QCD without Matrices:

the Color-Flow Decomposition

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I. Motivation

II. Color decomposition

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I. Introduction

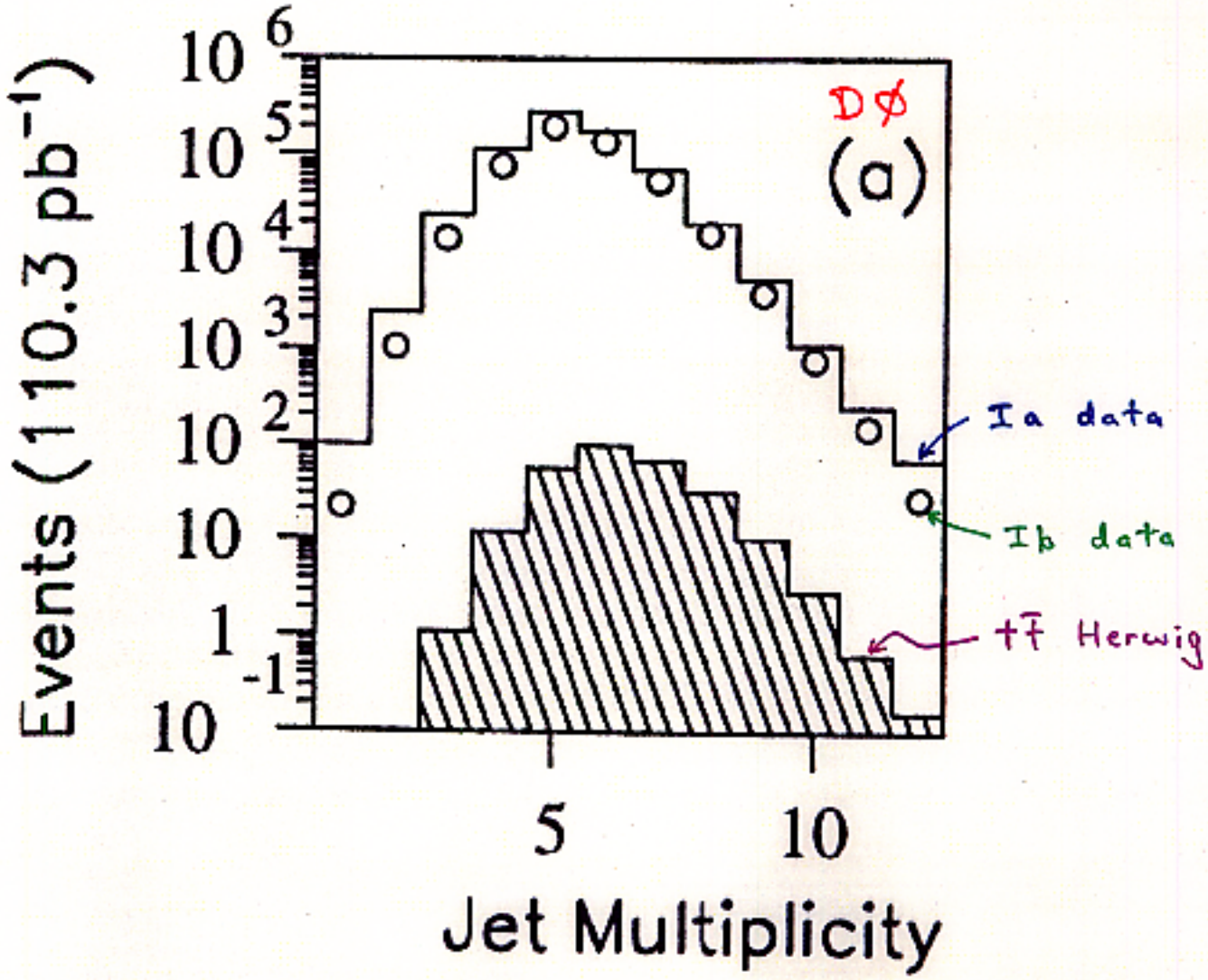


FIG. 4. Properties of jets with $\mathcal{R}$=0.3 cones. Data from the Ia (histograms) and Ib (circles) periods, and $t\bar{t}$ HERWIG for m_t=175 GeV/c^2 (shaded histograms). Only jets with E_T > 10 GeV and $|\eta|$ < 2 are included.

