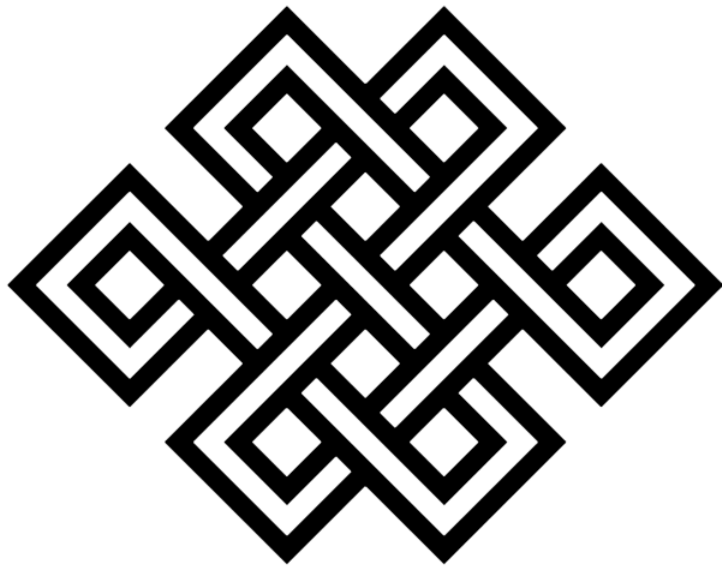
Dalefest!

Donna Kubik

Rolfsenfest

CIRM 2013

Interdependence of all things in the phenomenal world



"This knot (7_4 in the table) is one of the eight glorious emblems in Tibetan Buddhism. Just as a knot does not exist without reference to its embedding in space, this emblem is a reminder of the interdependence of all things in the phenomenal world."

Dale Rolfsen, *Knots and Links*

Interdependence of topology and cosmology

GOAL	METHOD	PROCEDURE	DIMENSION
Understand the large-structure of the universe	2D & 3D genus topology of large scale structure	Measure the genus of galaxy isodensity curves	2, 3
Understand the shape of space itself *Cosmic topology*	Cosmic crystallography	Look for repeating patterns in the distribution of clusters of galaxies	3
	Circles in the sky	Look for matching circles in the surface of last scattering	2, (3?)

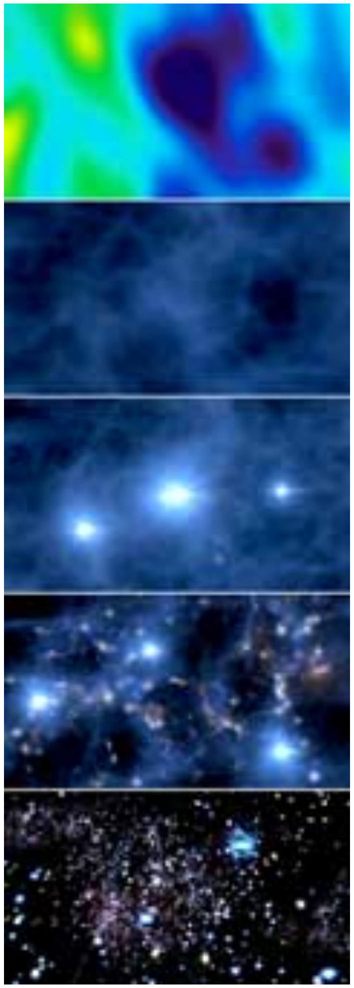
Genus topology of large scale structure

Density fluctuations evolve into structures we observe:
galaxies, clusters, super-clusters.

Density fluctuations

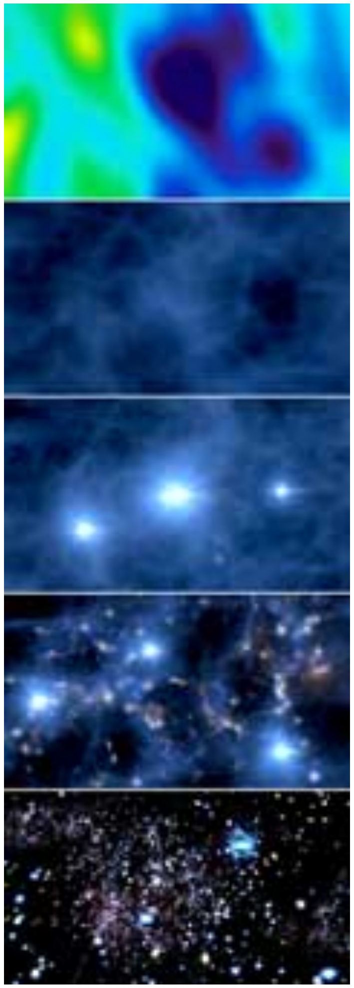
- According to the model of inflation, quantum fluctuations that existed when inflation began were amplified and formed the seed of all current observed structure.

- Measuring the genus as a function of density allows one to compare the topology observed with that expected for Gaussian random phase initial conditions, as those predicted in a standard big bang inflationary model where structure originates from random quantum fluctuations in the early universe.



Density fluctuations

- Topology of large-scale structure in the universe has been studied over the years through analyses of the *large-scale structure* of galaxies in two and three dimensions & compared to the theory of formation.

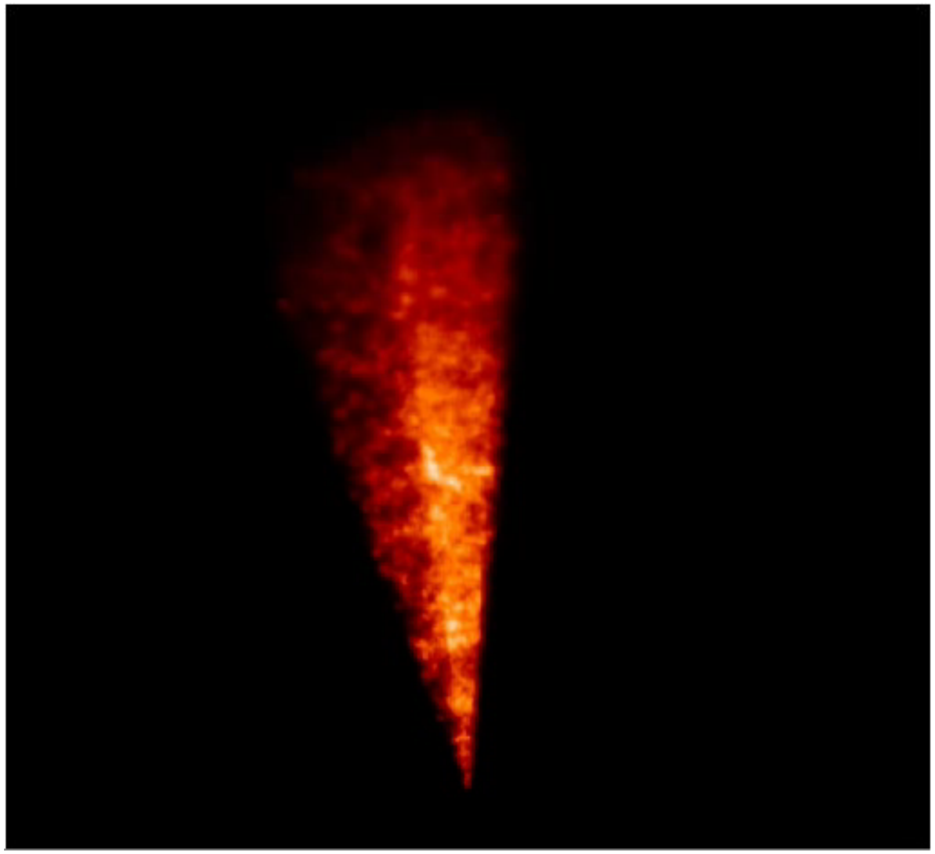


What is large-scale structure?

A look at large-scale structure

- Cosmologists use the term *large-scale structure* of the universe to refer to all structures bigger than individual galaxies.

- A map of the large-scale structure of the universe, as traced by the positions of galaxies, can be made by measuring the redshifts of a sample of galaxies and using the Hubble relation (later slide), to compute their distances from our own galaxy.

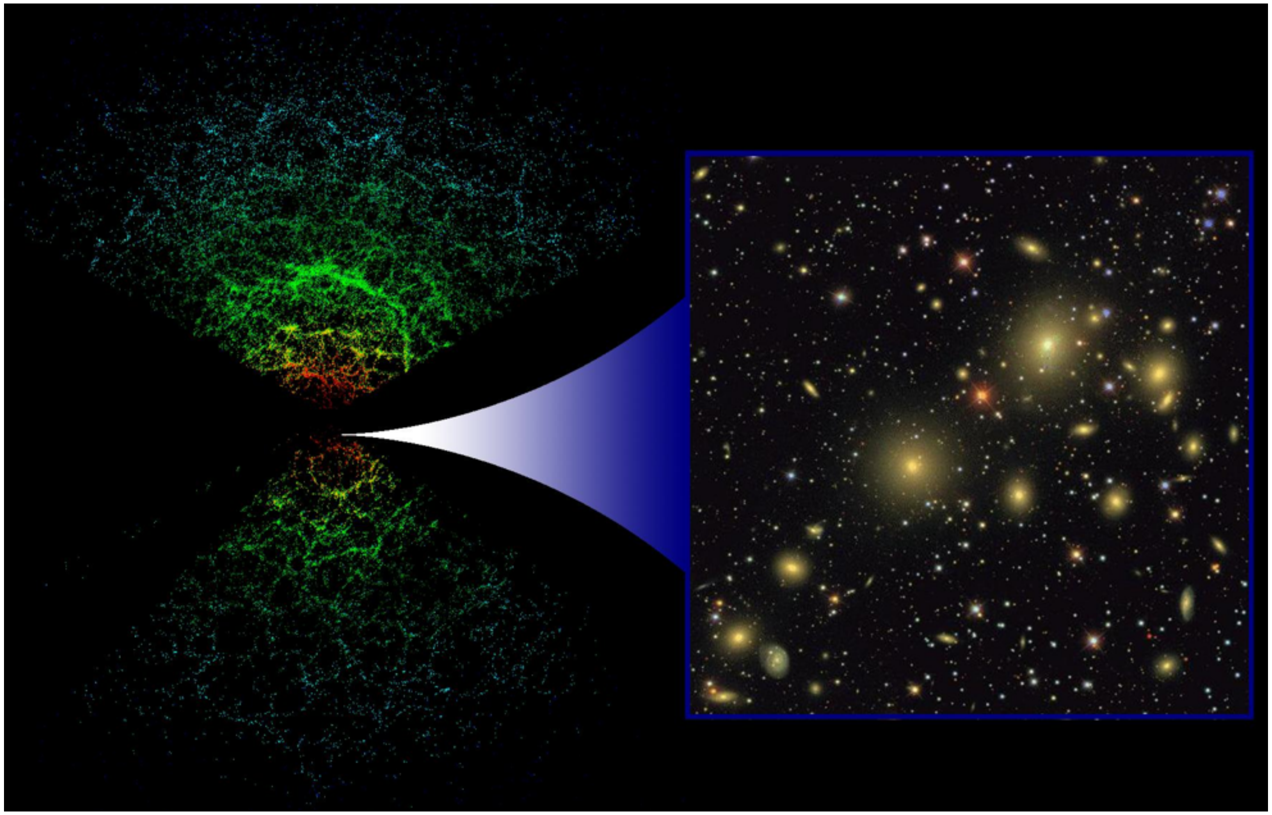


Each dot is a galaxy; Earth is at the apex.
Image from 2dF Redshift Survey
Click on image to start animation.

To map and quantify large-scale structure
(and compare with the theoretical predictions),
we need redshift surveys: mapping the 3-D distribution of galaxies in space.

Redshift surveys

- A redshift survey is two separate surveys in one: galaxies are identified in 2D images (right), then have their distance determined from their spectrum to create a 3D map (left) where each galaxy is shown as a single point.



Redshift is a proxy for distance

- If we have the spectrum of a galaxy, we can measure its redshift, *z*,

$$z = \frac{\lambda_o - \lambda'}{\lambda'}.$$

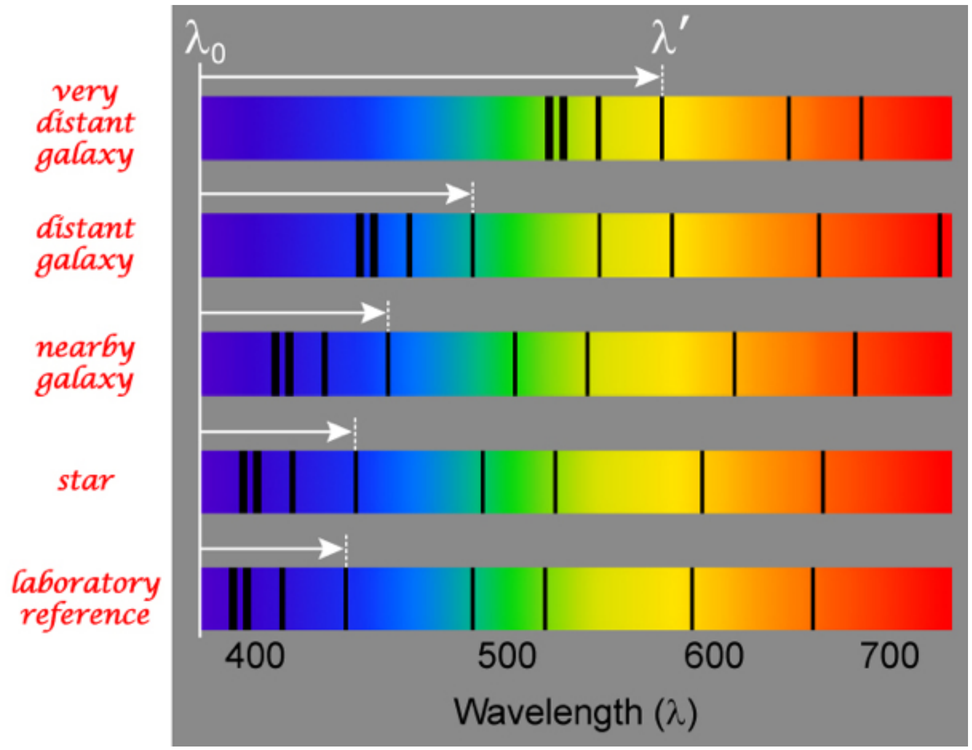
- We can then use the Hubble's Law to compute the distances (*d*) from our own galaxy.

$$d \cong \left(\frac{c}{H_o}\right) z$$

$$c = \text{speed of light}$$

$$H_o = \text{Hubble's constant} \approx 70 \text{ km/(s} \cdot \text{Mpc)}$$

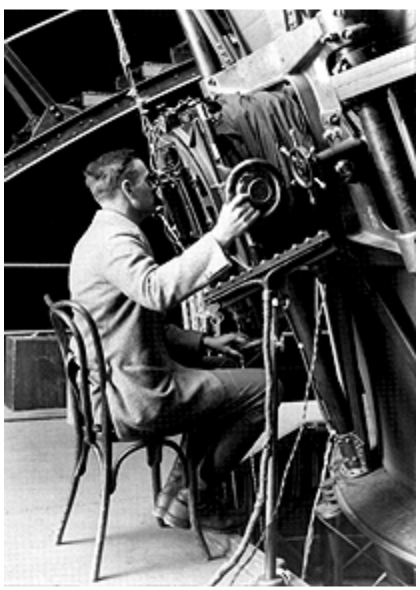
- So we can use redshift as a proxy for distance.

