Minimal Universal Extra Dimensions in CalcHEP/CompHEP

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We present implementation of minimal universal extra dimensions (MUED) in CalcHEP/CompHEP. We include not only all KK number conserving interactions of level-1 and level-2 Kaluza-Klein (KK) particles but also KK number violating interactions as well. The 1-loop corrected mass spectrum and 2-body decay widths of KK particles are automatically calculated as constraints for a given set of inputs, i.e., radius of extra dimension and cut-off scale of the model. The effect on running coupling constants due to the existence of extra dimensions has been incorporated properly. We have done extensive cross-checks with various publications numerically and analytically. Our model file has been used for relevant physics studies and provided for experimental groups.

I. INTRODUCTION

We assume that users are already familiar with CalcHEP/CompHEP [1, 2] and will only explain our implementation, convention of new particle names, and parameters/constraints that users need to know. In section II, we discuss implementation of MUED in CalcHEP/CompHEP considering each *.mdl file. Section III is reserved for the discussion about cross-checks we have done. Appendix includes relevant Lagrangian and Feynman rules for level 1 KK particles.

II. MODEL FILES

We take the simplest version of universal extra dimensions compactified on $S_1/Z_2$ orbifold [3] and the mass spectrum is evaluated based on 1-loop calculations given in [4]. We have checked numerical values of KK masses with authors’ private code. We also include KK number violating interaction of $f_0 \gamma_\mu A_0^\mu P_L/R f_0$ (or boundary interaction) in [4]. The Weinberg angles for KK states are small [4] and we ignore them in our implementation. So KK-photon ($\gamma_n$) is the hypercharge gauge boson ($B_n$) and KK-Z boson ($Z_n$) is $W_n^{\mu 3}$. There exists a mixing between $SU(2)_W$-doublet and $SU(2)_W$-singlet KK fermions and we ignore this mixing angle since it is suppressed by $vR$ where $v$ is the vacuum expectation value of the standard model higgs and $R$ is the radius of extra dimension (See Appendix for details.).

A. Particles

We show a list of KK particles which are newly introduced in the model file. The superscript represents KK number, either $n = 1$ or $n = 2$ and subscript represents either Lorentz index ($\mu$) for vector particles or chirality ($L$ or $R$) for fermions. One should keep in mind that all KK fermions are vector-like and the chirality represents their transformation property under $SU(2)_W$. There are also standard model (SM) particles which are not shown in the list below. Names of particles can be easily understood. KK-lepton whose name ends with ‘L’ (‘R’) and KK-quark whose name starts with ‘D’ (‘S’) are doublets (singlets) under $SU(2)_W$. 
TABLE I: KK gauge bosons

<table>
<thead>
<tr>
<th>Name</th>
<th>A</th>
<th>A+</th>
<th>2*spin</th>
<th>mass</th>
<th>width</th>
<th>color</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g^1_{\mu}$</td>
<td>KG</td>
<td>KG</td>
<td>2</td>
<td>MKG</td>
<td>wKG</td>
<td>8</td>
</tr>
<tr>
<td>$B_{\mu}$</td>
<td>B1</td>
<td>B1</td>
<td>2</td>
<td>MB1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$Z^1_{\mu}$</td>
<td>Z1</td>
<td>Z1</td>
<td>2</td>
<td>MZ1</td>
<td>wZ1</td>
<td>1</td>
</tr>
<tr>
<td>$W^1_{\mu}$</td>
<td>~W+</td>
<td>~W-</td>
<td>2</td>
<td>MW1</td>
<td>wW1</td>
<td>1</td>
</tr>
<tr>
<td>$g^2_{\mu}$</td>
<td>~G2</td>
<td>~G2</td>
<td>2</td>
<td>MKG2</td>
<td>wKG2</td>
<td>8</td>
</tr>
<tr>
<td>$B_{\mu}$</td>
<td>B2</td>
<td>B2</td>
<td>2</td>
<td>MB2</td>
<td>wB2</td>
<td>1</td>
</tr>
<tr>
<td>$Z^2_{\mu}$</td>
<td>Z2</td>
<td>Z2</td>
<td>2</td>
<td>MZ2</td>
<td>wZ2</td>
<td>1</td>
</tr>
<tr>
<td>$W^2_{\mu}$</td>
<td>~W2</td>
<td>~w2</td>
<td>2</td>
<td>MW2</td>
<td>wW2</td>
<td>1</td>
</tr>
</tbody>
</table>

TABLE II: KK leptons

<table>
<thead>
<tr>
<th>Name</th>
<th>A</th>
<th>A+</th>
<th>2*spin</th>
<th>mass</th>
<th>width</th>
<th>color</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^1_L$</td>
<td>~eL</td>
<td>~EL</td>
<td>1</td>
<td>DMe</td>
<td>wDe1</td>
<td>1</td>
</tr>
<tr>
<td>$\mu^1_L$</td>
<td>~mL</td>
<td>~ML</td>
<td>1</td>
<td>DMm</td>
<td>wDe2</td>
<td>1</td>
</tr>
<tr>
<td>$e^1_R$</td>
<td>~eR</td>
<td>~ER</td>
<td>1</td>
<td>SME</td>
<td>wSe1</td>
<td>1</td>
</tr>
<tr>
<td>$\mu^1_R$</td>
<td>~mR</td>
<td>~MR</td>
<td>1</td>
<td>SMm</td>
<td>wSe2</td>
<td>1</td>
</tr>
<tr>
<td>$\nu^1_{\mu}$</td>
<td>~n3</td>
<td>~N3</td>
<td>1</td>
<td>DMt1</td>
<td>wDn3</td>
<td>1</td>
</tr>
<tr>
<td>$\nu^1_{\nu}$</td>
<td>~n2</td>
<td>~N2</td>
<td>1</td>
<td>DMm1</td>
<td>wDn2</td>
<td>1</td>
</tr>
</tbody>
</table>

B. Parameters

In MUED, there are only two additional parameters introduced, radius of extra dimension and cut-off scale of the model. For convenience, inverse radius $R^{-1}$ and the number of KK states $\Lambda R$ are used in the model file. There are a couple of other variables that one can in principle change but these are not model parameters. $R_G$ is used to turn on and off the running of coupling constants and scale $\mu$ is the energy scale at which coupling are evaluated. A few other parameters are introduced for proper running of coupling constants. We summarized parameters in table IV.

C. Constraints

In constraints, we have coded masses and widths of KK particles. Therefore they are automatically computed by CalcHEP/CompHEP when numerical session calculates values of variables defined as constraints. The names of variables for these masses and 2 body decay widths are already summarized in section II A. Partial list of 2 body decay widths can be found in [5–7] and our formulas in constraints agree with their expressions. Defining widths as constraints is very useful since we do not need to launch separate numerical session to calculate widths that are necessary for the calculation.

The table V shows values for masses and widths of $n = 1$ KK particles for $R^{-1} = 500\text{ GeV}$ and $\Lambda R = 20$. The values given here are evaluated with $\alpha_e = 1/128$, $\alpha_s = 0.118$, $M_Z = 91.1882\text{ GeV}$, $M_W = 80.419\text{ GeV}$, $G_F = 1.1663910^{-5}\text{ GeV}^{-2}$, $m_t = 175\text{ GeV}$ and $m_b = 4.2\text{ GeV}$. All the widths calculated by CalcHEP/CompHEP are 2 body widths. While these are enough for all KK particles, it may not be the case for top-quark singlet($t^3_1$). For low $R^{-1}$ (such that $\Delta m = m_{t^3_1} - m_{B_1} < m_t$) where the decay $t^3_1 \rightarrow tB_1$ is disallowed, we need calculate 3 body (or higher) decay widths manually by launching separate CalcHEP/CompHEP sessions and supply them as an input parameter to CalcHEP/CompHEP. Values of other variables are also defined in constraints for convenience. They are summarized in table VI.
The number of KK states

