Neutrinos,
In and Beyond the Standard Model:

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Fermilab

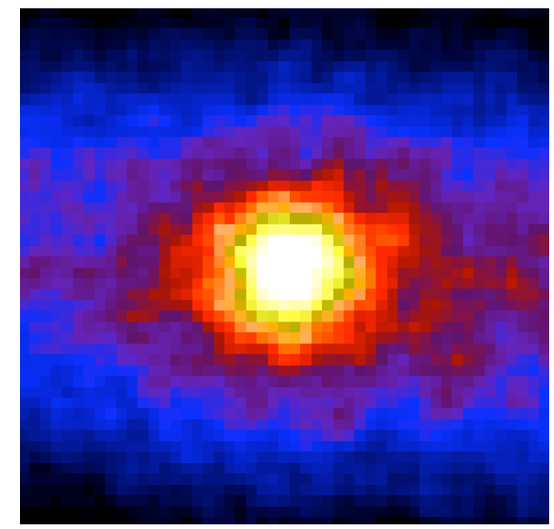
- Standard Model
- Addition of Neutrino Mass
- Implications

Neutron Decay:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Solar Engine:

$$p + p \rightarrow d + e^+ + \nu_e$$

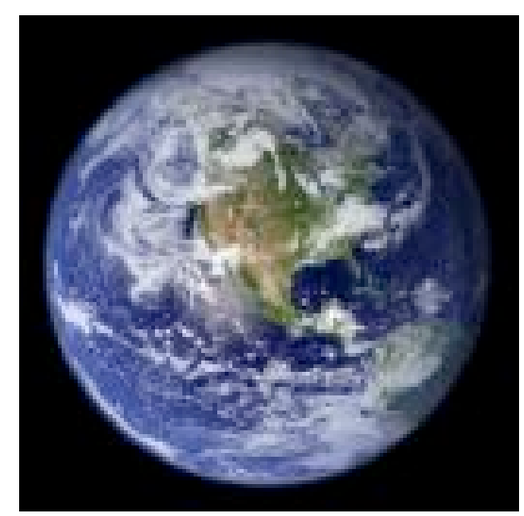


SuperNova Cooling:

$$e^+ + e^- \rightarrow \nu + \bar{\nu}$$

Leptogenesis:

???



Standard Model of Particle Physics

Gauge Theory based on the group:

$$SU(3) \times SU(2) \times U(1)$$

$SU(3) \Rightarrow$ Quantum Chromodynamics

Strong Force (Quarks and Gluons)

$SU_L(2) \times U(1) \Rightarrow$ ElectroWeak Interactions broken to $U_{EM}(1)$

by HIGGS (Tevatron, LHC)

$SU_L(2) \times U_Y(1) \Rightarrow U_{EM}(1)$

Force Carriers: $W^{\pm}$, Z^0 and γ masses: 80, 91 and 0 GeV

quark, SU(2) doublets: $\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$

up-quark, SU(2) singlets: u_R, c_R, t_R

down-quark, SU(2) singlets: d_R, s_R, b_R

lepton, SU(2) doublets: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$

neutrino, SU(2) singlets: $- - -$

charge lepton, SU(2) singlets: e_R, μ_R, τ_R

Electron mass

comes from a term of the form

$$\bar{L}\phi e_R$$

Absence of ν_R

forbids such a mass term (dim 4)

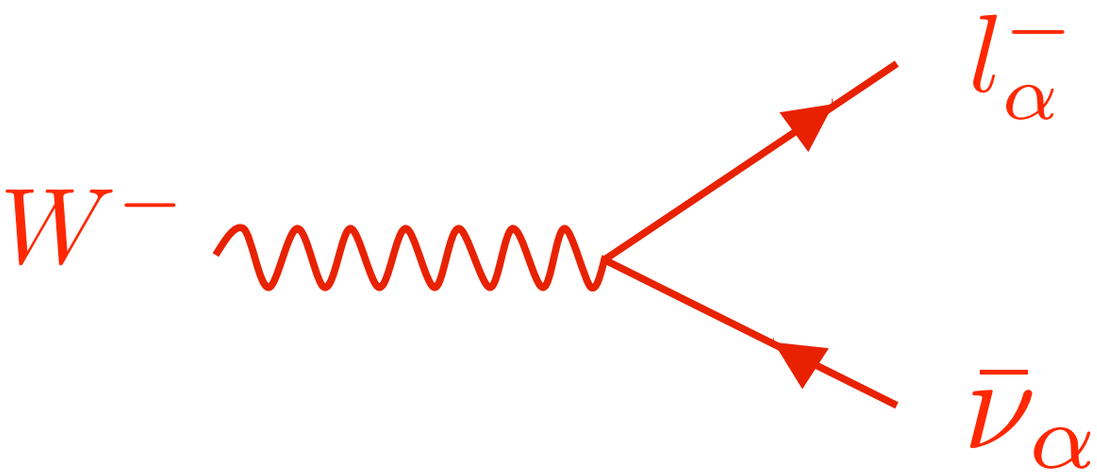
for the Neutrino

Therefore in the SM neutrinos are massless

and hence travel at speed of light.

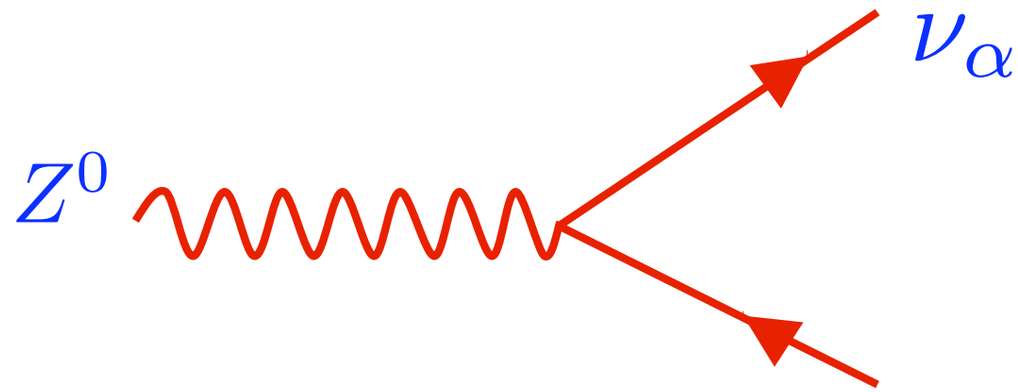
Interactions:

