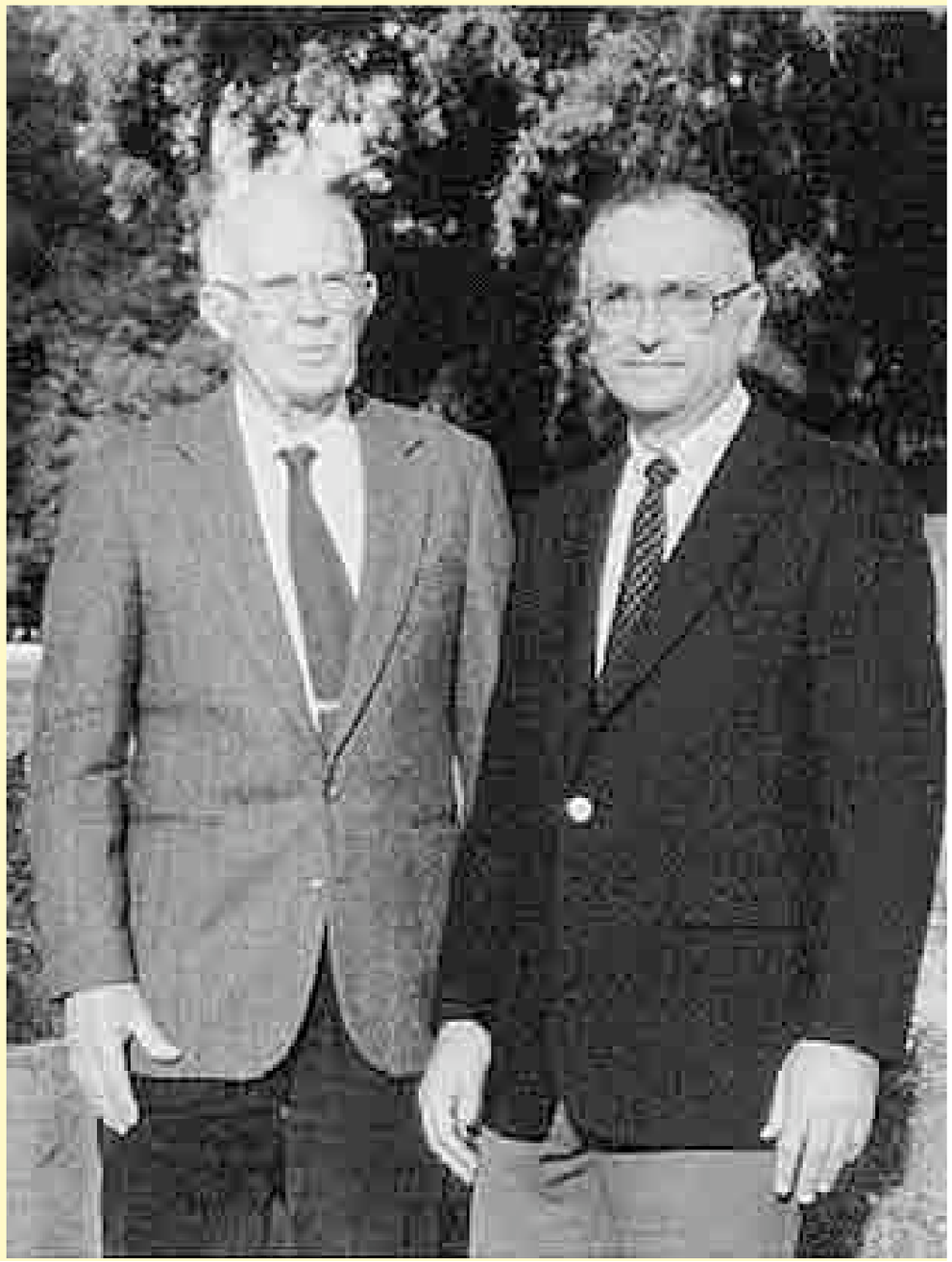
Neutrinos,

In and Beyond the Standard Model:

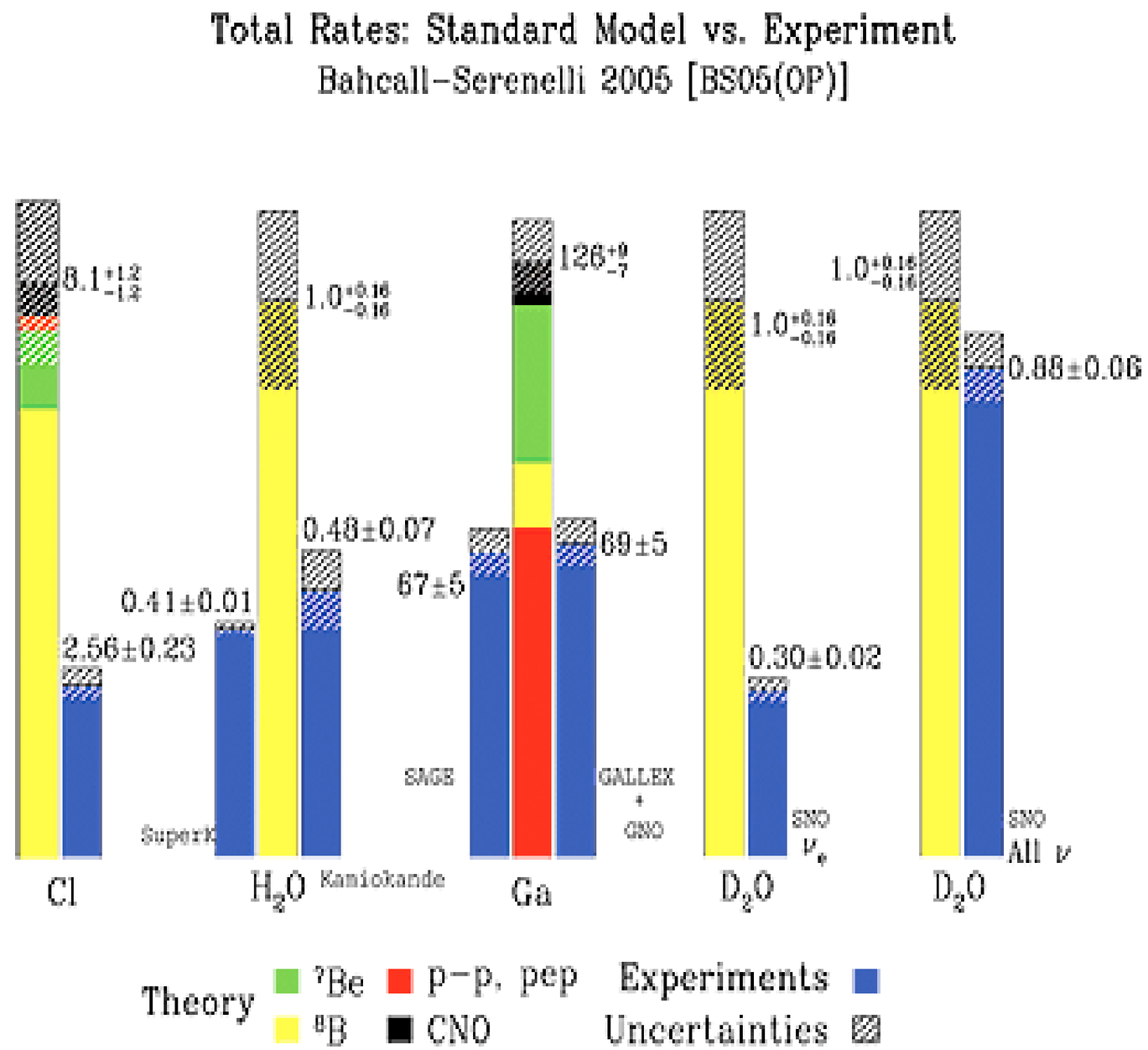
Stephen Parke
Fermilab

Solar Neutrinos

Nu Standard Model:



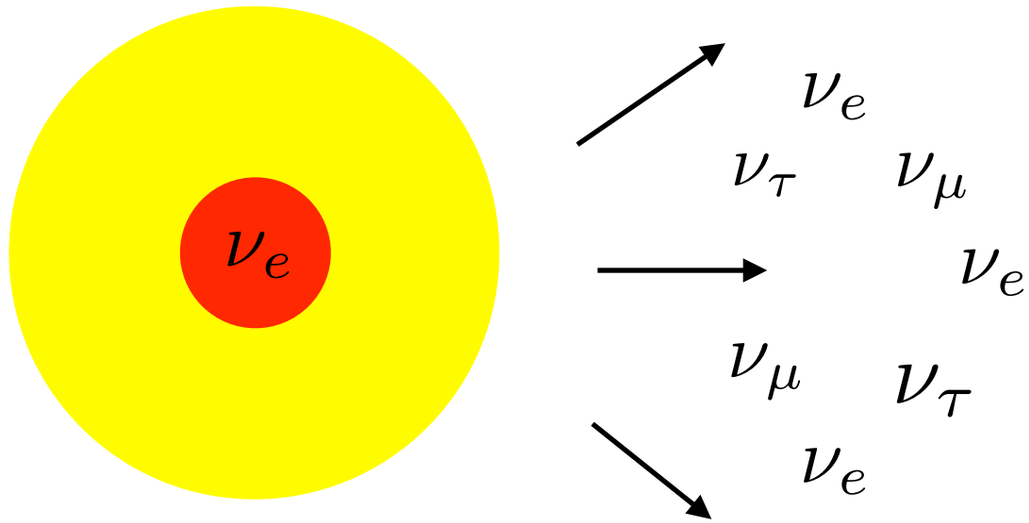
Ray Davis & John Bahcall



Theory v Exp.

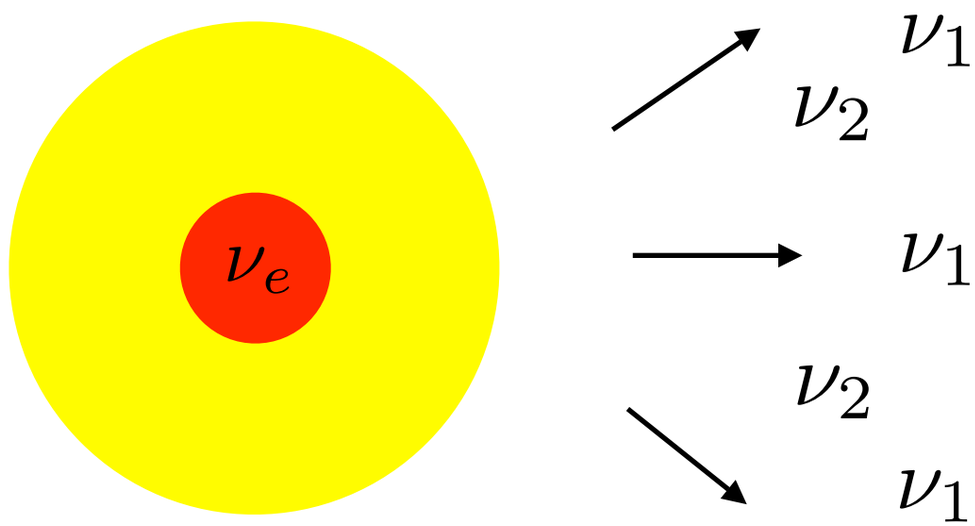
Neutrino Flavor Transistions!!!