TABLE III: KK quarks

<table>
<thead>
<tr>
<th>Name</th>
<th>A+</th>
<th>2*spin</th>
<th>mass</th>
<th>width</th>
<th>color</th>
<th>Name</th>
<th>A+</th>
<th>2*spin</th>
<th>mass</th>
<th>width</th>
<th>color</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_L</td>
<td>Du</td>
<td>DU</td>
<td>1</td>
<td>DMu</td>
<td>wDu</td>
<td>3</td>
<td>u_L^2</td>
<td>Du</td>
<td>~ DU</td>
<td>1</td>
<td>DMu2</td>
</tr>
<tr>
<td>d_L</td>
<td>Dd</td>
<td>DD</td>
<td>1</td>
<td>DMd</td>
<td>wDd</td>
<td>3</td>
<td>d_L^2</td>
<td>DD</td>
<td>~ DD</td>
<td>1</td>
<td>DMd2</td>
</tr>
<tr>
<td>c_L</td>
<td>Dc</td>
<td>DC</td>
<td>1</td>
<td>DMc</td>
<td>wDc</td>
<td>3</td>
<td>c_L^2</td>
<td>DC</td>
<td>~ DC</td>
<td>1</td>
<td>DMc2</td>
</tr>
<tr>
<td>s_L</td>
<td>Ds</td>
<td>DS</td>
<td>1</td>
<td>DMs</td>
<td>wDs</td>
<td>3</td>
<td>s_L^2</td>
<td>DS</td>
<td>~ DS</td>
<td>1</td>
<td>DMs2</td>
</tr>
<tr>
<td>t_L</td>
<td>Dt</td>
<td>DT</td>
<td>1</td>
<td>DMtop</td>
<td>wDt</td>
<td>3</td>
<td>t_L^2</td>
<td>DT</td>
<td>~ DT</td>
<td>1</td>
<td>DMtop2</td>
</tr>
<tr>
<td>b_L</td>
<td>Db</td>
<td>DB</td>
<td>1</td>
<td>DMb</td>
<td>wDb</td>
<td>3</td>
<td>b_L^2</td>
<td>DB</td>
<td>~ DB</td>
<td>1</td>
<td>DMb2</td>
</tr>
<tr>
<td>u_R</td>
<td>Su</td>
<td>SU</td>
<td>1</td>
<td>SMu</td>
<td>wSu</td>
<td>3</td>
<td>u_R^2</td>
<td>Su</td>
<td>~ Su</td>
<td>1</td>
<td>SMu2</td>
</tr>
<tr>
<td>d_R</td>
<td>Sd</td>
<td>SD</td>
<td>1</td>
<td>SMD</td>
<td>wSd</td>
<td>3</td>
<td>d_R^2</td>
<td>SD</td>
<td>~ SD</td>
<td>1</td>
<td>SMD2</td>
</tr>
<tr>
<td>c_R</td>
<td>Sc</td>
<td>SC</td>
<td>1</td>
<td>SMc</td>
<td>wSc</td>
<td>3</td>
<td>c_R^2</td>
<td>SC</td>
<td>~ SC</td>
<td>1</td>
<td>SMc2</td>
</tr>
<tr>
<td>s_R</td>
<td>Ss</td>
<td>SS</td>
<td>1</td>
<td>SMs</td>
<td>wSs</td>
<td>3</td>
<td>s_R^2</td>
<td>SS</td>
<td>~ SS</td>
<td>1</td>
<td>SMs2</td>
</tr>
<tr>
<td>t_R</td>
<td>St</td>
<td>ST</td>
<td>1</td>
<td>SMtop</td>
<td>wSt</td>
<td>3</td>
<td>t_R^2</td>
<td>ST</td>
<td>~ ST</td>
<td>1</td>
<td>SMtop2</td>
</tr>
<tr>
<td>b_R</td>
<td>Sb</td>
<td>SB</td>
<td>1</td>
<td>SMb</td>
<td>wSb</td>
<td>3</td>
<td>b_R^2</td>
<td>SB</td>
<td>~ SB</td>
<td>1</td>
<td>SMb2</td>
</tr>
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TABLE IV: Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Default values</th>
<th>Symbols</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rinv</td>
<td>500</td>
<td>$R^{-1}$</td>
<td>Inverse of the extra dimensional radius</td>
</tr>
<tr>
<td>LR</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RG</td>
<td></td>
<td></td>
<td>The number of KK states</td>
</tr>
<tr>
<td>scaleN</td>
<td>2</td>
<td>n</td>
<td>Renormalization scale, $\mu = \frac{n}{R}$</td>
</tr>
<tr>
<td>n</td>
<td>2</td>
<td>1</td>
<td>turn on the effect of the running coupling constants</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>turn it off</td>
</tr>
<tr>
<td>cb1</td>
<td>6.8333</td>
<td>$b_1$</td>
<td>The coefficient of the SM $\beta$-function for $U(1)_Y$</td>
</tr>
<tr>
<td>cb2</td>
<td>-3.1667</td>
<td>$b_2$</td>
<td>The coefficient of the SM $\beta$-function for $SU(2)_W$</td>
</tr>
<tr>
<td>cb3</td>
<td>-7</td>
<td>$b_3$</td>
<td>The coefficient of the KM $\beta$-function for $SU(3)_c$</td>
</tr>
<tr>
<td>cb1t</td>
<td>6.8333</td>
<td>$\tilde{b}_1$</td>
<td>The coefficient of the KK $\beta$-function for $U(1)_Y$</td>
</tr>
<tr>
<td>cb2t</td>
<td>-2.8333</td>
<td>$\tilde{b}_2$</td>
<td>The coefficient of the KK $\beta$-function for $SU(2)_W$</td>
</tr>
<tr>
<td>cb3t</td>
<td>-6.5</td>
<td>$\tilde{b}_3$</td>
<td>The coefficient of the KM $\beta$-function for $SU(3)_c$</td>
</tr>
<tr>
<td>c1MZ</td>
<td>98.4151</td>
<td>$\alpha_1^{-1}$</td>
<td>$\alpha_1^{-1}(\mu = M_Z)$</td>
</tr>
<tr>
<td>c2MZ</td>
<td>29.5846</td>
<td>$\alpha_2^{-1}$</td>
<td>$\alpha_2^{-1}(\mu = M_Z)$</td>
</tr>
<tr>
<td>c3MZ</td>
<td>8.53244</td>
<td>$\alpha_3^{-1}$</td>
<td>$\alpha_3^{-1}(\mu = M_Z)$</td>
</tr>
</tbody>
</table>

D. Vertices: 4-point interaction with gluons

Our model file includes all interactions of level-1 and level-2 KK particles except KK-Higgses. The phenomenology of KK-Higgses can be very different depending on their masses and SM higgs mass. However it is not clear how to calculate mass corrections, and also in principle a bulk mass can exist and change the mass of KK-higgs (see [4] for detail). Therefore we ignore interaction involving KK-Higgses. The Lagrangian for UED can easily derived and we include detailed description in the Appendix. In this section, we only point out how to deal with 4-point interaction with gluons in CalcHEP/CompHEP since this vertex requires a special treatment.

The Lagrangian for the 4-gluon interactions is the following

$$\mathcal{L}_4 = -\frac{1}{4}g^2 f^{abc} f^{ade} G^{0,b,c} G^{d,e} - \frac{g^2}{2} f^{abc} f^{ade} G^{0,d,c} G^{b,e} G^{1} - \frac{g^2}{4} \left(f^{abc} (G^{0,b,c} G^{d,e} + G^{0,c,e} G^{b,d})\right)^2 - \frac{1}{4} \frac{3}{2} g^2 f^{abc} f^{ade} \tilde{G}^{1,b,c} \tilde{G}^{d,e} G^{1} G^{1}. \tag{1}$$

The color structure of this 4-point interaction cannot be directly written down in the CalcHEP/CompHEP format. Hence to implement this vertex in CalcHEP/CompHEP, we use the following trick. We introduce three auxiliary tensor fields
<table>
<thead>
<tr>
<th>Bosons</th>
<th>Mass (GeV)</th>
<th>Width (Formula, GeV)</th>
<th>Width (by CalcHEP/CompHEP, GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1$</td>
<td>641.34</td>
<td>4.2102</td>
<td>4.207153</td>
</tr>
<tr>
<td>$g_2$</td>
<td>1223.5</td>
<td>12.574</td>
<td>12.56804</td>
</tr>
<tr>
<td>$B_1$</td>
<td>500.89</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$B_2$</td>
<td>1000.3</td>
<td>0.29402</td>
<td>0.292527</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>535.63</td>
<td>0.075725</td>
<td>0.07567860</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>1048.6</td>
<td>0.6292</td>
<td>0.631493</td>
</tr>
<tr>
<td>$W^+_1$</td>
<td>535.31</td>
<td>0.073502</td>
<td>0.07345485</td>
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<td>$W^+_2$</td>
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<td>0.63078</td>
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<table>
<thead>
<tr>
<th>Quark Doublets</th>
</tr>
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<tbody>
<tr>
<td>$u^D_1$</td>
</tr>
<tr>
<td>$d^D_1$</td>
</tr>
<tr>
<td>$s^D_1$</td>
</tr>
<tr>
<td>$c^D_1$</td>
</tr>
<tr>
<td>$b^D_1$</td>
</tr>
<tr>
<td>$t^D_1$</td>
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</table>

<table>
<thead>
<tr>
<th>Quark Singlets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^S_1$</td>
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<td>$d^S_1$</td>
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<tr>
<td>$s^S_1$</td>
</tr>
<tr>
<td>$c^S_1$</td>
</tr>
<tr>
<td>$b^S_1$</td>
</tr>
<tr>
<td>$t^S_1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lepton Doublets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^D_1$</td>
</tr>
<tr>
<td>$\mu^D_1$</td>
</tr>
<tr>
<td>$\tau^D_1$</td>
</tr>
<tr>
<td>$\nu^e_1$</td>
</tr>
<tr>
<td>$\nu^\mu_1$</td>
</tr>
<tr>
<td>$\nu^\tau_1$</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Lepton Singlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^S_1$</td>
</tr>
<tr>
<td>$\mu^S_1$</td>
</tr>
<tr>
<td>$\tau^S_1$</td>
</tr>
</tbody>
</table>

**TABLE V:** Masses and 2-body decay widths of $n = 1$ KK particles for $R^{-1} = 500$ GeV and $\Lambda R = 20$.

$t^a_{\mu\nu}$, $s^a_{\mu\nu}$, and $u^a_{\mu\nu}$ in the same line as the original CalcHEP/CompHEP approach for SM gluons. So with them, the Lagrangian looks like

$$
\mathcal{L} = -\frac{1}{2} g^a_{\mu\nu} a^a_{\mu\nu} + i \frac{g}{\sqrt{2}} f^a_{\mu\nu} f^{abc} G^{0b\mu} G^{0c\nu} + i \frac{g}{\sqrt{2}} f^a_{\mu\nu} f^{abc} G^{1b\mu} G^{1c\nu} - \frac{1}{2} s^a_{\mu\nu} s^{a\mu\nu} + \frac{g}{2} s^a_{\mu\nu} f^{abc} G^{1b\mu} G^{1c\nu} \\
- \frac{1}{2} u^a_{\mu\nu} u^{a\mu\nu} + i \frac{g}{\sqrt{2}} u^a_{\mu\nu} f^{abc} \left( G^{0b\mu} G^{1c\nu} + G^{1b\mu} G^{0c\nu} \right) \\
- \frac{1}{2} \left( f^a_{\mu\nu} - ig \frac{1}{\sqrt{2}} f^{abc} G^{0b\mu} G^{0c\nu} - ig \frac{1}{\sqrt{2}} f^{abc} G^{1b\mu} G^{1c\nu} \right)^2 \\
- \frac{1}{4} g^2 f^{abc} f^{ade} \left( G^{0b\mu} G^{0c\nu} + G^{1b\mu} G^{1c\nu} \right) \left( G^{0d\mu} G^{0e\nu} + G^{1d\mu} G^{1e\nu} \right) \\
- \frac{1}{2} \left( s^a_{\mu\nu} - ig \frac{1}{\sqrt{2}} f^{abc} G^{1b\mu} G^{1c\nu} \right)^2 - \frac{1}{8} g^2 f^{abc} f^{ade} G^{1b\mu} G^{1c\nu} G^{1d\mu} G^{1e\nu} \\
- \frac{1}{2} \left( u^a_{\mu\nu} - ig \frac{1}{\sqrt{2}} f^{abc} G^{0b\mu} G^{0c\nu} \right)^2 - \frac{1}{4} g^2 \left( f^{abc} \left( G^{0b\mu} G^{1c\nu} + G^{1b\mu} G^{0c\nu} \right) \right)^2,
$$

We can find that the functional integration over three auxiliary tensor fields reproduces the terms for 4-gluon interactions. In our model file, two additional auxiliary fields ($s^a_{\mu\nu}$ and $u^a_{\mu\nu}$) are introduced.
E. Running of coupling constants

In the SM, the couplings

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z}, \quad (3)$$

are solutions of following simple differential equation

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{1}{2\pi} b_i, \quad (4)$$

where $M_Z$ is the mass of $Z$ and $\alpha_i = \frac{g_i^2}{4\pi}$, $t = \ln \mu$ and $l = 1, 2, 3$, corresponding to the gauge group $U(1)_Y \times SU(2)_W \times SU(3)_C$. ($\alpha_1 = \frac{2}{3}g_Y$ and $b_1 = \frac{2}{3}b_Y$) For any gauge group, the coefficients are given by

$$b = \frac{11}{3}C_{adj} + \frac{2}{3} \sum_f C_f + \frac{1}{6} \sum_h C_h, \quad (5)$$

where $C_{adj}$ is the Dynkin index of the adjoint representation of the gauge group, $C_f$ is the Dynkin index of the representation of the left-handed Weyl fermions and $C_h$ is that of the representation of the real Higgs field. $C_{adj} = n$ for $SU(n)(n > 2)$ and $C_f = \frac{1}{2}$ for the fundamental representation of $SU(2)_W$ and $C_f = 2Y_f^2$ for $U(1)_Y$. Applying this formula for one Higgs doublet and $n_f$ chiral families, we find

$$b_1 = \frac{4}{3}n_f + \frac{1}{10}, \quad b_2 = -\frac{22}{3} + \frac{4}{3}n_f + \frac{1}{6}, \quad b_3 = -11 + \frac{4}{3}n_f. \quad (6)$$

In the standard model, with $n_f = 3$, the numerical values of these coefficients are $(b_1, b_2, b_3) = (\frac{41}{10}, -\frac{17}{6}, -\frac{7}{2})$.