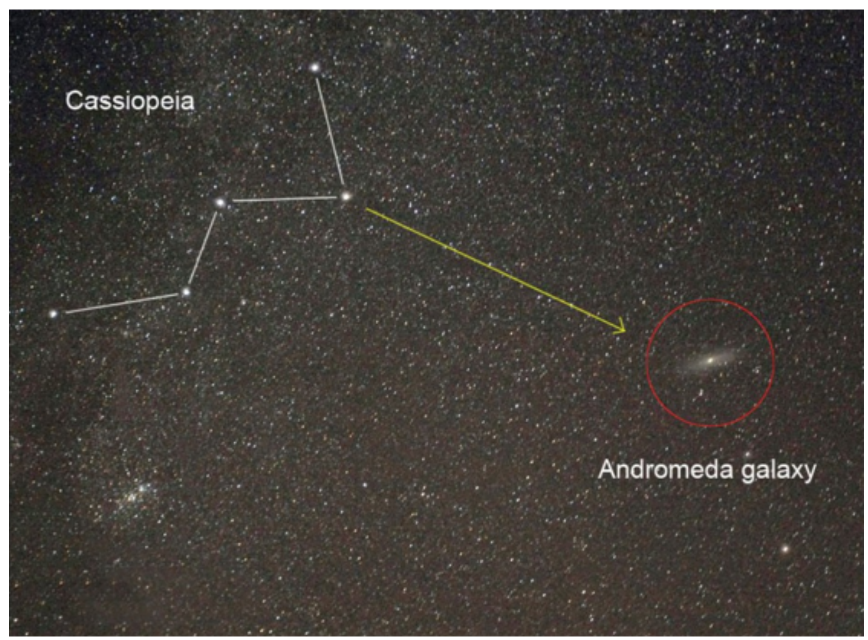


Hubble had to measure the spectrum of each galaxy, one by one.

- In the early 1920s, using photographic plates made with the Mt. Wilson 100” telescope, Edwin Hubble determined the distance to the Andromeda Galaxy, demonstrating the existence of other galaxies far beyond the Milky Way.

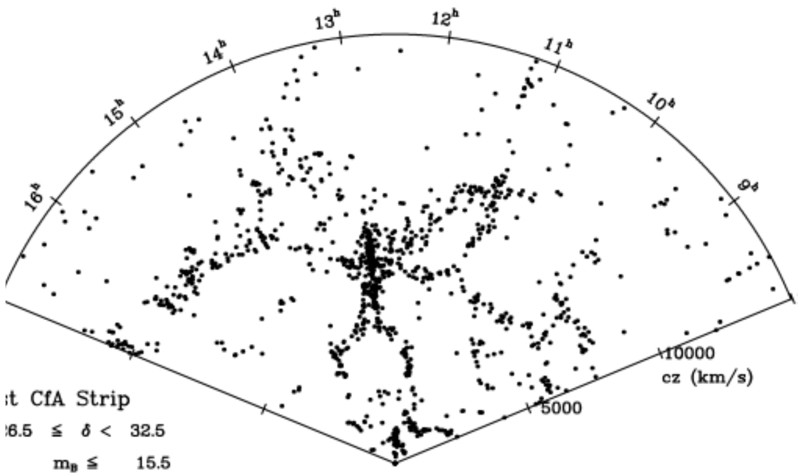
- So you could say that the first “redshift survey” was comprised of only one galaxy!



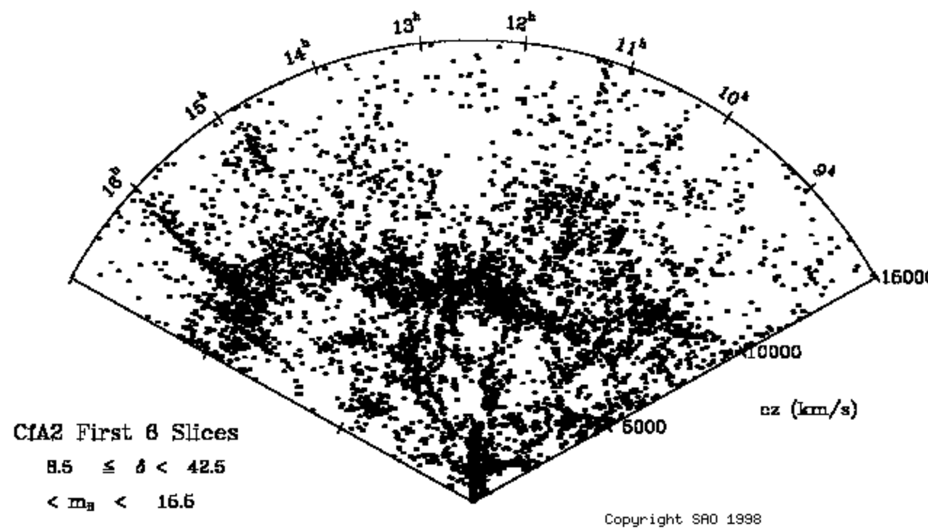


The first redshift surveys were only 2D but showed that galaxies were anything but randomly distributed!

- The initial map from the CfA survey (left) was quite surprising, showing that the distribution of galaxies in space is anything but random, with galaxies actually appearing to be distributed on surfaces, almost bubble like, surrounding large empty regions, or ``voids.''



Stick Man and Coma Cluster's "Finger of God" effect due to velocity dispersion in the cluster



Great Wall, 100 Mpc structure

Smithsonian Astrophysicsal Observatory Center for Astrophysics (CfA)
Huchra, Davis, Latham and Tonry, 1983, ApJS 52, 89 (laft)
Geller and Huchra 1989, Science 246, 897 (right)

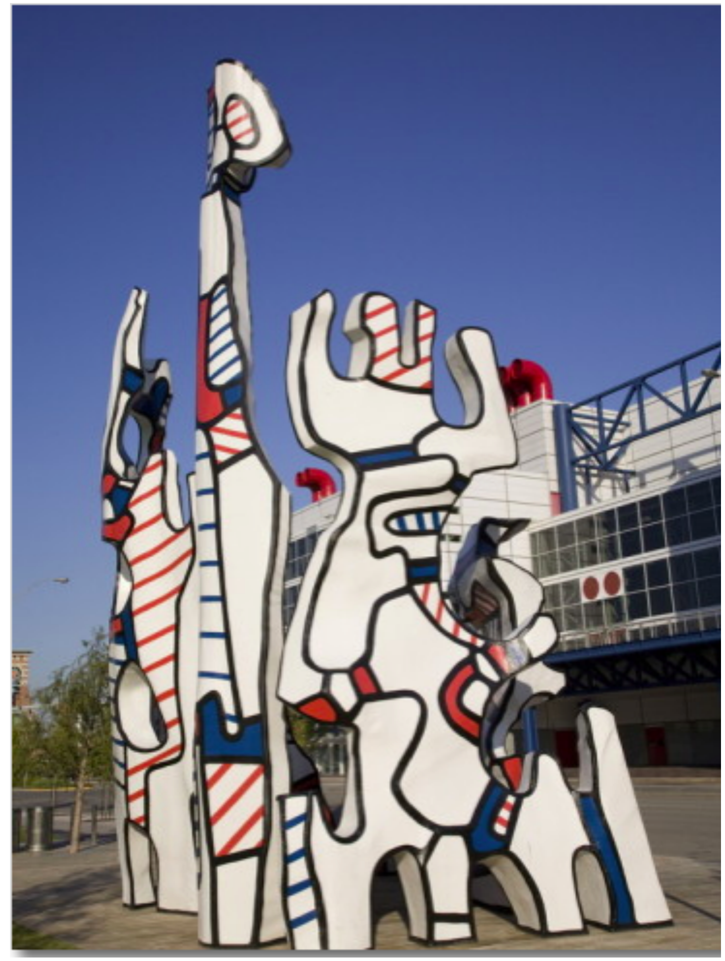
For scale: Milky Way diameter ~100 kly or 30 kpc
H_o=20 km/(s*Mly)

There is a lot of structure in Dubuffet's 2D paintings

La Vie de Famille, 1963, by Jean Dubuffet

There is much more information in Dubuffet's 3D sculptures

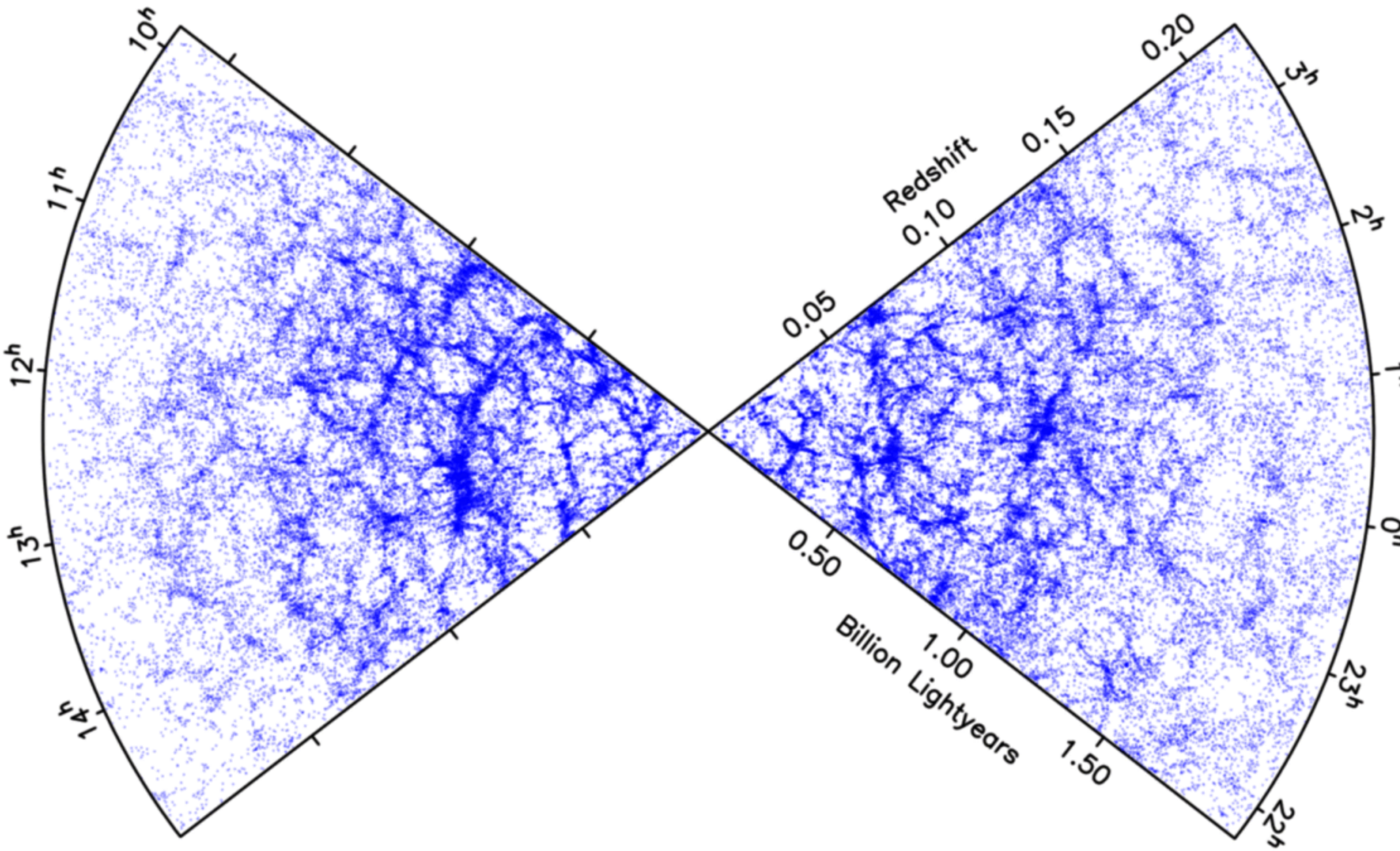
La Vie de Famille, 1963, by Jean Dubuffet



