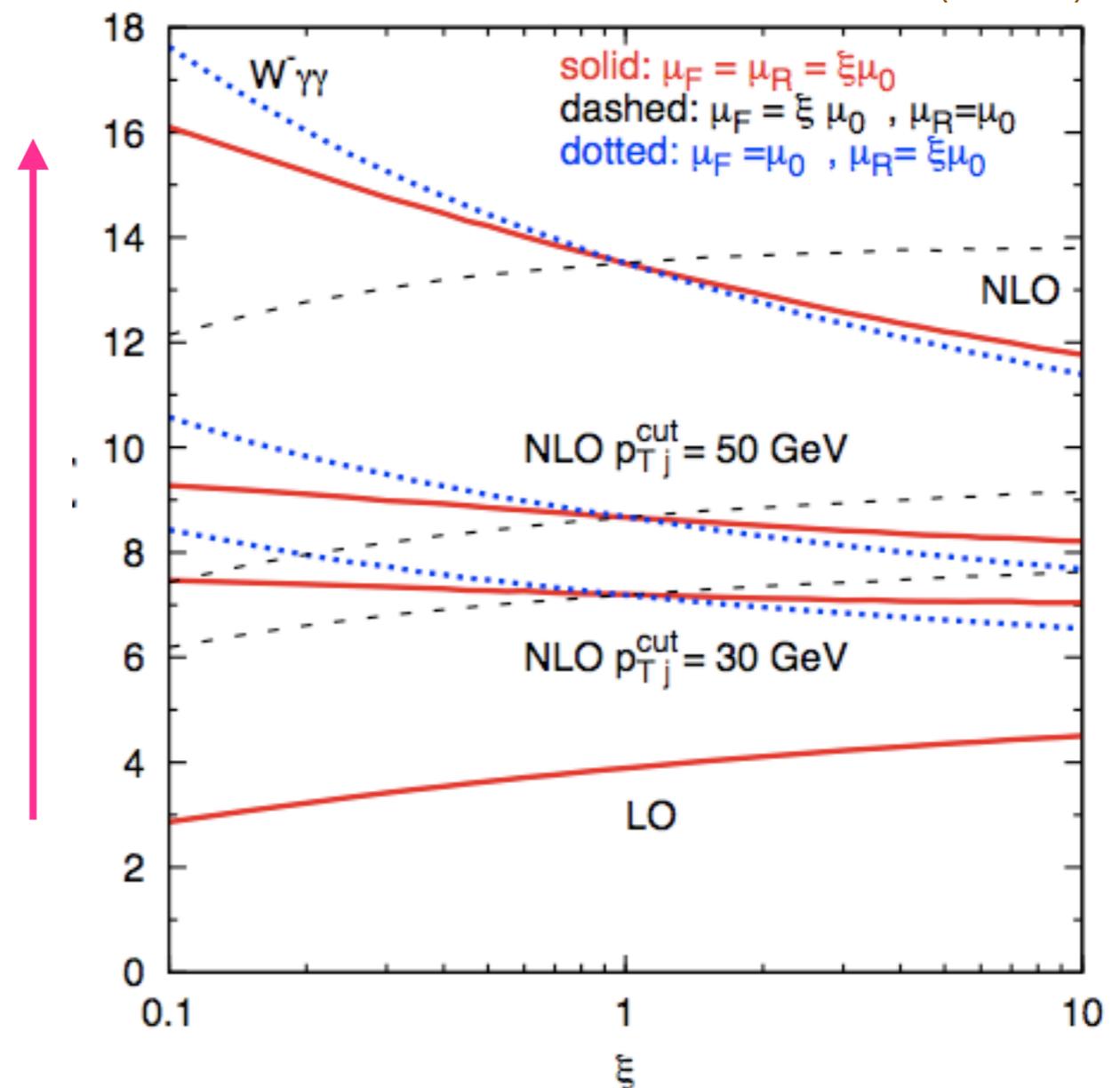
- Cross sections very small: after including decays and cuts, cross sections are in the region of tens of femtobarns (at most).
- All modes available in VBFNLO,  $Z\gamma\gamma$  in MCFM.
- Example:  $W\gamma\gamma$  scale dependence (VBFNLO).

Large enhancement due to gluon flux

Even after vetoing real radiation, significant enhancement due to RAZ

- Era of tri-boson measurements just beginning at LHC.

Bozzi et al (2011)



# Summary

- **The importance of multi-boson production.**
  - role of self-interactions (gauge structure) in taming high-energy behaviour
- **Review of selected di-boson phenomenology.**
  - radiation amplitude zero, jet-binning, aTGCs, interference
- **Beyond inclusive di-boson measurements.**
  - the importance of vector boson scattering