The gauge couplings run faster in extra dimensions and the previous differential equation is modified as follows \[8\].

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i - \tilde{b}_i}{2\pi} - \frac{\tilde{b}_i X_\delta}{2\pi} \left( \frac{\mu}{\mu_0} \right)^\delta, \quad (7)$$

where $\delta$ is the number of extra dimensions and $X_\delta = \frac{2e^\delta}{2!/(3/2)}$. The new beta function coefficients $\tilde{b}_i$ correspond to the contributions of the Kaluza-Klein states at each massive KK excitation level. The solution becomes

$$\alpha_i^{-1} = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z} - \frac{\tilde{b}_i}{2\pi} \ln \frac{\mu}{\mu_0} - \frac{\tilde{b}_i X_\delta}{2\pi \delta} \left[ \left( \frac{\mu}{\mu_0} \right)^\delta - 1 \right]. \quad (8)$$

In the minimal universal extra dimension, new $\beta$ functions \[9, 10\] are

$$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\frac{41}{10}, -\frac{17}{6}, \frac{13}{2}), \quad (9)$$
using
\[ b = -\frac{11}{3} C_{adj} + \frac{2}{3} \sum_f C_f + \frac{1}{6} \sum_h C_h + \frac{1}{6} C_{adj}. \]  

\((10)\)

alpha1 = \alpha_Y, alpha1 = \alpha_2 and alpha1 = \alpha_3 are in constraints.

### III. DISCUSSION

We have done extensive cross-checks for our implementation. We used a private code of authors of [4] to check our calculation of 1-loop corrected masses. To check correct implementation of interaction vertices, we have computed cross sections for selected diagrams in certain scattering processes and compared with analytic expressions from CalcHEP/CompHEP. We have checked the consistency between our calculations of 2 body decay widths and calculations by CalcHEP/CompHEP. Our analytic formulas for decay widths agree with the expression which can be found in [5–7]. Some of interaction vertices (especially self-interactions of gauge bosons) are checked with LanHEP [11].

We have compared various physical distributions such as cross-sections and invariant mass distributions with ones in published papers [5, 12–14]. We have used our codes for relevant physics studies such as precise calculation of Kaluza-Klein dark matter [15] and we have extensively checked analytic expressions for all (co)annihilation cross-sections of level 1 KK particles with [16–19]. We were able to reproduce all results. Our model file has been used for various collider studies [7, 15, 20–26], as well as PythiaUED [27, 28] to implement matrix elements in PYTHIA [29]. The current limit from the Tevatron on the compactification scale, set by searches for trilepton events using this PythiaUED, is around 280 GeV at 95% C.L. [30].

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**Appendix**

### A. Boundary conditions on fields

We consider universal extra dimensions in 5 dimensions with one extra dimension compactified [3]. In 5 dimension, we encounter two problems. First we can not write down the usual Dirac mass terms if chiral fermions are not properly introduced. Second we have the fifth component (pseudo scalar in 4D) of 5D vector field after the compactification so we have too many zero mode scalar particles in 4D. To introduce chiral fermions and project out unwanted parts we use \(Z_2\) symmetry on the \(S^1\). There are two fixed points in this geometry \(S^1/Z_2\) called the orbifold. We can see this geometry is still invariant under the exchange of two fixed points (\(Z_2\)). This symmetry is called the KK parity. We impose the following special boundary conditions for fermions and vector fields at the fixed points. We want scalar field to be either even or odd under the transformation \(P_5: y \rightarrow -y\) then at \(y = 0, \pi R\)

\[ \partial_5 \phi^+ = 0 \quad \text{for even fields} \]
\[ \phi^- = 0 \quad \text{for odd fields} \]

\(\partial_5\) is the derivative with respect to extra dimensional coordinate. These are Neumann and Dirichlet boundary conditions respectively at the fixed points. Associated KK expansions are

\[ \phi^+(x,y) = \frac{1}{\sqrt{2\pi R}} \phi^+_0 + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi^+_n(x) \cos \frac{ny}{R}, \]
\[ \phi^-(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi^-_n(x) \sin \frac{ny}{R}, \]  

(A.1)
where $x$ is the 4-dimensional spacetime coordinate $x^\mu = (t, x, y, z)$ and $y$ is the extra dimensional coordinate. $R$ is the size of the extra dimension and $n$ represents the KK-level. $n = 0$ is the SM mode. The orbifold compactification forces the first four components to be even under $P_5$ while the fifth component is odd:

$$
\begin{align*}
\partial_5 A^\mu &= 0 \\
A^5 &= 0
\end{align*}
$$

at the fixed points. Hence the KK expansion of a vector field is

$$
A_\mu(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ A^0_\mu(x) + \sqrt{2} \sum_{n=1}^\infty A^0_\mu(x) \cos \left( \frac{ny}{R} \right) \right\},
$$

$$
A_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^\infty A^n_5(x) \sin \left( \frac{ny}{R} \right).
$$

For fermions imposing following boundary conditions

$$
\begin{align*}
\partial_5 \psi^+_R &= 0 \\
\psi^+_L &= 0 \\
\partial_5 \psi^+_L &= 0 \\
\psi^+_R &= 0
\end{align*}
$$

at $y = 0$, $\pi R$ or

$$
\begin{align*}
\partial_5 \psi^-_R &= 0 \\
\psi^-_L &= 0 \\
\partial_5 \psi^-_L &= 0 \\
\psi^-_R &= 0
\end{align*}
$$

at $y = 0$, $\pi R$,

respective KK mode expansions are

$$
\begin{align*}
\psi^+(x, y) &= \frac{1}{\sqrt{2\pi R}} \psi^0_R(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^\infty \left( \psi^+_R(x) \cos \left( \frac{ny}{R} \right) + \psi^+_L(x) \sin \left( \frac{ny}{R} \right) \right),
\psi^-(x, y) &= \frac{1}{\sqrt{2\pi R}} \psi^0_L(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^\infty \left( \psi^-_R(x) \cos \left( \frac{ny}{R} \right) + \psi^-_L(x) \sin \left( \frac{ny}{R} \right) \right).
\end{align*}
$$

So the zero mode is either right handed or left handed. However KK modes come in chiral pairs. This chiral structure is a natural consequence of the orbifold boundary conditions.

### B. The Standard Model in 5D

The 5 dimensional SM fields are defined as follows

$$
\begin{align*}
H(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ H(x) + \sqrt{2} \sum_{n=1}^\infty H_n(x) \cos \left( \frac{ny}{R} \right) \right\},
B_\mu(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ B^0_\mu(x) + \sqrt{2} \sum_{n=1}^\infty B^n_\mu(x) \cos \left( \frac{ny}{R} \right) \right\},
B_5(x, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^\infty B^n_5(x) \sin \left( \frac{ny}{R} \right),
W_\mu(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ W^0_\mu(x) + \sqrt{2} \sum_{n=1}^\infty W^n_\mu(x) \cos \left( \frac{ny}{R} \right) \right\},
W_5(x, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^\infty W^n_5(x) \sin \left( \frac{ny}{R} \right),
G_\mu(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ G^0_\mu(x) + \sqrt{2} \sum_{n=1}^\infty G^n_\mu(x) \cos \left( \frac{ny}{R} \right) \right\},
G_5(x, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^\infty G^n_5(x) \sin \left( \frac{ny}{R} \right).
\end{align*}
$$
\[ Q(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ q_L(x) + v_\mu^2 \sum_{n=1}^{\infty} \left[ P_L Q_L^n(x) \cos\left(\frac{ny}{R}\right) + P_R Q_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\}, \]
\[ U(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ u_R(x) + v_\mu^2 \sum_{n=1}^{\infty} \left[ P_R u_R^n(x) \cos\left(\frac{ny}{R}\right) + P_L u_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\}, \]
\[ D(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ d_R(x) + v_\mu^2 \sum_{n=1}^{\infty} \left[ P_R d_R^n(x) \cos\left(\frac{ny}{R}\right) + P_L d_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\}, \]
\[ L(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ l_0(x) + v_\mu^2 \sum_{n=1}^{\infty} \left[ P_L l_L^n(x) \cos\left(\frac{ny}{R}\right) + P_R l_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\}, \]
\[ E(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ e_R(x) + v_\mu^2 \sum_{n=1}^{\infty} \left[ P_R e_R^n(x) \cos\left(\frac{ny}{R}\right) + P_L e_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\}, \]

where \( H(x, y) \) is the 5D scalar field and \((B_\mu(x, y), B_\nu(x, y)), (W_\mu(x, y), W_\nu(x, y)) \) and \((G_\mu(x, y), G_\nu(x, y)) \) are the 5D gauge fields for \( U(1)_Y, SU(2)_W \) and \( SU(3)_c \) respectively. \( Q(x, y) \) and \( L(x, y) \) are the \( SU(2)_W \) fermion doublets while \( U(x, y), D(x, y) \) and \( E(x, y) \) are respectively the generic singlet fields for the up-type quark, the down-type quark and the lepton. The \( SU(2)_W \) and \( SU(3)_c \) gauge fields are

\[ W_M = W_M^a \frac{\tau^a}{2}, \]
\[ G_M = G_M^A \frac{\lambda^A}{2}, \]

where \( \tau^a \)'s are usual Pauli's matrices and \( \lambda^A \)'s are usual Gell-Mann matrices. \( P_{L,R} = \frac{1 + \gamma_5}{2} \) and \( M = \mu, 5 \) and \( \mu = 0, 1, 2, 3 \). Now we write 5 dimensional Lagrangian invariant under \( SU(3)_c \times SU(2)_W \times U(1)_Y \) and compactify over the orbifold to get 4 dimensional effective Lagrangian.