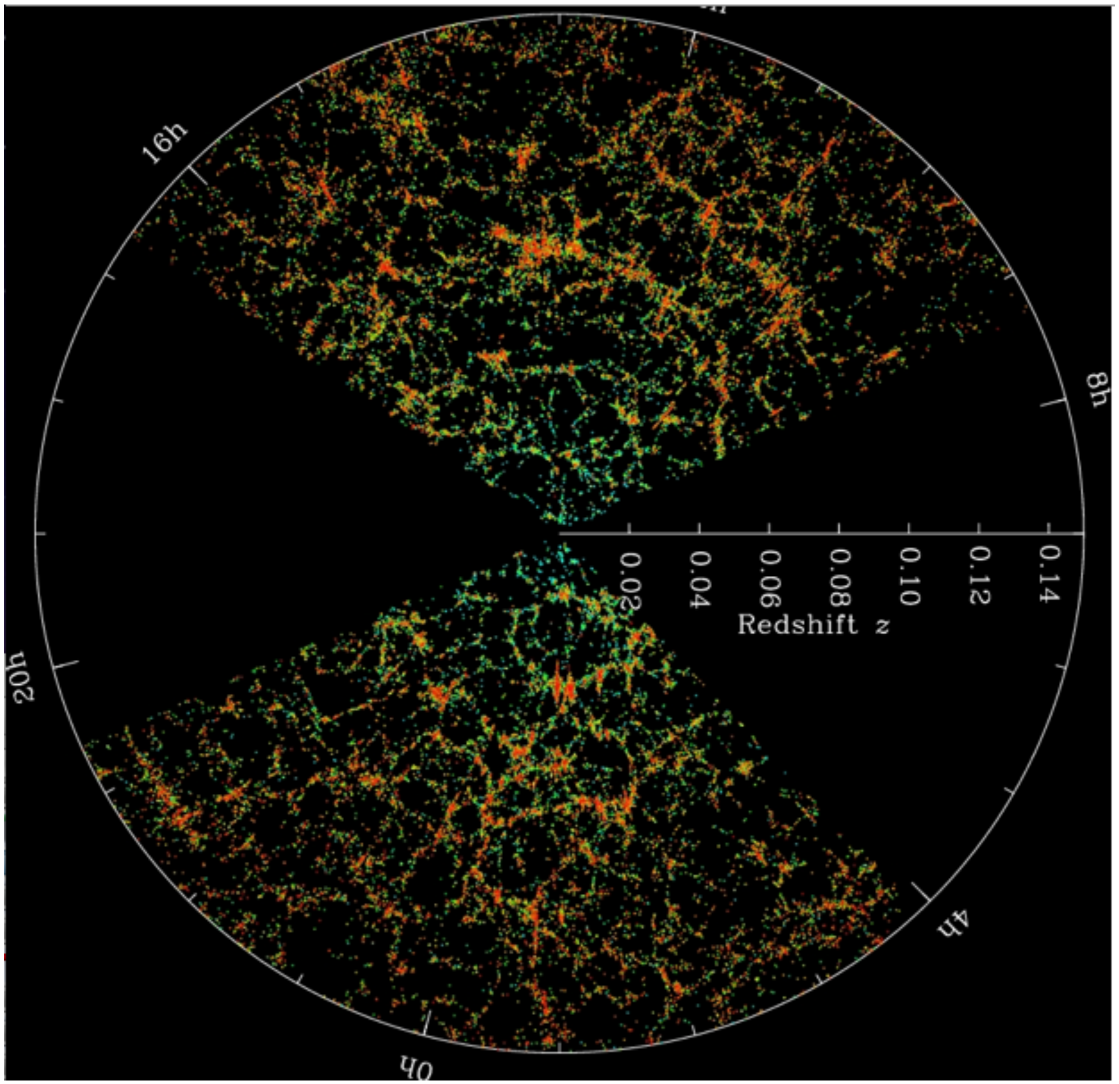
Monument au fantôme, 1983, by Jean Dubuffet
Interfirst Plaza, Houston, Texas (USA)

3D redshift surveys

Two-degree-Field Galaxy Redshift Survey (2dF)



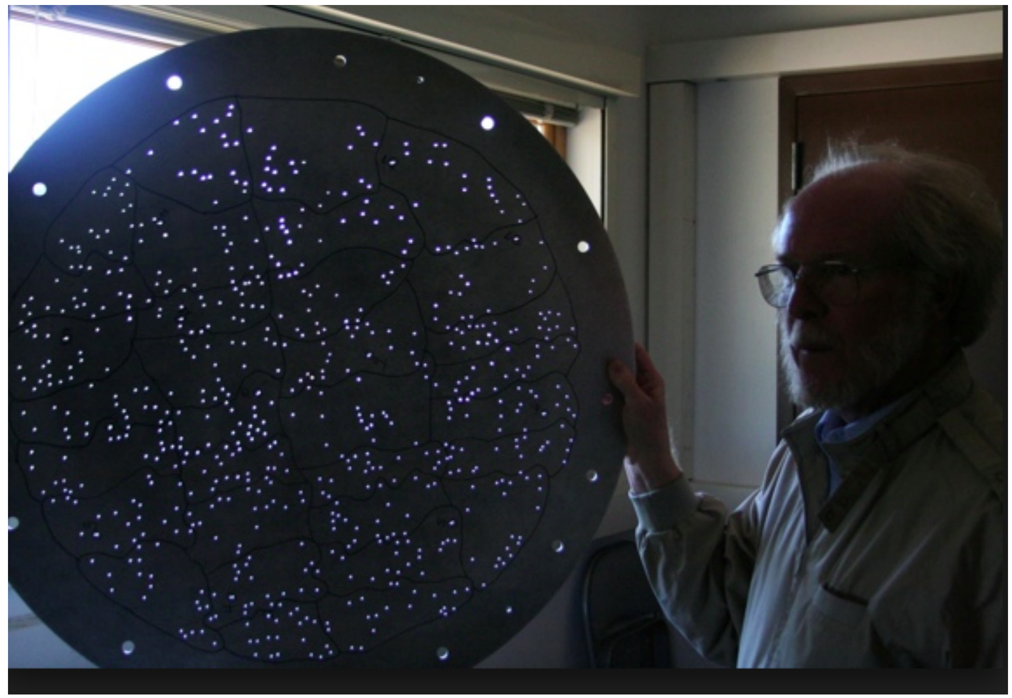
Sloan Digital Sky Survey (SDSS)



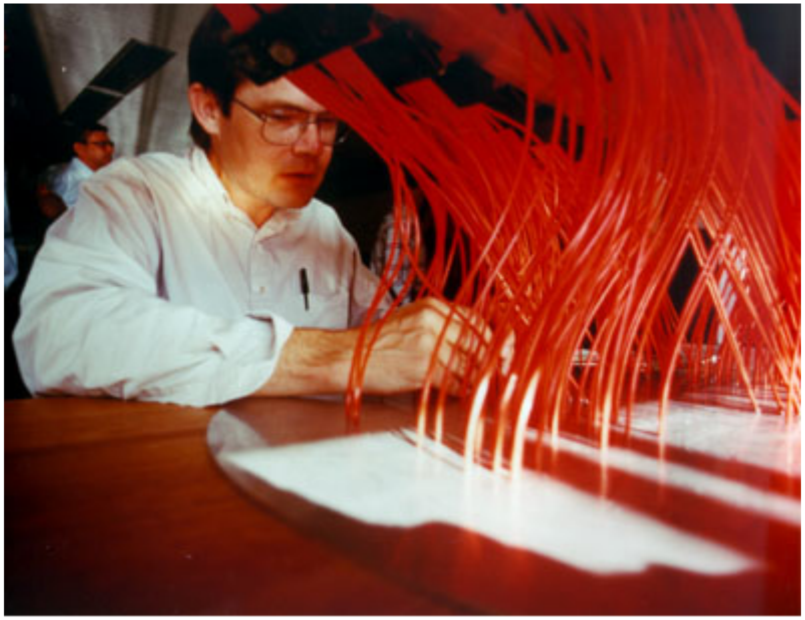
Today we have redshifts measured for millions of galaxies.

How do you measure redshifts of millions of galaxies?

- Multiobject spectrographs are used to measure the spectrum of >500 galaxies *simultaneously.*



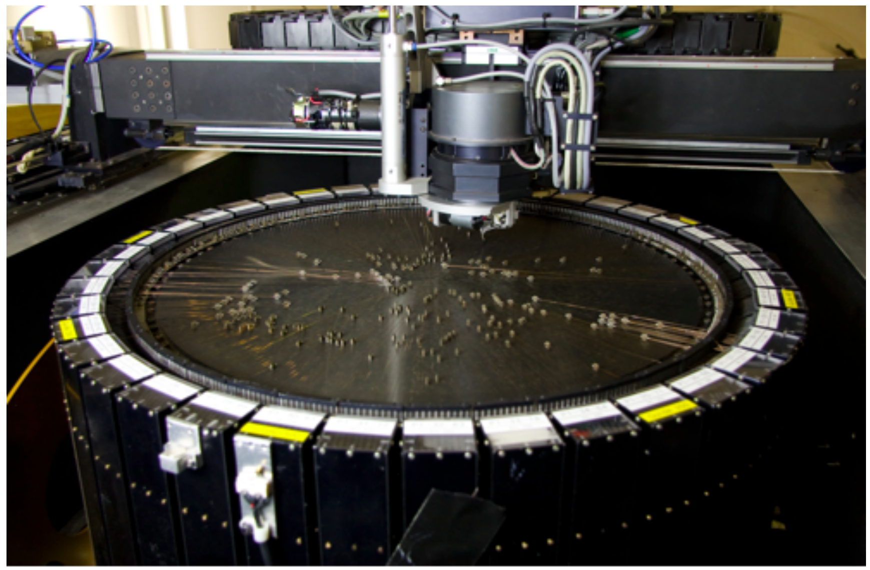
Very labor-intensive

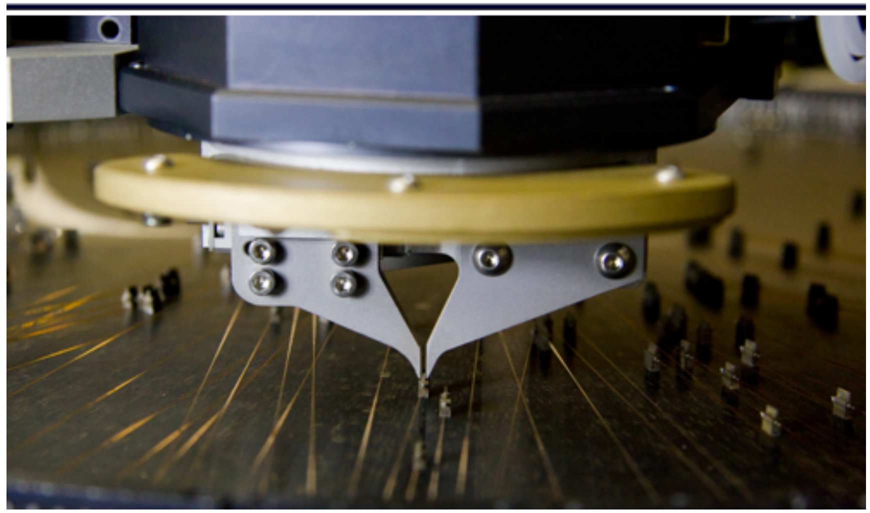


Insertion of optical fibers into a pre-drilled "plug-plate," part of the Sloan Digital Sky Survey's spectrographic system for determining the distances to stars, galaxies, and quasars.

Ouch!

Robotic fiber positioners are now being used for large redshift surveys



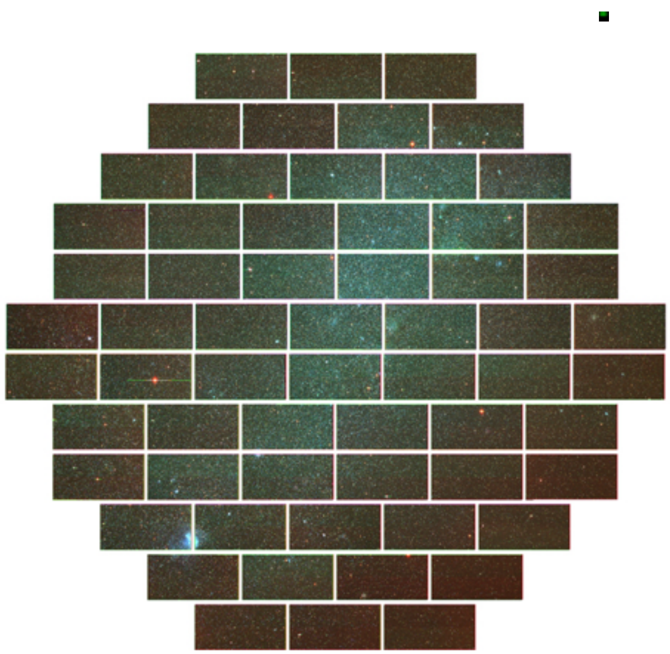


Alternative methods of obtaining redshift information

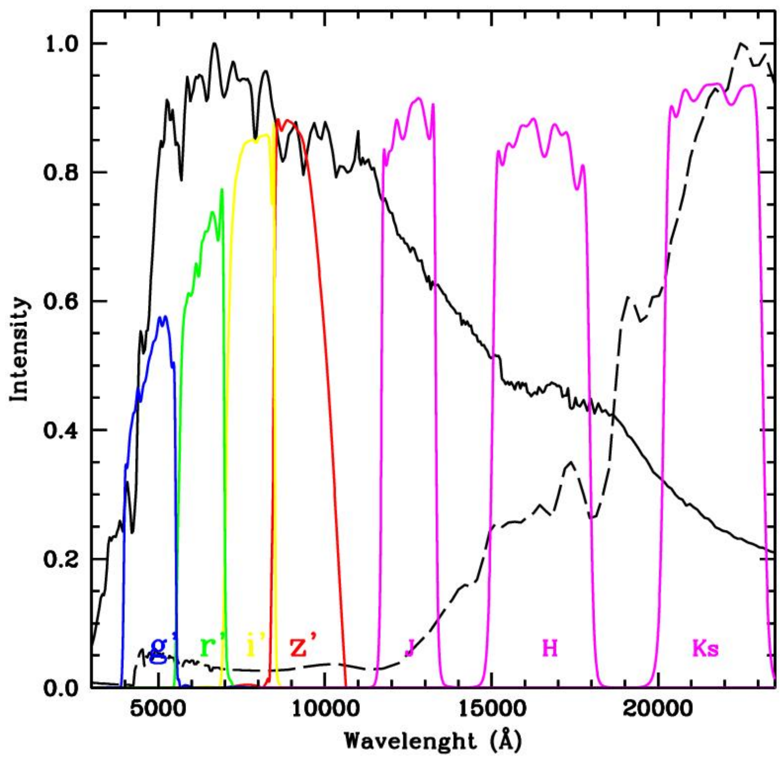
Photometric redshifts

- Dark Energy Survey expects to measure 300 million redshifts *uses photometric redshifts.*
- DECam (Dark Energy Camera) is a 500 Mpixel array of red-sensitive CCDs (Charge-Coupled Devices).

