What Fraction of ^{8}Boron Solar Neutrinos arrive at the Earth as a ν_2 mass eigenstate?[1]

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- 2 Neutrino Analysis - $\sin^2\theta_\odot$ and $\delta m^2_\odot$
- 3 Neutrino Analysis - $\sin^2\theta_{12}$ and δm^2_{21}

[1] Dedicated to the memory of John Bahcall who championed solar neutrinos for many lonely years.

• 2 Neutrino Analysis - $\sin^2\theta_\odot$ and $\delta m^2_\odot$

back of the envelope

• Let f_i are the detector energy response weighted fractions of ν_i mass eigenstate and $f_1 + f_2 = 1$.

SNO:

$$\begin{aligned} \frac{CC}{NC}|_{day} &= f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot \\ &= \sin^2\theta_\odot + f_1 \cos 2\theta_\odot \end{aligned}$$

- $\sin^2\theta_\odot$ is fraction of ν_2 that is ν_e.
- $\cos^2\theta_\odot$ is fraction of ν_1 that is ν_e.

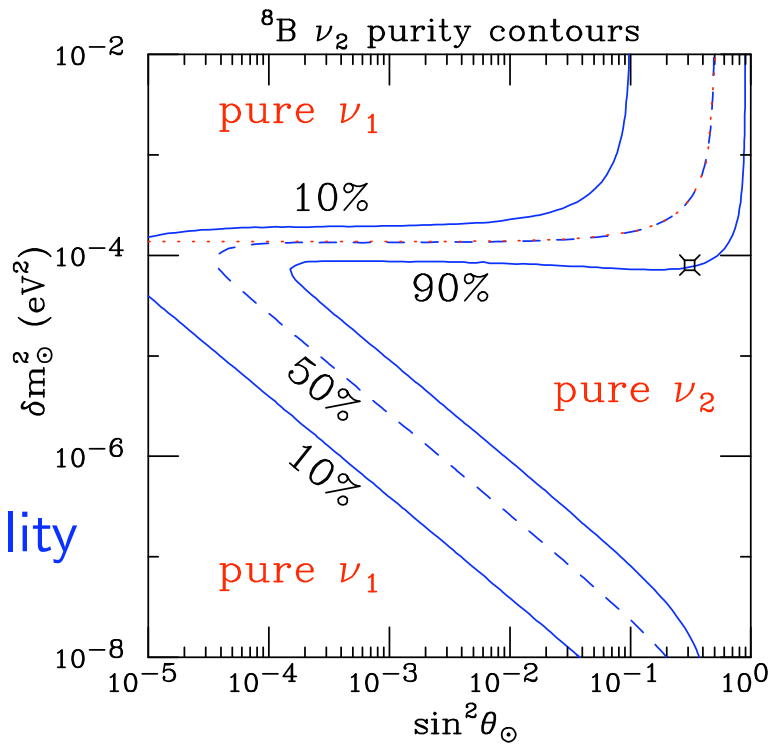
$$\begin{aligned} f_1 &= (\frac{CC}{NC}|_{day} - \sin^2\theta_\odot)/\cos 2\theta_\odot \\ &= (0.347 - 0.311)/0.38 \\ &\approx 10\% \ \pm ??\% \end{aligned}$$

Note, that if $f_1 = 0$ then $\frac{CC}{NC}|_{day} = \sin^2\theta_\odot$!!!

expectation from MSW

Following the analytical analysis of PRL,57,1275(1986)

$$f_2 \quad = \quad \langle \sin^2\theta_\odot^N + P_\times \cos 2\theta_\odot^N \rangle_{^8B}$$



where $\theta_\odot^N$ is mixing angle at the solar production point and $P_\times$ is the probability to "jump" from one mass eigenstate to the other during resonance crossing.

$$\sin^2\theta_\odot^N \;=\; \frac{1}{2}\left\{1 + \frac{(A - \delta m^2_\odot \cos 2\theta_\odot)}{\sqrt{(\delta m^2_\odot \cos 2\theta_\odot - A)^2 + (\delta m^2_\odot \sin 2\theta_\odot)^2}}\right\}$$