C. The \( \gamma \)-matrices, the metric, the gauge couplings and the covariant derivatives

First let us set up the convention for the ingredients that go into the Lagrangian. The gamma matrices in 5D

\[ \Gamma^M = (\gamma^\mu, i\gamma^5), \]  

satisfies the Dirac-Clifford algebra

\[ \{\Gamma^M, G^N\} = 2g^{MN}, \]

where \( g^{MN} \) is the 5D metric

\[ g_{MN} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}, \]

and \( g^{\mu\nu} = (+ - - - -) \) to be the usual 4D metric. The gauge couplings are denoted by

\[ g_i = g_i^{(5)} \sqrt{\pi R}, \]

where \( i = 1, 2, 3 \) stand for \( U(1)_Y, SU(2)_W \) and \( SU(3)_c \) and \( g_i^{(5)} \)'s are the 5-dimensional gauge couplings and \( g_i \)'s are the 4-dimensional gauge couplings.

The covariant derivative in 5D act on 5D fields in following ways.

\[ D_M L(x, y) = \left( \partial_M + ig_2^{(5)} W_M + i\frac{g_3^{(5)}}{2} g_1^{(5)} B_M \right) L(x, y), \]
\[ D_M E(x, y) = \left( \partial_M + i\frac{g_2^{(5)}}{2} g_1^{(5)} B_M \right) E(x, y), \]
\[ D_M Q(x, y) = \left( \partial_M + ig_3^{(5)} G_M + ig_2^{(5)} W_M + i\frac{g_3^{(5)}}{2} g_1^{(5)} B_M \right) Q(x, y), \]
\[ D_M U(x, y) = \left( \partial_M + ig_3^{(5)} G_M + i\frac{g_2^{(5)}}{2} g_1^{(5)} B_M \right) U(x, y), \]
\[ D_M D(x, y) = \left( \partial_M + ig_3^{(5)} G_M + i\frac{g_2^{(5)}}{2} g_1^{(5)} B_M \right) D(x, y). \]
The Lagrangian for five dimensional standard model is written as follows.

\[ \mathcal{L}_{\text{Gauge}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} W_{\mu}^{aMN} W^{a\mu MN} - \frac{1}{4} G_{\mu}^{A} G^{A\mu MN} \right\}, \]

\[ \mathcal{L}_{\text{GF}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ -\frac{1}{2} \left( \partial_{\mu} B_{\mu} - \xi \partial_{5} B_{5} \right)^{2} - \frac{1}{2 \xi} \left( \partial_{\mu} W_{\mu}^{a} - \xi \partial_{5} W_{5}^{a} \right)^{2} - \frac{1}{2 \xi} \left( \partial_{\mu} G_{\mu}^{A} - \xi \partial_{5} G_{5}^{A} \right)^{2} \right\}, \]

\[ \mathcal{L}_{\text{Leptons}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{L}(x,y) \Gamma^{M} D_{M} L(x,y) + i \bar{E}(x,y) \Gamma^{M} D_{M} E(x,y) \right\}, \]

\[ \mathcal{L}_{\text{Quarks}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{Q}(x,y) \Gamma^{M} D_{M} Q(x,y) + i \bar{U}(x,y) \Gamma^{M} D_{M} U(x,y) + i \bar{D}(x,y) \Gamma^{M} D_{M} D(x,y) \right\}, \]

\[ \mathcal{L}_{\text{Yukawa}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \lambda_{u} \bar{Q}(x,y) U(x,y) \tau^{2} H^{*}(x,y) + \lambda_{d} \bar{Q}(x,y) D(x,y) H(x,y) + \lambda_{e} \bar{L}(x,y) E(x,y) H(x,y) \right\}, \]

\[ \mathcal{L}_{\text{Higgs}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left[ \left( D_{M} H(x,y) \right)^{\dagger} \left( D^{M} H(x,y) \right) + \mu^{2} H^{\dagger}(x,y) H(x,y) - \lambda \left( H^{\dagger}(x,y) H(x,y) \right)^{2} \right]. \]

Now we integrate out the extra dimensional coordinate using the orthogonalties defined below.

E. The Orthogonality Relations of Trigonometric Functions

We expressed the 5 dimensional fields in terms of the trigonometric function due to the orbifold structure. Following orthogonality relations are very important when we compactify.

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos \left( \frac{m y}{R} \right) \cos \left( \frac{n y}{R} \right) = \frac{\pi R}{2} \delta_{m,n}, \]

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin \left( \frac{m y}{R} \right) \sin \left( \frac{n y}{R} \right) = \frac{\pi R}{2} \delta_{m,n}, \]

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos \left( \frac{m y}{R} \right) \cos \left( \frac{l y}{R} \right) \cos \left( \frac{k y}{R} \right) = \frac{\pi R}{4} \Delta_{mnl}^{2}, \]

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos \left( \frac{m y}{R} \right) \cos \left( \frac{l y}{R} \right) \sin \left( \frac{k y}{R} \right) = \frac{\pi R}{8} \Delta_{mnl}^{2}, \]

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin \left( \frac{m y}{R} \right) \sin \left( \frac{l y}{R} \right) \sin \left( \frac{k y}{R} \right) = \frac{\pi R}{8} \Delta_{mnl}^{3}, \]

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin \left( \frac{m y}{R} \right) \sin \left( \frac{l y}{R} \right) \cos \left( \frac{k y}{R} \right) = \frac{\pi R}{8} \Delta_{mnl}^{3}, \]

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos \left( \frac{m y}{R} \right) \sin \left( \frac{n y}{R} \right) = 0, \]

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin \left( \frac{m y}{R} \right) \sin \left( \frac{l y}{R} \right) = 0, \]

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos \left( \frac{m y}{R} \right) \cos \left( \frac{l y}{R} \right) \cos \left( \frac{k y}{R} \right) = 0, \]

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin \left( \frac{m y}{R} \right) \cos \left( \frac{n y}{R} \right) \cos \left( \frac{k y}{R} \right) = 0. \]
\[ \Delta^1_{mnk} = \delta_{m,n+} + \delta_{n,m+} + \delta_{m,n+} \]
\[ \Delta^2_{mnk} = \delta_{k,l+m+n} + \delta_{l,m+n+k} + \delta_{m,n+k+l} + \delta_{n,k+l+m} + \delta_{k+l,m+n} + \delta_{k+n,l+m} \]
\[ \Delta^3_{mnk} = -\delta_{k,l+m+n} - \delta_{l,m+n+k} - \delta_{m,n+k+l} - \delta_{n,k+l+m} - \delta_{k+l,m+n} - \delta_{k+n,l+m} \]
\[ \Delta^4_{mnk} = -\delta_{l,m+n} + \delta_{n,l+m} + \delta_{m,n+} \]
\[ \Delta^5_{mnk} = -\delta_{k,l+m+n} - \delta_{l,m+n+k} + \delta_{m,n+k+l} - \delta_{n,k+l+m} + \delta_{k+l,m+n} + \delta_{k+n,l+m} \]

F. The Lagrangian

1. The Gauge Sector

The 5D field strength tensors for \( U(1)_Y \), \( SU(2)_W \) and \( SU(3)_c \) are defined as follows

\[ B_{MN} = \partial_M B_N - \partial_N B_M , \]
\[ W^a_{MN} = \partial_M W^a_N - \partial_N W^a_M + g^2 \epsilon^{abc} W^a_b W^c_N , \]
\[ G^A_{MN} = \partial_M G^A_N - \partial_N G^A_M + g^3 f^{ABC} G^B_M G^C_N . \]

where \( \epsilon^{abc} \) and \( f^{ABC} \) are the structure constants for \( SU(2)_W \) and \( SU(3)_c \) respectively. The gauge kinetic terms and gauge fixing terms are given by

\[ \mathcal{L}_{Gauge} = \frac{1}{2} \int_{-\pi R}^{\pi R} \left\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} W^a_{MN} W^{aMN} - \frac{1}{4} G^A_{MN} G^{AMN} \right\} , \]
\[ \mathcal{L}_{GF} = \frac{1}{2} \int_{-\pi R}^{\pi R} \left\{ \frac{1}{2\xi} (\partial^\mu B_\mu - \xi \partial_5 B_5)^2 - \frac{1}{2\xi} (\partial^\mu W^a_\mu - \xi \partial_5 W^a_5)^2 - \frac{1}{2\xi} (\partial^\mu G^A_\mu - \xi \partial_5 G^A_5)^2 \right\} , \]

where \( \xi \) is the parameter in the generalized \( R_\xi \) gauge. For \( SU(N) \), we can split the field strength tensor into two parts.

\[ F^a_{MN} F^{aMN} = F^a_{\mu\nu} F^{a\mu\nu} + 2 F^a_{5\nu} F^{a5\nu} , \]
\[ F^a_{5\nu} = \partial_5 A^a_\nu - \partial_\nu A^a_5 + g^5 C^{abc} A^b_5 A^c_\nu , \]
\[ F^{a\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g^5 C^{abc} A^b_\mu A^c_\nu . \]

4 dimensional gauge coupling is defined as \( g = \frac{g_5}{\sqrt{\pi R}} \) and we integrate over the extra dimension.

\[ \frac{1}{2} \int_{-\pi R}^{\pi R} \left\{ (F^a_{5\nu} F^{a5\nu}) = -\sum_{n=1}^{\infty} \left( \frac{n}{\pi R} A^{n,a}_\mu + \partial_\mu A^{n,a}_5 + g C^{abc} A^m_b A^{n,c}_\mu \right)^2 \]
\[ + \sum_{m,n,l=1}^{\infty} \sqrt{2} g C^{abc} \left( \frac{n}{\pi R} A^{m,a}_\mu - \partial_\mu A^{m,a}_5 + g C^{abc} A^m_b A^{n,c}_\mu \right) A^{n,d}_\mu A^{l,\epsilon}_\mu \]
\[ - \frac{g_5^2}{2} C^{abc} A^{d\epsilon} \sum_{m,n,l,k=1}^{\infty} A^{m,b}_5 A^{n,c}_\mu A^{l,d}_5 A^{k,\epsilon}_\mu \Delta_{mnkl} \right\} . \]
\[ + \sum_{n=1}^{\infty} g^2 (C^{abc}(A_{\mu}^{0,b}A_{\nu}^{0,c} + A_{\nu}^{0,c}A_{\mu}^{0,b}))^2 \]
\[ + \sqrt{2} g^2 \sum_{m,n,l=1}^{\infty} C^{abc} C^{ade} (A_{\mu}^{0,b}A_{\nu}^{0,c} + A_{\nu}^{0,c}A_{\mu}^{0,b}) A^{n,du} A^{i,cv} \Delta^1_{mn} \]
\[ + \sum_{m,n,l,k=1}^{\infty} \frac{g^2}{2} C^{abc} C^{ade} A_{\mu}^{m,b} A_{\nu}^{n,c} A^{l,du} A^{k,cv} \Delta^2_{mnlk} . \]

