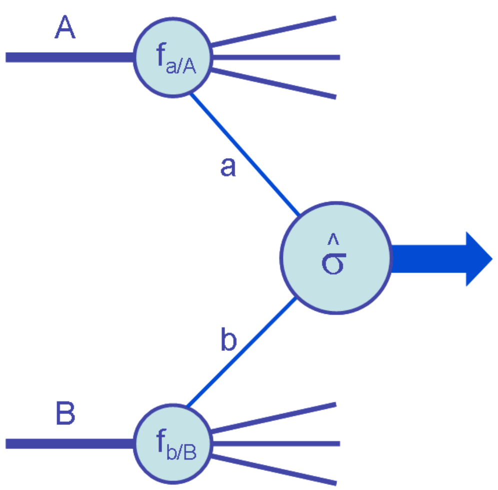
Hard Interactions

John Campbell, University of Glasgow

UK HEP Forum, “LHC Start-up”, April 12th 2007

Hard interactions

- Scattering processes at high-energy colliders can be classified as either hard or soft.
- The underlying theory is always QCD, but the approach and level of understanding differs considerably.
- For hard processes, rates and event properties can be predicted to good precision using perturbation theory.



$$\sigma_{AB} = \int dx_a dx_b \; f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \; \hat{\sigma}_{ab\to X}$$

Such *factorization* is a general feature of hard scattering processes.

Any hard scale (Q^2) will do, e.g. particle with large enough mass (Drell-Yan) or high E_T object (inclusive jets).

Soft processes

- For instance:
  - determining the total cross section;
  - predicting properties of the underlying event;
  - understanding multiple interactions.
- These are dominated by non-perturbative effects that are much more poorly understood.
- Of course, usually we must understand both soft and hard interactions to make definitive comparisons. **Fortunately, my charge here is to discuss the latter.**

Shameless plug

- A lot of material for this talk is borrowed from a recent review paper for the IOP.

Institute of Physics Publishing — Reports on Progress in Physics

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Hard interactions of quarks and gluons: a primer for LHC physics

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Recipe for a calculation

$$\sigma_{AB} = \int dx_a dx_b\, f_{a/A}(x_a, \mu_F^2)\, f_{b/B}(x_b, \mu_F^2)\ \times\ [\,\hat{\sigma}_0\ +\ \alpha_S(\mu_R^2)\,\hat{\sigma}_1\ +\ \dots]_{ab\to X}\ .$$

At any order in perturbation theory, the cross section is formally independent of the hard scale that separates long and short distance physics (μ_F) and the QCD coupling scale (μ_R).

(1) Identify the leading-order partonic process that contributes to the hard interaction producing X.

(2) Calculate the corresponding ME's, starting with $\hat{\sigma}_0$.

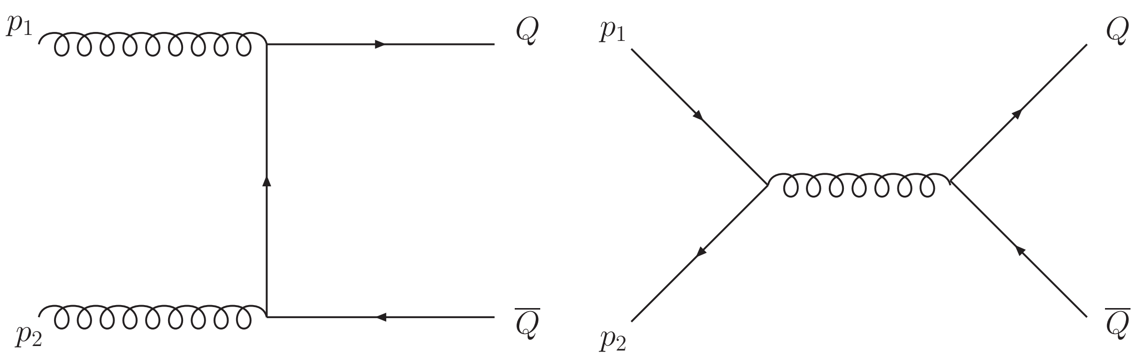
(3) Combine with appropriate combinations of pdfs for the initial-state partons a and b.

(4) Make a specific choice for the scales μ_F and μ_R.

(5) Perform a numerical integration over the energy fractions x_a, x_b and the phase-space for the state X.