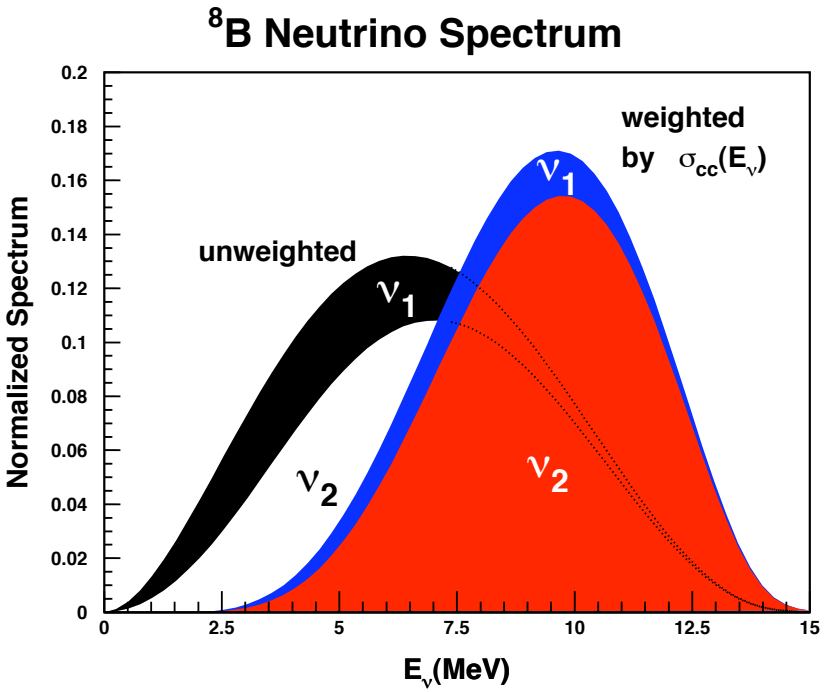
$$\text{where} \quad A \;=\; 2\sqrt{2} G_F N_e E_\nu = 1.23 \times 10^{-4} eV^2 \left(\frac{Y_e \rho}{80 g.cm^{-3}}\right)\left(\frac{E_\nu}{10 MeV}\right).$$

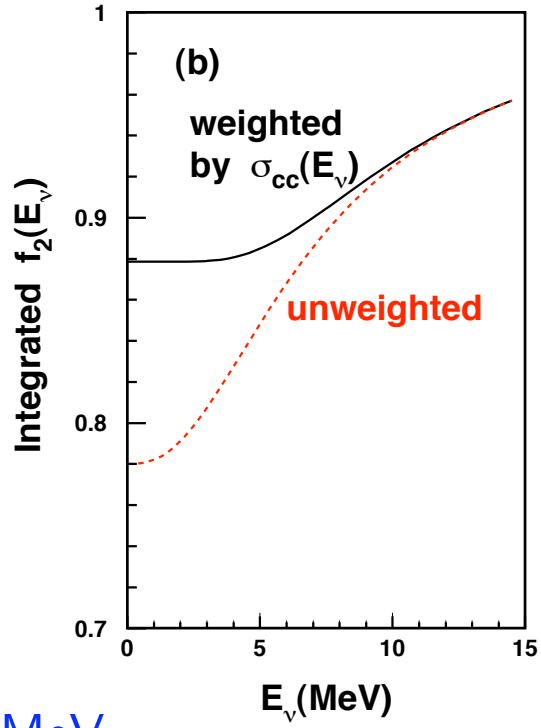
with precision

Using best fit point:

$\sin^2\theta_\odot = 0.31$ and $\delta m^2_\odot = 8.0 \times 10^{-5}\ eV^2$

^{8}B: ν_2 fraction

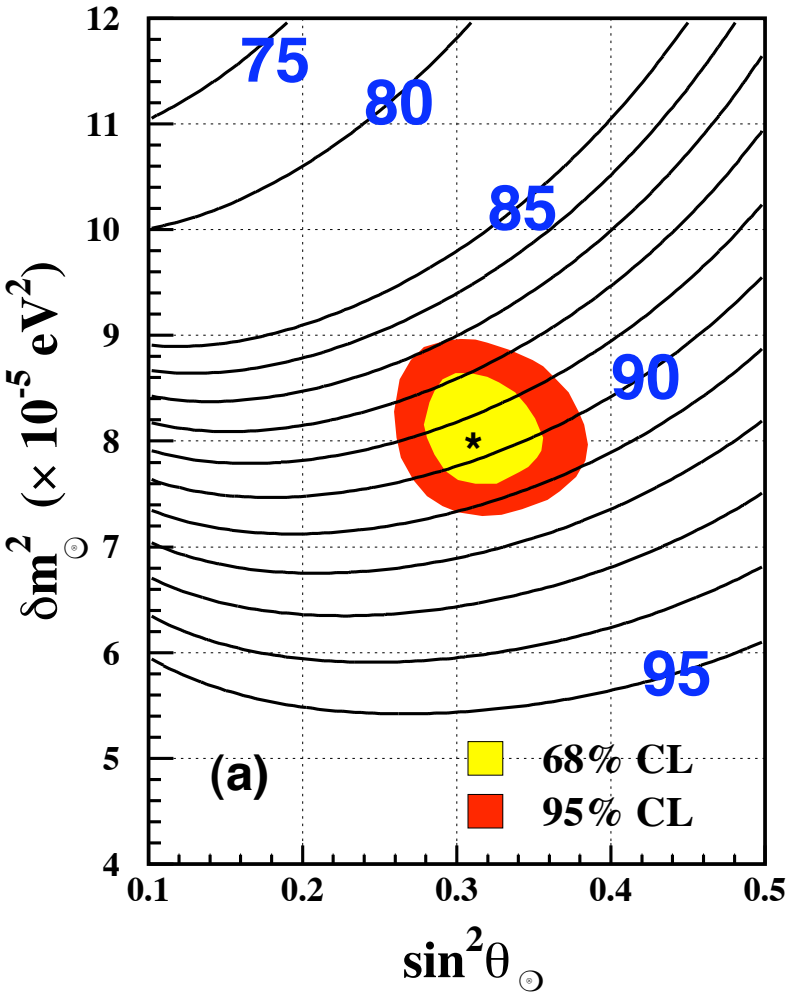




threshold $E_\nu = 6.5$ MeV

$$\sigma_{CC}(E_\nu) \sim E^{2.67}$$

^{8}B: ν_2 fraction (%)



$$f_2 = 89 \pm 2\%$$

At the best fit point

$$\xi \equiv \frac{\delta m^2_\odot \sin 2\theta_\odot}{(A(^8B) - \delta m^2_\odot \cos 2\theta_\odot)} - \frac{3}{4} \approx 0$$

$$f_2 \approx \frac{9}{10} - \frac{24}{125}\xi + \mathcal{O}(\xi^2)$$

using $A(^8B)_{eff} = 1.26 \times 10^{-4} eV^2$
this corresponds to
$Y_e \rho E_\nu = 0.823\ kg\ cm^{-3} MeV.$