Here we used the orthogonality relations of the trigonometric functions. We can notice the gauge boson \( A_{\mu}^{n,a} \) gets the mass \( \frac{n}{R} \). For general group \( SU(N) \), the gauge sector up to 1st KK level is

\[
\mathcal{L}_{SU(N)} = - \frac{1}{4} F_{\mu \nu}^a F^{a,\mu \nu} - \frac{1}{4} \left( \partial_\mu A_{\nu}^{a} - \partial_\nu A_{\mu}^{a} \right)^2 - \frac{g}{2} C^{abc} F_{\mu \nu}^a A_{\mu}^{1,b} A_{\nu}^{1,c} \]
\[ - \frac{1}{2} g^2 C^{abc} (\partial_\mu A_{\nu}^{a} - \partial_\nu A_{\mu}^{a}) (A_{\mu}^{0,b} A_{\nu}^{1,c} + A_{\nu}^{0,c} A_{\mu}^{1,b}) - \frac{g^2}{4} (C^{abc}(A_{\mu}^{0,b} A_{\nu}^{1,c} + A_{\nu}^{0,c} A_{\mu}^{1,b}))^2 \] \[ + \frac{1}{2} \left( \frac{n}{R} \right)^2 (B_{\mu}^1)^2 + \ldots . \] \[ \text{For } U(1), \text{ keeping terms only up to } n = 1 \]
\[ \mathcal{L}_{U(1)} = - \frac{1}{4} B_{\mu \nu}^0 B_{0,\mu \nu} - \frac{1}{4} B_{\mu \nu}^1 B_{1,\mu \nu} - \frac{1}{2} \left( \frac{n}{R} \right)^2 (B_{\mu}^1)^2 + \ldots . \] \[ \text{ (A.22)} \]

2. The Lepton Sector

The Lagrangian for KK-leptons is written as

\[
\mathcal{L}_{\text{leptons}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{L}(x, y) \Gamma^M D_M L(x, y) + i \bar{E}(x, y) \Gamma^M D_M E(x, y) \right\} , \] \[ \text{ (A.23)} \]

where \( L(x, y) \) is the lepton doublet and \( E(x, y) \) is lepton singlet for one generation. Using the field definitions and the orthogonality, we can integrate over extra dimension \( y \).

- \( SU(2)_W \) Lepton Doublet

\[
\mathcal{L}_L = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{L}(x, y) \Gamma^M \left[ \partial_\mu + \frac{iy}{2} g_1 \gamma_5 B_\mu + i g_2 W_\mu \right] L(x, y) \right\} \]
\[ = i \bar{L}_0 \gamma^\mu \left( \partial_\mu + \frac{iy}{2} g_1 \gamma_5 B_\mu + i g_2 W_\mu \right) L_0 - \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{L}_R L^1_L + \bar{L}_L L^1_R \right] \]
\[ + \sum_{n=1}^{\infty} \left[ i \bar{L}_R^\mu \gamma^\mu \left( \partial_\mu + \frac{iy}{2} g_1 B_\mu + ig_2 W_\mu \right) L_R^1 - \frac{g_2}{2 \sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ L_0 \gamma^\mu W_\mu^1 L^1_L + \bar{L}_0 i \gamma^5 W_5^1 L^1_R \right] \right] \]
\[ - \frac{g_2}{2 \sqrt{2 \pi R}} \sum_{n=1}^{\infty} \left[ L_0^1 \gamma^\mu B_\mu^1 L_L^1 \Delta^1_{mnl} + L_0^1 \gamma^\mu B_\mu^1 L_R^1 \Delta^4_{mn} + L_0^1 \gamma^5 B_5^1 L_L^1 \Delta^4_{l \mu} \right] \]
\[ \text{ (A.24)} \]

\[
\mathcal{L}_E = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{E}(x, y) \Gamma^M \left[ \partial_\mu + \frac{iy}{2} g_1 B_\mu + i g_2 W_\mu \right] E(x, y) \right\} \]

\[
= i \bar{E}_0 \gamma^\mu \left( \partial_\mu + \frac{iy}{2} g_1 B_\mu + i g_2 W_\mu \right) E_0 - \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{E}_R E^1_L + \bar{E}_L E^1_R \right] \]
\[ + \sum_{n=1}^{\infty} \left[ i \bar{E}_R^\mu \gamma^\mu \left( \partial_\mu + \frac{iy}{2} g_1 B_\mu + ig_2 W_\mu \right) E_R^1 - \frac{g_2}{2 \sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ E_0 \gamma^\mu W_\mu^1 E^1_L + \bar{E}_0 i \gamma^5 W_5^1 E^1_R \right] \right] \]
\[ - \frac{g_2}{2 \sqrt{2 \pi R}} \sum_{n=1}^{\infty} \left[ \bar{E}_0 \gamma^\mu B_\mu^1 E^1_L \Delta^1_{mnl} + \bar{E}_0 \gamma^\mu B_\mu^1 E^1_R \Delta^4_{mn} + \bar{E}_0 \gamma^5 B_5^1 E^1_L \Delta^4_{l \mu} \right] \]
It’s important to notice that the term \(- \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{L}_R \gamma^\mu B_\mu^{n l} \bar{L}_L + \bar{L}_L \gamma^\mu B_\mu^{n l} L_R \right] \) exactly looks like the usually Dirac mass. However if \( n = 0 \), there is no such mass. So this mass only apply for nonzero mode (KK-mode). As usual the SM particles get masses from the Yukawa coupling.

**SU(2)\(_W\) Lepton Singlet**

\[
\mathcal{L}_E = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{e}(x, y) \Gamma^M \left[ \partial_M + \frac{i y_2 g_1(5)}{2} B_M \right] E(x, y) \right\} = i \bar{e}_R \gamma^\mu \left( \partial_\mu + \frac{i y_2 g_1(5)}{2 \sqrt{\pi R}} B_\mu \right) e_R
\]

\[
+ \sum_{n=1}^{\infty} \frac{n}{\sqrt{\pi R}} \left[ \bar{e}_R \gamma^\mu B_\mu^{n l} e_L + \bar{e}_L \gamma^\mu B_\mu^{n l} \right] = \sum_{n=1}^{\infty} n \left[ \bar{e}_R \gamma^\mu B_\mu^{n l} e_L + \bar{e}_L \gamma^\mu B_\mu^{n l} \right]
\]

\[
L_{\text{quarks}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{Q}(x, y) \Gamma^M D_M Q(x, y) + i \bar{U}(x, y) \Gamma^M D_M U(x, y) + i \bar{D}(x, y) \Gamma^M D_M D(x, y) \right\}
\]

where \( Q(x, y) \) is the quark doublet and \( U(x, y) \) is the up-type quark singlet, \( D(x, y) \) is the down-type quark singlet for one generation.

**SU(2)\(_W\) Quark Doublet**

\[
\mathcal{L}_Q = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{Q}(x, y) \Gamma^M \left[ \partial_M + \frac{i y_3 g_3(5)}{2} B_M + \frac{i y_2 g_1(5)}{2 \sqrt{\pi R}} W_\mu^0 + \frac{i y_1 g_1(5)}{2 \sqrt{\pi R}} G_\mu \right] Q(x, y) \right\} = i \bar{q}_L \gamma^\mu \left( \partial_\mu + \frac{i y_3 g_3(5)}{2 \sqrt{\pi R}} B_\mu + \frac{i y_2 g_1(5)}{2 \sqrt{\pi R}} W_\mu^0 + \frac{i y_1 g_1(5)}{2 \sqrt{\pi R}} G_\mu \right) q_L - \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{Q}_R \gamma^\mu Q^L_R + \bar{Q}^L_R \gamma^\mu Q_R \right]
\]
\[
\begin{align*}
&+ \sum_{n=1}^{\infty} \left( i\bar{q}_L^{\gamma_\mu} (\partial_\mu + \frac{ig_3 g_1}{2\sqrt{\pi R}} B_\mu^0 + \frac{ig_2}{\sqrt{\pi R}} W_\mu^0 + \frac{ig_3}{\sqrt{\pi R}} G_\mu^0) Q_R^n \right) Q_R^n \right] \\
&- \frac{g_2}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ \bar{q}_L \gamma_\mu B_\mu^0 Q_L^n + \bar{q}_L i\gamma^5 B_5^n Q_R^n \right] \\
&- \frac{g_3}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ \bar{q}_L \gamma_\mu G_\mu^0 Q_L^n + \bar{q}_L i\gamma^5 G_5^n Q_R^n \right] \\
&- \frac{g_2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \left[ \bar{Q}_L^m \gamma_\mu B_\mu^0 Q_L^i \Delta_{mn}^1 + \bar{Q}_R^m \gamma_\mu B_\mu^0 Q_R^i \Delta_{mn}^4 + \bar{Q}_L^m i\gamma^5 B_5^m Q_R^i \Delta_{mn}^4 \right] \\
&- \frac{g_2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \left[ \bar{Q}_L^m \gamma_\mu W_\mu^0 Q_L^i \Delta_{mn}^1 + \bar{Q}_R^m \gamma_\mu W_\mu^0 Q_R^i \Delta_{mn}^4 + \bar{Q}_L^m i\gamma^5 W_5^m Q_R^i \Delta_{mn}^4 \right] \\
&= \bar{q}_L \gamma_\mu \left( \partial_\mu + \frac{1}{2} g_1 B_\mu^0 + ig_2 W_\mu^0 + ig_3 G_\mu^0 \right) q_L - \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{Q}_R^n Q_L^n + \bar{Q}_L^n Q_R^n \right] \\
&+ \sum_{n=1}^{\infty} \left( i\bar{Q}_R^n \gamma_\mu (\partial_\mu + \frac{1}{2} g_1 B_\mu^0 + ig_2 W_\mu^0 + ig_3 G_\mu^0) Q_R^n \right] \\
&+ \sum_{n=1}^{\infty} \left( i\bar{Q}_L^n \gamma_\mu (\partial_\mu + \frac{1}{2} g_1 B_\mu^0 + ig_2 W_\mu^0 + ig_3 G_\mu^0) Q_L^n \right] \\
&- \frac{g_3}{2} \sum_{n=1}^{\infty} \left[ \bar{q}_L \gamma_\mu B_\mu^0 Q_L^n + \bar{q}_L i\gamma^5 B_5^m Q_R^n \right] \\
&- \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \bar{U}(x, y) \Gamma_M \left[ \partial_M + \frac{1}{2} g_4 g_1 B_M + ig_3 G_M \right] U(x, y) \right\} \\
&= \bar{u}_R \gamma_\mu \left( \partial_\mu + \frac{1}{2} g_4 g_1 B_\mu^0 + \frac{ig_3}{\sqrt{\pi R}} G_\mu^0 \right) u_R + \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{u}_R^n u_L^n + \bar{q}_L^n u_R^n \right]
\end{align*}
\]