Top pair production

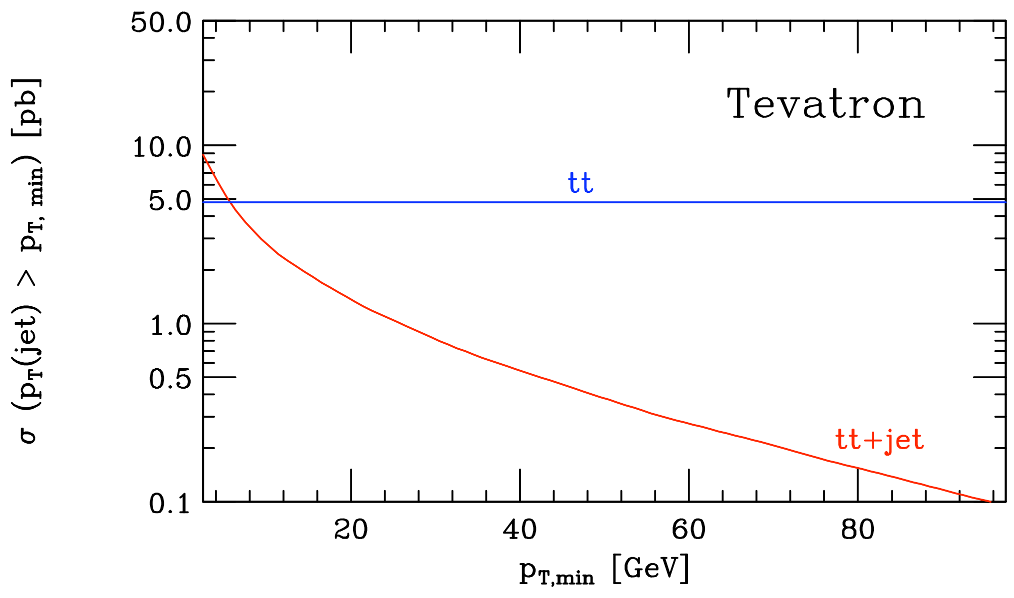
- An archetypal hard process, with the top quark mass setting the scale.

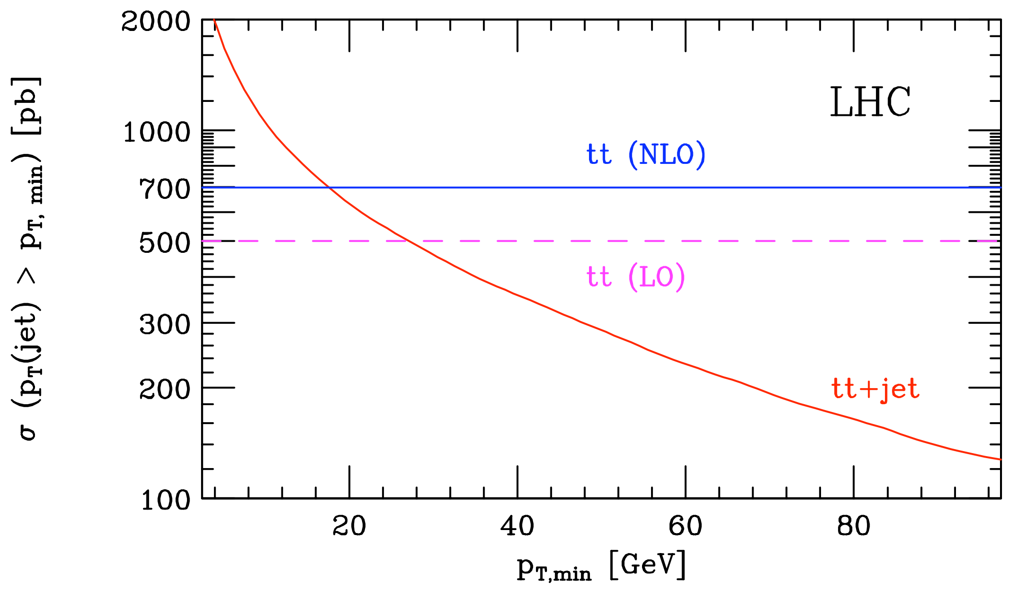


- One event per second at the LHC - great for studying top quark properties.
- Bad for backgrounds, e.g. Higgs production, especially through weak boson fusion.
- With so many of these events, we will also need to understand the production of this state with additional jets (especially for the ttH search).

How hard is hard?

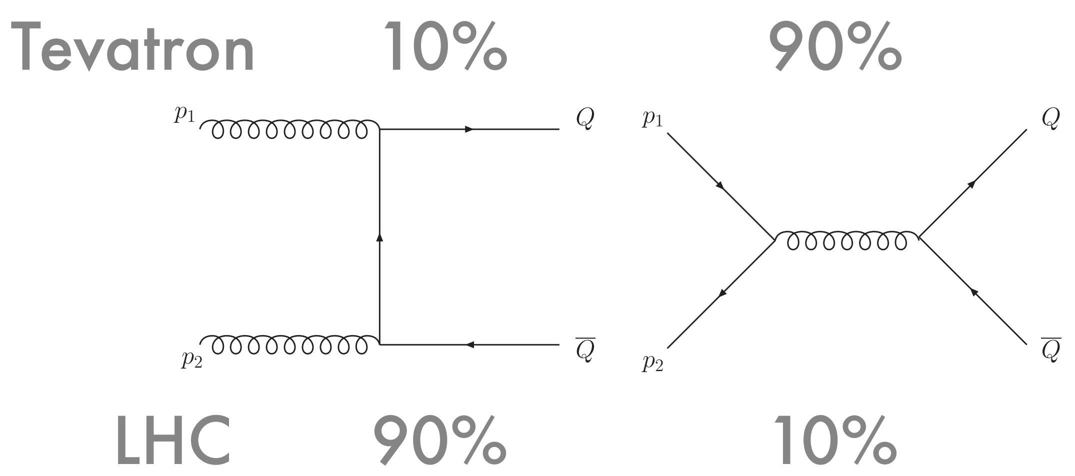
- Adding jets introduces another hard scale, below which collimated energy deposits are not considered jets.
- Required by theoretical (parton-level) predictions.
- Typical jet transverse momenta at the Tevatron are ~10-20 GeV.
- Such cuts don't seem so good at the LHC - not hard enough?