Lower Bound on Solar $Y_e\rho$

$$f_2 \equiv \sin^2\langle\theta_\odot^N\rangle = 0.895$$

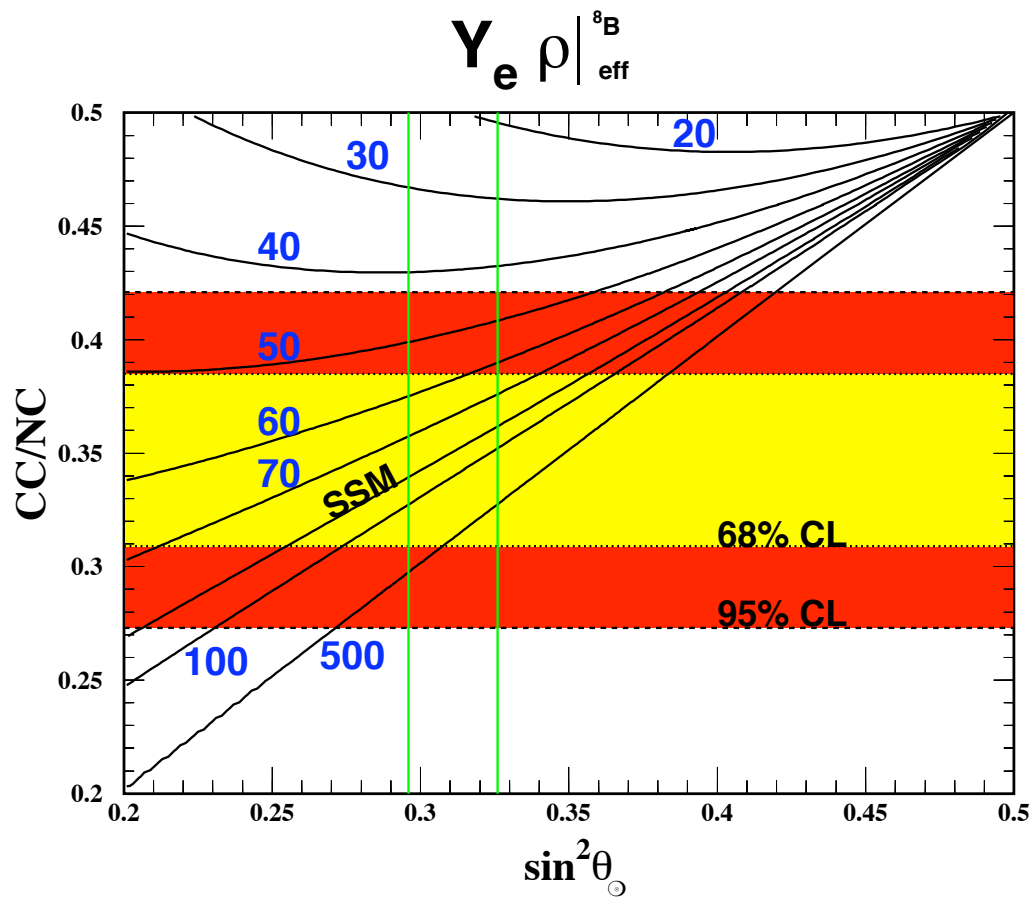
$$Y_e\rho\,\Big|_{eff}^{^8B} \equiv \frac{m_N}{2\sqrt{2}G_F\langle E\rangle_{^8B}}\,\delta m_\odot^2 \sin 2\theta_\odot \left[\underbrace{\cot 2\theta_\odot}_{=-\cot 2\langle\theta_\odot^N\rangle} + \frac{2f_2-1}{2\sqrt{f_2(1-f_2)}}\right]$$

but $f_2 = \dfrac{\cos^2\theta_\odot - \frac{CC}{NC}}{\cos 2\theta_\odot}$

$$Y_e\rho\,\Big|_{eff}^{^8B} > 43\ g\ cm^{-3}$$

at 95% C.L.

$Y_e\rho\,\Big|_{eff}^{^8B}$ for SSM is 86 g cm^{-3}



- 3 Neutrino Analysis - $\sin^2\theta_{12}$ and δm^2_{21}

  - KamLAND implies that: $\delta m^2_{21} = \delta m^2_{\odot}$
    to high accuracy as KamLAND
    averages over many δm^2_{atm} oscillations.

  - $\mathcal{F}_1$, $\mathcal{F}_2$ and $\mathcal{F}_3$ are the fraction of ν_1, ν_2 and ν_3
    ($\mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3 = 1$).

  - The value of $\sin^2\theta_{12}$ is determined, for a given $\sin^2\theta_{13}$,
    so as to hold some measured quantity fixed
    e.g. the SNO CC/NC ratio.

  - We will use a Taylor series expansion in $\sin^2\theta_{13}$
    about the two component analysis, $\sin^2\theta_{13} = 0$.

3 ν Details:

- The ν_3 fraction is given by $\mathcal{F}_3 = \left(1 \pm \frac{2A}{|\delta m^2_{31}|}\right) \sin^2\theta_{13} \approx \sin^2\theta_{13}$.
$\frac{2A}{|\delta m^2_{31}|} \sim 10\%$ and $\pm$ depends on the hierarchy !!!