Size of iPhone camera



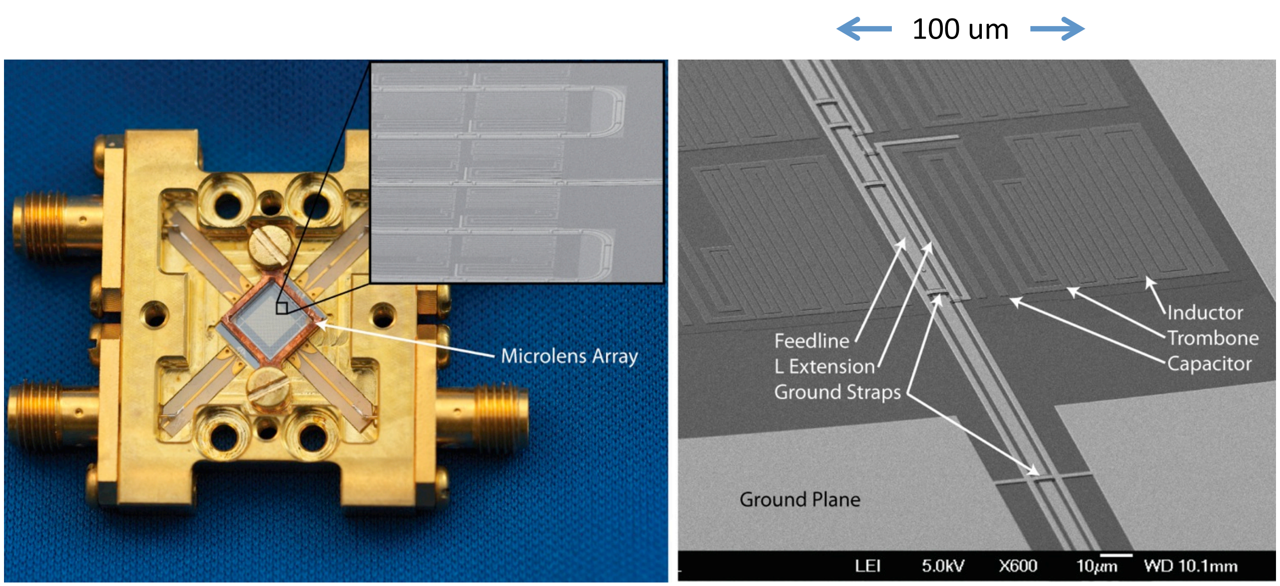
74 2kx4k CCDs



Microwave Kinetic Inductance Detectors (MKIDs)

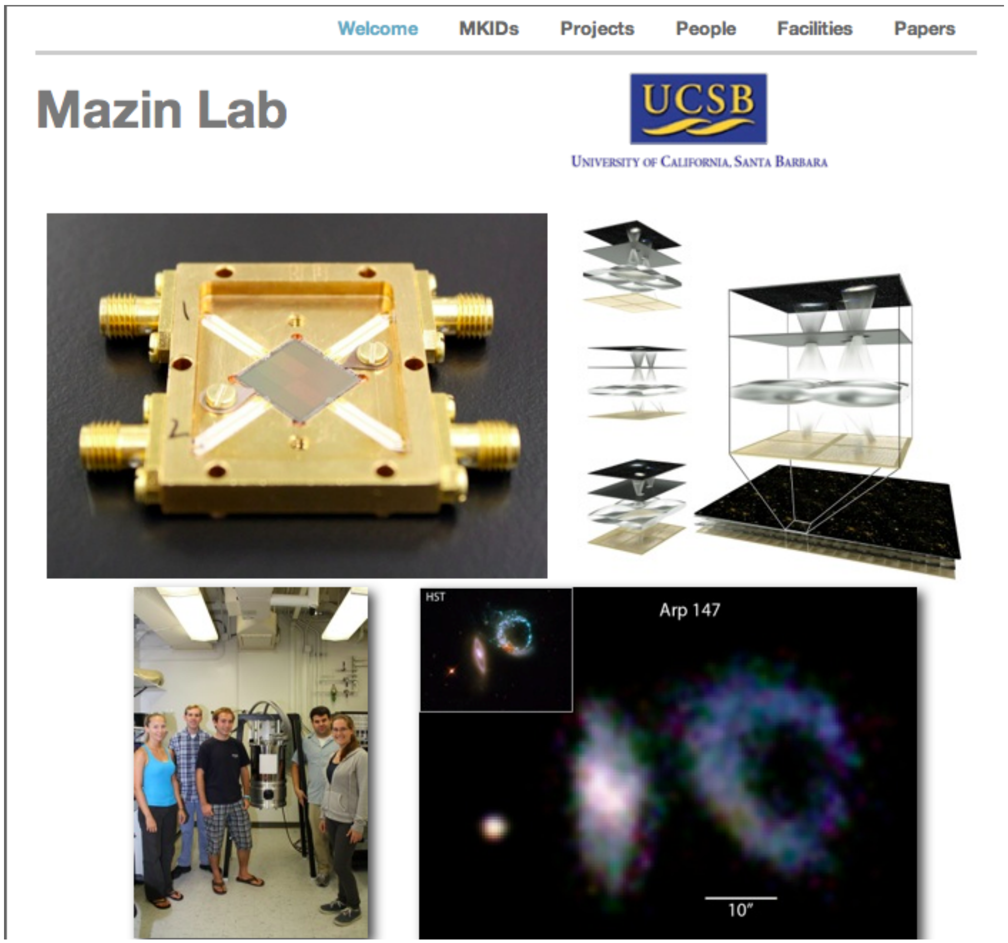
- An array of superconducting thin film microwave resonators (MKIDs) can be used to detect the location, the energy, and the arrival time of the incident photons – *simultaneously.*

- The two-survey aspect of a redshift survey could become one survey!



1024-pixel OLE MKID array with microlenses mounted in a microwave package. Pixels are ~100 um x 100 um. The grayscale inset is a scanning electron microscope image of the array to show the pixel design.

Fermilab collaboration with UCSB

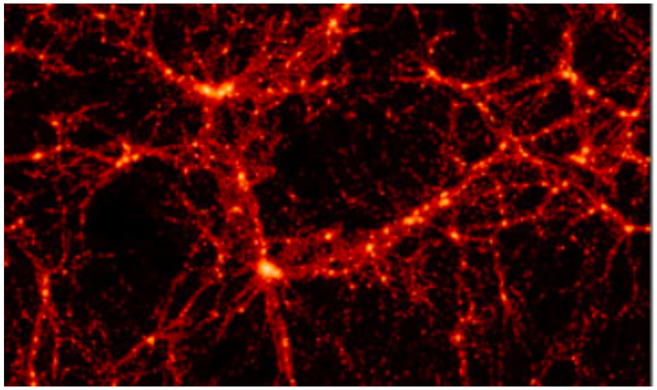


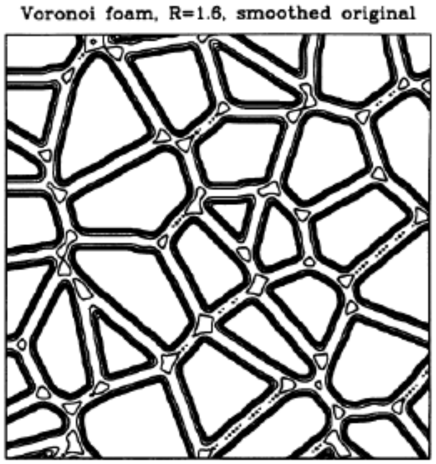
http://web.physics.ucsb.edu/~bmazin/Mazin_Lab/Welcome.html

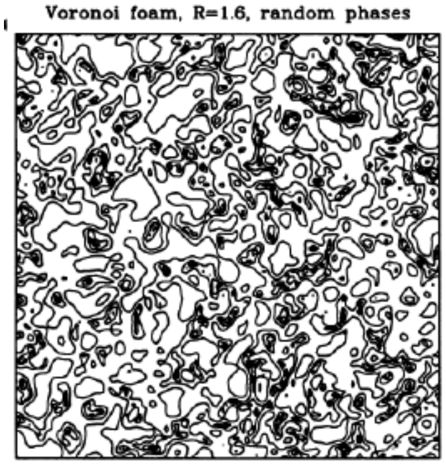
Statistical characterization of the large-scale distribution of galaxies is necessary to quantify the complex appearance of the observed distribution & to test models for the formation of structure.

Correlation function and power spectrum

- If galaxies are clustered, they are "correlated"
- This can be studied using the Fourier pair: *the 2-point correlation function & the power spectrum*
- However, the two images (right) have *identical power spectra.*
- The power spectrum alone does not capture the phase information: the coherence of cosmic structures (voids, walls, filaments …) shown below.







From Hirata and Djorgovski
http://www.astro.caltech.edu/~george/ay127/Ay127_LSS.pdf

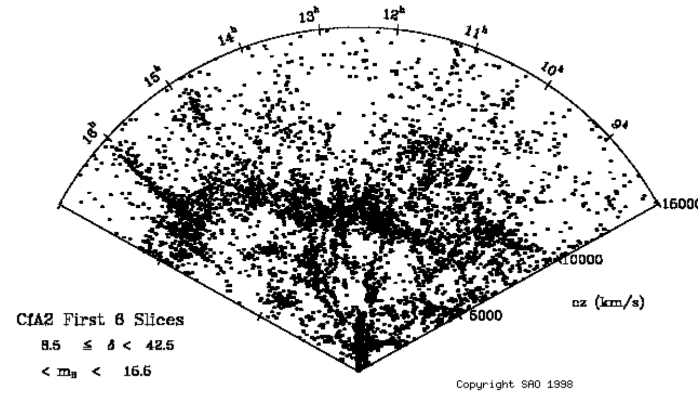
Genus topology to the rescue!

Genus topology

- First proposed by Gott, et al. (1986)
- The genus measures the connectivity rather than the dimensionality of isodensity surfaces; it does not characterize the geometry of the structure.
- When smoothed on a scale sufficient to remove small-scale nonlinearities of gravitational clustering*, the topology of the observed galaxy distribution should be isomorphic to the initial topology*.
- Thus the topology provides a test of *Gaussian initial conditions* for structure formation: expect topology at the median density level to be *sponge-like topology*.
- Inflation, for example, provides a mechanism for such conditions
- Opposing topologies are
  - *Meatball topology*: isolated clusters growing in a low density connected background
  - *Swiss cheese topology*: isolated voids surrounded on all sides by walls.

Gott, et al. **The sponge-like topology of large-scale structure in the universe**, Astrophysical Journal, Part 1, vol. 306, July 15, 1986, p. 341-357.

How to measure the genus



- To quantify the topology of the galaxy distribution, smooth the distribution to obtain a continuous density field.
- Identify contours of constant density.
- Compute the genus of these isodensity surfaces using the Gauss-Bonnet theorem to convert from curvature to genus:

$$G = -\tfrac{1}{4\pi}\int \kappa \, dA$$

where

$$\kappa = \frac{1}{a_1 a_2}$$

where a_1 and a_2 are the principal radii of curvature at the point

- Repeat for various density thresholds, ν, to obtain G=f(ν).

Vogeley, et al. **Topological analysis of the CfA redshift survey**, 1994ApJ...420..525V

Computing the genus

- With this definition, genus is defined as: G = number of holes − number of isolated regions

- This is related to the usual definition of the genus, g, by G = g-1 and to the Euler-Poincare characteristic, χ, by G = -1/2 χ

- *Hole* is in the sense of doughnut hole and *isolated regions* may be either high or low density excursions.

- With this definition, the genus of a sphere is -1, a toroid has genus 0, and two isolated spheres have a genus of -2 (as do two isolated voids within a high density region)

- A multiply-connected structure, like a sponge, has many holes and therefore a large positive genus.

Vogeley, et al. **Topological analysis of the CfA redshift survey**, 1994ApJ...420..525V

Apology to Euler, et al.

- From personal correspondence with Michael Vogeley:
- "Using this slightly different definition, the genus has the simple and easily understandable relation to the structure of a surface:

  G = (# holes) - (# isolated regions)"
- "There is no other mathematical or practical reason for this offset. The computer program could certainly keep track of that."
- "For the large galaxy data sets that we now analyze, the genus can be a few hundred, so the offset of one makes almost no difference."

Vogeley, et al. **Topological analysis of the CfA redshift survey**, 1994ApJ...420..525V

Analytic formula for mean genus/volume

- The analytic formula for the mean genus per unit volume of density contours in a random phase distribution is

$$g_s = \frac{1}{4\pi^2}\left(\frac{\langle k^2\rangle}{3}\right)^{3/2}(1-\nu^2)e^{1-\nu^2/2}$$

where

g_s is the mean genus/unit volume of contour

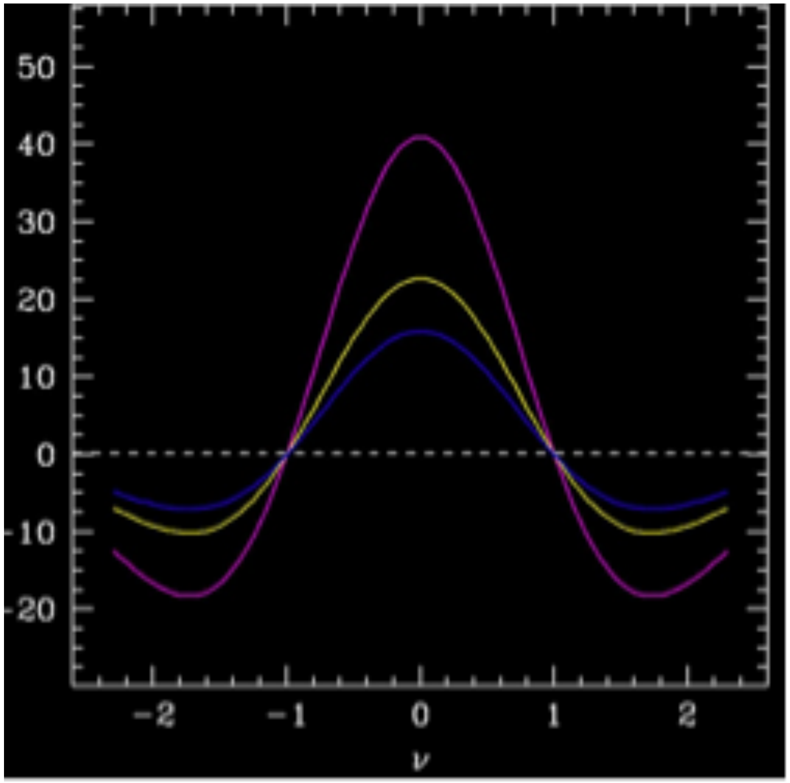
ν is the number of standard deviations the threshold density departs from the mean density

$\langle k^2\rangle$ is related to the Fourier transform between the correlation & power spectrum

- Resultant "W" shape: symmetric because high- and low-density regions are topologically equivalent.