Identical Solar Twins:



flavor eigenstates

?????



mass eigenstates

Kinematical Phase:

$$\delta m^2_\odot = 8.0 \times 10^{-5} eV^2$$

$$\sin^2\theta_\odot = 0.31$$

$$\Delta_\odot = \frac{\delta m^2_\odot L}{4E} = 1.27 \qquad \frac{8\times10^{-5}\ eV^2\ \cdot\ 1.5\times10^{11}\ m}{0.1-10\ MeV}$$

???

$$\Delta_\odot \approx 10^{7\pm1}$$

Effectively Incoherent !!!

Vacuum ν_e Survival Probability:

$$\langle P_{ee} \rangle = f_1 \cos^2 \theta_\odot + f_2 \sin^2 \theta_\odot$$

where f_1 and f_2 are the fraction of ν_1 and ν_2 at production.

In vacuum $f_1 = \cos^2 \theta_\odot$ and $f_2 = \sin^2 \theta_\odot$.

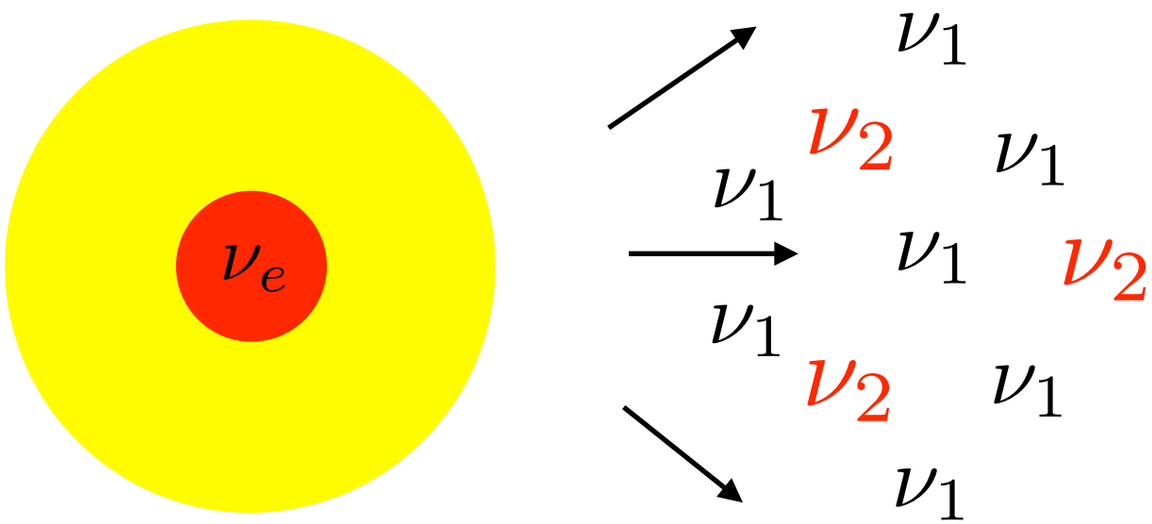
Note energy independence.

$$\langle P_{ee} \rangle = \cos^4 \theta_\odot + \sin^4 \theta_\odot = 1 - \frac{1}{2} \sin^2 2\theta_\odot$$

for pp and ^{7}Be this is approximately THE ANSWER.

$f_1 \sim 69\%$ and $f_2 \sim 31\%$ and $\langle P_{ee} \rangle \approx 0.6$

pp and ^{7}Be



$f_1 \sim 69\%$

$f_2 \sim 31\%$

$$\langle P_{ee} \rangle \approx 0.6$$

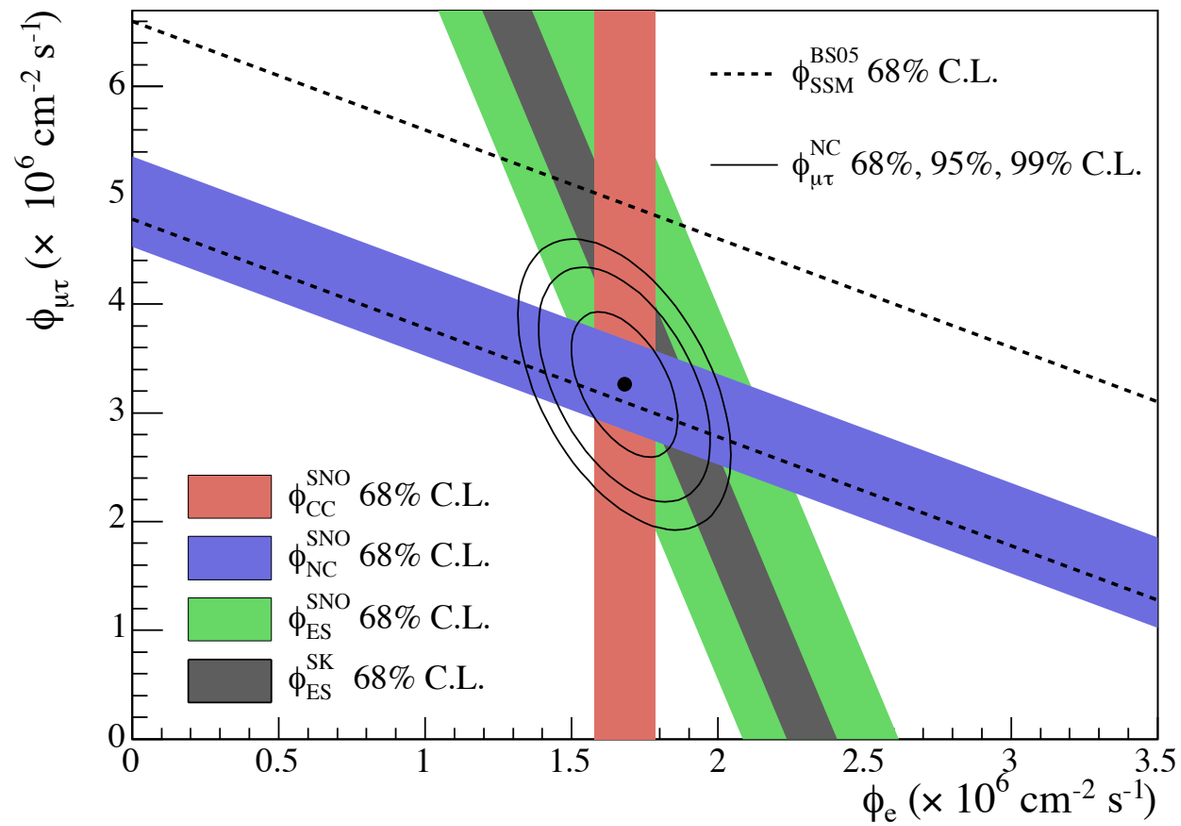
$f_3 = \sin^2\theta_{13} < 4\%$

What about ^8B ?

CC: $\nu_e + d \rightarrow e^- + p + p$

NC : $\nu_x + d \rightarrow \nu_x + p + n$

ES: $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$



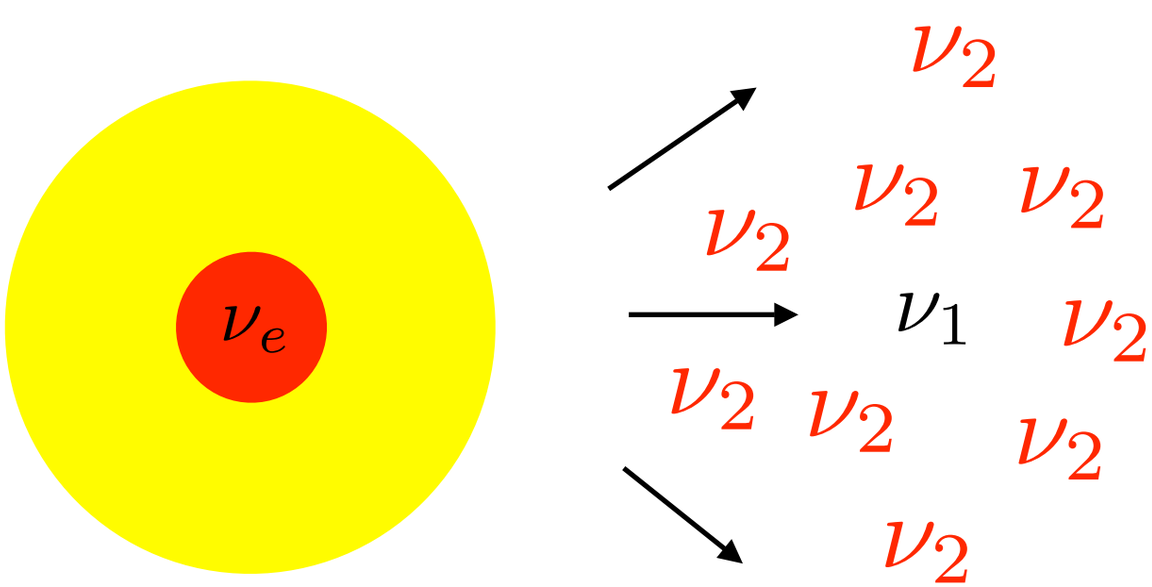
SNO's CC/NC

$$\frac{CC}{NC} = \langle P_{ee} \rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$$