from DØ $t\bar{t} \to$ jets paper

Multi-jet events are backgrounds to new physics

Examples: $t\bar{t} \to 6$ jets

$t\bar{t}h \to 8$ jets
$\hookrightarrow b\bar{b}$

$t\bar{t}h \to 10$ jets
$\hookrightarrow WW$

Need fast tree-level event generators for multi-jet events

State of the art:

NJETS	$p\bar{p} \to \leq 5$ jets	Berends, Giele, Kuijf
MadEvent	$p\bar{p} \to \leq 5$ jets	Maltoni, Stelzer
ALPGEN	$p\bar{p} \to$ jets not yet implemented	Mangano, Moretti, Piccinini, Pittau, Polosa

Multiparton amplitudes are difficult to calculate, even at tree level

One approach: <u>Color decomposition</u>

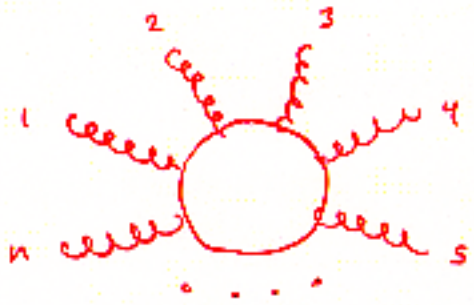
Berends + Giele

Mangano, Parke, Xu

Break amplitude up into smaller pieces that can be calculated efficiently

<u>Color-flow decomposition</u>:

- Most efficient decomposition
- Elegant, physical approach
- Ideal for merging with shower MC



n	# diagrams full amplitude
4	4
5	25
6	220
7	2485
8	34300
9	559405
10	10525900
11	224449225
12	5348843500

II. Color Decomposition

Example: n gluons

1. Fundamental-rep decomposition

Mangano, Parke, Xu

$$= \sum_{\substack{P(2,\dots,n) \\ \uparrow \\ (n-1)!}} \underbrace{\mathrm{Tr}\, \lambda^{a_1} \lambda^{a_2} \cdots \lambda^{a_n}}_{\text{color}} \underbrace{A(1,2,\dots,n)}_{\text{"partial amplitude"}}$$

- color independent

Similar for any number of $q\bar{q}$ and gluons

2. Adjoint-rep decomposition

Del Duca, Dixon, Frizzo, Maltoni

$$= \sum_{P(2,\dots,n-1) \leftarrow (n-2)!} (F^{a_2} F^{a_3} \cdots F^{a_{n-1}})^{a_1}{}_{a_n} \underbrace{A(1,2,\dots,n)}_{\text{same as above}}$$

where $(F^a)^b{}_c = -i f^{abc}$

Exists only for all-gluon amplitude

3. Color-flow decomposition

$$= \sum_{P(2,\dots,n) \leftarrow (n-1)!} \delta^{i_1}_{j_2} \delta^{i_2}_{j_3} \cdots \delta^{i_n}_{j_1} \underbrace{A(1,2,\dots,n)}_{\text{same as above}}$$

Similar for any number of $q\bar{q}$ and gluons

III. Color-Flow Decomposition

SU(N) gauge theory:

$$\mathcal{L} = \frac{1}{2g_s^2}\,\mathrm{Tr}\,F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\,(i\not{D} - m)\,\psi$$

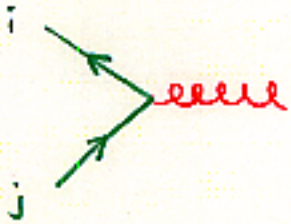
where

$$D_\mu = \partial_\mu + i g_s A_\mu$$

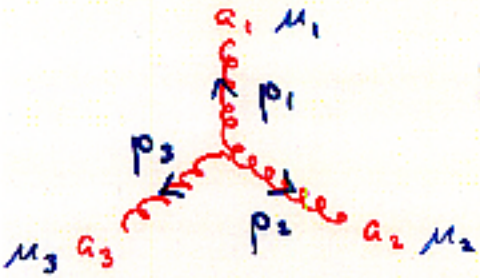
$$F_{\mu\nu} = [D_\mu, D_\nu]$$

Conventional description:

$$(A_\mu)^i{}_j = \frac{1}{\sqrt{2}} A_\mu^a (\lambda^a)^i{}_j \qquad a = 1, \ldots, N^2-1$$