Charge Current (CC)

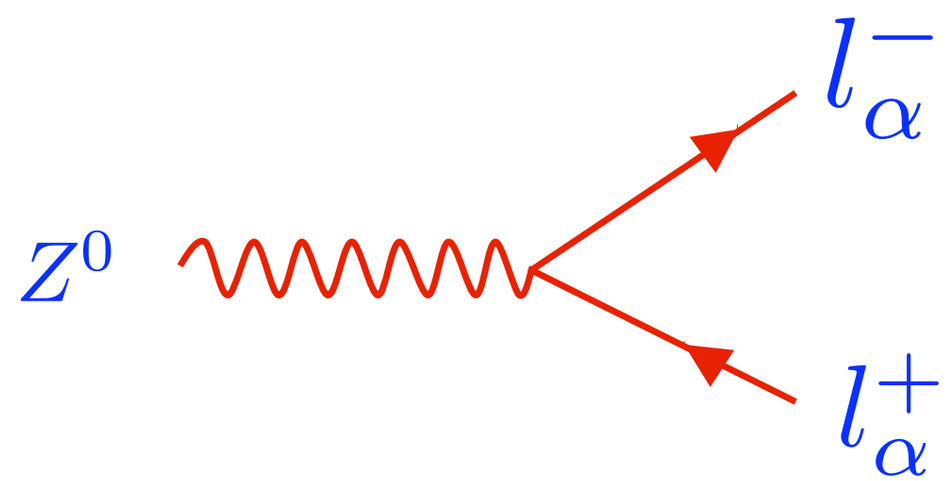


$W^- \rightarrow l_\alpha^- + \bar{\nu}_\alpha$

Neutral Current (NC)



$Z^0 \rightarrow \nu_\alpha + \bar{\nu}_\alpha$



$Z^0 \rightarrow l_\alpha^- + l_\alpha^+$

$$\alpha = e, \mu, \ or \ \tau$$

Invisible width of Z plus other data from LEP:

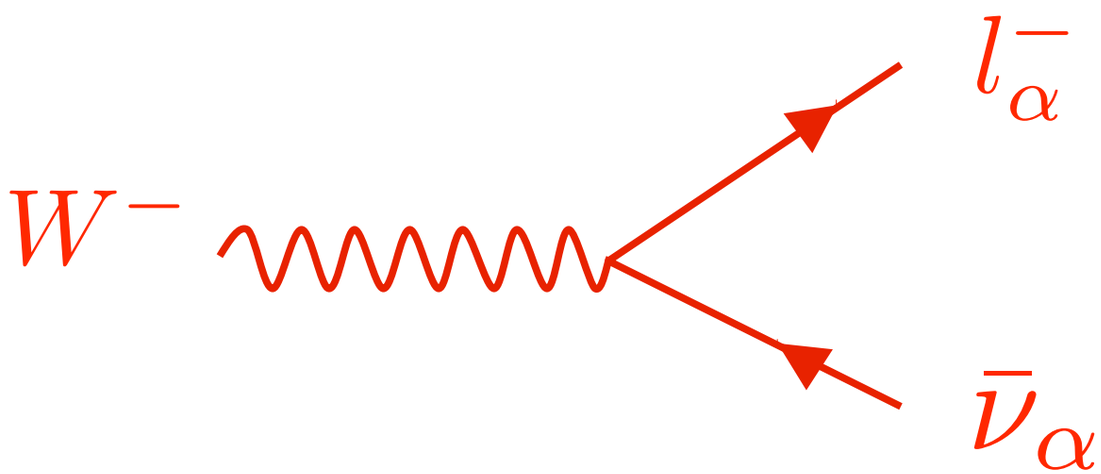
$$Z^0 \to \nu\bar{\nu}$$

Implies $N_\nu = 2.99 \pm 0.01$

Three Active Neutrinos!!!