$$f_1 = \left(\frac{CC}{NC} - \sin^2\theta_\odot\right) / \cos 2\theta_\odot$$

$$= (0.35 - 0.31)/0.4 \approx 10 \ \pm \ ???\%$$

^8B



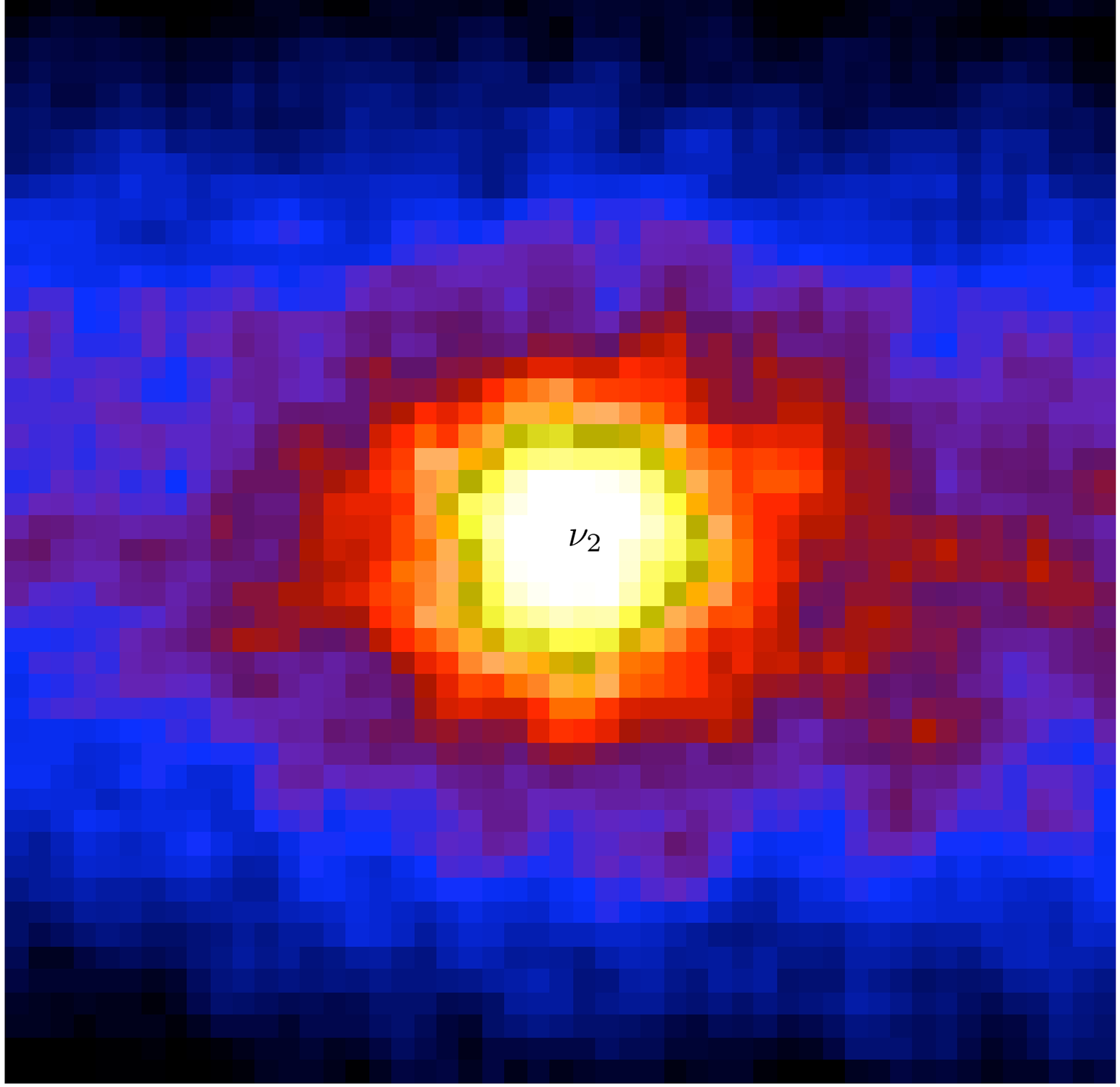
$f_2 \sim 90\%$

$f_1 \sim 10\%$

$$\langle P_{ee} \rangle = \sin^2\theta + f_1 \cos 2\theta_\odot \approx \sin^2\theta_\odot = 0.31$$

Wow!!! How did that happen???

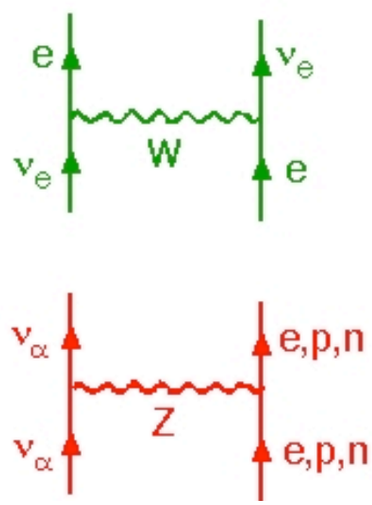
energy dependence!!!



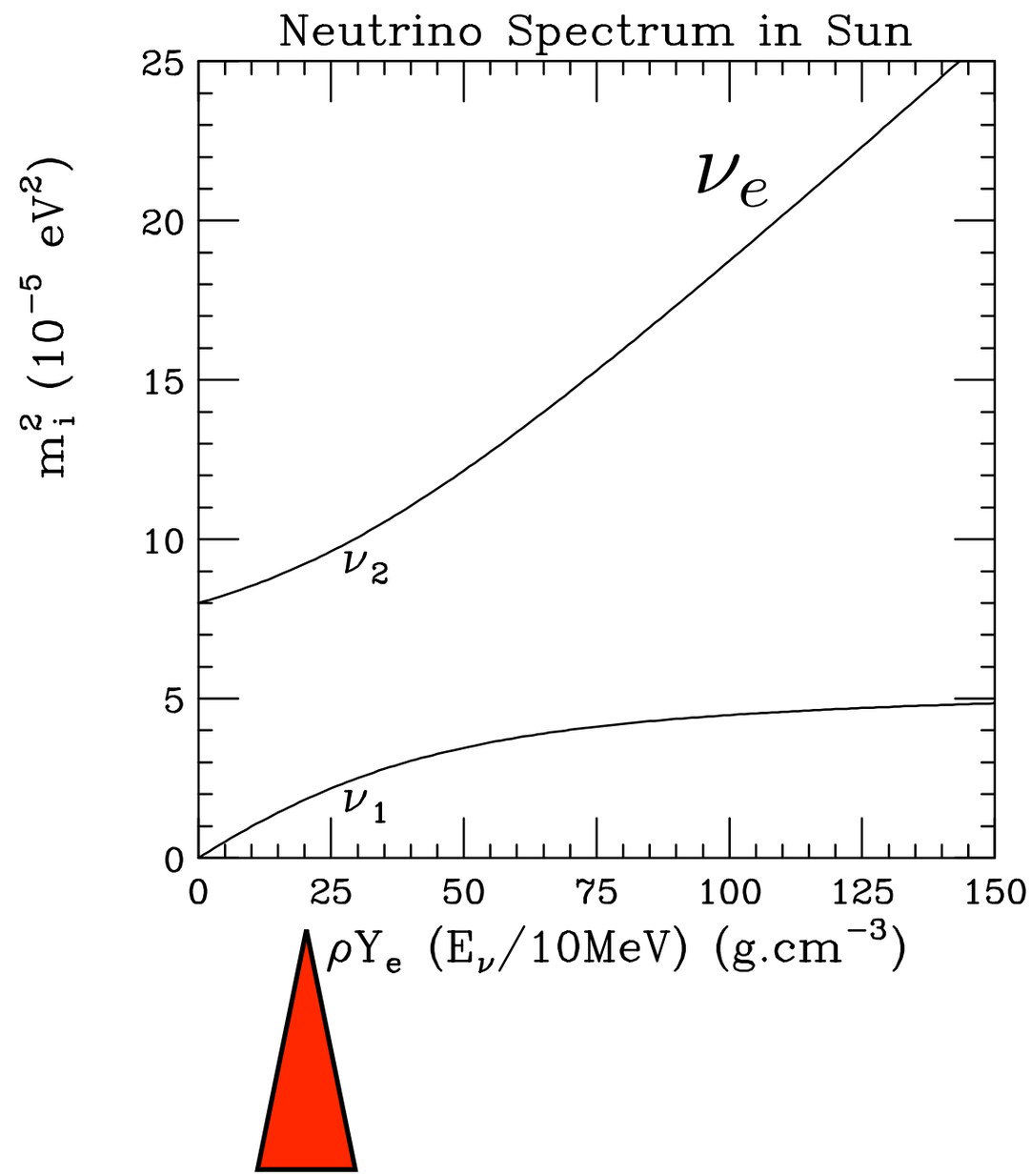
These are ν_2 Neutrinos !!!

MSW

Coherent Forward Scattering:



Wolfenstein '78



Mikheyev + Smirnov Resonance WIN '85

MATTER EFFECTS CHANGE THE NEUTRINO MASSES AND MIXINGS

Neutrino Evolution:

$$-i\frac{\partial}{\partial t}\nu = H\nu$$

in the mass eigenstate basis

$$\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \text{ and } H = \begin{pmatrix} \sqrt{p^2+m_1^2} & 0 \\ 0 & \sqrt{p^2+m_2^2} \end{pmatrix}$$

$$E = \sqrt{p^2+m^2}$$

$$H = (p + \frac{m_1^2+m_2^2}{4p})I + \frac{1}{4E}\begin{pmatrix} -\delta m^2 & 0 \\ 0 & \delta m^2 \end{pmatrix}$$

$$\delta m^2 = m_2^2 - m_1^2 > 0$$

in the flavor basis

$$\nu \to U\nu \text{ and } H \to UHU^\dagger$$

where $\nu = \begin{pmatrix} \nu_e \\ \nu_\sigma \end{pmatrix}$ and $U = \begin{pmatrix} \cos\theta_\odot & \sin\theta_\odot \\ -\sin\theta_\odot & \cos\theta_\odot \end{pmatrix}$

$$0 < \theta_\odot < \frac{\pi}{2}$$

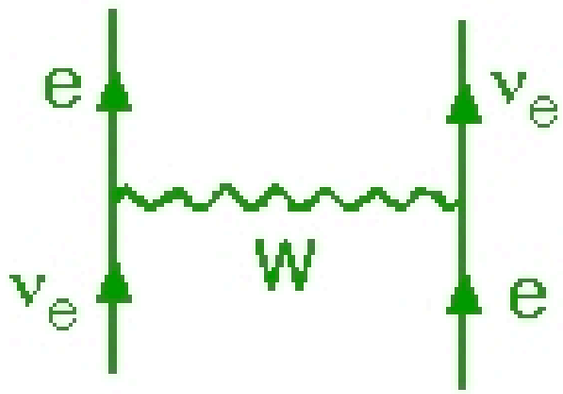
and therefore in flavor basis

$$H = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}$$

i.e. $\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}_{mass} \Rightarrow \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}_{flavor}$

Coherent Forward Scattering:

dimensions $[G_F N_e] = M^{-2} L^{-3} = M$

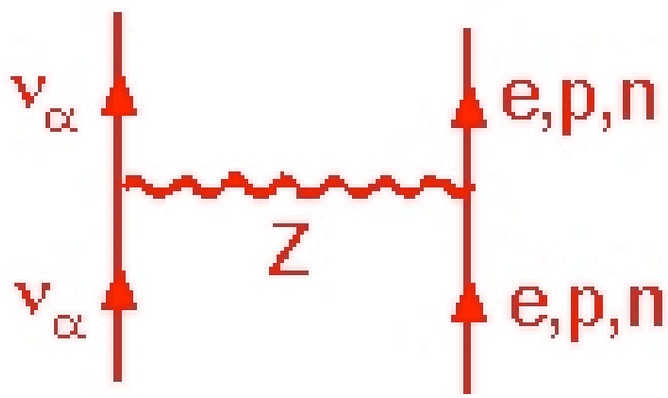


$$\pm\sqrt{2} G_F N_e \; \delta_{ee}$$

N_e is number density of electrons

+(-) for neutrinos (anti-neutrinos)