- \textbf{SU}(2)_W Up-type Quark Singlet

\[
\mathcal{L}_U = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \bar{U}(x, y) \Gamma_M \left[ \partial_M + \frac{1}{2} g_4 g_1 B_M + ig_3 G_M \right] U(x, y) \right\}
\]

\[
= \bar{u}_R \gamma_\mu \left( \partial_\mu + \frac{1}{2} g_4 g_1 B_\mu^0 + \frac{ig_3}{\sqrt{\pi R}} G_\mu^0 \right) u_R + \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{u}_R^n u_L^n + \bar{q}_L^n u_R^n \right]
\]
\[ \begin{align*}
&+ \sum_{n=1}^{\infty} \left[ i \bar{u}^\gamma R^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 \sqrt{\pi R}} B^0_\mu + \frac{i g_3}{\sqrt{\pi R}} G^0_\mu \right) u^\mu_R + i \bar{u}^\gamma L^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 \sqrt{\pi R}} B^0_\mu + \frac{i g_3}{\sqrt{\pi R}} G^0_\mu \right) u^\mu_L \right] \\
&- \frac{y_5 g_1}{2 \sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - \frac{g_3}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \\
&- \frac{y_5 g_1}{2 \sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - \frac{g_3}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \\
&= \bar{u}^\gamma R^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_R + \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{u}^\gamma R^\mu L^\mu \bar{D}^n_R + \bar{u}^\gamma L^\mu R^\mu \bar{D}^n_L \right] \\
&+ \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_R + \bar{u}^\gamma L^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_L \right] \\
&- \frac{y_5}{2 g_1} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - g_3 \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \\
&- \frac{y_5}{2 g_1} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - g_3 \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \\
&= \bar{u}^\gamma R^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_R + \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{u}^\gamma R^\mu L^\mu \bar{D}^n_R + \bar{u}^\gamma L^\mu R^\mu \bar{D}^n_L \right] \\
&+ \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_R + \bar{u}^\gamma L^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_L \right] \\
&- \frac{y_5}{2 g_1} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - g_3 \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \\
&- \frac{y_5}{2 g_1} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - g_3 \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \\
&= \bar{u}^\gamma R^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_R + \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{u}^\gamma R^\mu L^\mu \bar{D}^n_R + \bar{u}^\gamma L^\mu R^\mu \bar{D}^n_L \right] \\
&+ \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_R + \bar{u}^\gamma L^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_L \right] \\
&- \frac{y_5}{2 g_1} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - g_3 \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \\
&- \frac{y_5}{2 g_1} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - g_3 \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \
&= \bar{u}^\gamma R^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_R + \sum_{n=1}^{\infty} \frac{n}{R} \left[ \bar{u}^\gamma R^\mu L^\mu \bar{D}^n_R + \bar{u}^\gamma L^\mu R^\mu \bar{D}^n_L \right] \\
&+ \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_R + \bar{u}^\gamma L^\mu \left( \partial_\mu + \frac{i y_5 \gamma_5}{2 g_1 B^0_\mu + i g_3 G^0_\mu} \right) u^\mu_L \right] \\
&- \frac{y_5}{2 g_1} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - g_3 \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \\
&- \frac{y_5}{2 g_1} \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu B^0_\mu u^\mu_R + \bar{u}^\gamma R^5 B^5_\mu u^\mu_L \right] - g_3 \sum_{n=1}^{\infty} \left[ \bar{u}^\gamma R^\mu G^\mu u^\mu_R + \bar{u}^\gamma R^5 G^5_\mu u^\mu_L \right] \
\end{align*}\]
We can check all KK particles (nonzero mode) have the Dirac masses up to sign.

4. The Yukawa Sector

The Yukawa couplings to give their fermions masses are follows

$$\mathcal{L}_{Yukawa} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \lambda_u \bar{Q}(x, y) U(x, y) i\tau^2 H^*(x, y) + \lambda_d \bar{Q}(x, y) D(x, y) H(x, y) + \lambda_e \bar{L}(x, y) E(x, y) H(x, y) \right\} \tag{A.30}$$

There is no mass term for lower component of $SU(2)_W$ doublet since there is no right handed partner.

$$\frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \lambda_u \bar{Q}(x, y) U(x, y) i\tau^2 H^* \right\} = \frac{\lambda_u}{\sqrt{\pi R}} \bar{q}_L u_R i\tau^2 H^*$$
$$+ \frac{\lambda_u}{\pi R} \sum_{n=1}^\infty \left[ \bar{Q}_L^n u_R^n i\tau^2 H^* + \bar{Q}_R^n u_L^n i\tau^2 H^* \right] \tag{A.31}$$

$$+ \frac{\lambda_u}{\sqrt{\pi R}} \sum_{n=1}^\infty \left[ \bar{q}_L u_R^n i\tau^2 H_n^* + \bar{Q}_L^n u_R i\tau^2 H_n^* \right]$$
$$+ \frac{\lambda_u}{\sqrt{2\pi R}} \sum_{n=1}^\infty \left[ \bar{Q}_L^n u_R i\tau^2 H_n^* \Delta^1_{mnl} + \bar{Q}_R^n u_L^m i\tau^2 H_n^* \Delta^4_{mnl} \right],$$

$$\frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \lambda_d \bar{Q}(x, y) D(x, y) H(x, y) \right\} = \frac{\lambda_d}{\sqrt{\pi R}} \bar{q}_L d_R H$$
$$+ \frac{\lambda_d}{\pi R} \sum_{n=1}^\infty \left[ \bar{Q}_L^n d_R^n H + \bar{Q}_L^n d_R H \right] \tag{A.32}$$
$$+ \frac{\lambda_d}{\sqrt{\pi R}} \sum_{n=1}^\infty \left[ \bar{q}_L d_R^n H_n + \bar{Q}_L^n d_R H_n \right]$$
$$+ \frac{\lambda_d}{\sqrt{2\pi R}} \sum_{n=1}^\infty \left[ \bar{Q}_L^n d_R^n H_1 \Delta^1_{mnl} + \bar{Q}_L^n d_R^m H_1 \Delta^4_{mnl} \right],$$

$$\frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \lambda_e \bar{L}(x, y) E(x, y) H(x, y) \right\} = \frac{\lambda_e}{\sqrt{\pi R}} \bar{l}_0 e_R H$$
$$+ \frac{\lambda_e}{\pi R} \sum_{n=1}^\infty \left[ \bar{L}_L^n e_R^n H + \bar{L}_R^n e_R^n H \right] \tag{A.33}$$
$$+ \frac{\lambda_e}{\sqrt{\pi R}} \sum_{n=1}^\infty \left[ \bar{l}_0 e_R^n H_n + \bar{L}_L^n e_R H_n \right]$$
$$+ \frac{\lambda_e}{\sqrt{2\pi R}} \sum_{n=1}^\infty \left[ \bar{L}_L^n e_R^m H_1 \Delta^1_{mnl} + \bar{L}_R^n e_R^m H_1 \Delta^4_{mnl} \right].$$

5. The Higgs Sector

$$\mathcal{L}_{Higgs} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left[ (D_M H(x, y))^\dagger (D_M H(x, y)) + \mu^2 H^\dagger(x, y) H(x, y) - \lambda (H^\dagger(x, y) H(x, y))^2 \right]$$
\[
\left[ \partial_\mu + ig_2^{(5)} W^0_\mu + \frac{ig_1^{(5)}}{2} B^0_\mu \right] H \right] + \left[ \left( \partial_\mu + ig_2^{(5)} W^{0\mu} + \frac{ig_1^{(5)}}{2} B^{0\mu} \right) H \right]
\]
\[
+ \sum_{n=1}^{\infty} \left[ \left( \partial_\mu + ig_2^{(5)} W^{0\mu} + \frac{ig_1^{(5)}}{2} B^{0\mu} \right) H_n \right] + \left[ \left( \partial_\mu + ig_2^{(5)} W^{0\mu} + \frac{ig_1^{(5)}}{2} B^{0\mu} \right) H_n \right]
\]
\[
+ \frac{ig_2^{(5)}}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \left[ (\partial^\mu H) W^1_\mu H_n + (\partial^\mu H_n) W^1_\mu H - H_n^1 W^1_\mu (\partial^\mu H) - H^1 W^1_\mu (\partial^\mu H_n) \right]
\]
\[
+ \frac{ig_2^{(5)}}{2\sqrt{2\pi R}} \sum_{m,n,l=1}^{\infty} \left[ (\partial^\mu H) B^1_\mu H_n + (\partial^\mu H_n) B^1_\mu H - H_n^1 B^1_\mu (\partial^\mu H) - H^1 B^1_\mu (\partial^\mu H_n) \right]
\]
\[
+ \frac{ig_1^{(5)}}{2\sqrt{2\pi R}} \sum_{m,n,l=1}^{\infty} \left[ (\partial^\mu H) B^1_\mu H_n + (\partial^\mu H_n) B^1_\mu H - H_n^1 B^1_\mu (\partial^\mu H) - H^1 B^1_\mu (\partial^\mu H_n) \right]
\]
\[
+ \frac{g_2^{(5)}}{\pi R} \sum_{n=1}^{\infty} \left[ H^1 W^{01}_\mu W^{1\mu} H_n + H^1 W^{01}_\mu W^{1\mu} H_n + H^1 W^{01}_\mu W^{1\mu} H_n + H^1 W^{01}_\mu W^{1\mu} H - H_n^1 W^{01}_\mu W^{1\mu} H \right]
\]
\[
+ \frac{g_2^{(5)}}{\sqrt{2\pi R}} \sum_{m,n,l=1}^{\infty} \left[ H^1 W^{01}_\mu W^{1\mu} H_n + H^1 W^{01}_\mu W^{1\mu} H_n + H^1 W^{01}_\mu W^{1\mu} H_n + H^1 W^{01}_\mu W^{1\mu} H - \Delta^1_{nnnl} \right]
\]
\[
+ \frac{2g_2^{(5)}}{\pi R} \sum_{m,n,l,k=1}^{\infty} \left[ [H^1 W^{1\mu}_{\mu} W^{1\mu} H_k] + \Delta^2_{nnnkk} \right]
\]
\[
+ \frac{g_2^{(5)}}{2\pi R} \sum_{n=1}^{\infty} \left[ H^1 W^{01}_\mu B^{1\mu} H_n + H^1 W^{01}_\mu B^{1\mu} H_n + H^1 W^{01}_\mu B^{1\mu} H_n + H^1 W^{01}_\mu B^{1\mu} H_n + H^1 W^{01}_\mu B^{1\mu} H_n \right]
\]
\[
+ \frac{g_2^{(5)}}{2\pi R} \sum_{m,n,l=1}^{\infty} \left[ H^1 W^{01}_\mu B^{1\mu} H_n + H^1 W^{01}_\mu B^{1\mu} H_n + H^1 W^{01}_\mu B^{1\mu} H_n + H^1 W^{01}_\mu B^{1\mu} H_n + H^1 W^{01}_\mu B^{1\mu} H_n \right]
\]
\[
+ \frac{g_1^{(5)}}{4\pi R} \sum_{m,n,l,k=1}^{\infty} \left[ [H^1 W^{1\mu}_{\mu} B^{1\mu} H_k] + \Delta^2_{nnnkk} \right]
\]
\[
+ \frac{g_2^{(5)}}{\pi R} \sum_{n=1}^{\infty} \left[ 2H^1 B^{0\mu} B^{0\mu} H_n + 2H^1 B^{0\mu} B^{0\mu} H_n + H^1 B^{0\mu} B^{0\mu} H_n \right]
\]
\[
+ \frac{g_2^{(5)}}{4\sqrt{2\pi R}} \sum_{m,n,l=1}^{\infty} \left[ 2H^1 B^{0\mu} B^{0\mu} H_n + 2H^1 B^{0\mu} B^{0\mu} H_n + H^1 B^{0\mu} B^{0\mu} H_n \right]
\]
\[
+ \frac{g_2^{(5)}}{4\pi R} \sum_{m,n,l=1}^{\infty} \left[ 2H^1 B^{0\mu} B^{0\mu} H_n + 2H^1 B^{0\mu} B^{0\mu} H_n + H^1 B^{0\mu} B^{0\mu} H_n \right]
\]
\[
+ \mu^2 \left[ H^1 H + \sum_{n=1}^{\infty} H^1 H_n \right] - \sum_{n=1}^{\infty} \left( \frac{\pi}{R} \right)^2 H^1 H_n
\]
\[
- \frac{1}{\sqrt{2\pi R}} \sum_{m,n,l=1}^{\infty} \left[ H^1 H_n \left( \frac{ig_1^{(5)}}{2} B_5^m + ig_2^{(5)} W^1_5 \right) \right]
\]
\[
+ \frac{1}{\sqrt{2\pi R}} \sum_{m,n,l=1}^{\infty} \left[ H^1 H_n \left( \frac{ig_1^{(5)}}{2} B_5^m + ig_2^{(5)} W^1_5 \right) \right]
\]
\[
\Delta^4_{nnnkl} \]
\[ + \frac{1}{2\pi R} \sum_{m,n,l,k=1}^{\infty} H_l \left( \frac{i\alpha_l^5}{2} - D_5^m + i\alpha_2^5 W_5^m \right) \left( \frac{i\alpha_l^5}{2} - D_5^n + i\alpha_2^5 W_5^n \right) H_k \Delta_{mnlk} \]
\[ + \frac{\lambda}{\pi R} (H^\dagger H)^2 + \frac{\lambda}{2\pi R} \sum_{m,n,l,k=0}^{\infty} H_l^\dagger H_n H_l^\dagger H_k \Delta_{mnlk}. \]