From Tevatron to the LHC

- Even when producing quite heavy objects the typical momentum fractions of partons in the proton are small, leading to a dominance of gluon (and sea quark) scattering.
- Top production provides a classic example.

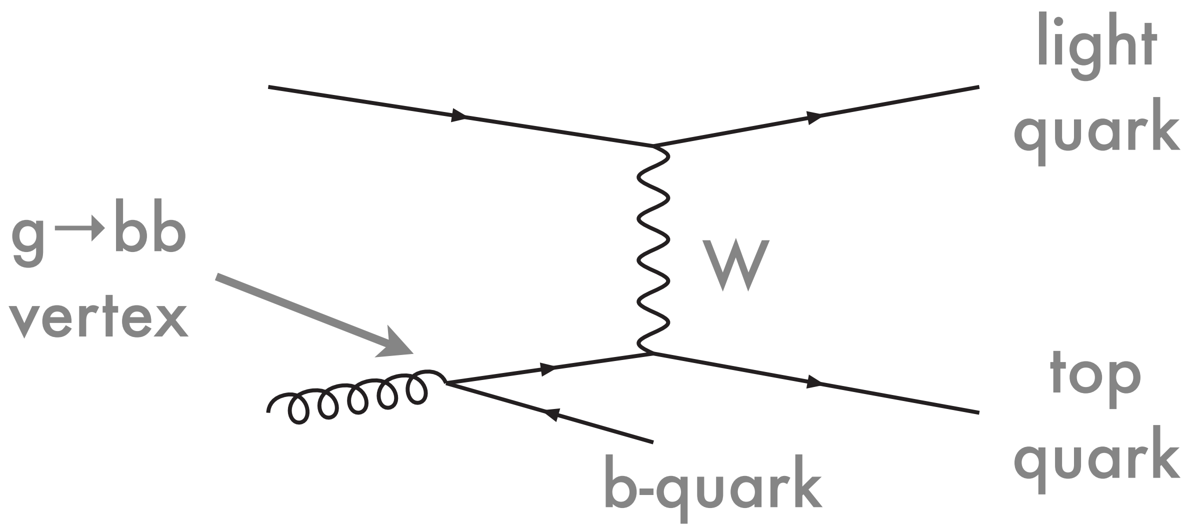


- This has important consequences for the phenomonology of hard interactions at the LHC.
- Scattering at the LHC is not simply *rescaled* scattering at the Tevatron.

Step One

(1) Identify the leading-order partonic process that contributes to the hard interaction producing X.

- This seems simple enough, by equating jets to partons.
- However, even this can cause headaches.
- Example: single top production (one every 2s).
- The primary mode is via t-channel exchange of a W:



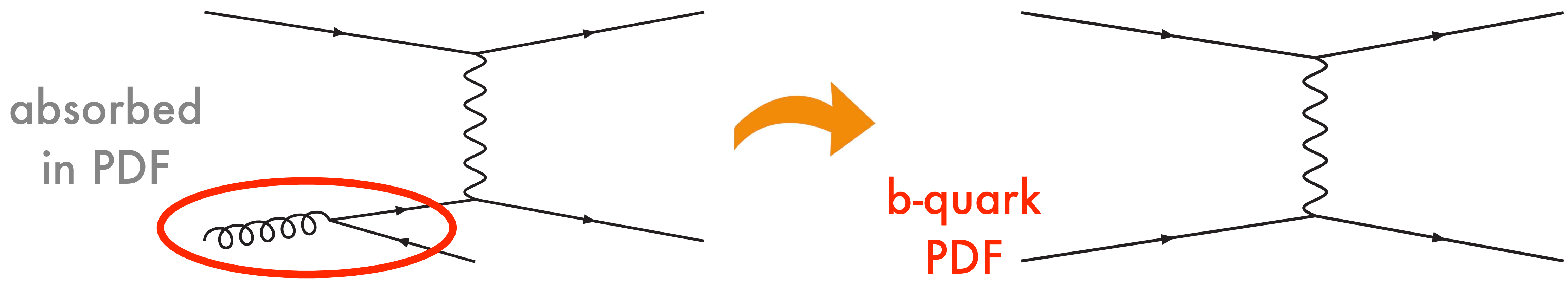
Usually, we are not interested in the fate of the b-quark and so integrate it out.

This is possible because the b-mass regulates the collinear divergence.

Collinear region still dominates the rate.