Wolfenstein '78



Same for all active flavors, therefore overall phases

$$\begin{pmatrix} +\sqrt{2} G_F N_e & 0 \\ 0 & 0 \end{pmatrix} \rightarrow \cancel{\frac{G_F N_e}{\sqrt{2}} I_2} + \frac{1}{2} \begin{pmatrix} +\sqrt{2} G_F N_e & 0 \\ 0 & -\sqrt{2} G_F N_e \end{pmatrix}$$

Including Matter Effects in the Flavor Basis:

$$H_{flavor} = \frac{1}{4E_\nu}\begin{pmatrix} -\delta m^2 \cos 2\theta_\odot + 2\sqrt{2}G_F N_e E_\nu & \delta m^2 \sin 2\theta_\odot \\ \delta m^2 \sin 2\theta_\odot & \delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu \end{pmatrix}$$

Diagonalize by identifying with

$$H_{flavor} = \frac{1}{4E_\nu}\begin{pmatrix} -\delta m_N^2 \cos 2\theta_\odot^N & \delta m_N^2 \sin 2\theta_\odot^N \\ \delta m_N^2 \sin 2\theta_\odot^N & \delta m_N^2 \cos 2\theta_\odot^N \end{pmatrix}$$

Masses and Mixings in MATTER: δm_N^2 and $\theta_\odot^N$

$$\boxed{\begin{aligned} \delta m_N^2 \cos 2\theta_\odot^N &= \delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu \\ \delta m_N^2 \sin 2\theta_\odot^N &= \delta m^2 \sin 2\theta_\odot \end{aligned}}$$

Notice:
(1) Possible zero when $\delta m^2 \cos 2\theta_\odot = 2\sqrt{2}G_F N_e E_\nu$
(2) the invariance of the product $\delta m^2 \sin 2\theta_\odot$

ν_e disappearance in Loooong Block of Lead:

$$1 - P(\nu_e \to \nu_e) = \sin^2 2\theta_\odot^N \sin^2 \Delta_N$$

$$\Delta_N = \frac{\delta m_N^2 L}{4E}$$

same form as vacuum

The Solution:

$$\delta m_N^2 = \sqrt{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2} G_F N_e E_\nu)^2 + (\delta m^2 \sin 2\theta_\odot)^2}$$

$$\sin^2 \theta_\odot^N = \frac{1}{2}\left(1 - \frac{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2} G_F N_e E_\nu)}{\delta m_N^2}\right)$$

$$\theta_\odot^N > \theta_\odot$$

pp and ^{7}Be

Quasi-Vacuum: $2\sqrt{2} G_F N_e E_\nu \ll \delta m^2 \cos 2\theta_\odot$

$$\delta m_N^2 = \delta m^2 \text{ and } \theta_\odot^N = \theta_\odot$$

Resonance (Mikheyev + Smirnov '85): $2\sqrt{2} G_F N_e E_\nu = \delta m^2 \cos 2\theta_\odot$

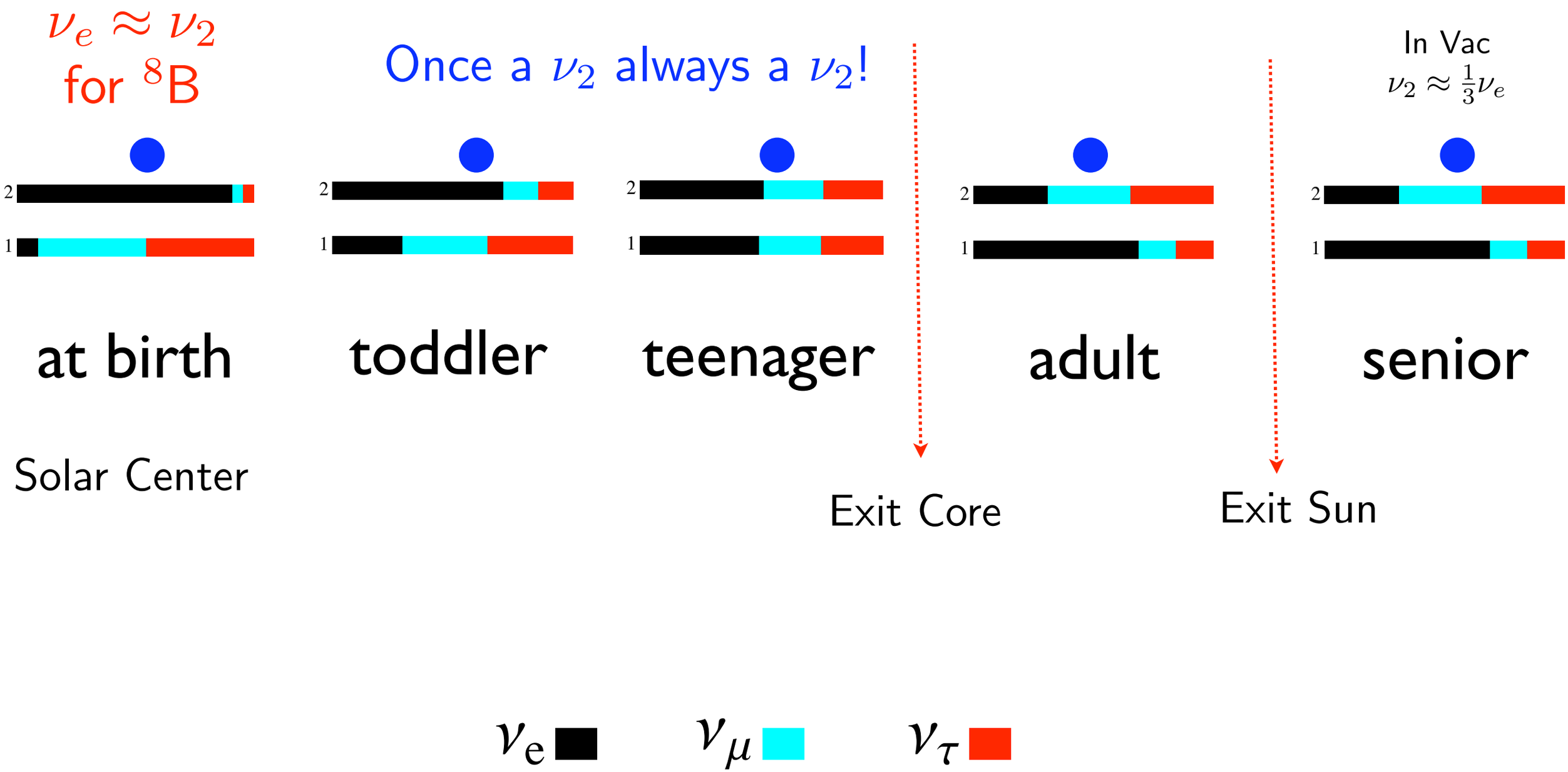
$$\delta m_N^2 = \delta m^2 \sin 2\theta_\odot \text{ and } \theta_\odot^N = \pi/4$$

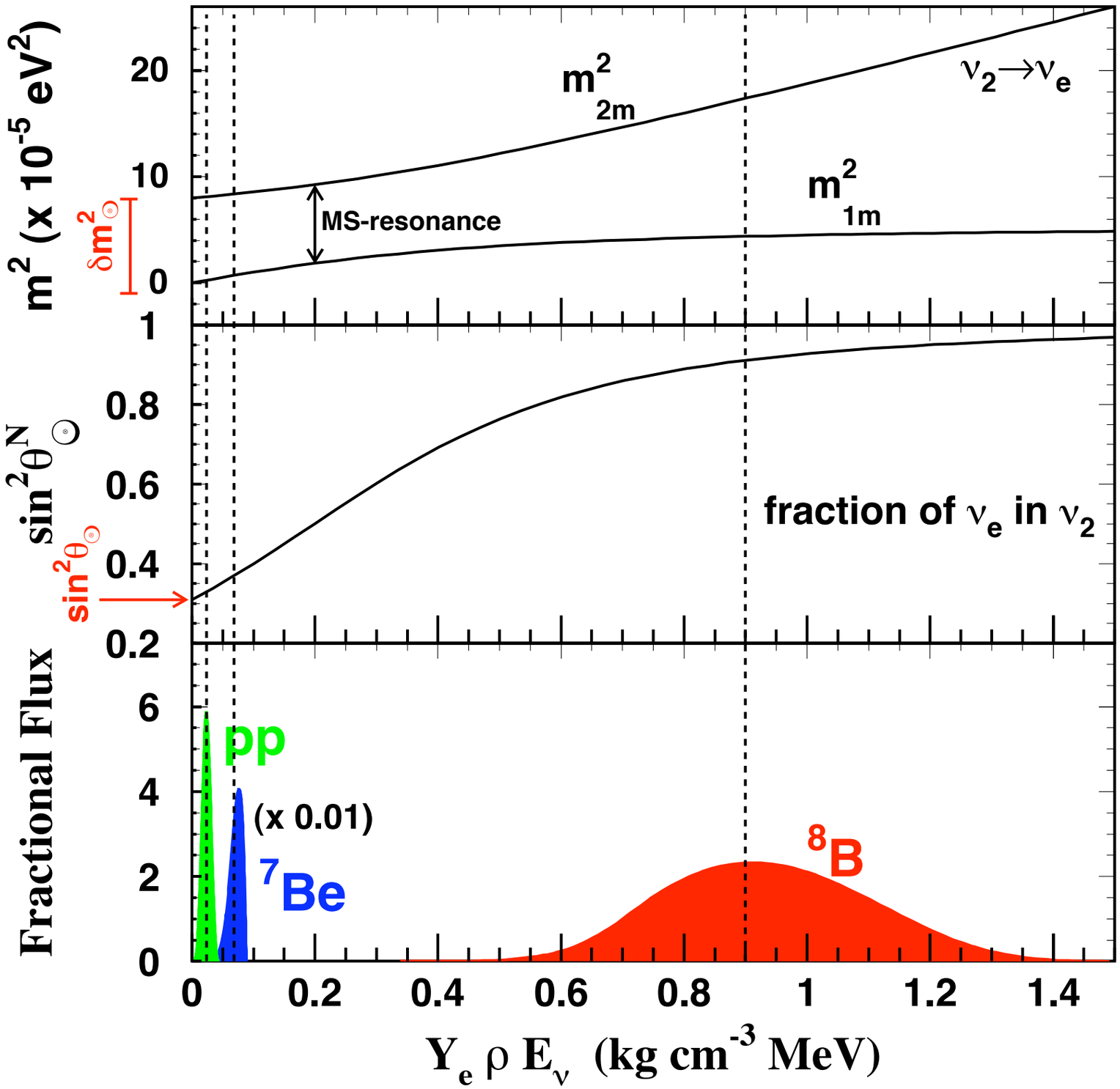
Matter Dominated: $2\sqrt{2} G_F N_e E_\nu \gg \delta m^2 \cos 2\theta_\odot$