Sterile Neutrinos don't couple to Z^0

Note That



$$W^- \to l_\alpha^- + \bar{\nu}_\alpha$$

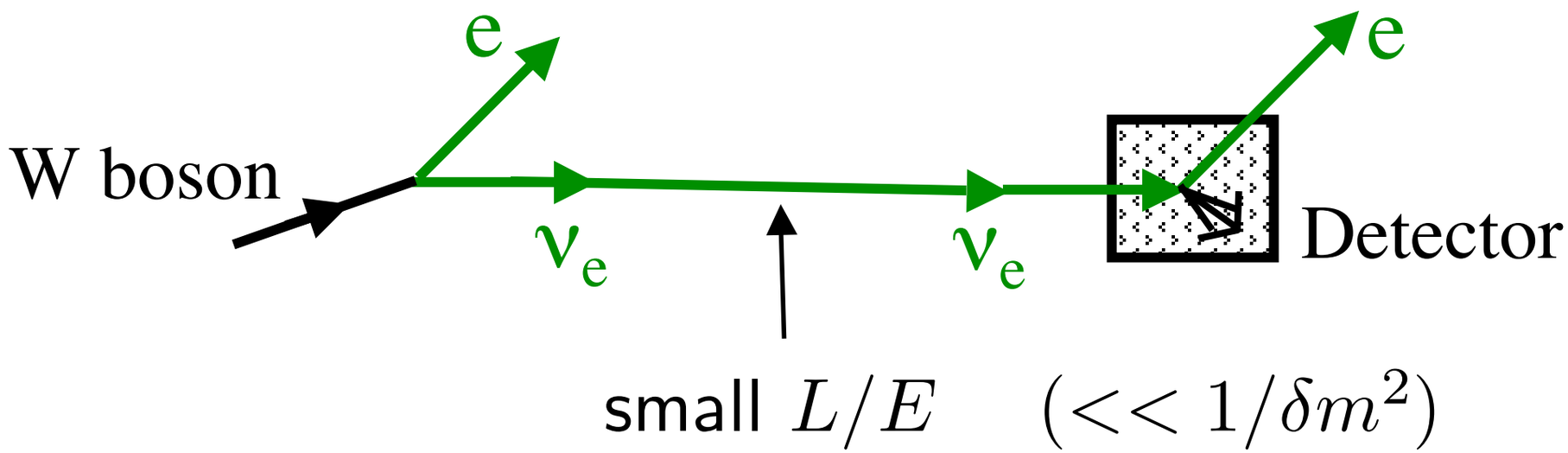
Implies

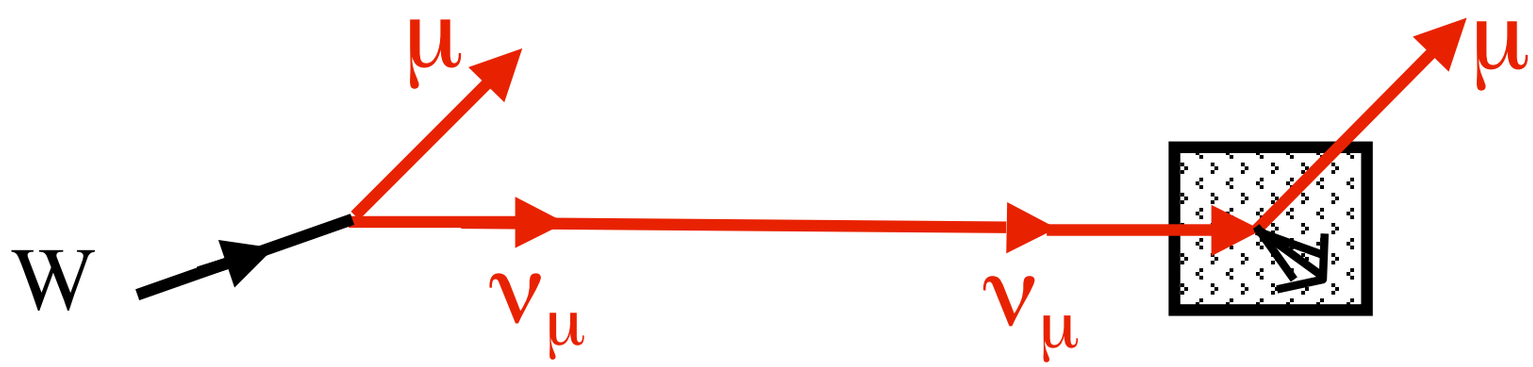
$$\nu_e \;,\; \nu_\mu \;,\; \nu_\tau$$

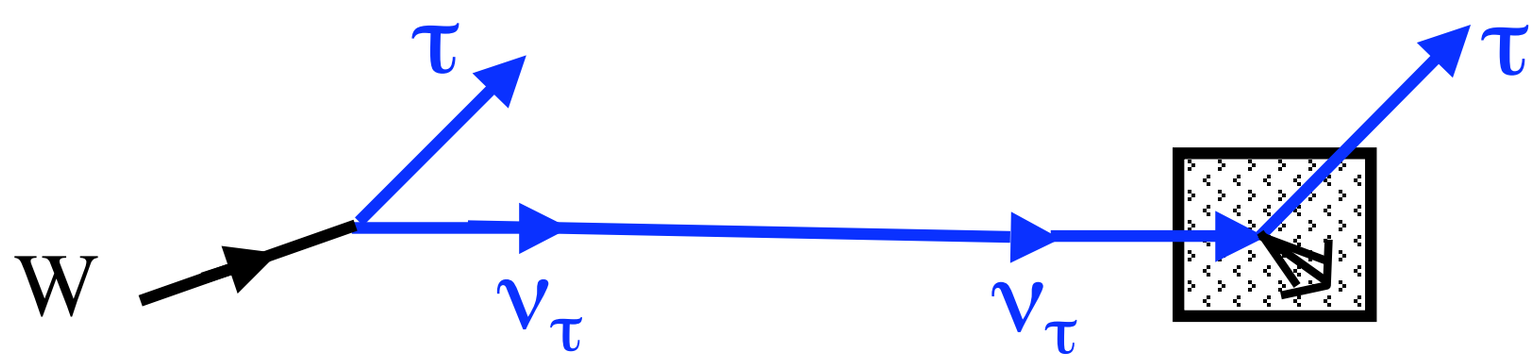
$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$e \;,\; \mu \;,\; \tau$$