Alternative formulation

- The effect of this collinear region can be factored out of the matrix elements and instead put into the PDF.



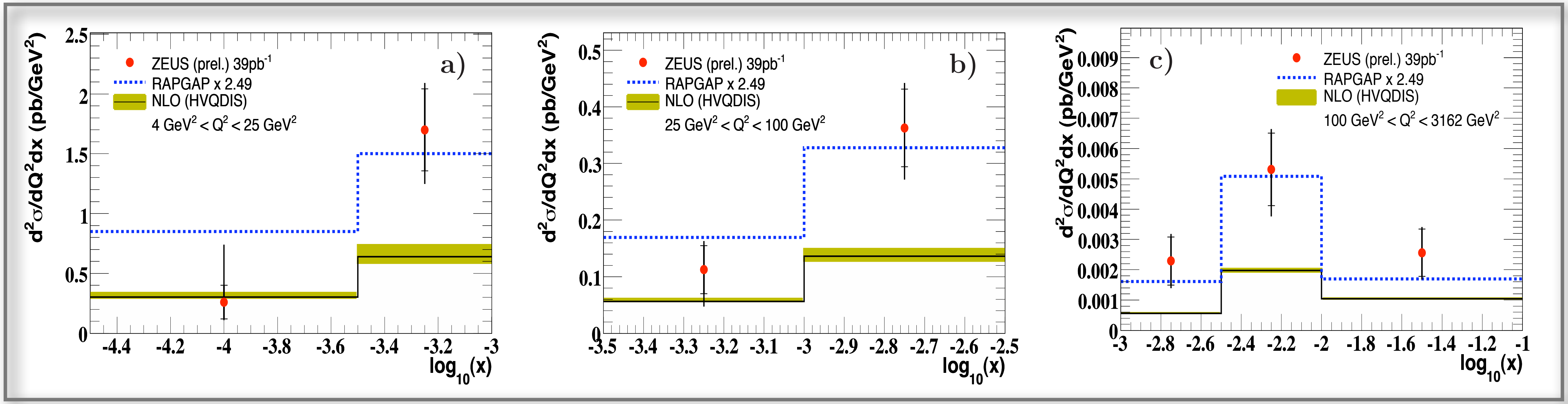
- The enhancement is logarithmic ~ $\alpha_S \log(m_t / m_b)$ and so can be resummed to all orders in the DGLAP approach.

  ☑ more convergent perturbative expansion;

  ☑ simpler calculation.

b-content of the proton

- Many calculations for the LHC make use of the b-pdf approach, e.g. associated Wt, H/W/Z + b's.
- They rely on the perturbatively-derived b PDF.
- Steps towards measurement presented by ZEUS in 2006.



- The LHC should be able to improve on this.

Step two

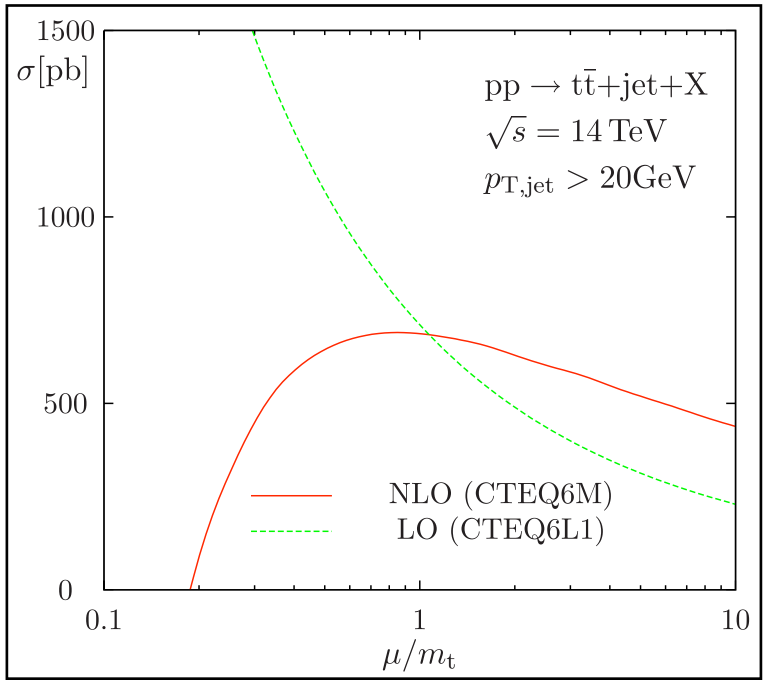
(2) Calculate the corresponding ME's, starting with $\hat{\sigma}_0$.

- The leading-order partonic matrix elements are nowadays readily calculated numerically.
- Practical limit (about 8 particles) due to the growth in the number of diagrams and calculation of colour.