^8B

$$\delta m_N^2 \to 2\sqrt{2} G_F N_e E_\nu \text{ and } \theta_\odot^N \to \pi/2$$

Life of a Boron-8 Solar Neutrino:





In Vacuum

$$\delta m^2_\odot = 8.0 \pm 0.4 \times 10^{-5}\ eV^2$$

$$\sin^2\theta_\odot = 0.31 \pm 0.03$$

Whereas for ^{8}B at center of Sun

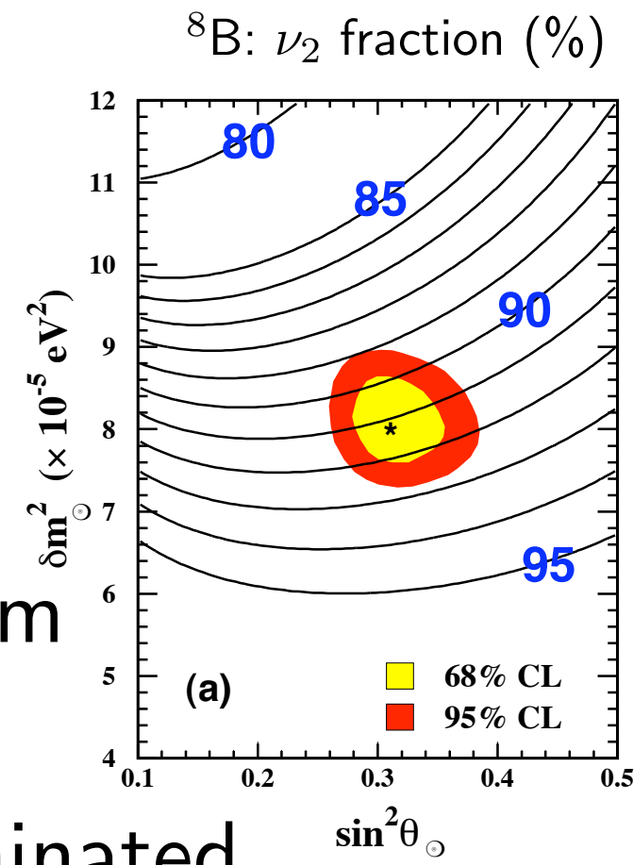
$$\delta m^2_N = 14 \times 10^{-5}\ eV^2$$

$$\sin^2\theta^N_\odot = 0.91$$

hep-ph/0601198

Mass Eigenstate Purity:

	$\langle f_1 \rangle$ (%)	$\langle f_2 \rangle$ (%)
vac	69 ± 3	31 ∓ 3
pp	67 ± 4	33 ∓ 4
^{7}Be	63 ± 4	37 ∓ 4
^{8}B	9 ∓ 2	91 ± 2



quasi-vacuum

matter dominated

$f_1 = \cos^2\theta_\odot^N$ and $f_2 = \sin^2\theta_\odot^N$

$\langle P_{ee} \rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$

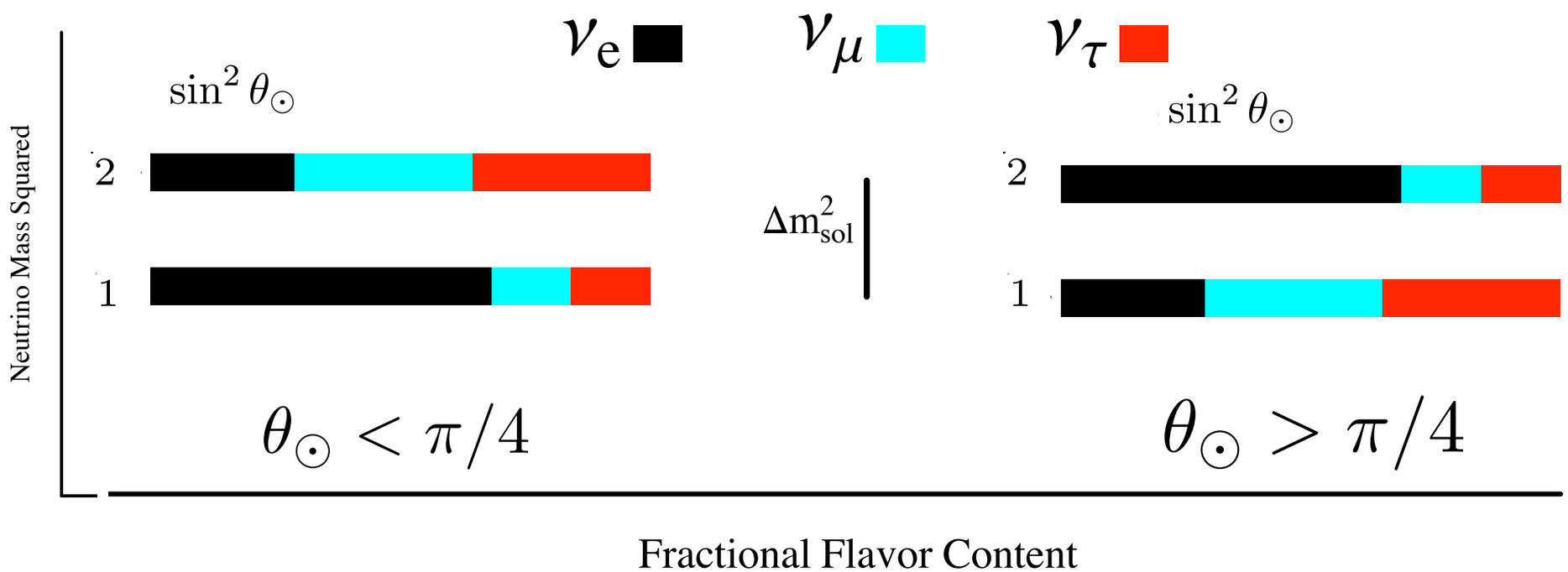
vac pp ^{7}Be ^{8}B

$N_e E_\nu$

$\langle P_{ee} \rangle = \cos^4\theta_\odot + \sin^4\theta_\odot$

$\Rightarrow \sin^2\theta_\odot$

Solar Pair Mass Hierarchy:



KamLAND doesn't care since $\sin^2 2\theta_\odot$ same for $\theta_\odot$ and $\frac{\pi}{2} - \theta_\odot$

but SNO does !!!

for neutrino in matter
$\theta_\odot^N > \theta_\odot$

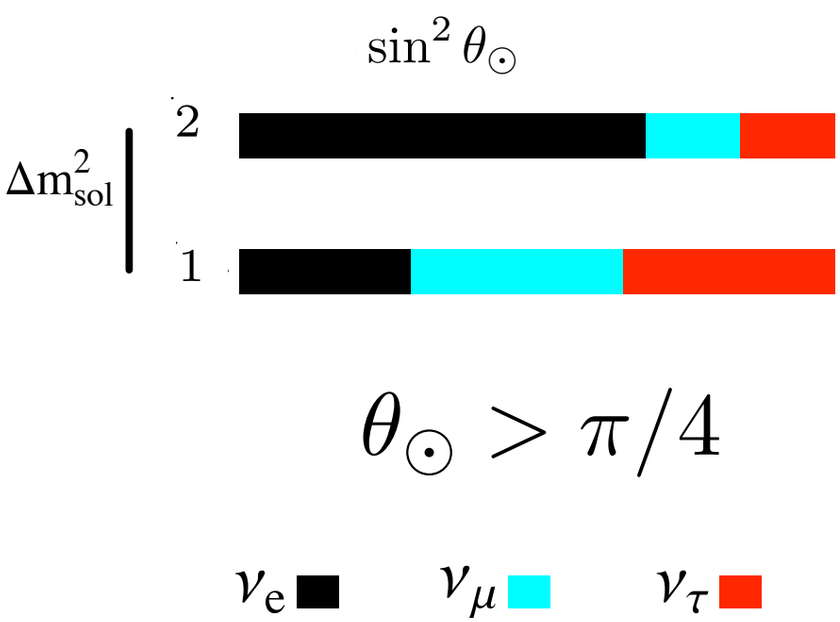
$$\langle P_{ee} \rangle = \cos^2\theta_\odot^N \cos^2\theta_\odot + \sin^2\theta_\odot^N \sin^2\theta_\odot = \frac{1}{2} + \frac{1}{2}\cos 2\theta_\odot^N \cos 2\theta_\odot$$

if $\theta_\odot < \pi/4$
$\langle P_{ee} \rangle \geq \sin^2\theta_\odot$

if $\theta_\odot > \pi/4$
$\langle P_{ee} \rangle \geq \frac{1}{2}(1 + \cos^2 2\theta_\odot) \geq \frac{1}{2}$

SNO: $\langle P_{ee} \rangle_{day} = 0.347 \pm 0.038$

Solar Hierarchy Determined !!!



Solar matter effects put more of the neutrino into ν_2.
This raises the survival probability above vacuum value since ν_2 has more ν_e.
But the minimum of P_{ee} in vacuum is $1/2$.

For this hierarchy $\boldsymbol{P_{ee}^{matter} \geq P_{ee}^{vac} \geq 1/2}$

But $\boldsymbol{P_{ee}^{SNO} = 0.347 \pm 0.038 < 1/2}$

This solar hierarchy EXCLUDED !!!.