- With this approximation:

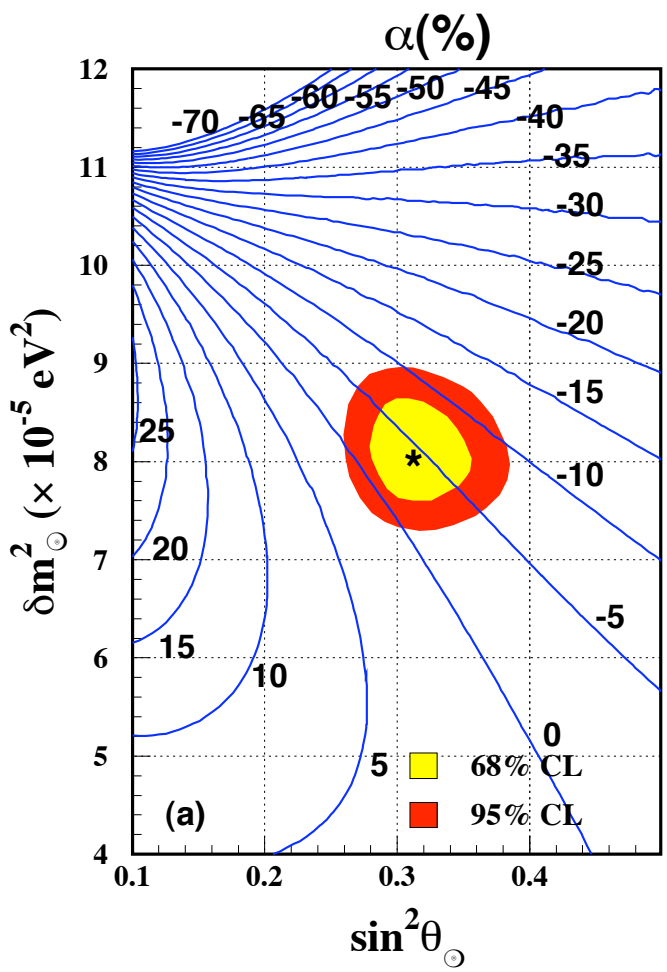
$$\mathcal{F}_1 = \cos^2\theta_{13}\ \langle\cos^2\theta^N_{12}\rangle_{^8B} \quad and \quad \mathcal{F}_2 = \cos^2\theta_{13}\ \langle\sin^2\theta^N_{12}\rangle_{^8B}.$$

also $A \to A\cos^2\theta_{13}$.

$\mathcal{F}_1$:

$$\mathcal{F}_1 = f_1 + \alpha \sin^2\theta_{13}$$

where $\alpha \equiv \left.\frac{d\mathcal{F}_1}{d\sin^2\theta_{13}}\right|_{\sin^2\theta_{13}=0}$ holding SNO's CC/NC fixed.



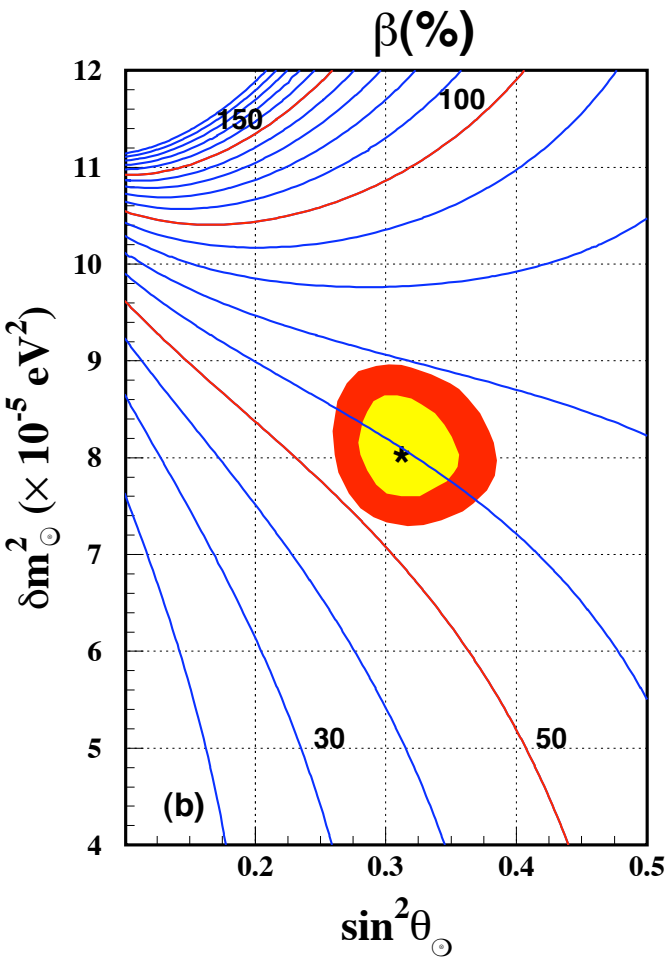
$$\mathcal{F}_1 = f_1 - 0.04\,\sin^2\theta_{13}$$
$$\approx 0.11$$