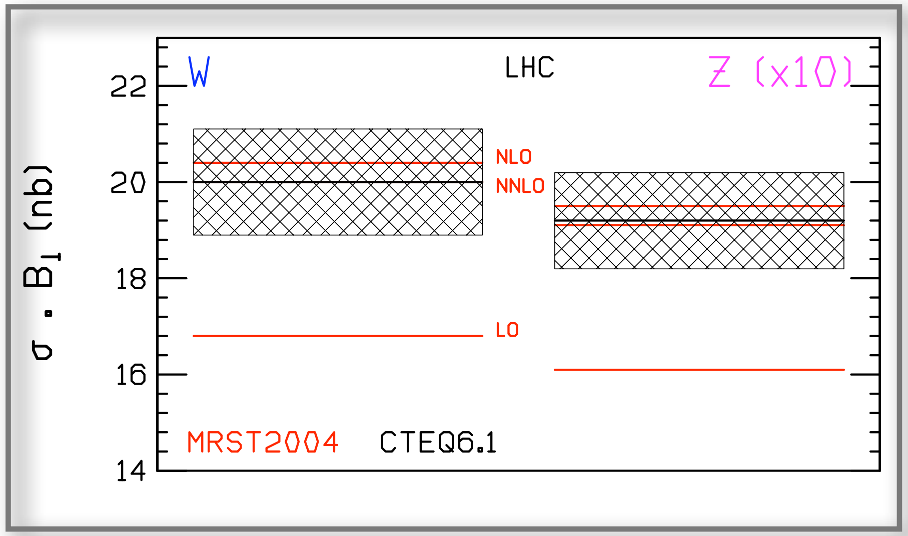
  can extend using "twistor-inspired" analytic methods.
- A number of different packages complete the calculation with efficient phase-space sampling integration.
- Very useful, but limited by the usual failings of LO.

Beyond LO

- Logarithms from the evolution of the pdfs and the running of α_s begin to be explicitly cancelled.
  - ☑ reduced scale dependence.
- Check of the reliability of perturbation theory.
- Better sensitivity to real physics.
  - ☑ more proton content, more realistic jets.

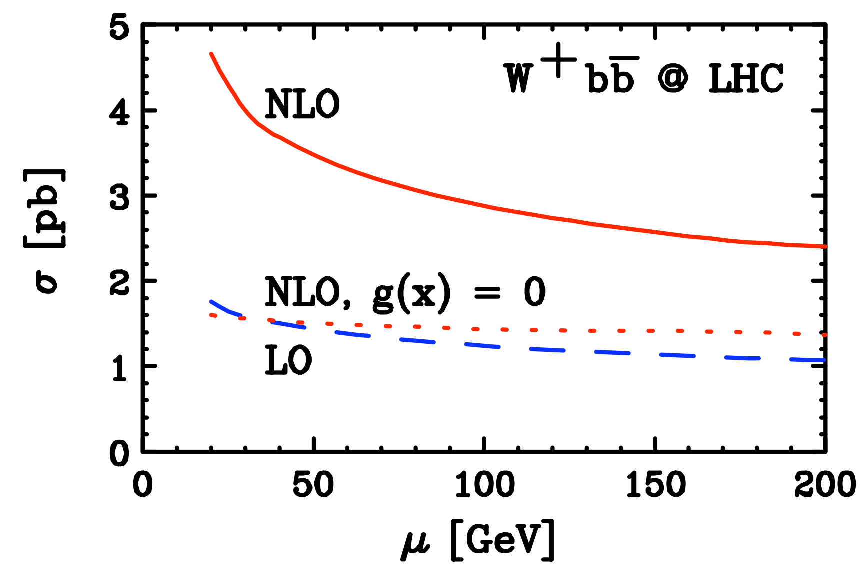


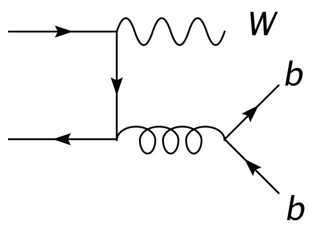
Dittmaier, Uwer & Weinzierl, hep-ph/0703120



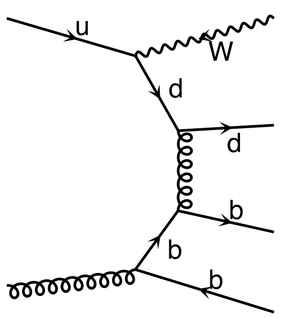
Caveat emptor

- Beware: this is not always the case.
- Example 1: additional large contributions that only appear at higher orders.
- Example 2: scale variation at LO is unusually small.