- The mean density contour (ν=0) has maximum genus.

- Contours more than 1σ from the mean density are negative (break up into isolated regions).



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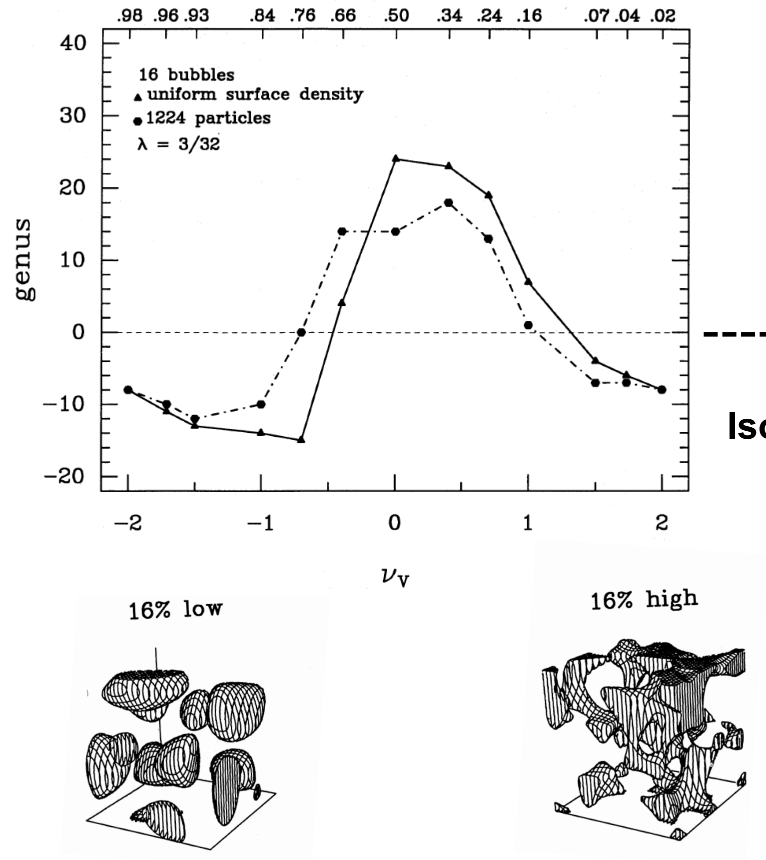
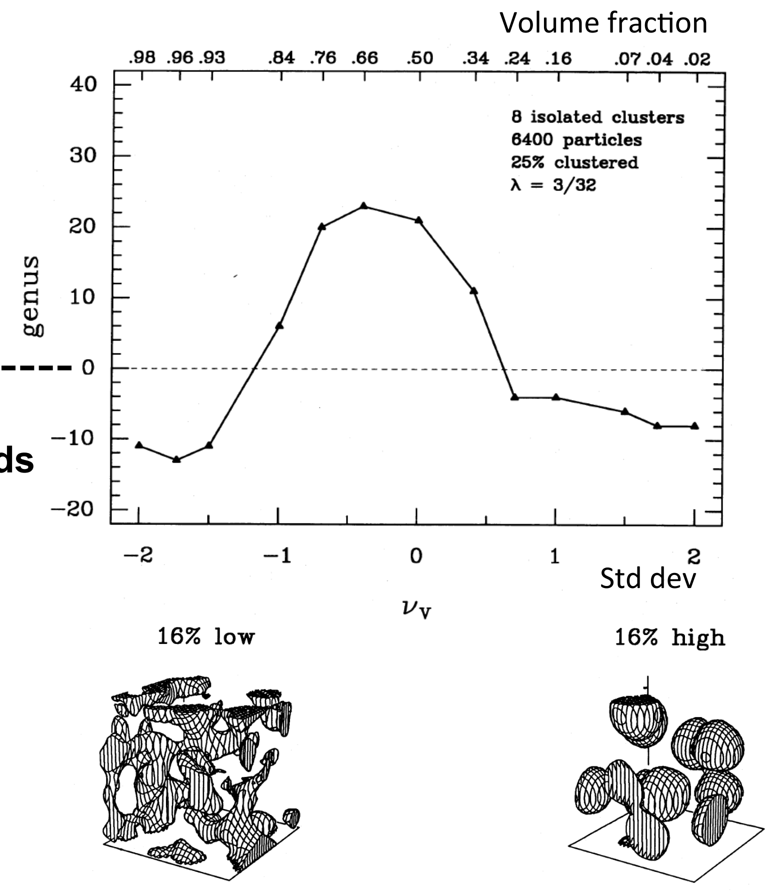
What do genus curves look like for small scales?



Many holes
Multiply connected

Isolated clusters & voids

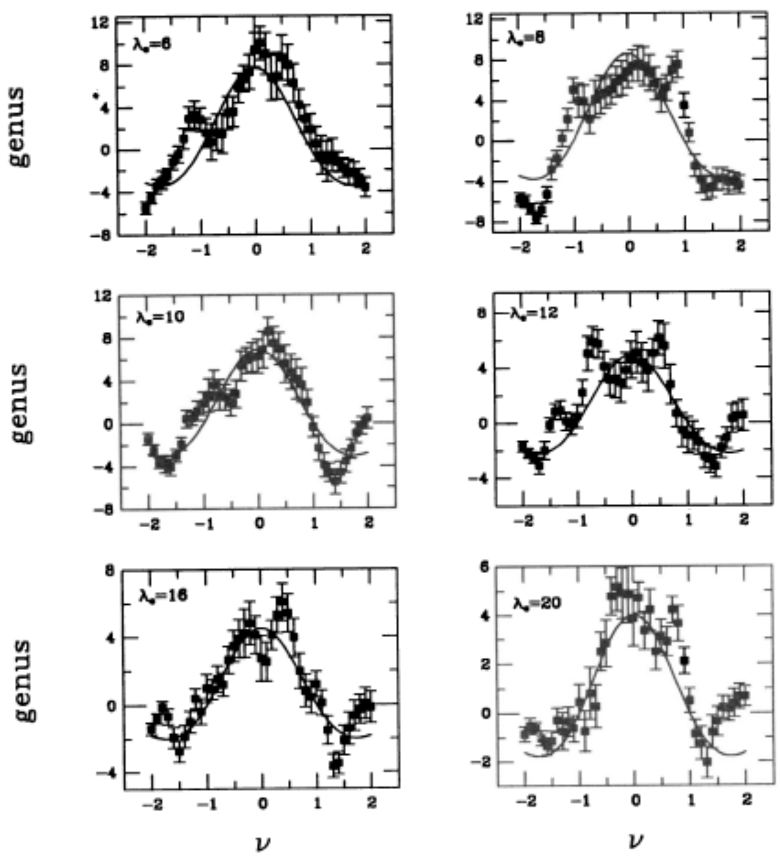


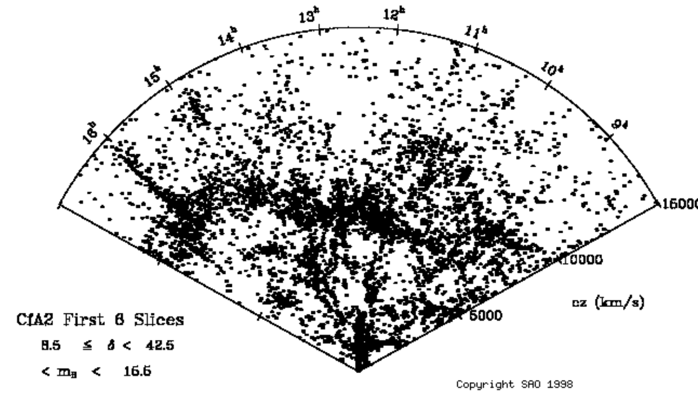
“Swiss cheese” topology

“Meatball” topology

Weinberg, D. H., et al. **The Topology of Large-Scale Structure. I. Topology and the Random Phase Hypothesis**, 1987, ApJ, 321, 2

Examples for small scales

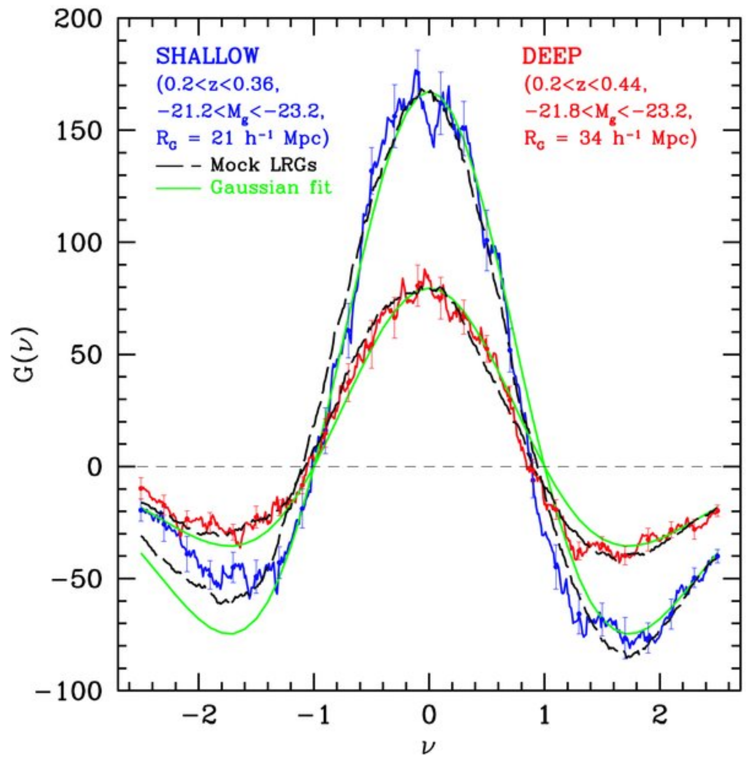




Vogeley, et al. **Topological analysis of the CfA redshift survey**, 1994ApJ...420..525V

Where is the field today?

- Gott, et al. (2009) have measured the 3D genus topology of large scale structure using luminous red galaxies (LRGs) in the SDSS and find it consistent with the Gaussian random phase initial condition expected from the simplest scenarios of inflation.
- They studied 3D topology on the largest scales ever obtained.
- Compared to simulations.
- The topology is sponge-like, strongly supporting the predictions of inflation.



Genus curves for observed (two noisy curves) and simulated (two long-dashed curves) LRGs. Gaussian fits (solid lines) are also shown. The genus curves with higher amplitudes are for the SHALLOW sample, and those with lower amplitudes are for the DEEP sample. Median volume fraction contour (ν=0)

Gott, et al., **Three-Dimensional Genus Topology of Luminous Red Galaxies**
The Astrophysical Journal Letters, Volume 695, Issue 1, pp. L45-L48 (2009).

Interdependence of topology and cosmology

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Understand the large-structure of the universe	2D & 3D genus topology of large scale structure	Measure the genus of galaxy isodensity curves	2, 3
Understand the shape of space itself *Cosmic topology*	Cosmic crystallography	Look for repeating patterns in the distribution of clusters of galaxies	3
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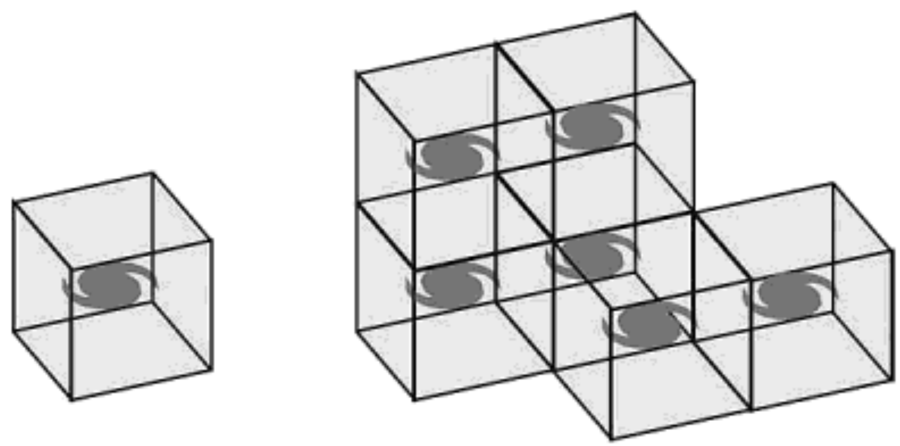
Cosmic crystallography

Cosmic Crystallography is another application for the redshift surveys!

How to detect a multiply connected universe

- In 1900 Schwartzchild had already imagined that our Galaxy could repeat itself endlessly within a regular cubic framework thus giving the illusion that space is far vaster than it really is.
- Problem: It's hard to recognize the image of the Galaxy because it has evolved or is in a different orientation.