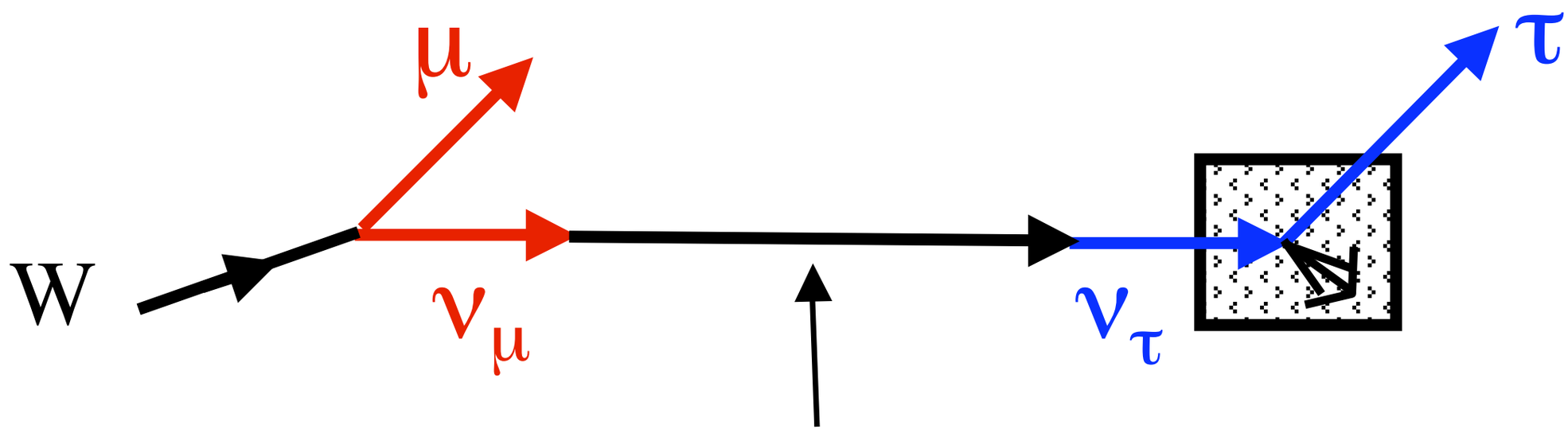
Observed


e
W boson
ν_e
ν_e
e
Detector
small L/E $(<< 1/\delta m^2)$


μ
W
ν_μ
ν_μ
μ

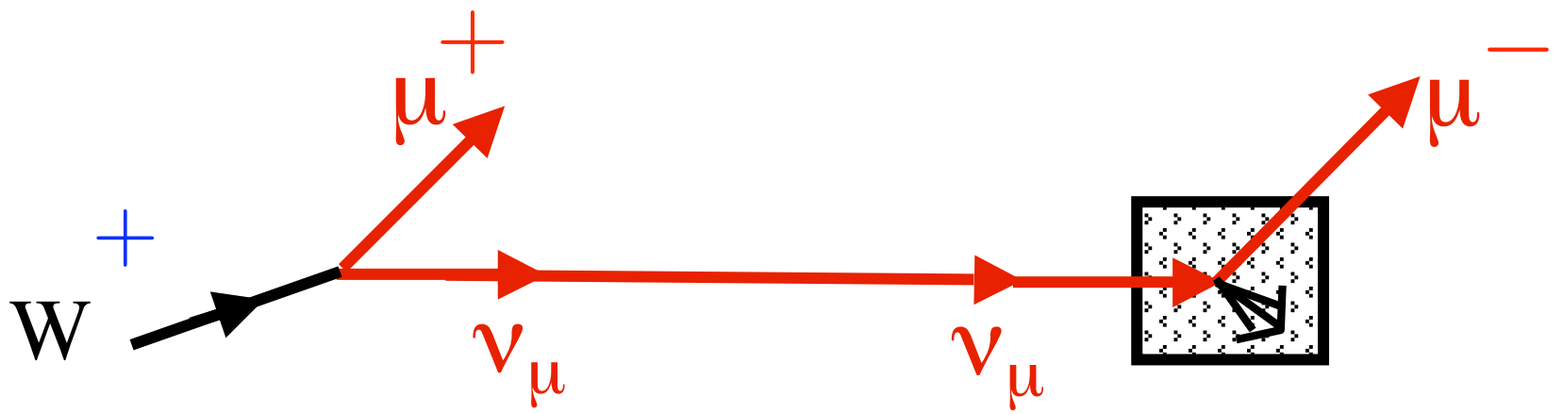

τ
W
ν_τ
ν_τ
τ

Not Observed



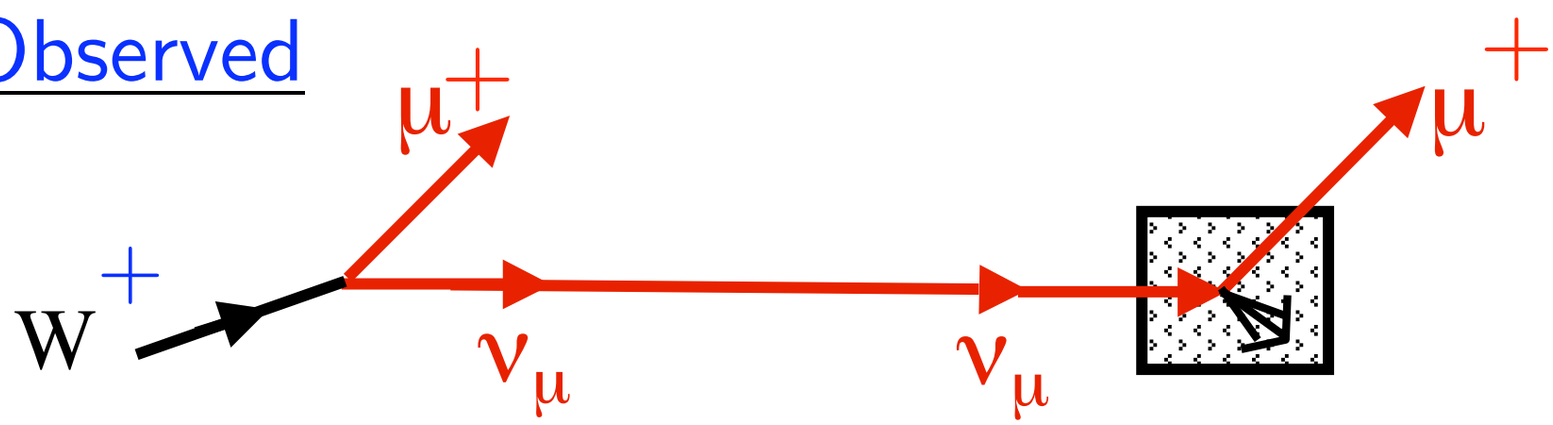
small $L/E \quad (<< 1/\delta m^2)$

Observed



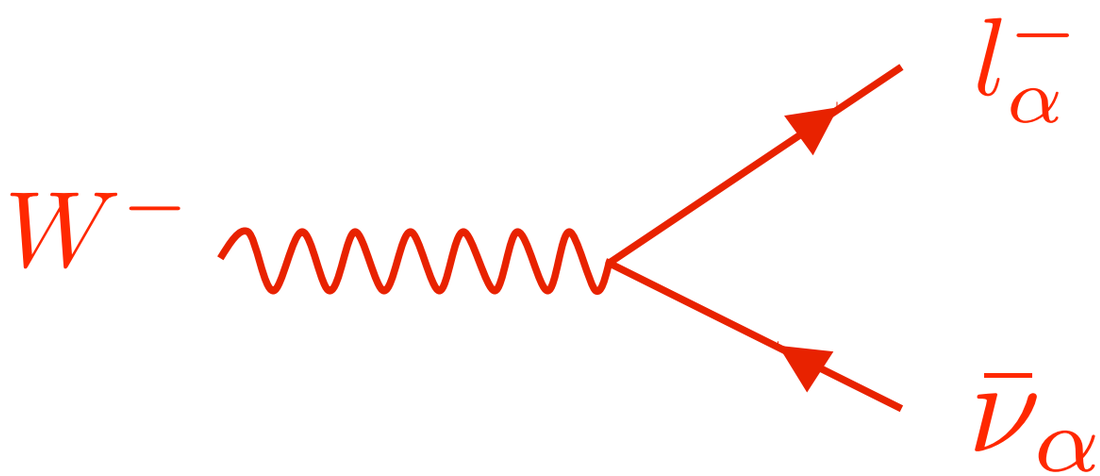
neutrino beam (not anti-neutrino beam)