G. KK Fermions and KK Gauge Bosons

We summarize the fermion contents of this theory in table A.1.

<table>
<thead>
<tr>
<th>SU(2)_W Symmetry</th>
<th>SM mode</th>
<th>KK mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark doublet</td>
<td>$q_L(x) = \left( \begin{array}{c} U_L(x) \ D_L(x) \end{array} \right)$</td>
<td>$Q_L^R(x) = \left( \begin{array}{c} U_R^0(x) \ D_R^0(x) \end{array} \right)$, $Q_R(x) = \left( \begin{array}{c} U_R^0(x) \ D_R^0(x) \end{array} \right)$</td>
</tr>
<tr>
<td>Lepton doublet</td>
<td>$L_0(x) = \left( \begin{array}{c} \nu_L(x) \ E_L(x) \end{array} \right)$</td>
<td>$L_L^R(x) = \left( \begin{array}{c} \nu_R^0(x) \ E_R^0(x) \end{array} \right)$, $L_R(x) = \left( \begin{array}{c} \nu_R^0(x) \ E_R^0(x) \end{array} \right)$</td>
</tr>
<tr>
<td>Quark Singlet</td>
<td>$u_R(x)$</td>
<td>$u_R^0(x)$, $u_R^2(x)$</td>
</tr>
<tr>
<td>Quark Singlet</td>
<td>$d_R(x)$</td>
<td>$d_R^0(x)$, $d_R^2(x)$</td>
</tr>
<tr>
<td>Lepton Singlet</td>
<td>$e_R(x)$</td>
<td>$e_R^0(x)$, $e_R^2(x)$</td>
</tr>
</tbody>
</table>

TABLE A.1: Fermion content

As we can see from the table, there are two KK particles corresponding to one SM particles. And from the Lagrangian we see their quantum numbers easily. The following table explains the quantum numbers for the mass eigenstates when we ignore the mass term from the Yukawa coupling. We show KK-fermions after compactification and their quantum numbers in table A.2.

<table>
<thead>
<tr>
<th>KK Fermions</th>
<th>$I_3$</th>
<th>$Y$</th>
<th>$Q = I_3 + \frac{2}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark</td>
<td>$U_u = U_u^0(x) + U_u^2(x)$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>Doublet</td>
<td>$D_u = D_u^0(x) + D_u^2(x)$</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>Quark</td>
<td>$u_u = u_u^0(x) + u_u^2(x)$</td>
<td>$0$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>Singlet</td>
<td>$d_u = d_u^0(x) + d_u^2(x)$</td>
<td>$0$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>Lepton</td>
<td>$\nu = \nu_L^0(x) + \nu_L^2(x)$</td>
<td>$\frac{1}{3}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Doublet</td>
<td>$E_u = E_u^0(x) + E_u^2(x)$</td>
<td>$\frac{2}{3}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Lepton</td>
<td>$e = e_L^0(x) + e_L^2(x)$</td>
<td>$0$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Singlet</td>
<td>no KK singlet $\nu$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

TABLE A.2: Quantum numbers

The standard model particles are shown in table A.3

Note that in UED, the Dirac fermions $F^u(x) = F_{u}^0(x) + F_{u}^2(x)$, as shown above, are constructed out of $F_{u}^0(x)$ and $F_{u}^2(x)$ which have same $SU(2)_W \times U(1)_Y$ quantum numbers. Contrast this with a SM Dirac fermion “$f” which is constructed out of $f_L(x)$ and $f_R(x)$ which have different $SU(2)_W \times U(1)_Y$ quantum numbers. This difference shows up in the processes involving gauge-bosons couplings with fermions.

The mass eigenstates of KK photons and Z can be obtained by diagonalizing following mass matrix in $W_n^3$ and $B_n$ basis

\[
\begin{pmatrix}
\frac{n^2}{m_{B_n}^2} + \delta m_{W_n}^2 + \frac{1}{4} g_1 g_2 v^2 \\
\frac{1}{4} g_1 g_2 v^2 & \frac{n^2}{m_{B_n}^2} + \delta m_{W_n}^2 + \frac{1}{4} g_1 g_2 v^2
\end{pmatrix}
\]

(A.35)
TABLE A.3: Standard model particles

<table>
<thead>
<tr>
<th>SM Fermions</th>
<th>SM Gauge Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u = U_L(x) + u_R(x) )</td>
<td>( W^\pm_{\mu} = \frac{W^1_{\mu} + i W^2_{\mu}}{\sqrt{2}} )</td>
</tr>
<tr>
<td>( d = D_L(x) + d_R(x) )</td>
<td>( Z_{\mu} = -\sin \theta_W B_{\mu} + \cos \theta_W W^3_{\mu} \approx W^3_{\mu} )</td>
</tr>
<tr>
<td>( e = E_L(x) + e_R(x) )</td>
<td>( A_{\mu} = \cos \theta_W B_{\mu} + \sin \theta_W W^3_{\mu} \approx W^3_{\mu} )</td>
</tr>
<tr>
<td>( \nu_L = \nu_L(x) )</td>
<td>( e )</td>
</tr>
</tbody>
</table>

The Lagrangian for the EW gauge bosons can be obtained using the following definitions

\[
W^\pm_{\mu} = \frac{W^1_{\mu} + i W^2_{\mu}}{\sqrt{2}},
\]

\[
Z_{\mu} = -\sin \theta_W B_{\mu} + \cos \theta_W W^3_{\mu} \approx W^3_{\mu},
\]

\[
A_{\mu} = \cos \theta_W B_{\mu} + \sin \theta_W W^3_{\mu} \approx W^3_{\mu}.
\]

The neutral gauge boson eigenstates become approximately pure \( B_n \) and \( W^3_n \) since Weinberg angles for KK are small.

The tree level mass of the \( n \)th KK mode is

\[
m^2_n = \frac{n^2 R^2 + m_0^2}{R^2}.
\]

H. The Mixing between Singlet and Doublet Fermions

The mass terms for the KK singlets appear with wrong sign in the fermion Lagrangian [3]. For example, the mass term for the KK quarks leads to the following structure for the mass matrix

\[
(\bar{U}^n(x), \bar{u}^n(x)) \begin{pmatrix}
\frac{n}{R} & \frac{m}{R} \\
\frac{m}{R} & -\frac{n}{R}
\end{pmatrix} \begin{pmatrix}
U^n(x) \\
u^n(x)
\end{pmatrix}.
\]

The diagonal terms are the masses induced by kinetic terms in extra dimension, while the off-diagonal terms are the contributions from Higgs VEV corresponding to SM fermion masses. The corresponding mass eigenstates \( U^n \) and \( u^n \) have the mass

\[
M_n = \sqrt{\left( \frac{n}{R} \right)^2 + m^2}.
\]

These are the masses before any contribution from the radiative correction which lifts the degeneracy. The radiative corrections to the masses are enlisted in Section VI. The KK eigenstates are related to their mass eigenstates by

\[
\begin{pmatrix}
U^n \\
u^n
\end{pmatrix} = \begin{pmatrix}
\cos \alpha & \gamma_5 \sin \alpha \\
\sin \alpha & -\gamma_5 \cos \alpha
\end{pmatrix} \begin{pmatrix}
U'^n \\
u'^n
\end{pmatrix},
\]

where \( \alpha \) is the mixing angle given by

\[
\tan 2\alpha = \frac{m}{n/R},
\]

which is very small except for the top quark. However even with \( \alpha = 0 \), the effect of this wrong sign shows up in the Yukawas through the redefinition of the field \( (u^n \rightarrow -\gamma^5 u^n) \). It does not affect the gauge-fermion couplings since they cannot flip the fermion chirality.

I. The Boundary Terms

The Lagrangian for the boundary term

\[
-\frac{g}{\sqrt{2}} \left( \delta m^2_{\Delta A} \frac{\delta m_{f_2}}{m^2_2} - 2 \frac{\delta m_{f_2}}{m^2_2} \right) \bar{\psi}_0 \gamma^\mu T^a P_+ \psi_0 A_{2\mu},
\]

is depicted in Fig. 1 and it is related to the mass corrections (\( \delta 's \)) that can be found in [4].
We summarize this interaction in table A.4. (\(J_3\) is the isospin of the fermion which couples to level 2 gauge boson.)