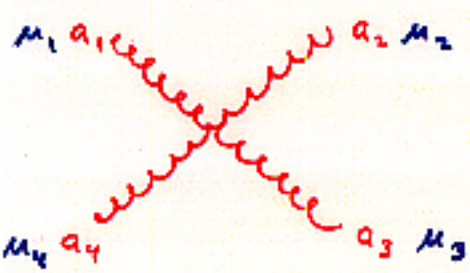


$$= -i\,\frac{g_s}{\sqrt{2}}\,(\lambda^a)^i{}_j\,\gamma^\mu$$



$$= \frac{g_s}{\sqrt{2}}\,f^{a_1a_2a_3}$$ ← from $[\lambda^{a_1}, \lambda^{a_2}] = i f^{a_1a_2a_3}\lambda^{a_3}$

$$\times\left[(p_1-p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2-p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3-p_1)_{\mu_2} g_{\mu_1\mu_3}\right]$$

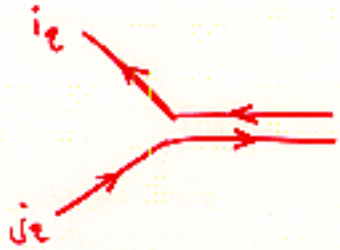


$$= -i\,\frac{g_s^2}{2}\Big[f^{a_1a_2b} f^{a_3a_4b}\left(g_{\mu_1\mu_4} g_{\mu_2\mu_3} - g_{\mu_1\mu_3} g_{\mu_2\mu_4}\right)$$
$$+ f^{a_1a_3b} f^{a_2a_4b}\left(g_{\mu_1\mu_2} g_{\mu_3\mu_4} - g_{\mu_1\mu_4} g_{\mu_2\mu_3}\right)$$
$$+ f^{a_1a_4b} f^{a_2a_3b}\left(g_{\mu_1\mu_2} g_{\mu_3\mu_4} - g_{\mu_1\mu_3} g_{\mu_2\mu_4}\right)\Big]$$

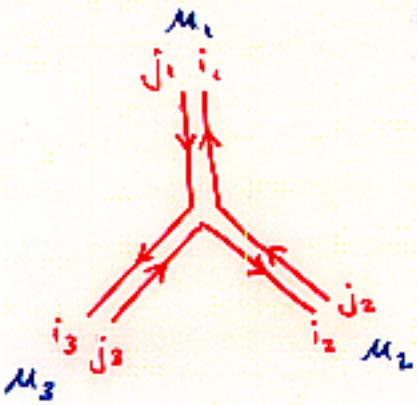
Color-flow description:

$(A_\mu)^i{}_j \Rightarrow$

$\psi^i \Rightarrow$

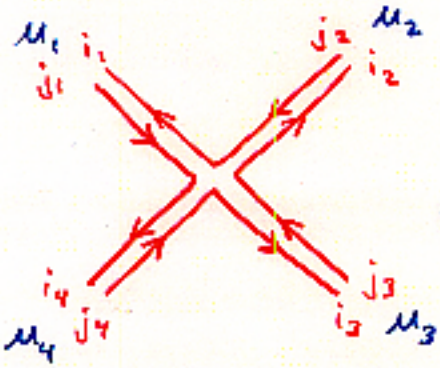


$$= i\,\frac{g_s}{\sqrt{2}}\,\delta^{i_2}_{j_1}\,\delta^{i_1}_{j_2}\,\gamma^\mu$$



$$= i\,\frac{g_s}{\sqrt{2}}\,\delta^{i_1}_{j_2}\,\delta^{i_2}_{j_3}\,\delta^{i_3}_{j_1}$$

$$\times\left[(p_1-p_2)_{\mu_3}\,g_{\mu_1\mu_2}+(p_2-p_3)_{\mu_1}\,g_{\mu_2\mu_3}+(p_3-p_1)_{\mu_2}\,g_{\mu_1\mu_3}\right]$$



$$= i\,\frac{g_s^2}{2}\,\delta^{i_1}_{j_2}\,\delta^{i_2}_{j_3}\,\delta^{i_3}_{j_4}\,\delta^{i_4}_{j_1}$$

$$\times\left[2\,g_{\mu_1\mu_3}\,g_{\mu_2\mu_4}-g_{\mu_1\mu_2}\,g_{\mu_3\mu_4}-g_{\mu_1\mu_4}\,g_{\mu_2\mu_3}\right]$$

Conservation of color made manifest!