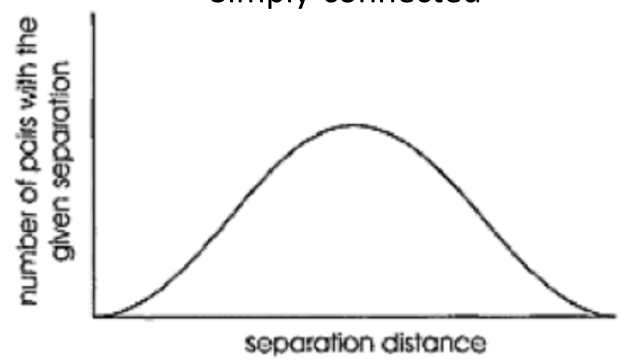


Shape of Space, Jeff Weeks
The Wraparound Universe, Jean-Pierre Luminet

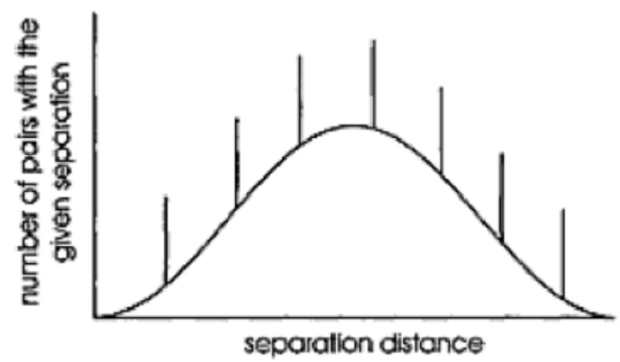
Cosmic crystallography

- Compute the distances between every pair of galaxies.
- In a simply-connected universe, will get a Poisson distribution.
- In a multiply-connected universe, certain distances may occur more frequently than random chance.
- This is called *Cosmic Crystallography.*

Simply-connected



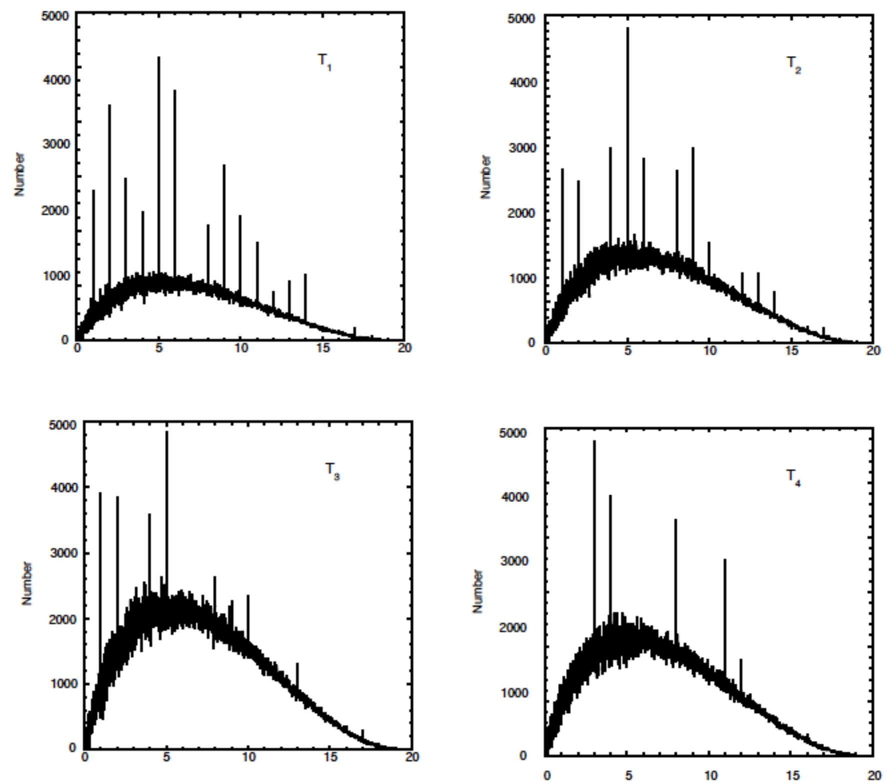
Multiply-connected



Shape of Space, Jeff Weeks
The Wraparound Universe, Jean-Pierre Luminet

Which topology?

- In these numerical simulations:
  - the presence of peaks indicates a multiply connected topology
  - the positions of the peaks reflect the size of the fundamental polyhedron
  - the relative heights of the peaks characterize the holonomy group
- A downside of cosmic crystallography is that it doesn't work for all manifolds



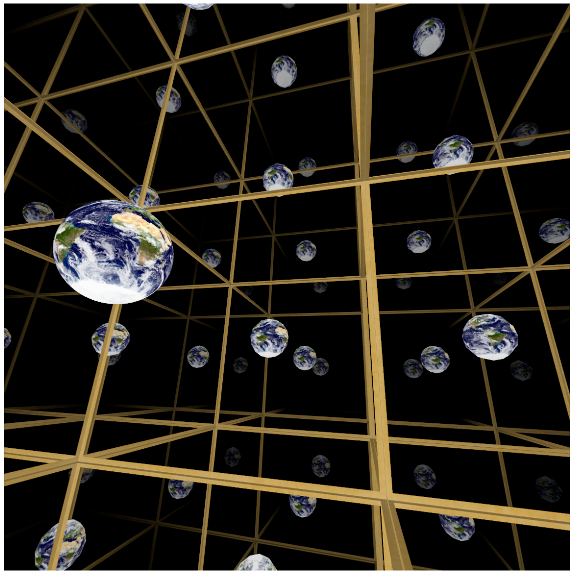
Histograms of pairwise separations for four multiply connected toroidal universes

Shape of Space, Jeff Weeks
The Wraparound Universe, Jean-Pierre Luminet

Current status of cosmic crystallography

- The main limitation of cosmic crystallography is that the presently available catalogs of observed sources are not complete enough to perform convincing tests.

- More extensive catalogs from ongoing and future redshift surveys will offer better opportunities to detect the shape of space via cosmic crystallography.



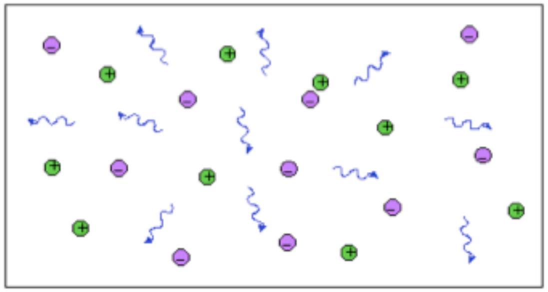
Shape of Space, Jeff Weeks
The Wraparound Universe, Jean-Pierre Luminet

Fortunately, the topology of a small universe may also be detected through its effect on the Cosmic Microwave Background (CMB)

Circles on the sky

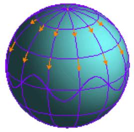
Cosmic Microwave Background (CMB)

- During the first 300,000 years after the big bang, temperatures were extremely high; photons kept the gas ionized & scattered off the charged particles in all directions, making the universe opaque.

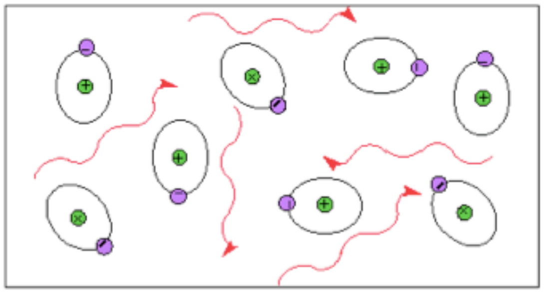


- The wavelength of light, λ, increases as it traverses the expanding universe: as the universe expanded, it cooled:

$$E = \frac{hc}{\lambda}$$



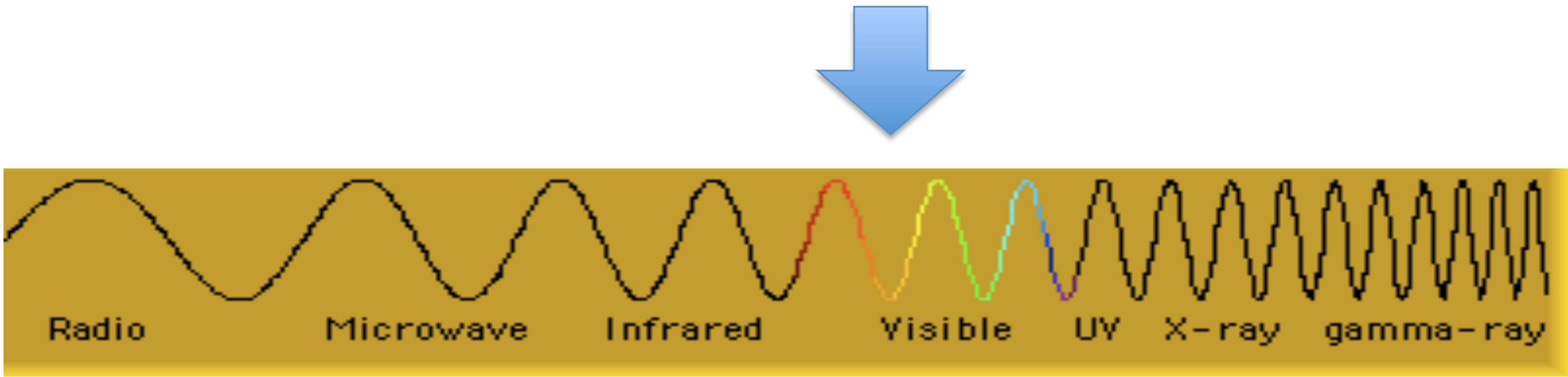
- Photons no longer had enough energy to ionize the gas; protons and electrons could now combine to form atoms & photons could travel long distances without being scattered



- These are the photons we see as the CMB!

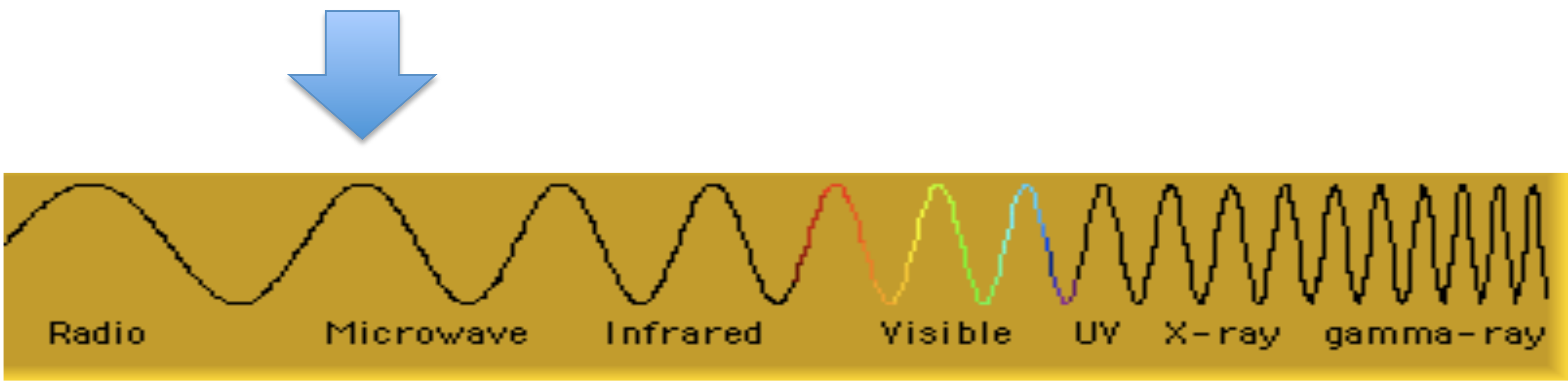
CMB then

- When it reached about 3000 K, the average energy of the photons was decreased to the point where they could no longer ionize hydrogen.
- 3000 K corresponds to optical wavelengths.



CMB now

- Between then and now, the universe has expanded by a factor of ~1100.
- We see the CMB photons 1100 times cooler, at about 3K, at wavelengths 1100 times longer at microwave wavelengths.



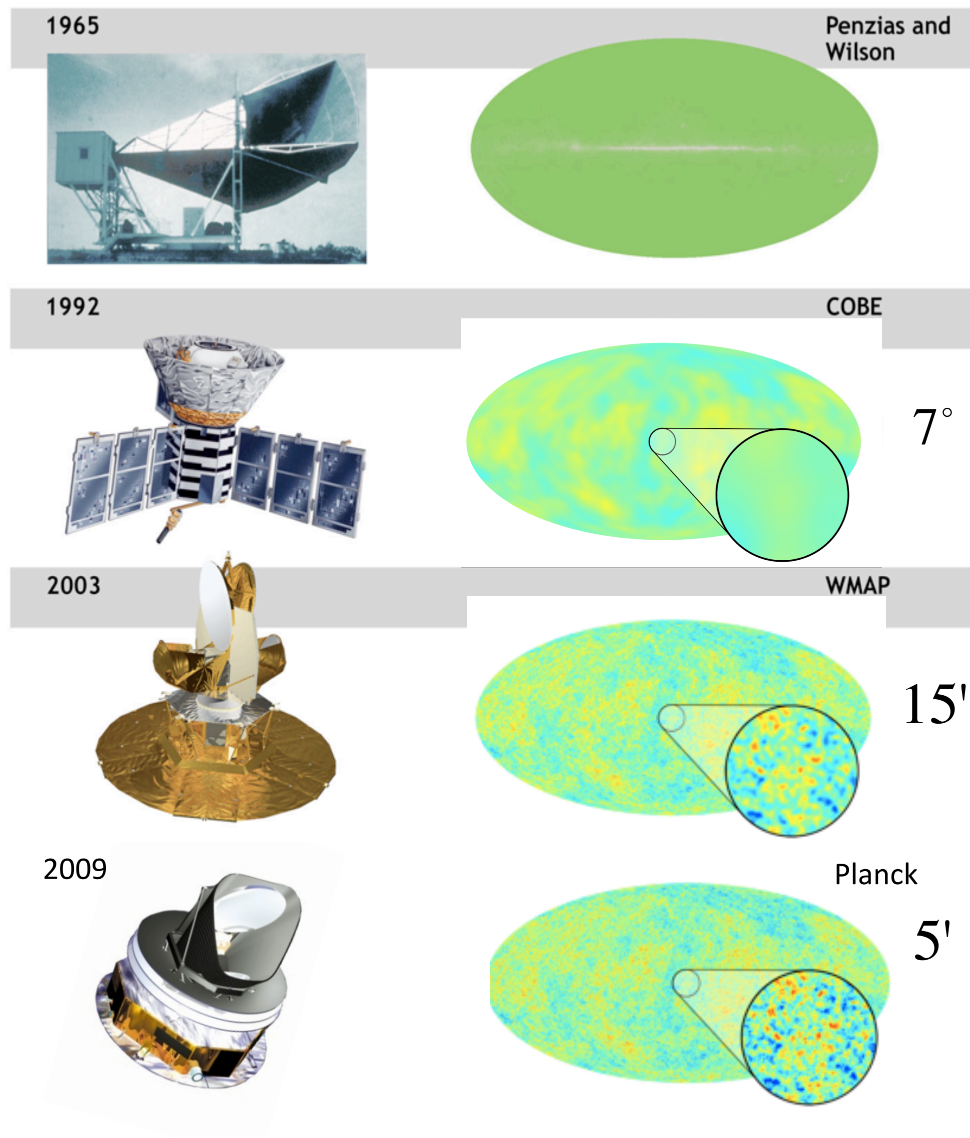
Observations of the CMB

- As the resolution of telescopes looking at the CMB got higher, we learned that CMB is not uniform.