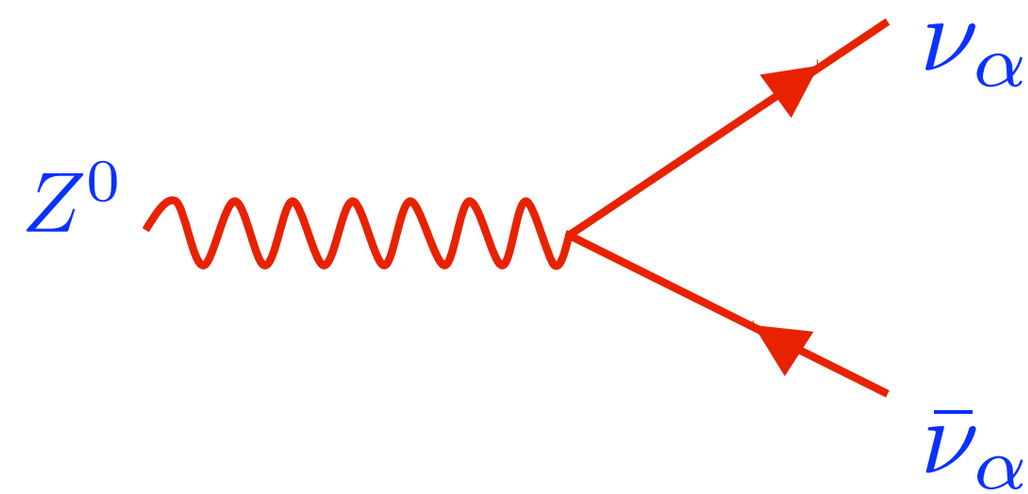
Not Observed



large E $(>> m_\nu)$

Standard Model



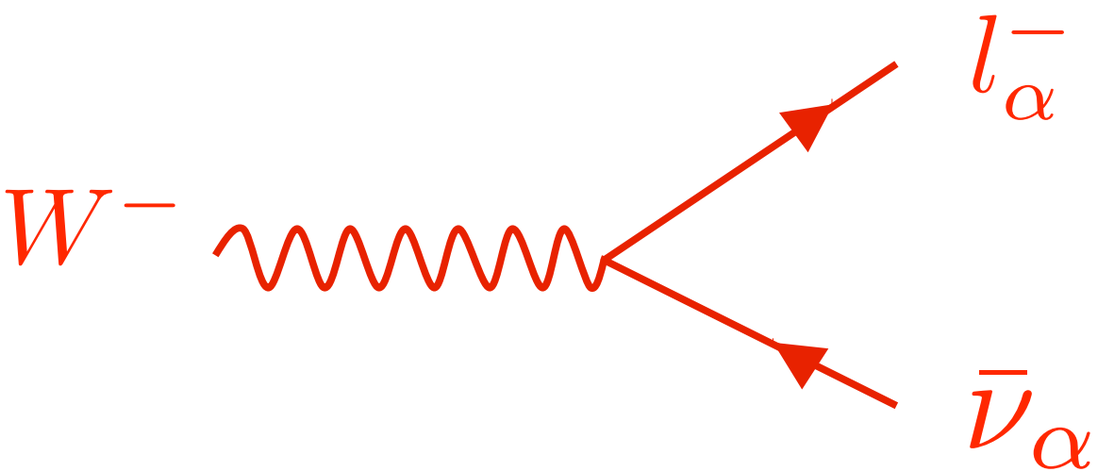


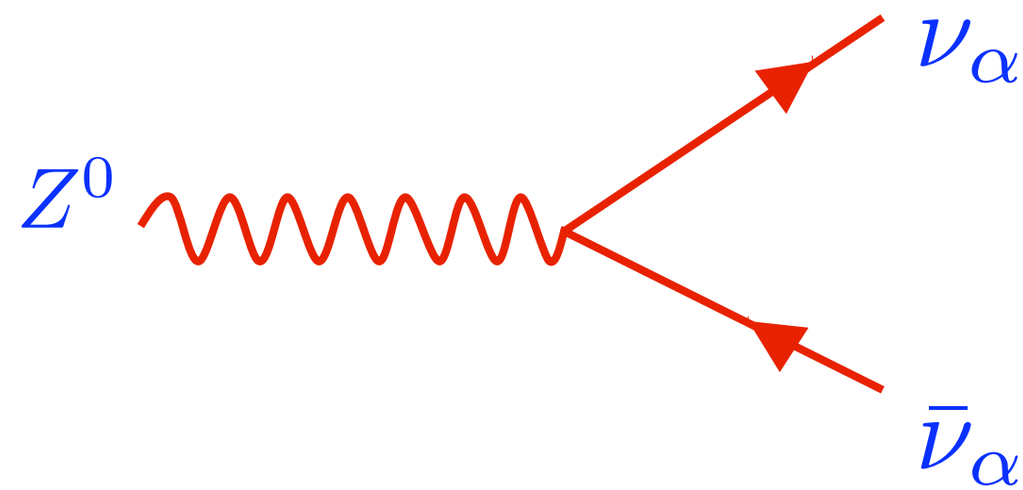
couplings conserve the Lepton Number L
defined by—