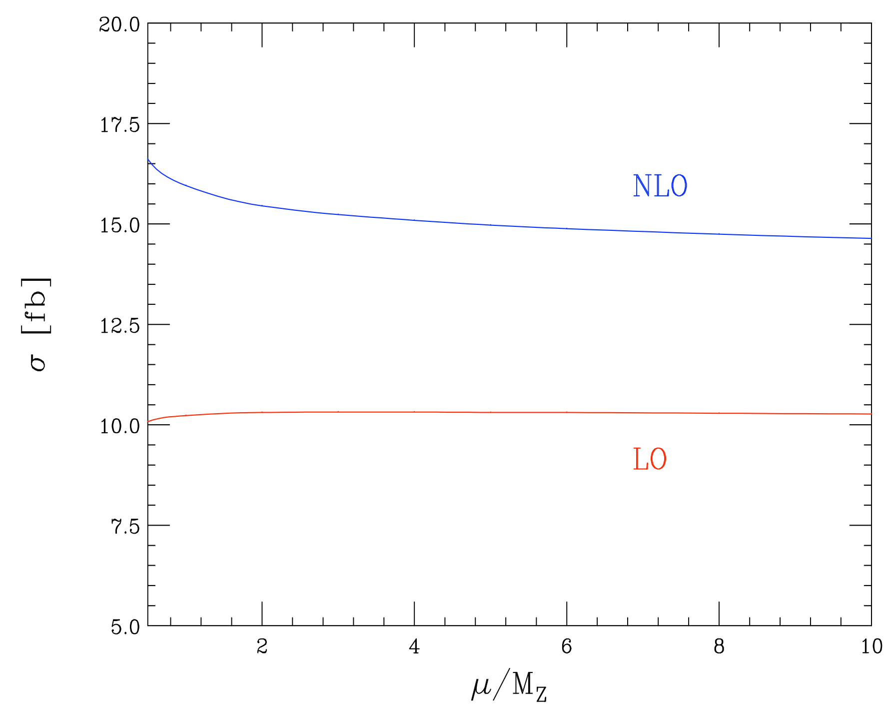




LO



NLO

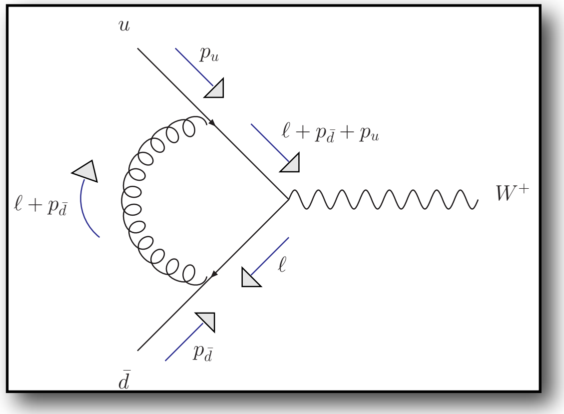


pp → ZZZ at the LHC

Lazopoulos & Melnikov, hep-ph/0703273

Loops and legs

- The bottleneck for NLO and beyond has always been the evaluation of loop diagrams.
  - dependence on particle masses and external momenta
  - historically, calculations performed case-by-case and by brute force



$$\int \frac{d^4\ell \quad \mathcal{N}}{\ell^2(\ell + p_{\bar{d}})^2(\ell + p_{\bar{d}} + p_u)^2}$$

- No complete NLO QCD calculation for a process involving more than 5 particles.
- Further progress unlikely to be made in this fashion.

Particle multiplicity

- With the increased c.o.m. energy of the LHC and the extended coverage of the CMS and ATLAS detectors, the number of particles seen in each event will be high.
- Pure QCD processes will be backgrounds to many signals of new physics.
- The best way to normalize a background cross-section is to measure it (e.g. sideband analysis) but that may not always be possible - especially during the early years.
- In that case, higher orders in QCD are the best option.

Wishlists

- One may concoct extravagant wishlists of backgrounds that one would like to know at NLO.
- Limited manpower necessitates prioritization by necessity and feasibility.

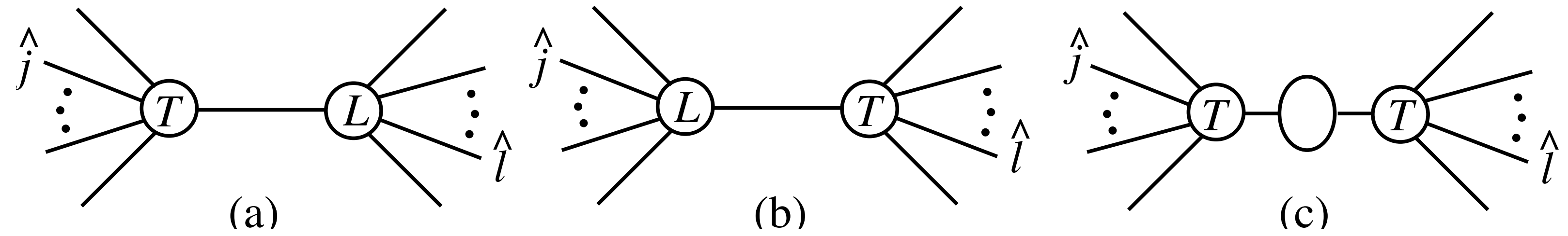
process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$+jet	$t\bar{t}H$, new physics
☑ $pp \to H + 2$ jets	H production by vector boson fusion (VBF)
3. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
☑ $pp \to t\bar{t} +$ ~~2~~ 1 jets	$t\bar{t}H$
5. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
7. $pp \to V + 3$ jets	various new physics signatures
☑ $pp \to$ ~~$V\,V\,V$~~ ZZZ	SUSY trilepton searches

Les Houches workshop, 2005

Dominated by final states containing b-quarks, high p_T leptons and missing energy.