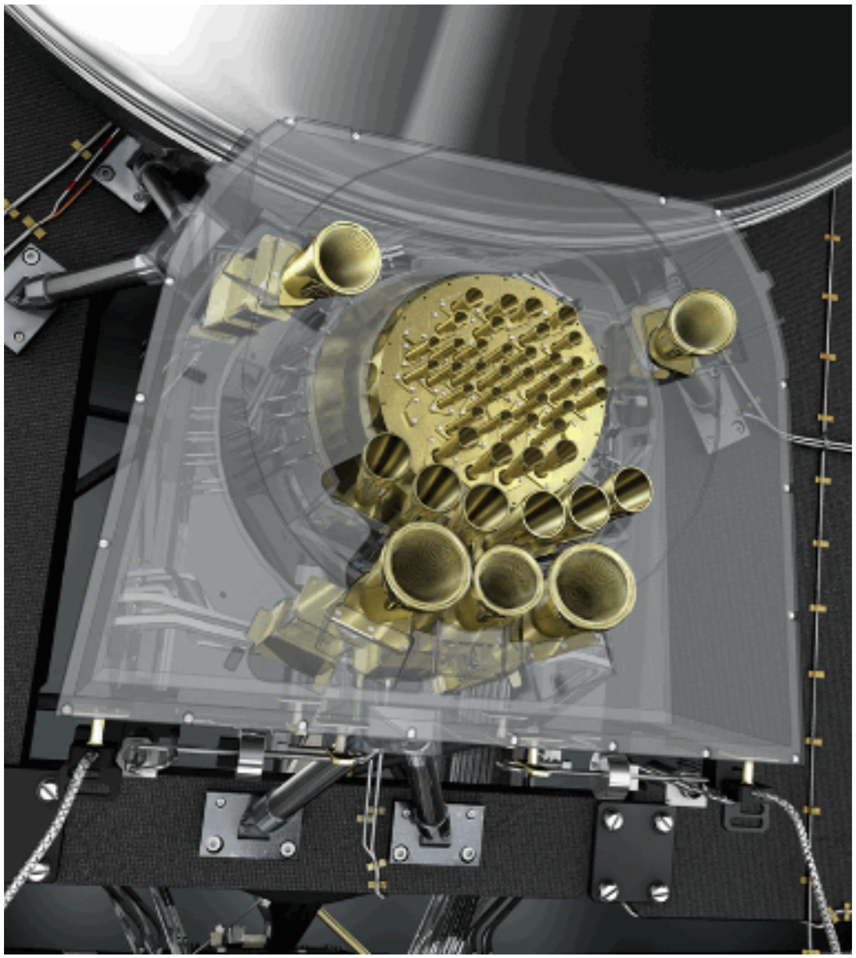
$$resolution = \frac{\lambda}{D}$$
$$\lambda = \text{wavelength of observations}$$
$$D = \text{diameter of telescope}$$

- The colors indicate temperature variations which correspond to density variations in the CMB.

- Perhaps patterns in the CMB can indicate the topology of the universe!



How does Planck achieve higher resolution?



- Larger diameter (for details, see next slide)

$$resolution = \frac{\lambda}{D}$$

λ = wavelength of observations

D = diameter of telescope

- Observes at 9 frequencies to sort out foregrounds.

- For more information on the history of CMB space antennas see:

  Space Antenna Handbook, Imbriale, et al. 2012

  Chapter 16 *Space Antennas for Radio Astronomy*, Paul F. Goldsmith (JPL)

Planck's larger diameter(s)

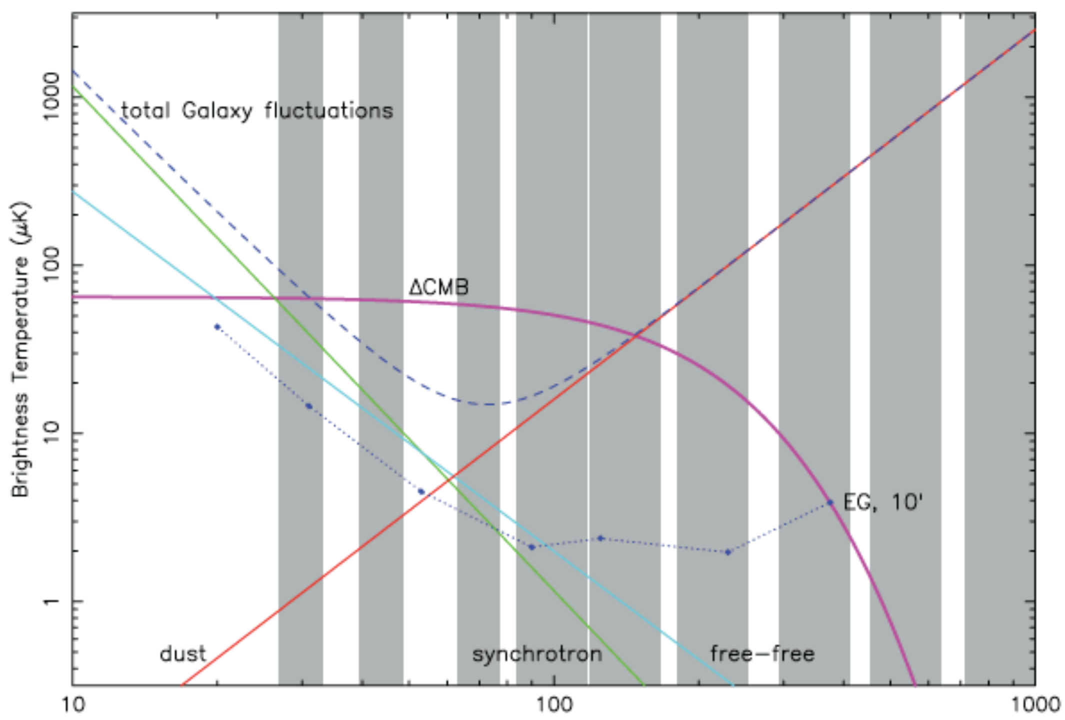
- From personal correspondence with Charles Lawrence, Planck Project Scientist, JPL:
- "The primary mirror is "under-illuminated''. Only the central part of the primary is used to collect light. The rest acts as a shield against light coming in from other directions."
- Using published beam widths, the calculated *effective diameters* of Planck and WMAP are shown in the table. Note the effective diameter is f(frequency).

	Frequency (GHz)	Wavelength (mm)	(Calculated) Diameter (m)	(Published) beam FWHM (arcmin)	number	polarization
COBE	31	9.645		7 degrees	2	n
	53	5.642		7 degrees	2	n
	90	3.322		7 degrees	2	n
WMAP	23	13.000	0.86	52.80	2	y
	33	9.061	0.80	39.60	2	y
	41	7.293	0.84	30.60	4	y
	61	4.902	0.82	21.00	4	y
	94	3.181	0.84	13.20	8	y
PLANCK	28.4	10.561	1.14	32.38	2	y
	44.1	6.807	0.88	27.10	3	y
	70.4	4.260	1.12	13.30	6	y
	100	3.000	1.09	9.65	0/8	n/y
	143	2.098	1.01	7.25	4/8	n/y
	217	1.382	0.97	4.99	4/8	n/y
	353	0.850	0.62	4.82	4/8	n/y
	545	0.550	0.41	4.68	4/0	n/y
	857	0.350	0.28	4.33	4/0	n/y

Planck 2013 results. I. Overview of products and scientific results
Planck Collaboration, et.al., http://arXiv.org/abs/1303.5062