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1.$$

Actually L_e, L_μ, and L_τ
separately

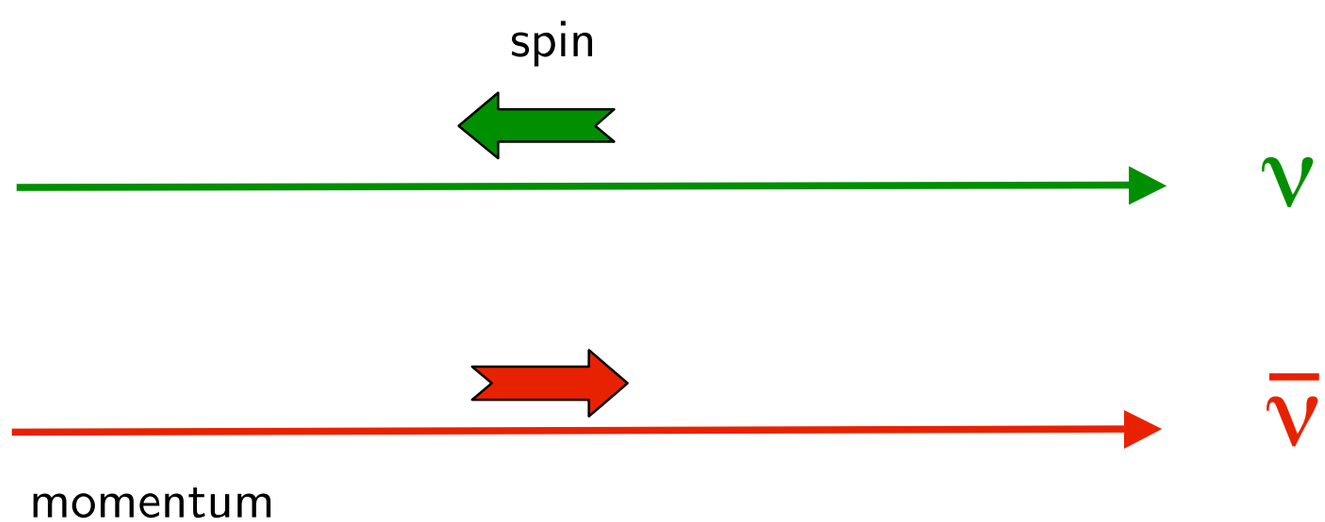
Left Handed Nature of The Neutrino





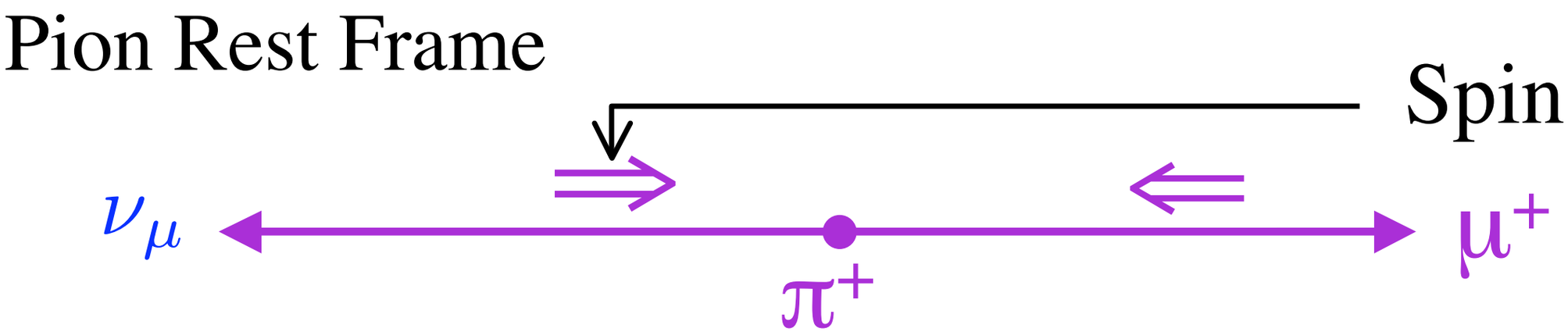
Produce Left-Handed Neutrinos

and Right-Handed Anti-Neutrinos



What about the RH neutrinos and LH anti-neutrino ????

π^+ decay



Left Handed anti-fermion

Suppressed by powers of m_f

Why $\pi^+ \to \mu^+\nu_\mu$ >99% and $\pi^+ \to e^+\nu_e$ is 0.01%.

Summary of ν's in SM:

Three flavors of massless neutrinos

$$W^- \rightarrow l^-_\alpha + \bar{\nu}_\alpha$$

$$W^+ \rightarrow l^+_\alpha + \nu_\alpha$$

$$\alpha = e, \mu, \ or \ \tau$$

Anti-neutrino, $\bar{\nu}_\alpha$, has $+$ve helicity, Right Handed

Neutrino, ν_α, has -ve helicity, Left Handed

ν_L and $\bar{\nu}_R$ are CPT conjugates

massless implies helicity $=$ chirality

Beyond the SM

What if Neutrino have a MASS?

speed is less than c therefore time can pass

and

Neutrinos can change character!!!

What are the stationary states?

How are they related to the interaction states?

NEUTRINO OSCILLATIONS:

Two Flavors

flavor eigenstates $\neq$ mass eigenestates

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

W's produce ν_μ and/or ν_τ's

but ν_1 and ν_2 are the states

that change by a phase over time, mass eigenstates.

$$|\nu_j\rangle \longrightarrow e^{-ip_j\cdot x}|\nu_j\rangle \qquad p_j^2 = m_j^2$$

$\alpha, \beta \ldots$ flavor index $\qquad$ $i, j \ldots$ mass index

Production:

$$|\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

Propogation:

$$\cos\theta e^{-ip_1\cdot x}|\nu_1\rangle + \sin\theta e^{-ip_2\cdot x}|\nu_2\rangle$$

Detection:

$$|\nu_1\rangle = \cos\theta|\nu_\mu\rangle - \sin\theta|\nu_\tau\rangle$$

$$|\nu_2\rangle = \sin\theta|\nu_\mu\rangle + \cos\theta|\nu_\tau\rangle$$

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

Same E, therefore $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$

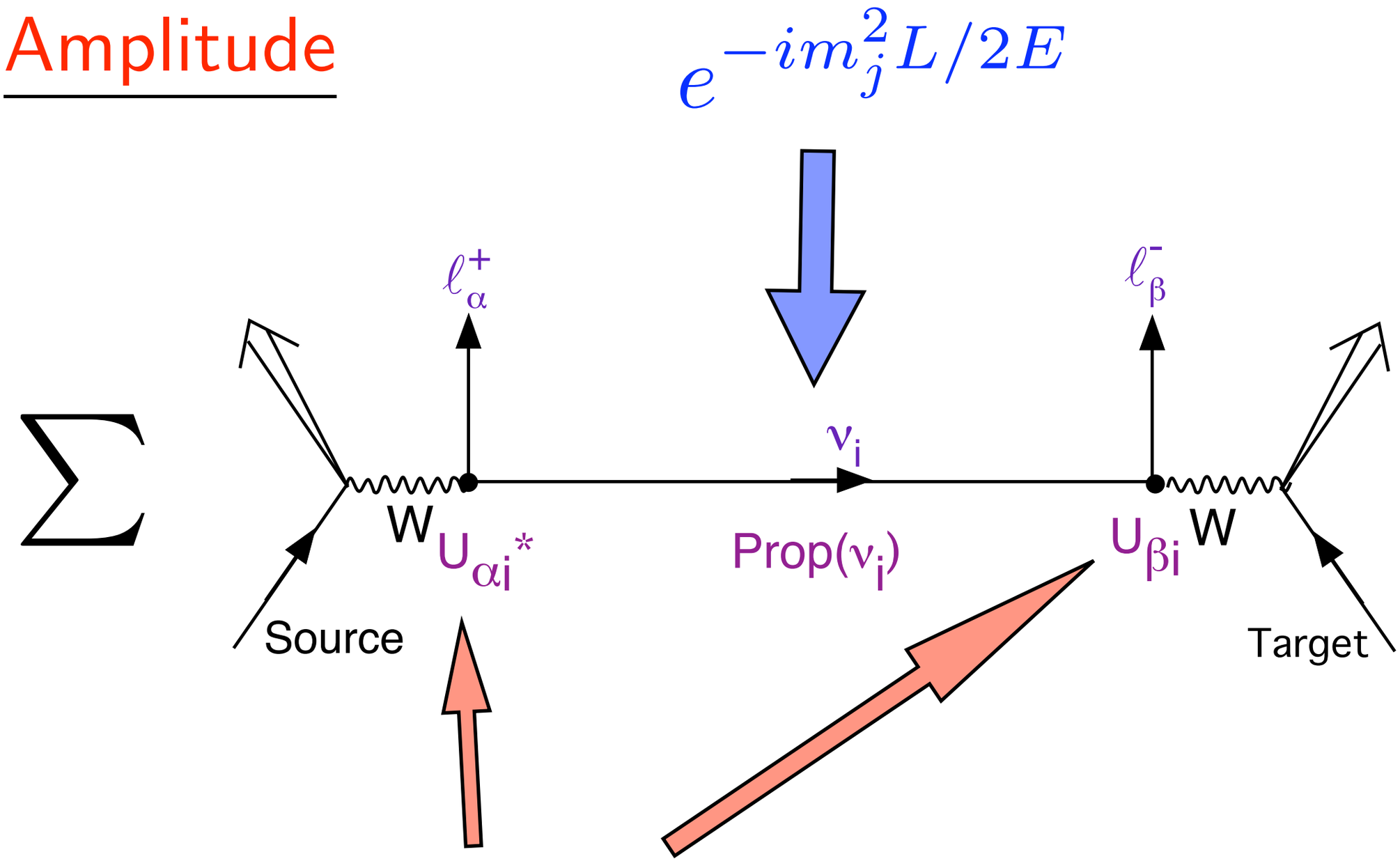
$$e^{-ip_j\cdot x} = e^{-iEt}e^{-ip_jL} \approx e^{-i(Et-EL)} \quad e^{-im_j^2L/2E}$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2\theta\cos^2\theta|e^{-im_2^2L/2E} - e^{-im_1^2L/2E}|^2$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$\delta m^2 = m_2^2 - m_1^2$ and $\frac{\delta m^2 L}{4E} \equiv \Delta$ kinematic phase:

Amplitude



$$U_{\alpha j} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Appearance:

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

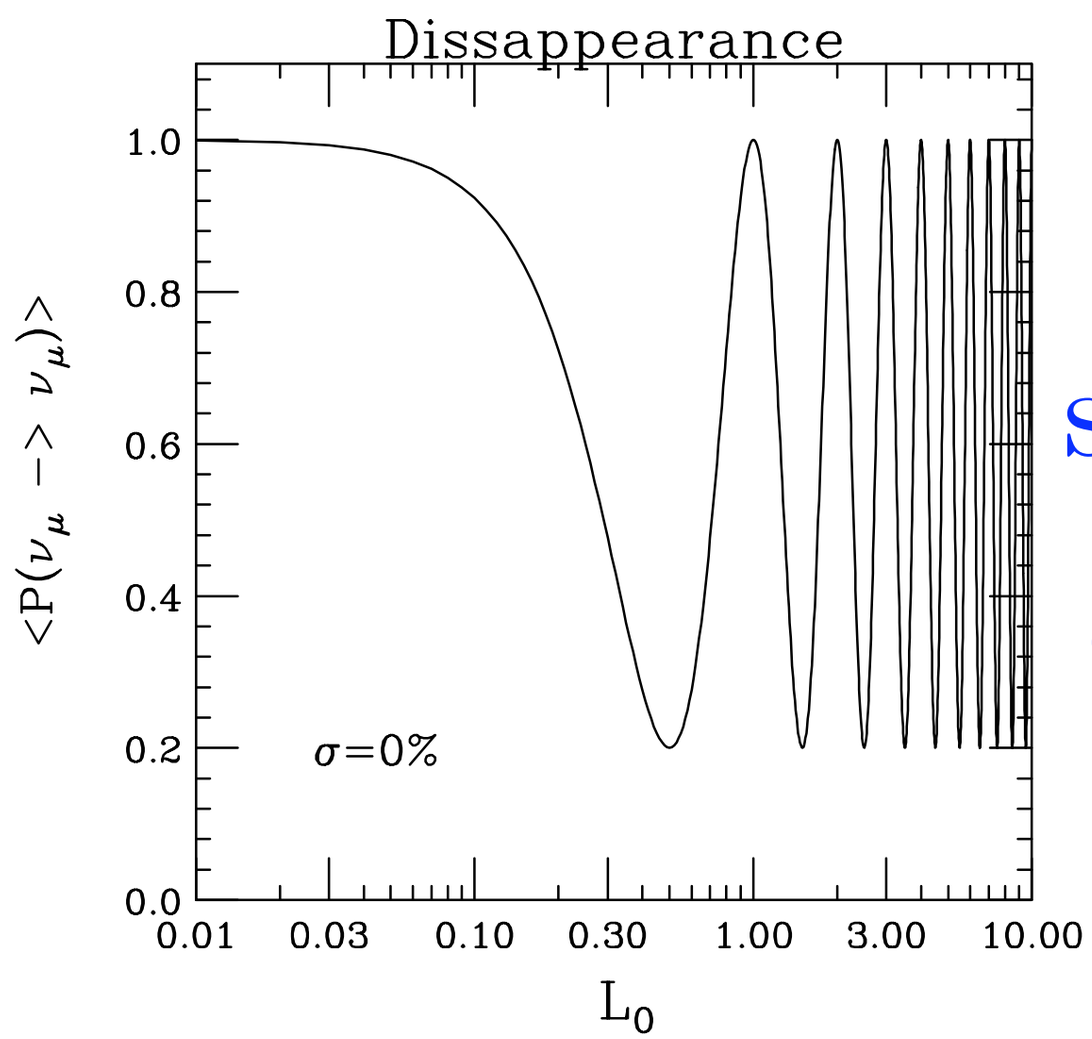
Disappearance:

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

small L/E: $P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \, (\frac{\delta m^2 L}{4E})^2$

Fixed E_ν



Amplitude of Oscillation

$\Uparrow$

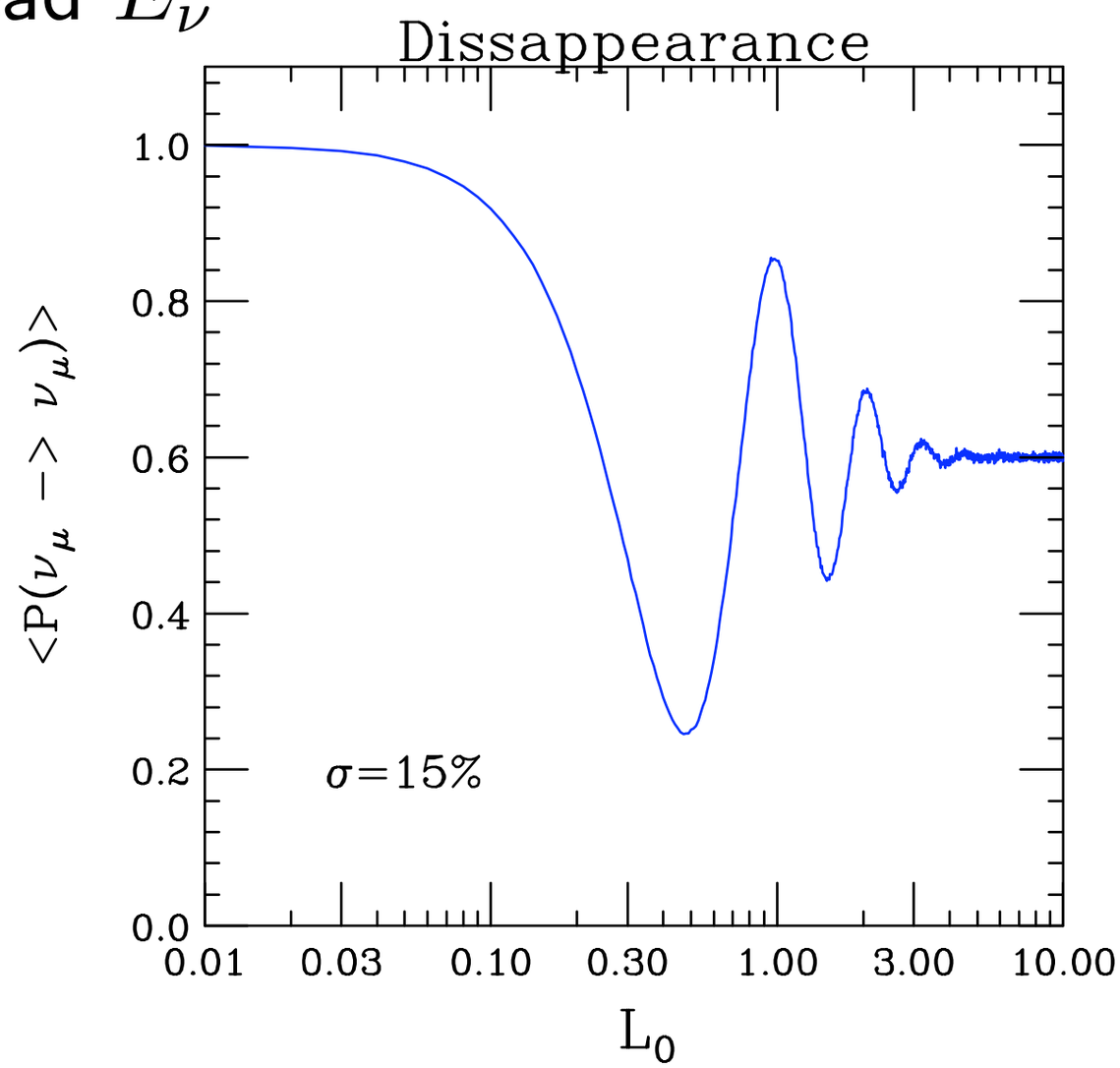
$\sin^2 2\theta$

$\Downarrow$

Oscillation Length $L_0 = 4\pi E/\delta m^2$

$$\langle P(\nu_\mu \to \nu_\mu)\rangle = 1 - \sin^2 2\theta \left\langle \sin^2 \frac{\delta m^2 L}{4E} \right\rangle$$

Spread E_ν



$$1 - \sin^2 2\theta(\tfrac{1}{2}) = \cos^4\theta + \sin^4\theta$$

flavor

flavor oscillations

effectively incoherent
mass eigenstates

$$\langle P(\nu_\mu \to \nu_\mu)\rangle = 1 - \sin^2 2\theta \left\langle \sin^2 \frac{\delta m^2 L}{4E} \right\rangle$$

Averaging over many oscillations $\Rightarrow$

$$1 - \sin^2 2\theta(\tfrac{1}{2}) = \cos^4\theta + \sin^4\theta$$

Understood in terms of probability

$W^+ \to \mu^+ + \nu_1$ probability $\cos^2\theta$

$W^+ \to \mu^+ + \nu_2$ probability $\sin^2\theta$

probability ν_1 contains ν_μ is $\cos^2\theta$

probability ν_2 contains ν_μ is $\sin^2\theta$

What happens to the neutrino oscillation length in the semi-classical limit, $\hbar \to 0$?

- $L_{osc} \to \infty$
- $L_{osc} \to 0$
- Other