<table>
<thead>
<tr>
<th>(n = 2) KK boson</th>
<th>(n = 0) SM fermion</th>
<th>Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U(1)_Y) gauge boson (B_2)</td>
<td>Lepton</td>
<td>(i g_1 \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16 \pi^2} \ln \left(\frac{\Delta}{\mu}\right)^2 \left[ \frac{g_1^2}{2} P_L \left( \frac{3}{8} g_1^2 + \frac{7}{8} g_2^2 \right) + \frac{g_2^2}{2} P_R \left( \frac{15}{8} g_1^2 \right) \right] )</td>
</tr>
<tr>
<td></td>
<td>Quark (up)</td>
<td>(i g_1 \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16 \pi^2} \ln \left(\frac{\Delta}{\mu}\right)^2 \left[ \frac{g_1^2}{2} P_L \left( \frac{3}{8} g_1^2 + \frac{7}{8} g_2^2 + 6g_3^2 \right) + \frac{g_2^2}{2} P_R \left( \frac{15}{8} g_1^2 + 6g_3^2 \right) \right] )</td>
</tr>
<tr>
<td></td>
<td>Quark (down)</td>
<td>(i g_1 \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16 \pi^2} \ln \left(\frac{\Delta}{\mu}\right)^2 \left[ \frac{g_1^2}{2} P_L \left( \frac{3}{8} g_1^2 + \frac{7}{8} g_2^2 + 6g_3^2 \right) + \frac{g_2^2}{2} P_R \left( \frac{15}{8} g_1^2 + 6g_3^2 \right) \right] )</td>
</tr>
<tr>
<td>(SU(2)_W) gauge boson (Z_2)</td>
<td>Lepton</td>
<td>(i g_3 \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16 \pi^2} \ln \left(\frac{\Delta}{\mu}\right)^2 \left[ \frac{g_2^2}{2} P_L \left( \frac{11}{8} g_1^2 - \frac{3}{8} g_2^2 \right) \right] )</td>
</tr>
<tr>
<td></td>
<td>Quark</td>
<td>(i g_3 \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16 \pi^2} \ln \left(\frac{\Delta}{\mu}\right)^2 \left[ \frac{g_2^2}{2} P_L \left( \frac{11}{8} g_1^2 - \frac{3}{8} g_2^2 + 6g_3^2 \right) \right] )</td>
</tr>
<tr>
<td>(SU(2)_W) gauge boson (W_2)</td>
<td>Lepton</td>
<td>(i g_3 \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16 \pi^2} \ln \left(\frac{\Delta}{\mu}\right)^2 \left[ \frac{g_2^2}{2} P_L \left( \frac{11}{8} g_1^2 - \frac{3}{8} g_2^2 \right) \right] )</td>
</tr>
<tr>
<td></td>
<td>Quark</td>
<td>(i g_3 \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16 \pi^2} \ln \left(\frac{\Delta}{\mu}\right)^2 \left[ \frac{g_2^2}{2} P_L \left( \frac{11}{8} g_1^2 - \frac{3}{8} g_2^2 + 6g_3^2 \right) \right] )</td>
</tr>
<tr>
<td>(SU(3)_c) gauge boson (G_2)</td>
<td>Quark (up)</td>
<td>(i g_3 \lambda_3 \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16 \pi^2} \ln \left(\frac{\Delta}{\mu}\right)^2 \left[ P_L \left( \frac{1}{8} g_1^2 + \frac{7}{8} g_2^2 + \frac{11}{2} g_3^2 \right) + P_R \left( 2g_1^2 - \frac{11}{2} g_3^2 \right) \right] )</td>
</tr>
<tr>
<td></td>
<td>Quark (down)</td>
<td>(i g_3 \lambda_3 \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16 \pi^2} \ln \left(\frac{\Delta}{\mu}\right)^2 \left[ P_L \left( \frac{1}{8} g_1^2 + \frac{7}{8} g_2^2 - \frac{11}{2} g_3^2 \right) + P_R \left( 2g_1^2 - \frac{11}{2} g_3^2 \right) \right] )</td>
</tr>
</tbody>
</table>

**TABLE A.4**: Boundary interactions with level 2 gauge bosons

For the top quark, we need to add the additional corrections \(\delta_m, m_{Q_{3a}}\) and \(\delta_m, m_{Q_{1a}}\) to \(\delta(m_{f_2})\) (see [4] for details).
J. Feynman Rules

\[ G^c = -i g_3 \gamma^\mu T^c_{ba} \]

\[ G_1^c = -i g_3 \gamma^\mu T^c_{ba} P_L \]

\[ G_1^c = -i g_3 \gamma^\mu T^c_{ba} P_R \]

\[ G^{\mu b} = i g_3 f^{abc} [(p - q)_\lambda g_{\mu \nu} + (q - r)_\mu g_{\lambda \nu} + (r - p)_\nu g_{\lambda \mu}] \]

\[ G^{\nu b} = -ig_3^2 \left[ f^{abc} f^{cde} (g_{\lambda \nu} g_{\mu \rho} - g_{\lambda \rho} g_{\mu \nu}) + f^{acd} f^{bde} (g_{\lambda \mu} g_{\nu \rho} - g_{\lambda \rho} g_{\mu \nu}) \right. \]

\[ + \left. f^{ade} f^{bce} (g_{\lambda \mu} g_{\nu \rho} - g_{\lambda \rho} g_{\mu \nu}) \right] \]

\[ G^{\nu b} = -ig_3^2 \left[ f^{abc} f^{cde} (g_{\lambda \nu} g_{\mu \rho} - g_{\lambda \rho} g_{\mu \nu}) + f^{acd} f^{bde} (g_{\lambda \mu} g_{\nu \rho} - g_{\lambda \rho} g_{\mu \nu}) \right. \]

\[ + \left. f^{ade} f^{bce} (g_{\lambda \mu} g_{\nu \rho} - g_{\lambda \rho} g_{\mu \nu}) \right] \]
\[ \gamma \rightarrow f_1 = -i Q_f e \gamma^\mu \]

\[ Z \rightarrow \bar{f}_1 = -i \frac{g_2}{2} \cos \theta_W c_L \gamma^\mu \]

\[ Z \rightarrow f_1^S = -i \frac{g_2}{2} \cos \theta_W c_{R} \gamma^\mu \]

\[ W^\pm \rightarrow \bar{f}_1^D = -i \frac{g_2}{\sqrt{2}} \gamma^\mu V_{f f'} \]

\[ B_1 \rightarrow \bar{f}_1^D = -i \frac{\gamma}{2} g_1 \gamma^\mu P_L \]

\[ B_1 \rightarrow \bar{f}_1^S = -i \frac{\gamma}{2} g_1 \gamma^\mu P_R \]

\[ Z_1 \rightarrow \bar{f}_0 = -i I_3 g_2 \gamma^\mu P_L \]

\[ W^\pm \rightarrow \bar{f}_0 = -i \frac{\gamma}{\sqrt{2}} \gamma^\mu P_L V_{f f'} \]

\[ B_2 \rightarrow \bar{f}_1^D = i \frac{\gamma}{2} \frac{g_1}{\sqrt{2}} \gamma^\mu \gamma^5 \]

\[ B_2 \rightarrow \bar{f}_1^S = -i \frac{\gamma}{2} \frac{g_1}{\frac{1}{2}} \gamma^\mu \gamma^5 \]

\[ Z_2 \rightarrow \bar{f}_1 = \frac{i I_3 g_2}{\sqrt{2}} \gamma^\mu \gamma^5 \]
\[ A_\mu \quad W_{\rho}^{1-} \quad \begin{align*} &= -i \cos \theta_W e^2 \left( 2 g^{\mu \nu} g^{\rho \sigma} - g^{\mu \rho} g^{\nu \sigma} - g^{\mu \sigma} g^{\nu \rho} \right) \\
Z_\nu \quad W_{\sigma}^{1+} \end{align*} \]

\[ A_\mu \quad W_{\rho}^{1-} \quad \begin{align*} &= -i \frac{1}{\sqrt{2} \sin \theta_W} e^2 \left( 2 g^{\mu \nu} g^{\rho \sigma} - g^{\mu \rho} g^{\nu \sigma} - g^{\mu \sigma} g^{\nu \rho} \right) \\
Z_\nu \quad W_{\sigma}^{1+} \end{align*} \]

\[ W^{+}_\mu \quad W_{\rho}^{1-} \quad \begin{align*} &= i g_2^2 \left( 2 g^{\mu \nu} g^{\rho \sigma} - g^{\mu \rho} g^{\nu \sigma} - g^{\mu \sigma} g^{\nu \rho} \right) \\
W^{+}_\nu \quad W_{\sigma}^{1-} \end{align*} \]

\[ W^{+}_\mu \quad Z_\rho \quad \begin{align*} &= -i g_2^2 \left( 2 g^{\mu \nu} g^{\rho \sigma} - g^{\mu \rho} g^{\nu \sigma} - g^{\mu \sigma} g^{\nu \rho} \right) \\
W^{-}_\nu \quad Z_{\sigma}^{1-} \end{align*} \]

\[ W^{+}_\mu \quad Z_\rho \quad \begin{align*} &= -i \cos \theta_W g_2^2 \left( 2 g^{\mu \nu} g^{\rho \sigma} - g^{\mu \rho} g^{\nu \sigma} - g^{\mu \sigma} g^{\nu \rho} \right) \\
W_{\nu}^{1-} \quad Z_{\sigma}^{1-} \end{align*} \]

\[ W^{+}_\mu \quad Z_\rho \quad \begin{align*} &= -i \frac{1}{\sqrt{2} \sin \theta_W} e^2 \left( 2 g^{\mu \nu} g^{\rho \sigma} - g^{\mu \rho} g^{\nu \sigma} - g^{\mu \sigma} g^{\nu \rho} \right) \\
W_{\nu}^{1-} \quad Z_{\sigma}^{2} \end{align*} \]
\[ W_{\mu}^{1+} \ W_{\rho}^{1-} = i^{3 \over 2} g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \]

\[ W_{\nu}^{1+} \ W_{\sigma}^{1-} = -i \cos^2 \theta_W g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \]

\[ W_{\mu}^{1+} \ Z_{\rho} = -i {1 \over \sqrt{2}} \cos \theta_W g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \]

\[ W_{\nu}^{1+} \ Z_{\sigma} = -i^{3 \over 2} g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \]

\[ W_{\mu}^{1+} \ Z_1 = -i^{1 \over 2} g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \]

\[ W_{\nu}^{1+} \ Z_2 = -i^{1 \over 2} g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \]


[27] P. Skands et al., “A repository for beyond-the-standard-model tools,”