$$\mathcal{F}_2 = f_2 - 0.96\,\sin^2\theta_{13}$$
$$\approx 0.89 - \sin^2\theta_{13}$$

$$\mathcal{F}_3 = \sin^2\theta_{13}$$

Chooz bound $\sin^2\theta_{13} < 0.04$

$|U_{e2}|^2$:



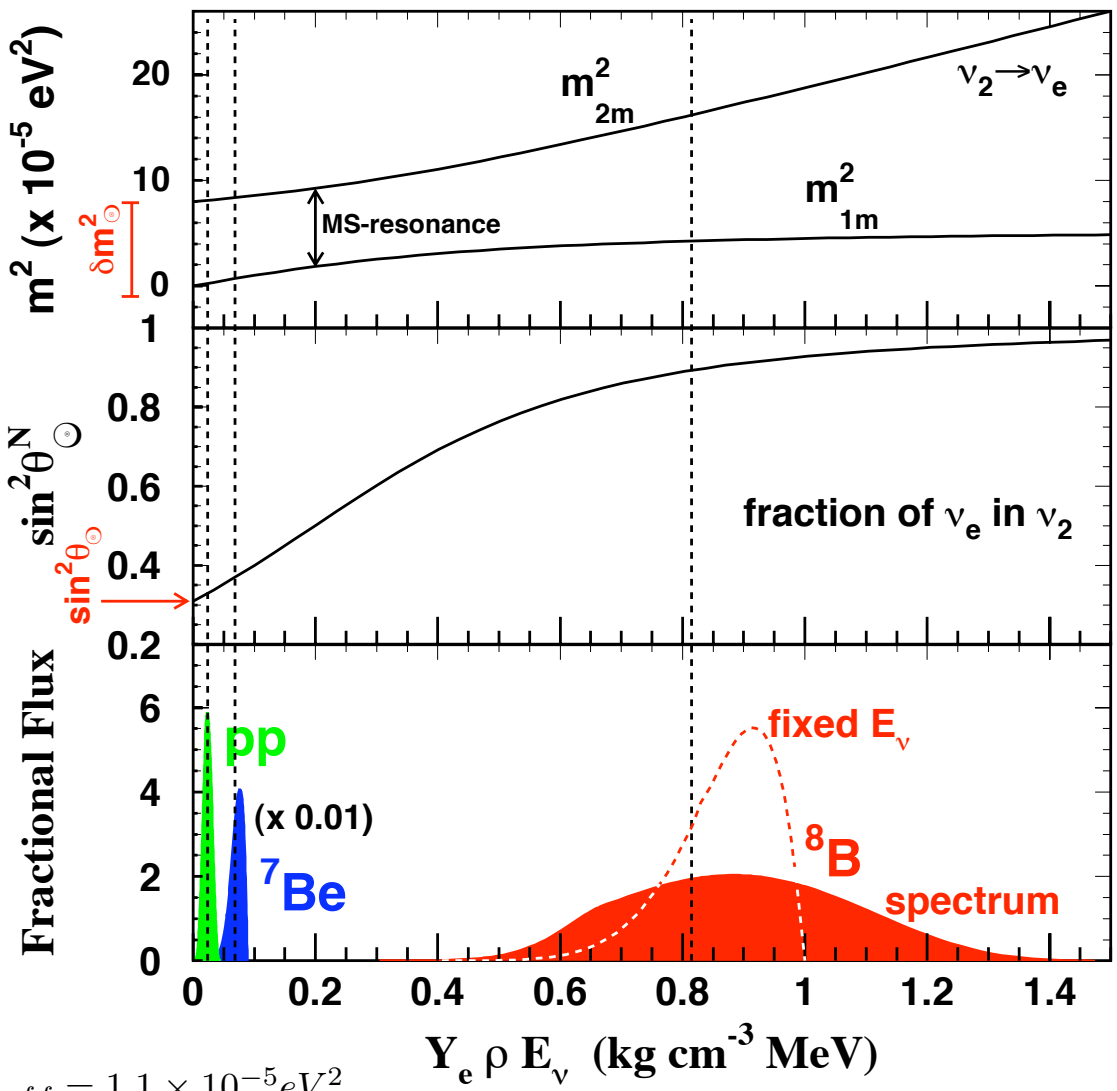
$$|U_{e2}|^2 = \cos^2\theta_{13}\sin^2\theta_{12} = \frac{(\frac{CC}{NC} - \cos^2\theta_{13}\mathcal{F}_1)}{(\cos^2\theta_{13} - 2\mathcal{F}_1)}$$