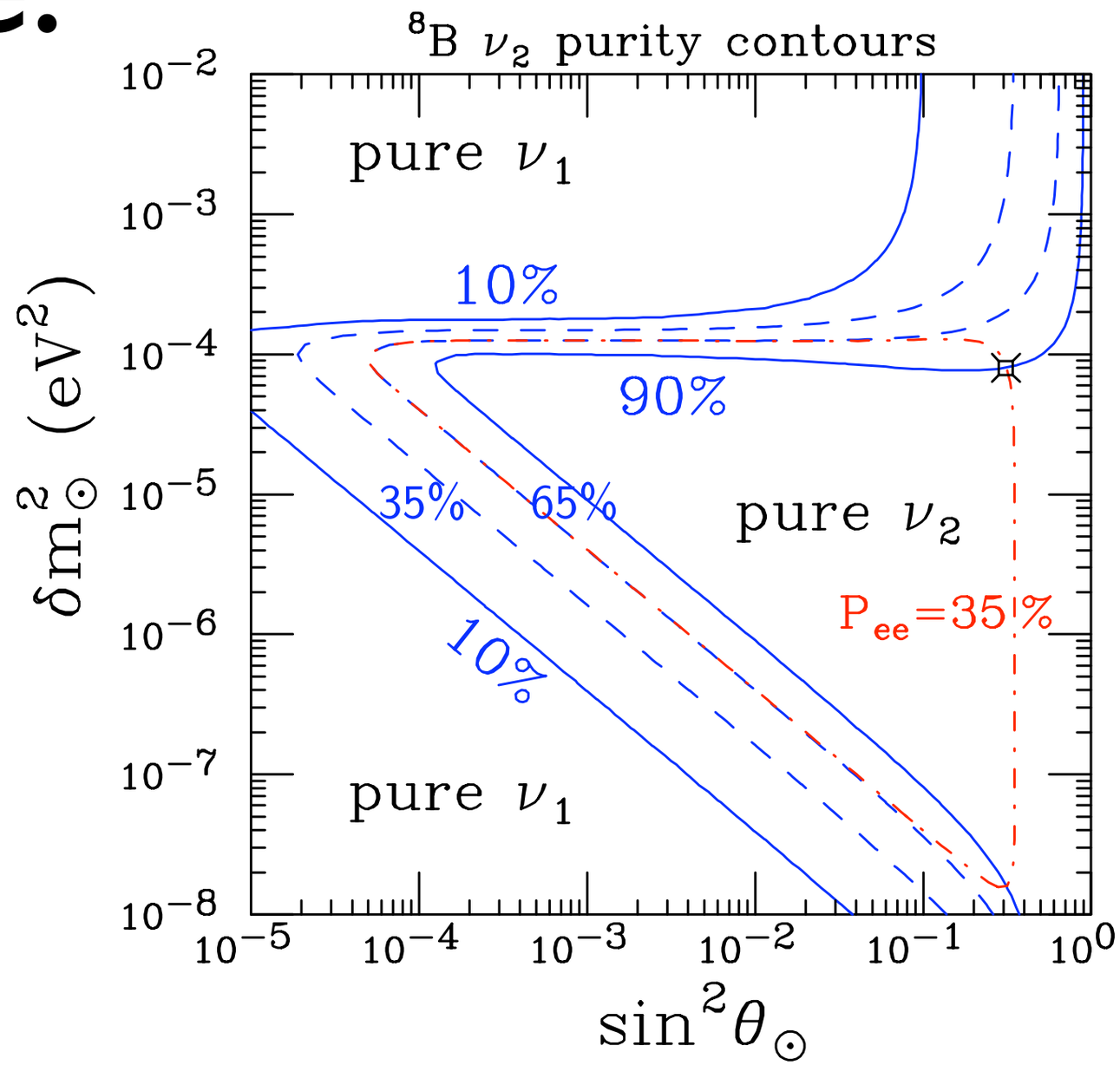
The Big Picture:

$$P_{ee} = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$$

$$f_1 = (1 - P_x)\cos^2\theta_\odot^N + P_x \sin^2\theta_\odot^N$$

$$f_2 = (1 - P_x)\sin^2\theta_\odot^N + P_x \cos^2\theta_\odot^N$$

P_x is the probability to jump from ν_2 to ν_1 (or ν_1 to ν_2) during MS-resonance crossing.



$$P_{ee} = \frac{1}{2} + \left(\frac{1}{2} - P_x\right)\cos 2\theta_\odot^N \cos 2\theta_\odot$$

Jump Probability:

$$P_x \approx \exp\left(-\pi \frac{Width\ of\ Resonance}{Oscillation\ Length}\right)$$

SP PRL, 57, 1275 and 2322 (1986)

Day/Night Asymmetry:

$\sin^2\theta_\odot \to \sin^2\theta_\oplus = \sin^2\theta_\odot + \frac{1}{2}\sin^2 2\theta_\odot \left(\frac{A_\oplus}{\delta m^2_\odot}\right)$ in the earth.

A=2(D-N)/(D+N) expected to be few %

SK:

$$A_{ES} = -1.8 \pm 1.6(\mathrm{stat})^{+1.3}_{-1.2}(\mathrm{syst})\%$$

Spectral Distortion:

A characteristic of matter effects is that the Fraction of ν_2 is energy dependent .

Smaller at smaller E.

Implies an increase in P_{ee} near threshold.

Maybe in SK with 4.5MeV threshold!!!

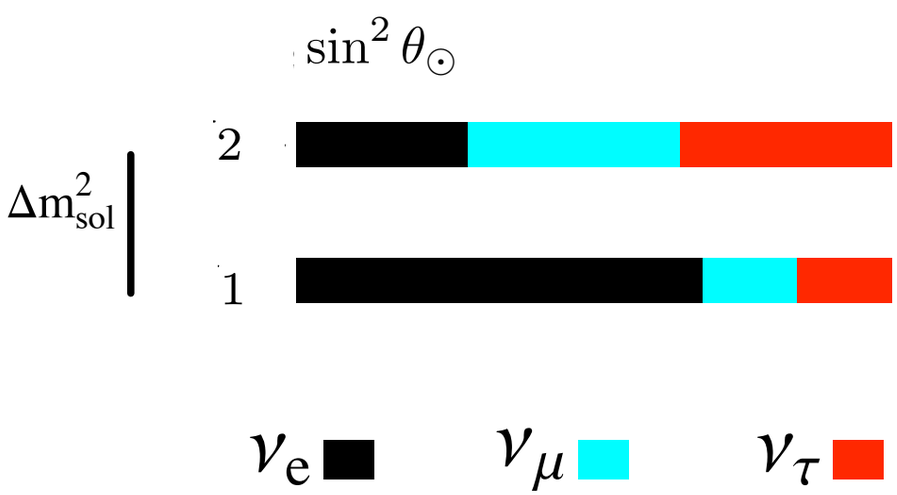
Summary:

The low energy pp and ^{7}Be Solar Neutrinos exit the sun as two thirds ν_1 and one third ν_2 due to (quasi-) vacuum oscillations.

$$f_1 = 65 \pm 2\%,\ f_2 = 35 \mp 2\% \text{ with } P_{ee} \approx 0.56$$

The high energy ^{8}B Solar Neutrinos exit the sun as "PURE" ν_2 mass eigenstates due to matter effects.

$$f_2 = 91 \pm 2\% \text{ and } f_1 = 9 \mp 2\% \text{ with } P_{ee} \approx 0.35.$$

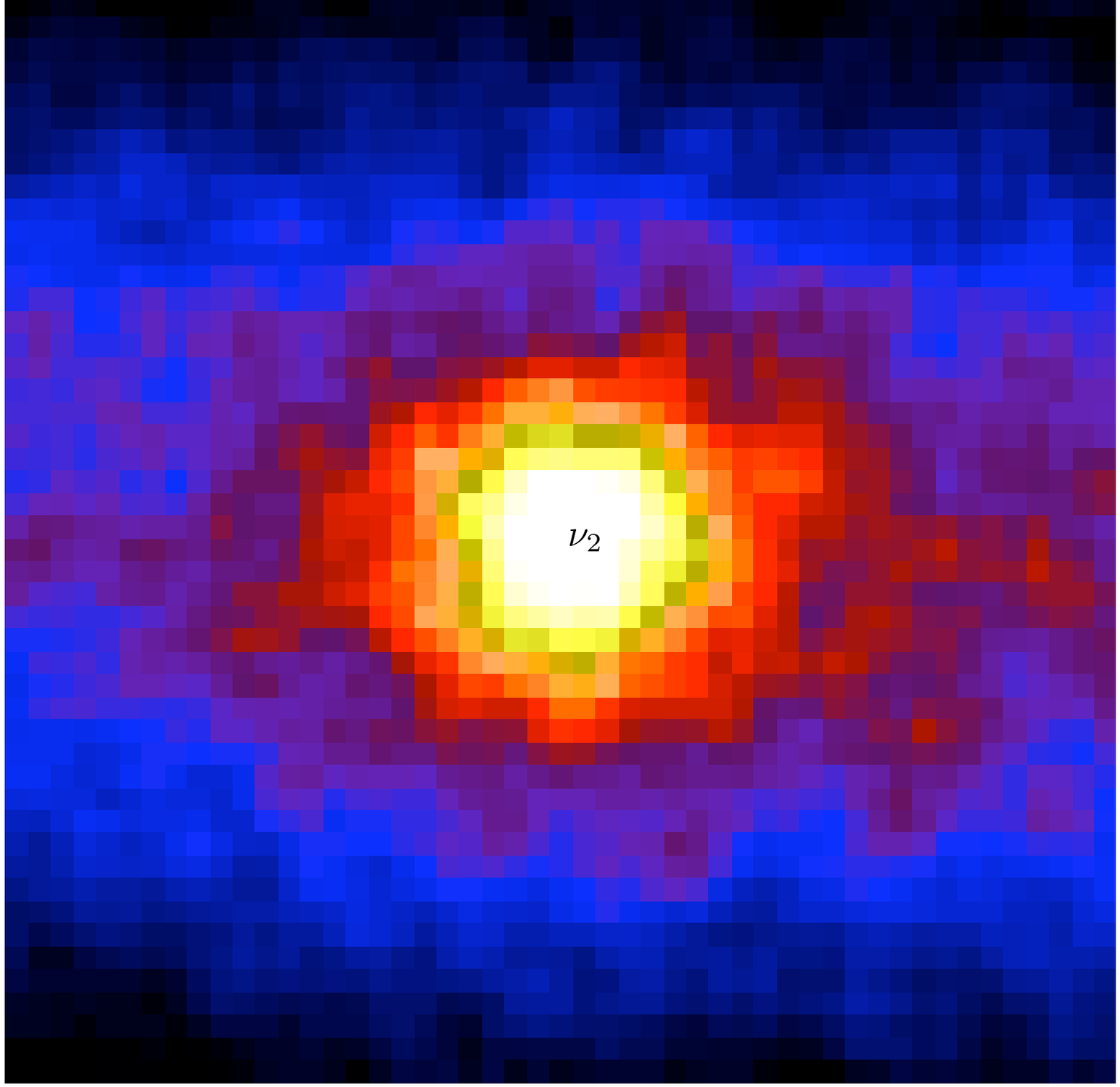


$$\delta m^2_\odot = 8.0 \pm 0.4 \times 10^{-5} eV^2$$

$$\sin^2\theta_\odot = 0.310 \pm 0.026$$

at 68% CL

SNO, KamLAND, SK/K, GNO/Gallex, SAGE, Cl



These are ν_2 Neutrinos !!!