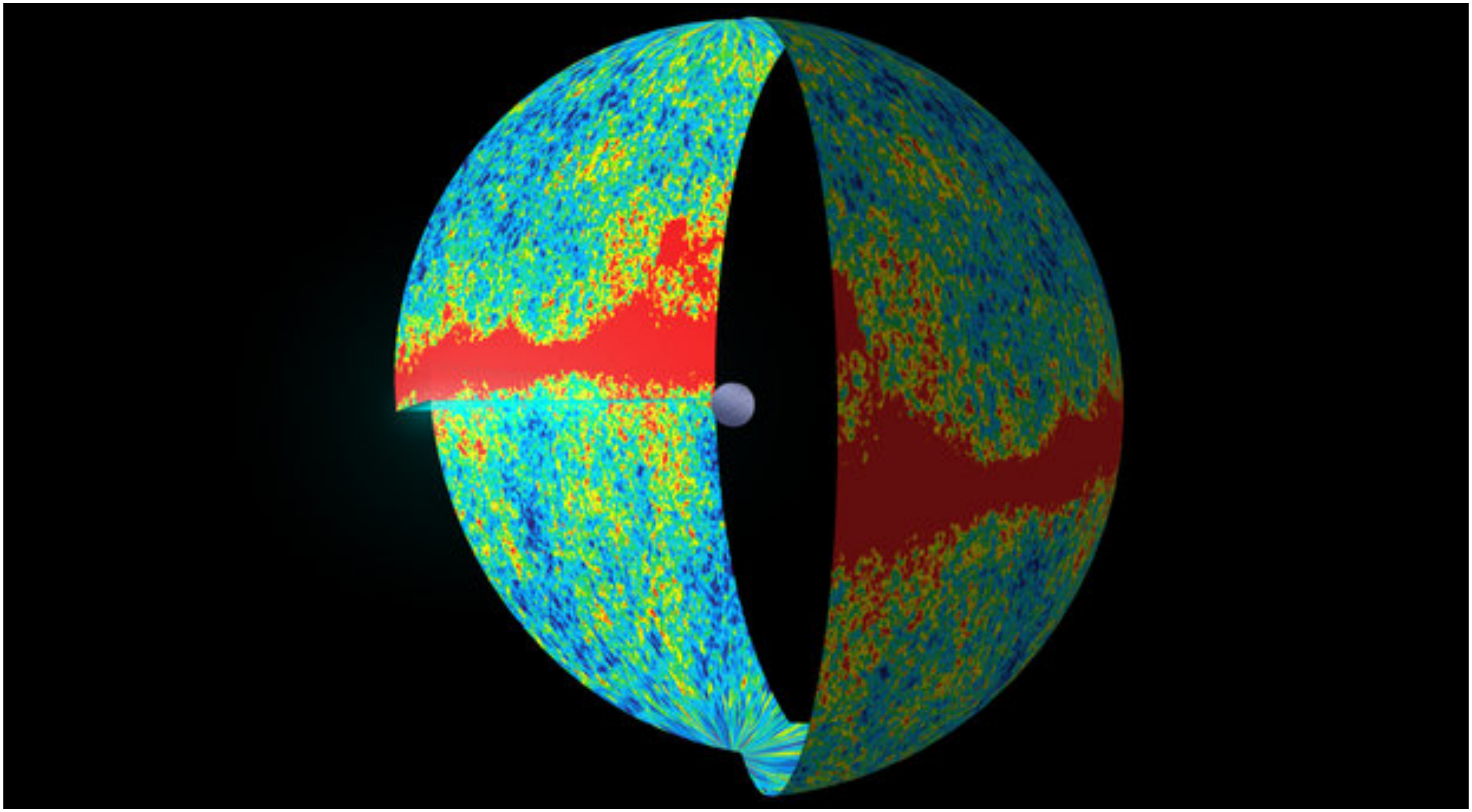
PLANCK observes at 9 frequencies

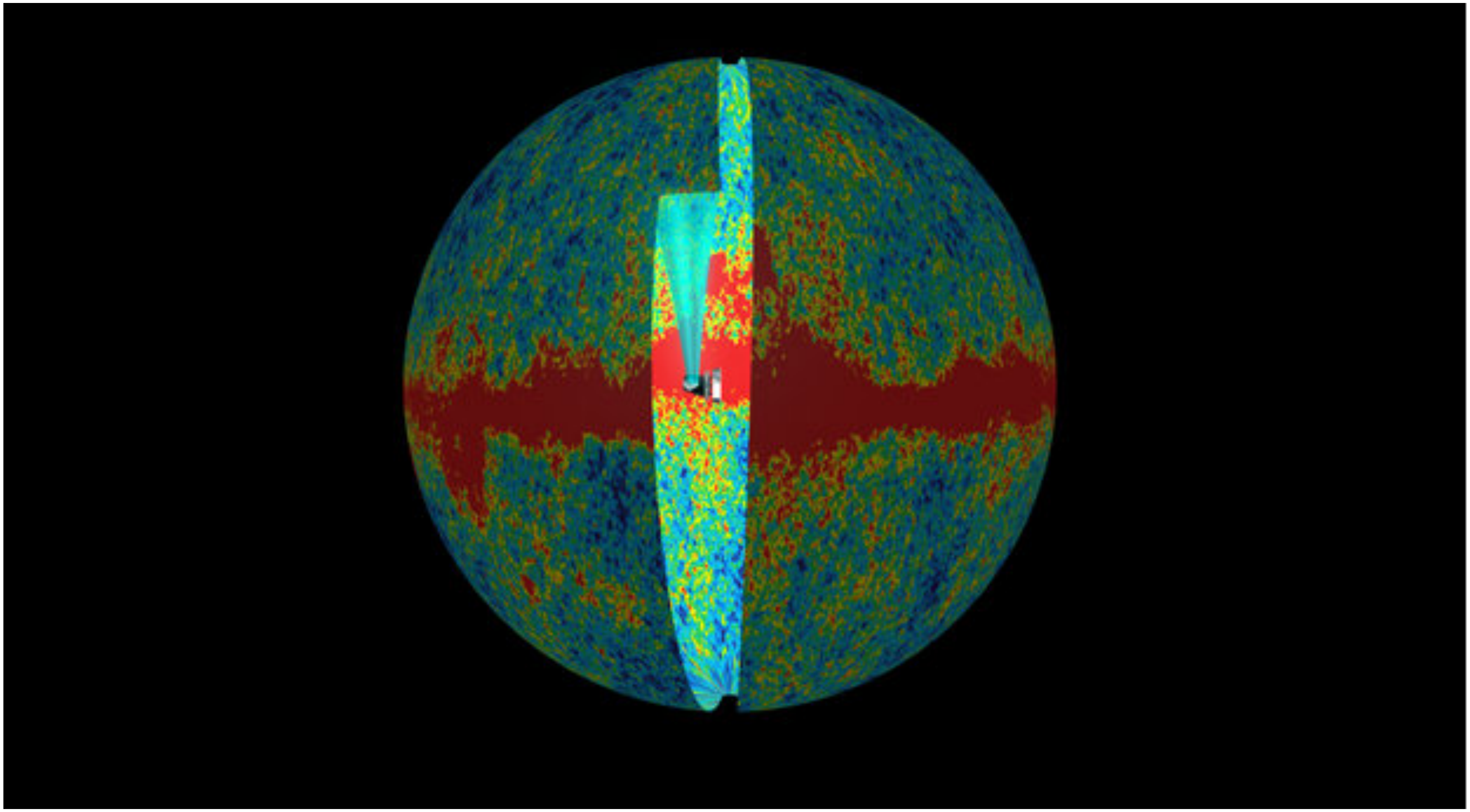
- Planck observes in 9 frequency bands with the goal of improving foreground removal.
- The dominant foreground depends on frequency.



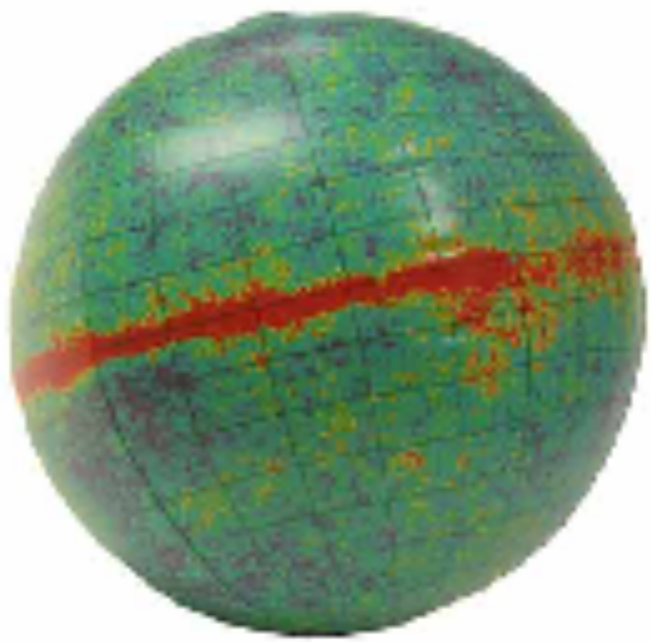
Planck Collaboration, et.al., http://www.rssd.esa.int/SA/PLANCK/docs/Bluebook-ESA-SCI(2005)1_V2.pdf

Mapping the CMB





The Inflatable Universe

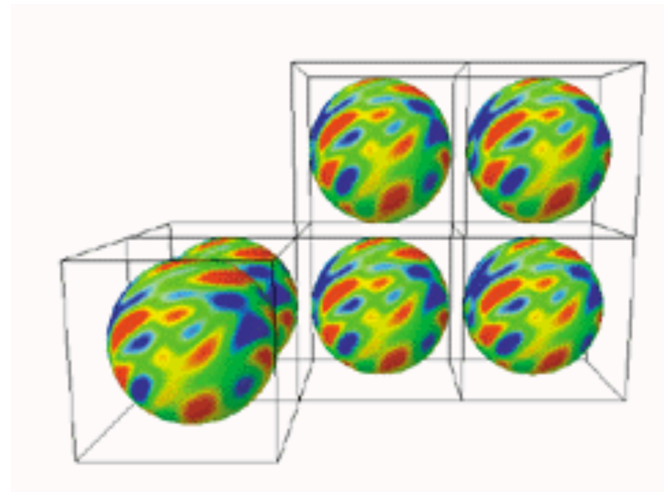


Circles in the sky

CMB in a three-torus

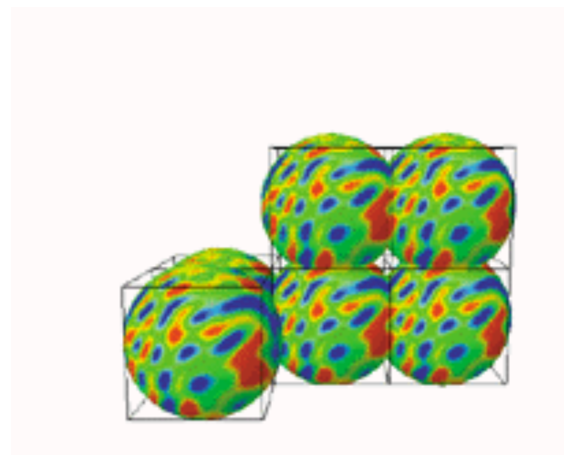
- There is only one last scattering surface (LSS), but we see multiple images of it.

- If the universe is slightly larger than the LSS, we learn nothing about its topology.



- If the universe is smaller that the LSS, the LSS wraps around the universe and intersects itself.

- Each self-intersection is a circle, creating pairs of matched circles.



Shape of Space, Jeff Weeks
The Wraparound Universe, Jean-Pierre Luminet

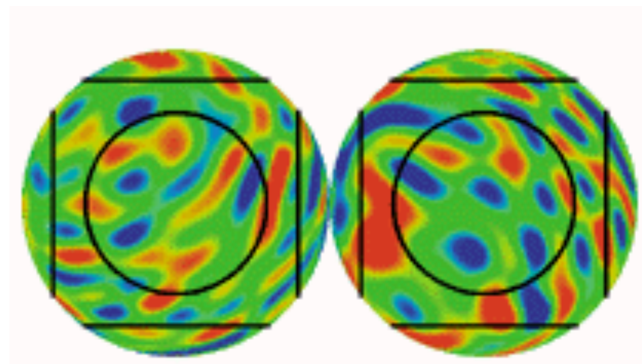
CMB in a three-torus

- If we sit at the center of the LSS, we can look to the west and see one of the circles of self-intersection and look to the east and see the *same* circle of self intersection.

- The same circle of points in space appears once in the western sky and once in the eastern sky.

- The overall temperature patterns in the two hemispheres are very different; the temperatures match only along the circles.

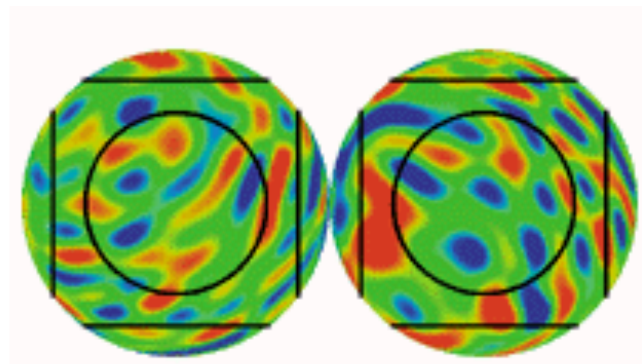
- The analysis is highly computationally intensive!



Shape of Space, Jeff Weeks
The Wraparound Universe, Jean-Pierre Luminet

Is the CMB really a surface? Or is it 3D?

- “Decoupling took place over roughly 115,000 years. Doesn’t this mean the CMB has a thickness; that it is not a single surface?"
- From personal correspondence with Jean-Pierre Luminet:
- “You're perfectly right, the CMB is technically 3-dimensional, and this should have (only slight) consequences on the strategies for detecting the topology in the CMB…. It would play a role when one looks at fluctuations on scales smaller than the projected width of the last scattering surface. In this case when looking in a given direction, one picks up fluctuations which are situated “on one side” of the last scattering surface, but for pairs of circles, one sees opposite sides of the last scattering surface.”
- “On larger scales, the effect is negligible as one averages temperature fluctuations on regions much larger than the thickness of the last scattering surface. ”
- “There is another 3D aspect in the problem of searching for the topology in CMB maps, much more important than the thickness of the last scattering surface : the integrated Sachs-Wolfe effect. ”



Latest results from Planck (March 20, 2013)

- The circles in the sky search show no evidence of a multiply-connected universe.
- Note that a null result is generic (i.e. not tied to a specific topology). But any detections must be calibrated with specific simulations for a chosen topology.
- We do not find any statistically significant correlation of circle pairs in any map. As seen in Fig. 6, the minimum radius at which the peaks expected for the matching statistic are larger than the false detection level is 20 degrees.
- Thus, we can exclude at the confidence level of 99% any topology that predicts matching pairs of back-to-back circles larger than this radius.

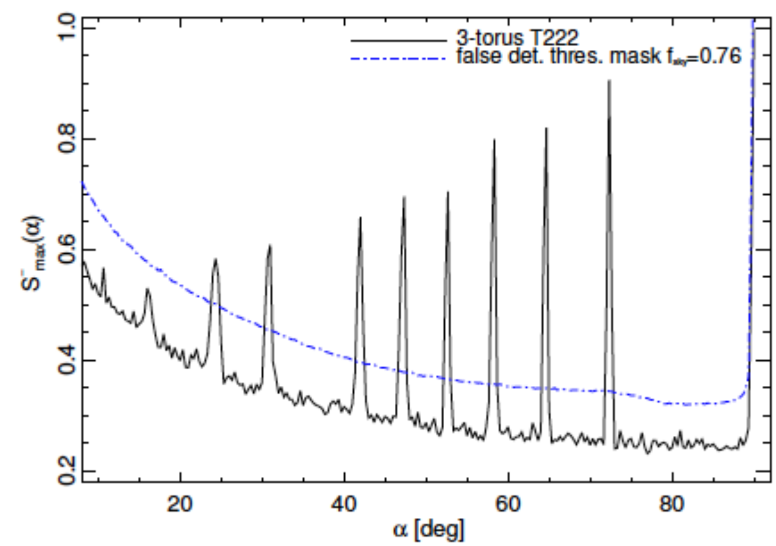


Fig. 6: An example of the S^-_{max} statistic as a function of circle radius α for a simulated CMB map (shown in Fig. 5) of a universe with the topology of a cubic 3-torus with dimensions $L = 2H_0^{-1}$ (solid line). The dash-dotted line show the false detection level established such that fewer than 1 % out of 300 Monte Carlo simulations of the CMB map, smoothed and masked in the same way as the data, would yield a false event.

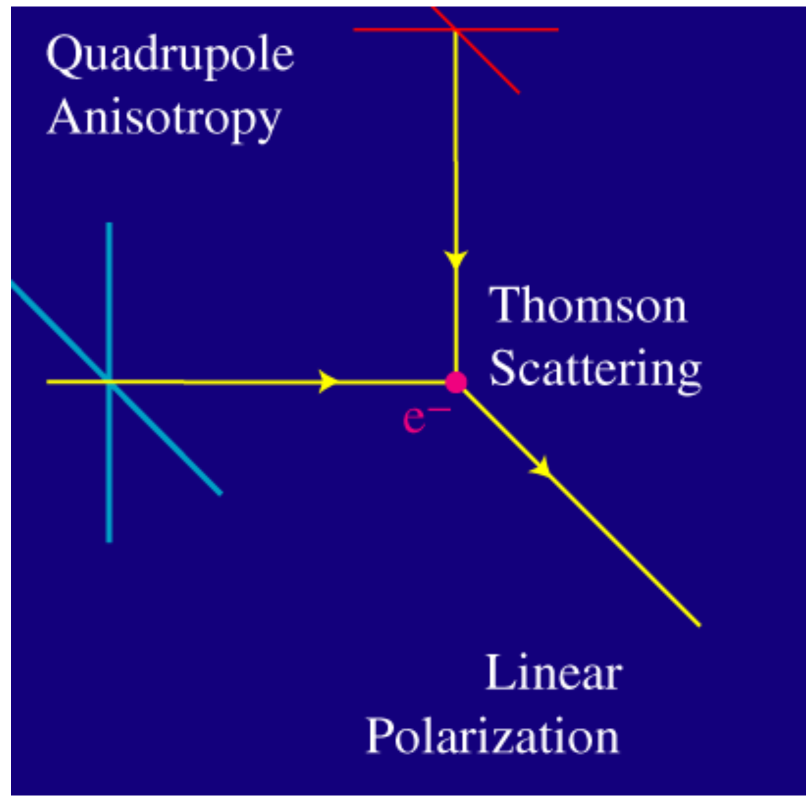
Planck 2013 results. XXVI. Background geometry and topology of the Universe
arXiv:1303.5086 [astro-ph.CO]

Polarization of CMB

- Future Planck measurement of CMB polarization will allow us to further test models of anisotropic geometries and non-trivial topologies and may provide more definitive conclusions.

Cooler, less dense
Fewer photons

Hotter, denser
More photons



Results in net vertical polarization

[Planck 2013 results. XXVI. Background geometry and topology of the Universe](#)
arXiv:1303.5086 [astro-ph.CO]

The best is yet to come!