$$|U_{e2}|^2 = \sin^2\theta_\odot + \beta\sin^2\theta_{13} + \mathcal{O}(\sin^4\theta_{13}),$$

$$with \quad \beta \equiv \left.\frac{d|U_{e2}|^2}{d\sin^2\theta_{13}}\right|_0 = \frac{(f_1 - \alpha) + (1 + 2\alpha)\sin^2\theta_\odot}{(1 - 2f_1)}$$

$$|U_{e2}|^2 \approx \sin^2\theta_\odot^{8B} + (0.6 \pm 0.1)\sin^2\theta_{13}$$

2 nu Summary



pp and ^{7}Be: $A \ll \delta m^2_\odot$

$$f_2 \approx \sin^2\theta_\odot + \tfrac{1}{2}\sin^2 2\theta_\odot \left(\frac{A}{\delta m^2_\odot}\right)$$

- ^{7}Be: $f_2 = 37 \pm 4\%$ implies $A(^7Be)_{eff} = 1.1 \times 10^{-5} eV^2$
- pp: $f_2 = 33 \pm 4\%$ implies $A(pp)_{eff} = 0.31 \times 10^{-5} eV^2$
- vacuum: A=0 $f_2 = 31 \pm 3\%$

pp and ^{7}Be

2 nu analysis: $A \ll \delta m^2_\odot$

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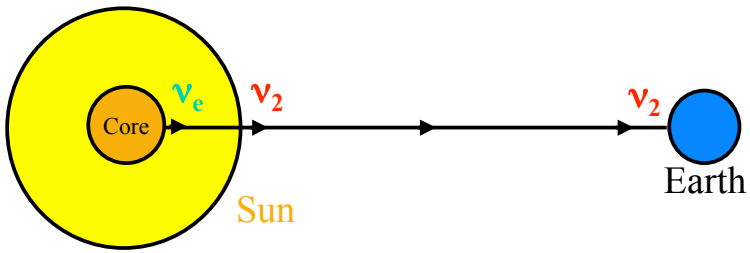
- vacuum: A=0 $f_2 = 31 \pm 3\%$

3 nu analysis: $A \ll \delta m^2_\odot$ and $\sin^2\theta_{13} \ll 1$

$$\begin{aligned}
\mathcal{F}_1 &\approx f_1 + \frac{\sin^2\theta_\odot}{\cos 2\theta_\odot}\sin^2\theta_{13} = f_1 + 0.82\sin^2\theta_{13} \\
\mathcal{F}_2 &\approx f_2 - \frac{\cos^2\theta_\odot}{\cos 2\theta_\odot}\sin^2\theta_{13} = f_2 - 1.8\sin^2\theta_{13} \\
\mathcal{F}_3 &\approx \sin^2\theta_{13}.
\end{aligned}$$

SUMMARY

^{8}Boron Neutrinos



Born as ν_e they exit the Sun as ν_2 and travel to the earth as ν_2 !!!

Fraction of

$$\nu_2 \approx 89\% - \sin^2\theta_{13}$$

$$\nu_1 \approx 11\%$$

$$\nu_3 = \sin^2\theta_{13}$$

- ^{8}Boron Solar Neutrinos give us the Purest known Mass Eigenstate Nu Beam !
- And $|U_{e2}|^2 \approx \sin^2\theta_{\odot}^{^8B} + (0.6 \pm 0.1)\sin^2\theta_{13}$ is the best known MNS matrix element.
- Lower bound on Solar $Y_e\rho$ equal to half SSM value.

Thanks John