Nu Standard Model:

The ν Standard Model

- 3 light (m_i <1 eV) Majorana Neutrinos:

  $\Rightarrow$ only 2 δm^2

  $|\delta m^2_{atm}| \sim 2.5 \times 10^{-3}$ eV^2 and $\delta m^2_{solar} \sim +8.0 \times 10^{-5}$ eV^2

- Only Active flavors (no steriles):

  $e,\ \mu,\ \tau$

- Unitary Mixing Matrix:

  3 angles ($\theta_{12},\ \theta_{23},\ \theta_{13}$), 1 Dirac phase ($\delta$),

  2 Majorana phases (α_2, α_3)

$$|\nu_\alpha\rangle_{flavor} = U_{\alpha i}|\nu_i\rangle_{mass}.$$

Atmos. L/E $\mu \to \tau$ 500km/GeV Atmos. L/E $\mu \leftrightarrow e$ Solar L/E $e \to \mu, \tau$ 15km/MeV $\beta\beta 0\nu$ decay

$$\begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}$$

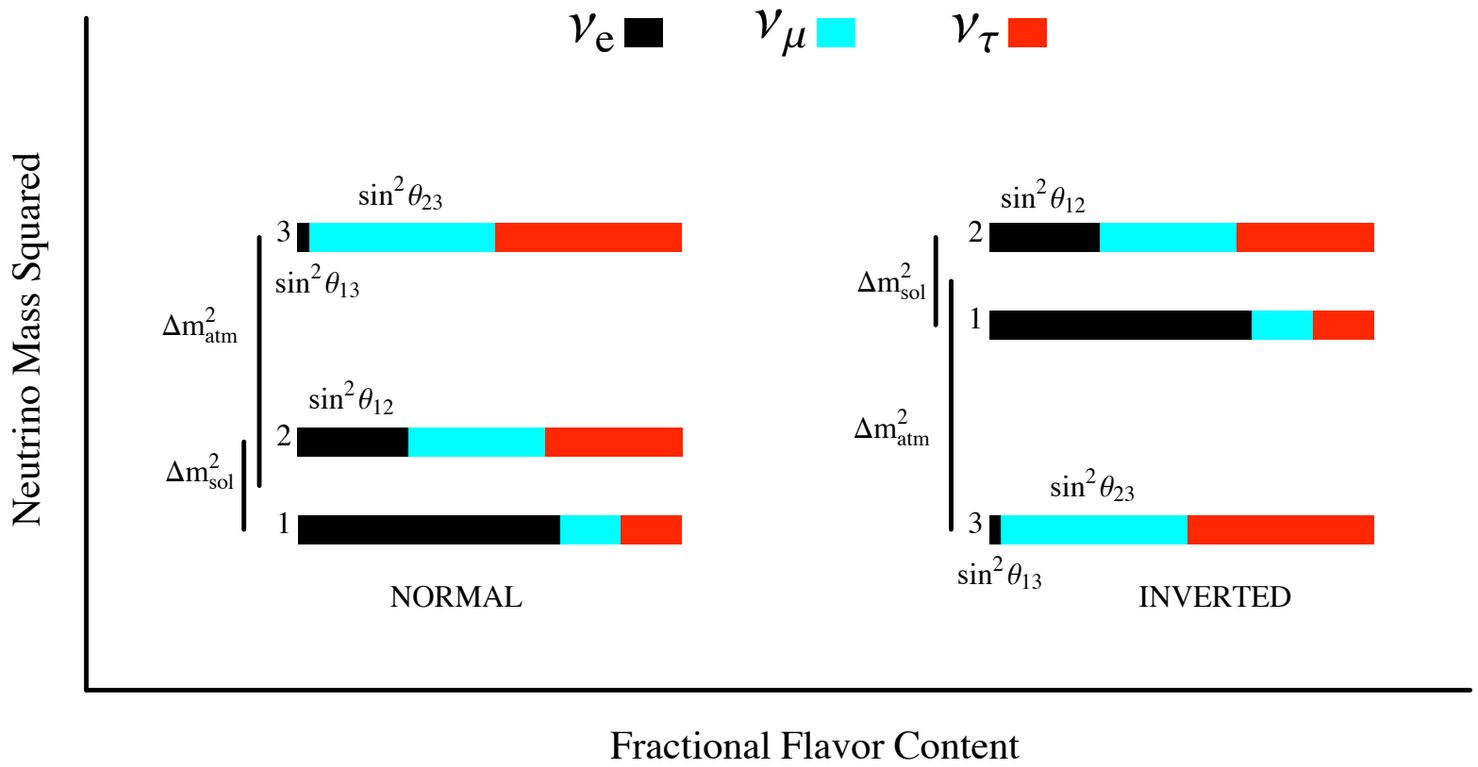
In oscillation phenomena,

the phases $\alpha_2,\ \alpha_3$ are unobservable $(U_{\alpha i}U^*_{\beta i})$

and also the value of m_{lite} is irrelevant (δm^2)

$$= \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

(12)-Sector:



(12) Parameters: SNO, KamLAND, SK

$$\delta m^2_{21} = +8.0 \pm 0.8 \times 10^{-5}\ eV^2$$

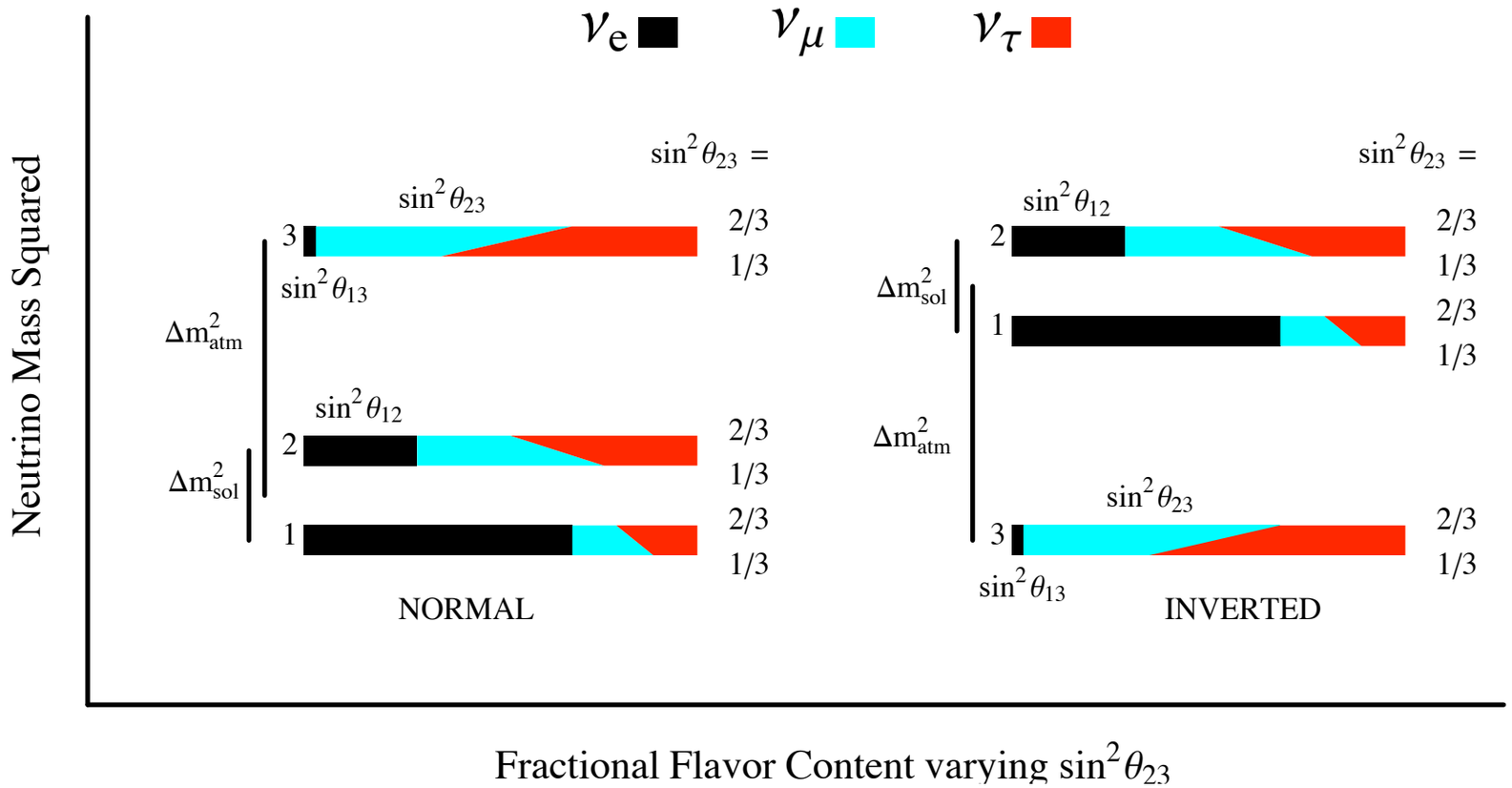
$$0.25 < \sin^2\theta_{12} < 0.37$$

$\sin^2\theta_{12} \geq \frac{1}{2}$ excluded at $> 5\ \sigma$!

sign of δm^2_{21} determined at this C.L.

(23)-Sector:

Mena + SP hep-ph/0312131



(23) Parameters: SK, K2K

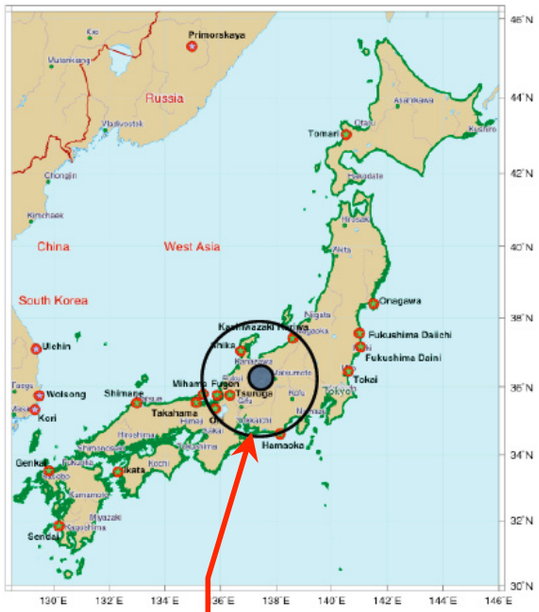
$$|\delta m^2_{32}| = 1.5 - 3.4 \times 10^{-3}\ eV^2$$

$$0.36 < \sin^2\theta_{23} < 0.64$$

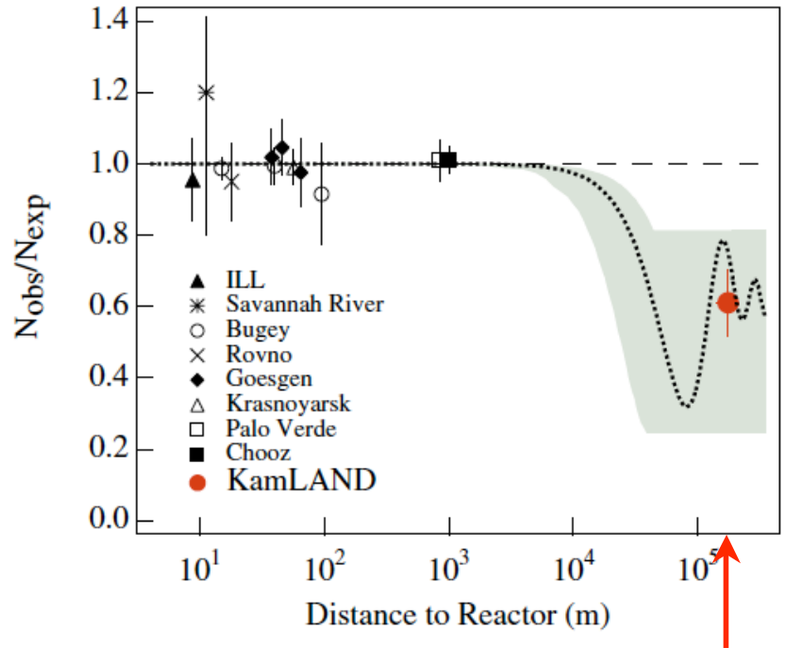
(obtained from $\sin^2 2\theta_{23} > 0.91$)

Magnitude of δm^2_{32} and $\sin^2\theta_{23}$ not as well known!