One-loop advances

- Massive progress from many approaches - “generalized unitarity” and different flavours of recursion relation.
  - together these enabled analytic results to be obtained for all helicity combinations of 6-gluon amplitudes.
- Extending the tree-level recursion relations (MHV, CSW, BCFW) to the one-loop level is difficult.



- The complications due to the analytic structure beyond LO (branch cuts) must be handled with care.

Recent progress

- One “bootstrap method” has been applied to obtain new results for a number of one-loop *n*-gluon and (*n*-2) gluon, 2 quark amplitudes. *Berger, Bern, Dixon, Forde, Kosower*
  - Mostly limited to specific helicities (e.g. MHV).
- Most of these methods apply only to gluons or (maybe) massless quarks.
- Very important contribution, but we’re often interested in the quark masses (e.g. in the wishlist).
- Implementation in a full MC is at least one step away.

Numerical approaches

- A completely different avenue has been taken by a number of numerical approaches to the loop integrals.

- One such method uses a "semi-numerical" calculation.

  Giele, Ellis, Zanderighi

internal masses

external momenta

$$\mathcal{M}_{\mathrm{virt}} = \sum_n c_n(m_i^2, p_i^2, \ldots) I_n(\epsilon, m_i^2, p_i^2, \ldots)$$

calculate numerically using recursion relations → coefficients

scalar integrals containing divergences

- Successfully included in a full calculation of H+2 jets.

Latest developments

- Another idea is to perform the loop integration itself numerically.
  - extract singularities using sector decomposition.
    Binoth, Heinrich
  - deform contours to avoid internal thresholds.
    Soper, Nagy
- This was the method used to calculate the NLO corrections to ZZZ production.
- Downside: slow!

  " ... ten thousand kinematic points required a few days of running on a cluster of several dozen processors."
- Has also been applied to a 2-loop calculation. *Anastasiou, Beerli, Daleo*

NNLO

- Calculation of 2-loop corrections has advanced considerably in the last ~10 years.
- Hadron collider matrix elements are now known for:
  - ☑ Drell Yan, Higgs production (gg fusion & with W/Z);
  - ☑ dijet, diphoton production;
  - ☑ production of a vector boson and a single jet.
- Particle multiplicity much smaller than at NLO.
- Very few of these have become full NNLO predictions.

Another bottleneck?

- Go back to the original list.
- I've talked about the first two points.
- R. Thorne has covered pdfs.
- I've discussed scale dependence.

Recipe for a calculation

$$\sigma_{AB} = \int dx_a dx_b \, f_{a/A}(x_a, \mu_F^2) \, f_{b/B}(x_b, \mu_F^2) \, \times \, [\hat{\sigma}_0 \, + \, \alpha_S(\mu_R^2) \, \hat{\sigma}_1 \, + \, \ldots]_{ab \to X} \, .$$

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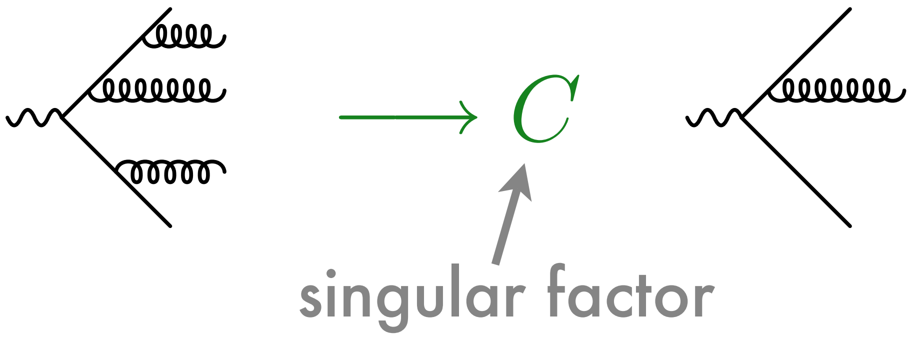
- ☑ Combine with appropriate combinations of pdfs for the initial-state partons a and b.
- ☑ Make a specific choice for the scales μ_F and μ_R.

(5) Perform a numerical integration over the energy fractions x_a, x_b and the phase-space for the state X.

The only step left - and the culprit - is the phase space integration.

Real radiation

- The loop diagrams are only part of the calculation.
- At NLO, machinery has been developed that extracts the *universal* soft and collinear singularities from the real radiation contribution (slicing, subtraction).
- This is important for *exclusive* cuts and measurements.
- At NNLO, the radiation can be triple collinear, doubly soft or collinear, or soft-collinear.



- Extracting the singularities without double-counting is a major hurdle that has to be overcome.

Strategies

- In the NNLO calculations that have been completed so far, the integrals have been computed directly using sector decomposition. *Anastasiou, Menilkov, Petriello*
  - 2→2 processes with only initial or final singularities.
- A more obvious approach is to try to generalize the NLO subtraction/slicing methods to NNLO.
- This has been successfully applied to the subleading in colour (QED-type) contributions to $e^+e^- \to 3$ jets. *Gehrmann-de Ridder, Gehrmann, Glover, Heinrich*
- With a full calculation, can try to extend to pp → V+jet.