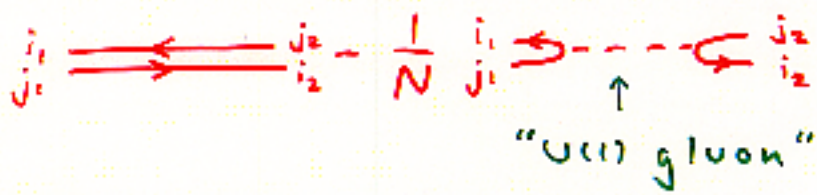
Gluon propagator:

<u>Conventional</u>:

$$\langle A_\mu^a A_\nu^b \rangle \sim \delta^{ab}$$

<u>Color-flow</u>:

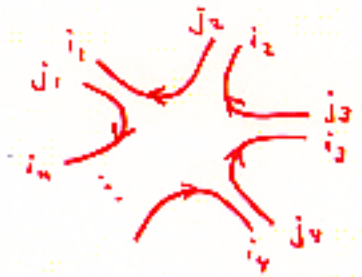
$$\langle (A_\mu)^{i_1}{}_{j_1} (A_\nu)^{i_2}{}_{j_2} \rangle \sim \delta^{i_1}_{j_2} \delta^{i_2}_{j_1} - \frac{1}{N} \delta^{i_1}_{j_1} \delta^{i_2}_{j_2}$$



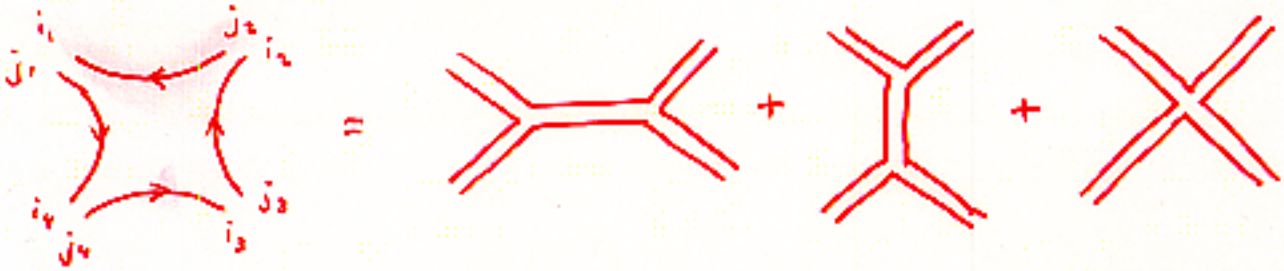
$$U(N) = SU(N) \times U(1)$$

IV. n gluons

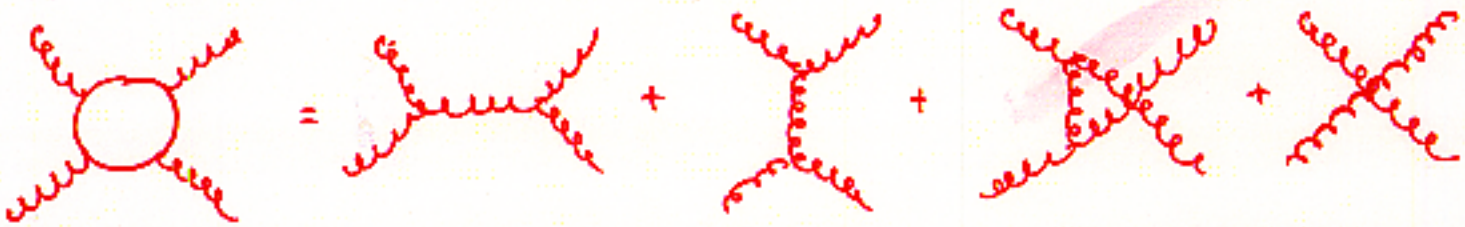
$$\mathcal{M} = \sum_{P(2,\dots,n)} \underbrace{\delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\cdots\delta^{i_n}_{j_1}} A(1,2,\dots,n)$$