From the archives ...

- Talk based on seminal paper calculating QCD corrections to the gluonic width of the upsilon.

CLNS 81/504
July 1981

QCD CORRECTIONS TO THE GLUONIC WIDTH OF THE Υ MESON

Paul B. Mackenzie* and G. Peter Lepage
Newman Laboratory of Nuclear Studies,
Cornell University, Ithaca, NY 14853

"Use of existing methods would require the assembly of a small army of Kinoshitas and Lindquists to devote many man-years of effort working on computer programs for many separate processes."

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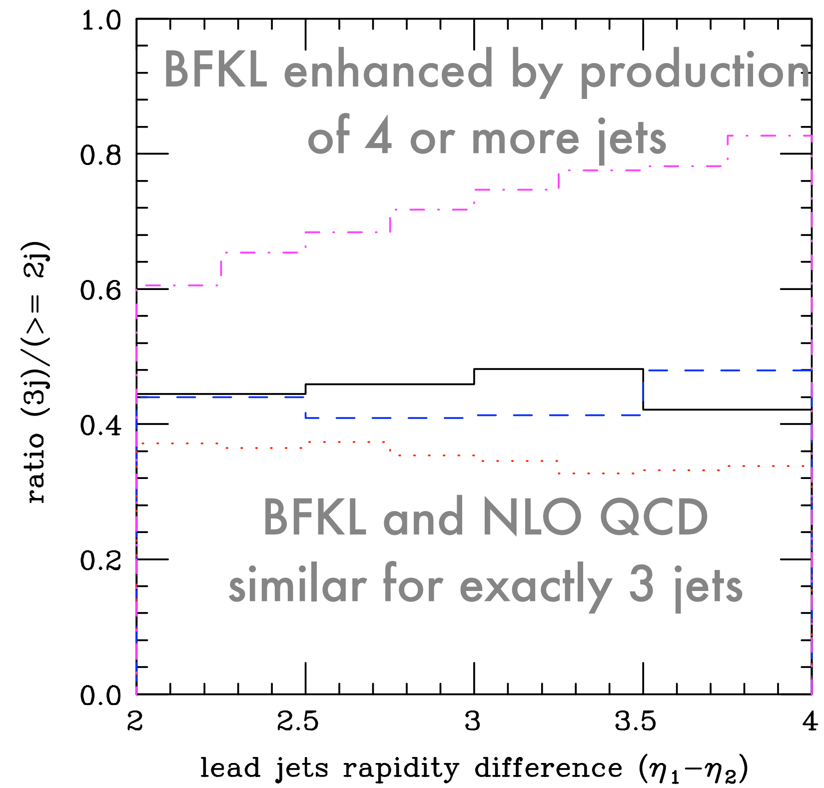
"The ideal would be the creation of a master program which for any desired process would generate the graphs, assign the momenta in the loops, evaluate the gamma matrix traces and colour algebra, and perform the integrals."

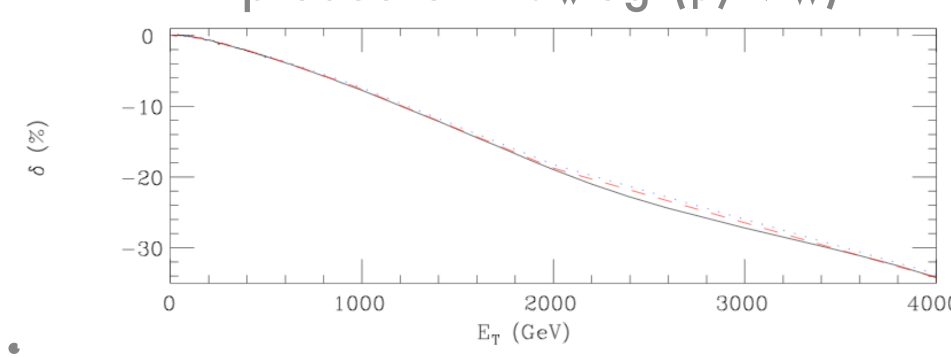
Other effects

- We may need to incorporate other descriptions of the data at the LHC.
- Example 1: the BFKL formalism predicts an enhanced rate of emission of jets between two widely-separated ones.
- Example 2: electroweak corrections may also be large, at least in certain kinematic regimes.



Andersen et al.

EW corrections to dijet production ~ $\alpha_W \log^2(\mu/M_W)$



Moretti, Nolten, Ross

Conclusions

- Understanding hard interactions more completely is essential to exploit the LHC to its fullest potential.
- In the early days, we should learn quite quickly how our current understanding of these hard interactions extrapolates to the new energy regime.
- We may find that we need to re-examine some of our rules-of-thumb and require more from pQCD.
- Thanks to a lot of hard work in recent years, an immense amount of progress has already been made: more on the way by the time data is rolling in.