Example: $gg \to gg$



Compare



$\Rightarrow$ Only planar diagrams contribute to $A(1,2,\dots,n)$

$A(1,2,\dots,n)$ is a color-flow amplitude

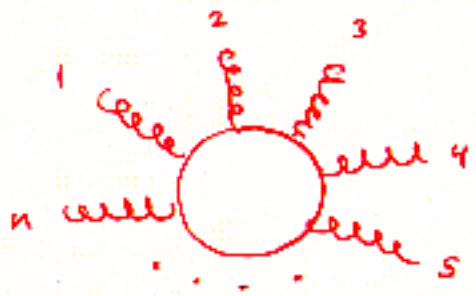
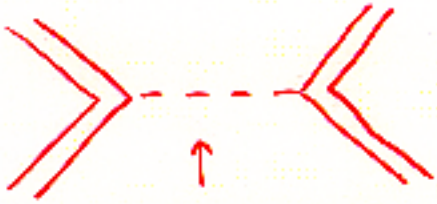


Table 1: Number of Feynman diagrams contributing to an n-gluon partial amplitude. The number grows approximately like 3.8^n. In contrast, the number of Feynman diagrams contributing to the full amplitude grows factorially, approximately $(2n)!$.

	# diagrams	
n	partial amplitude	full amplitude
4	3	4
5	10	25
6	36	220
7	133	2485
8	501	34300
9	1991	559405
10	7335	10525900
11	28199	224449225
12	108281	5348843500

What about



$= 0$

U(1) gluon

U(1) gluon doesn't couple to SU(N) gluons

For multi-jet cross section, must sum colors using <u>Monte-Carlo</u> methods

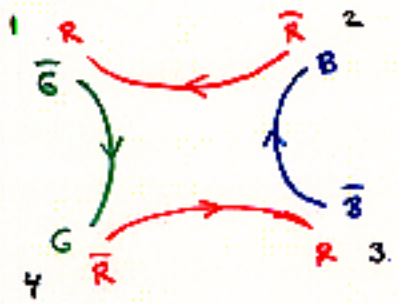
Caravaglios, Mangano, Moretti, Pittau
Draggiotis, Kleiss, Papadopoulos

Exemple: $gg \to gg$

a.)

	1	2	3	4
i	R	B	R	G
j	$\bar{G}$	$\bar{R}$	$\bar{B}$	$\bar{R}$

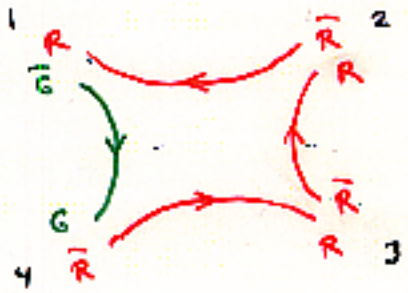
"color assignment"



$A(1,2,3,4)$

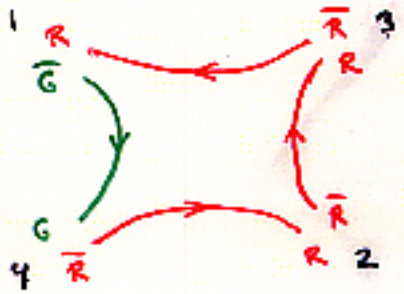
b.)

	1	2	3	4
i	R	R	R	G
j	$\bar{G}$	$\bar{R}$	$\bar{R}$	$\bar{R}$



$A(1,2,3,4)$

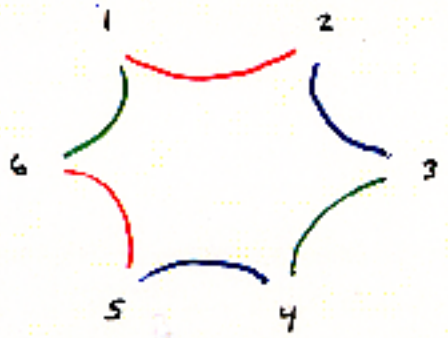
and



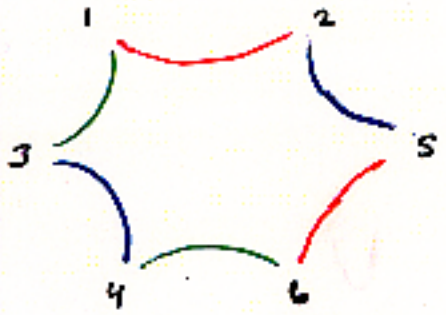
$A(1,3,2,4)$

A more pithy example: $gg \to gggg$

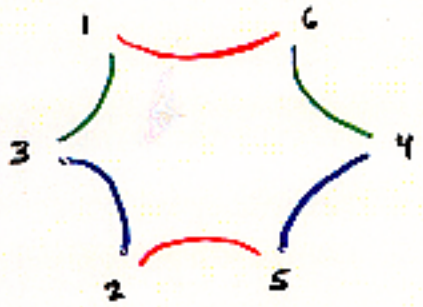
	1	2	3	4	5	6
i	R	B	G	B	R	G
j	$\bar{G}$	$\bar{R}$	$\bar{B}$	$\bar{G}$	$\bar{B}$	$\bar{R}$



$A(1,2,3,4,5,6)$



$A(1,2,5,6,4,3)$



$A(1,6,4,5,2,3)$

$\langle$ # partial amplitudes $\rangle$
color assignment

Table 2: Average number of partial amplitudes, for n-gluon scattering, that must be evaluated per nonvanishing color assignment in three different color decompositions: the fundamental-representation decomposition (using both the Gell-Mann matrices and the matrices used in Ref. [10]), the color-flow decomposition, and the adjoint-representation decomposition. The fundamental-representation decomposition is much more efficient using the matrices of Ref. [10]. The color-flow decomposition is much more efficient than the fundamental-representation decomposition, especially when n is large. The adjoint-representation decomposition is almost as efficient as the color-flow decomposition, but requires the multiplication of sparse 9×9 matrices.

	Decomposition			
	Fundamental			
n	Gell-Mann	Ref. [10]	Color-flow	Adjoint
4	4.83	3.02	1.28	1.15
5	15.2	7.26	1.83	1.52
6	56.5	20.6	3.21	2.55
7	251	68.0	6.80	5.53
8	1280	254	17.0	15.8
9	7440	1080	48.7	56.4
10	47800	4930	158	243
11	337000	25500	570	1210
12	2590000	148000	2250	6750

Ref. [10] = Caravaglios, Mangano, Moretti, Pittau

Color-flow decomp. is most efficient!

Putting the color-flow decomposition into practice:

	$\sqrt{\hat{s}}$ (GeV)		
$\hat{\sigma}(gg \to 8g)$	1500	0.70 pb	← Agrees w/ CMMP
$\hat{\sigma}(gg \to 9g)$	2000	0.30 pb	new
$\hat{\sigma}(gg \to 10g)$	2500	0.097 pb	new

↑ same phase-space efficiency (RAMBO)

$p_T > 60$ GeV, $|\eta| < 2$, $\Delta R > 0.7$

$A(1,2,\ldots,n)$ evaluated using Berends-Giele recursion relations and HELAS

Next step: Implement in MadEvent

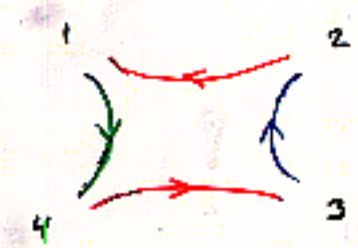
Merging with shower Monte Carlos

(HERWIG, PYTHIA)

Associate a color-flow with each event:

Examples:

a.)

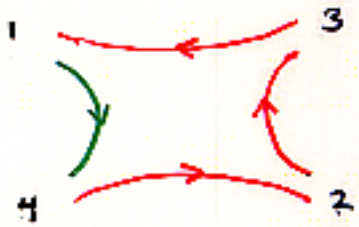


$A(1,2,3,4)$

unique color flow

b.)

and



$A(1,2,3,4)$ $\qquad$ $A(1,3,2,4)$

Which color flow do you choose?

$P_{1234} = |A(1,2,3,4)|^2 \qquad P_{1432} = |A(1,4,3,2)|^2$

Neglects interference $\mathcal{O}\left(\frac{1}{N^2}\right)$

V. ALPGEN

ALPGEN uses a hybrid approach:

majority of CPU

1. Calculate events using ALPHA algorithm (no color decomposition)

2. Calculate unweighted events using fundamental-rep decomposition (for merging with shower MC)

⇒ No big gain from color-flow decomp.

VI. Conclusions

Color-flow decomposition

- Based on color-flow diagrams
- Efficient
- Ideal for merging w/ shower MC's

Future

- Implement in MadEvent
- Also applicable to W + jets, Z + jets, ...
- Loop amplitudes

⋮

A step forward in our ability to calculate in perturbative QCD