Sign of δm^2_{32} Unknown !!!

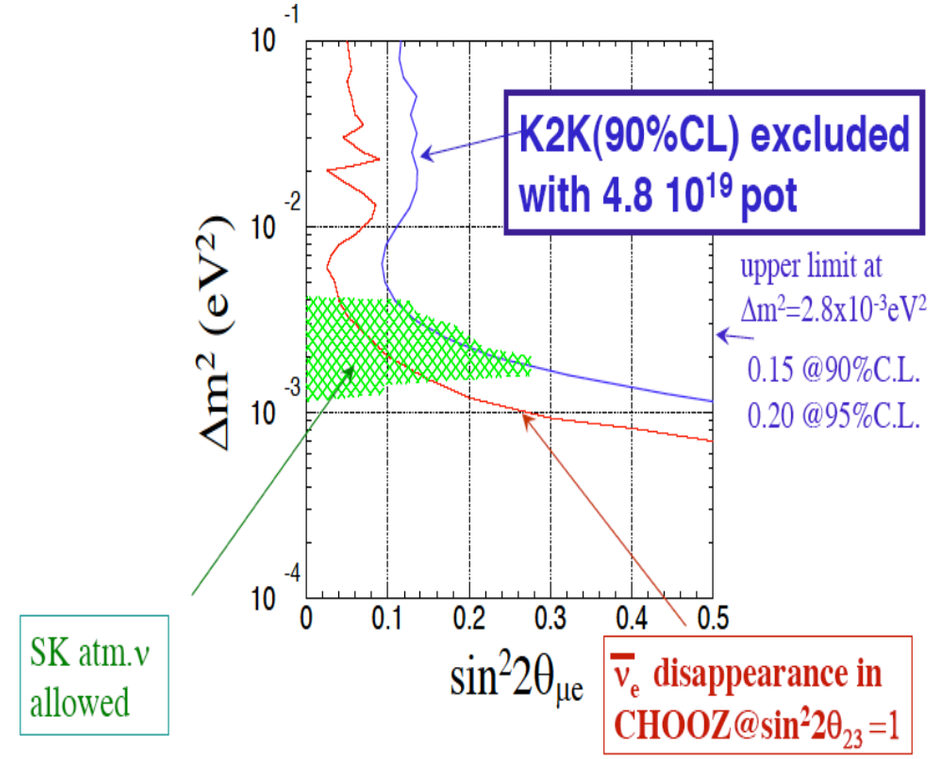


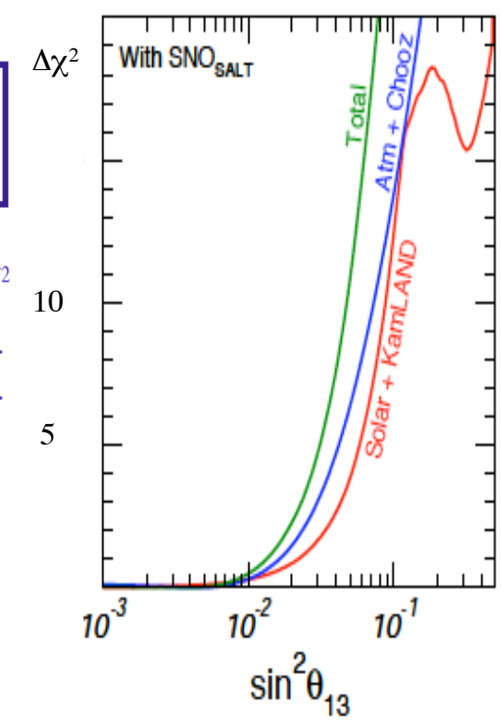
180 km



Disappearance: 99.

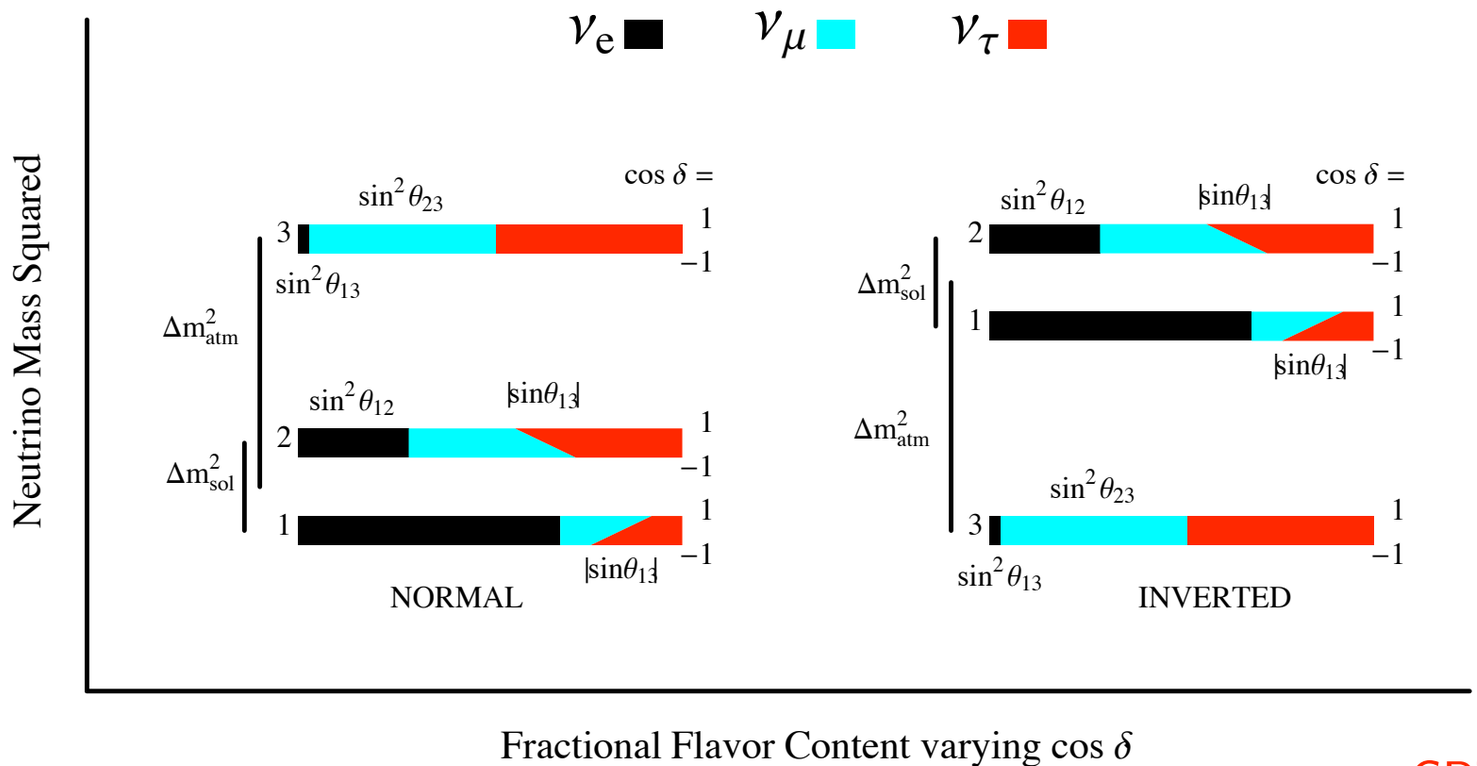
No indication yet of nonzero θ_{13} from atmospheric, solar and terrestrial ν





Maltoni et al.

(13)-Sector:



Less than
4% ν_e
in the 3 state!

CPT: $\delta \Leftrightarrow -\delta$ Invariant!

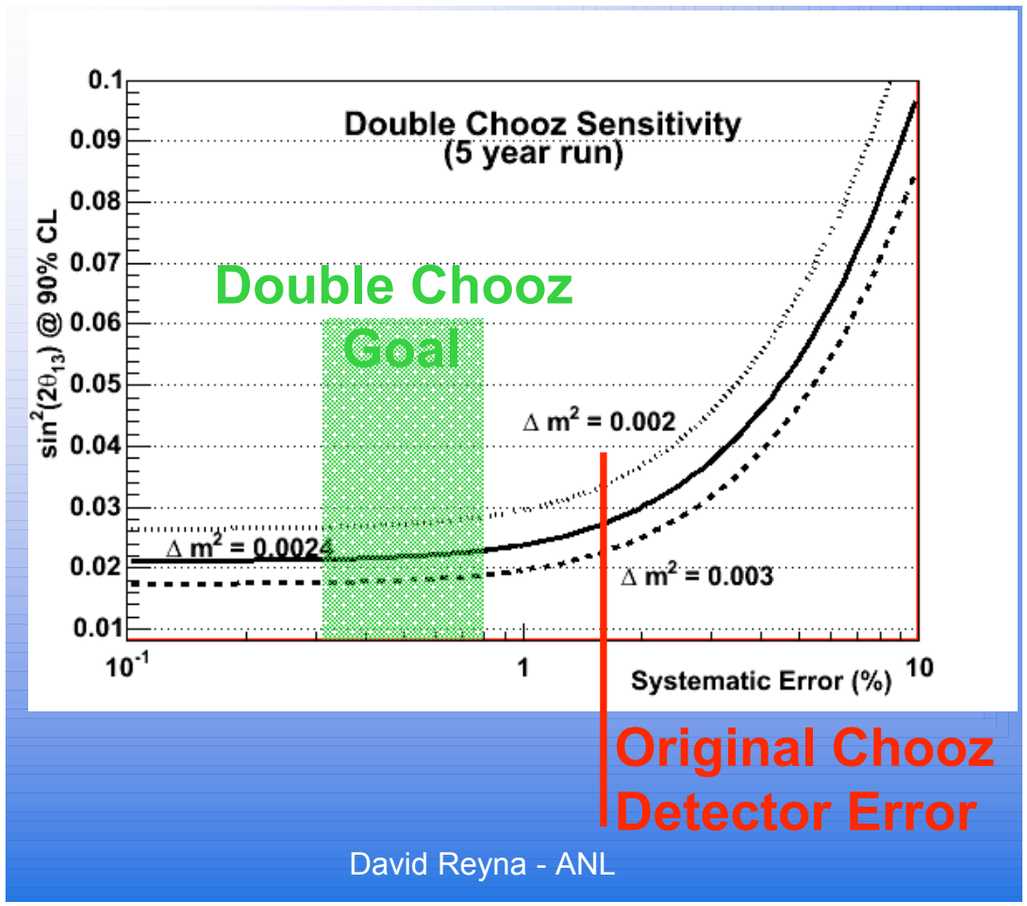
(13) Parameters: Chooz, SK, K2K

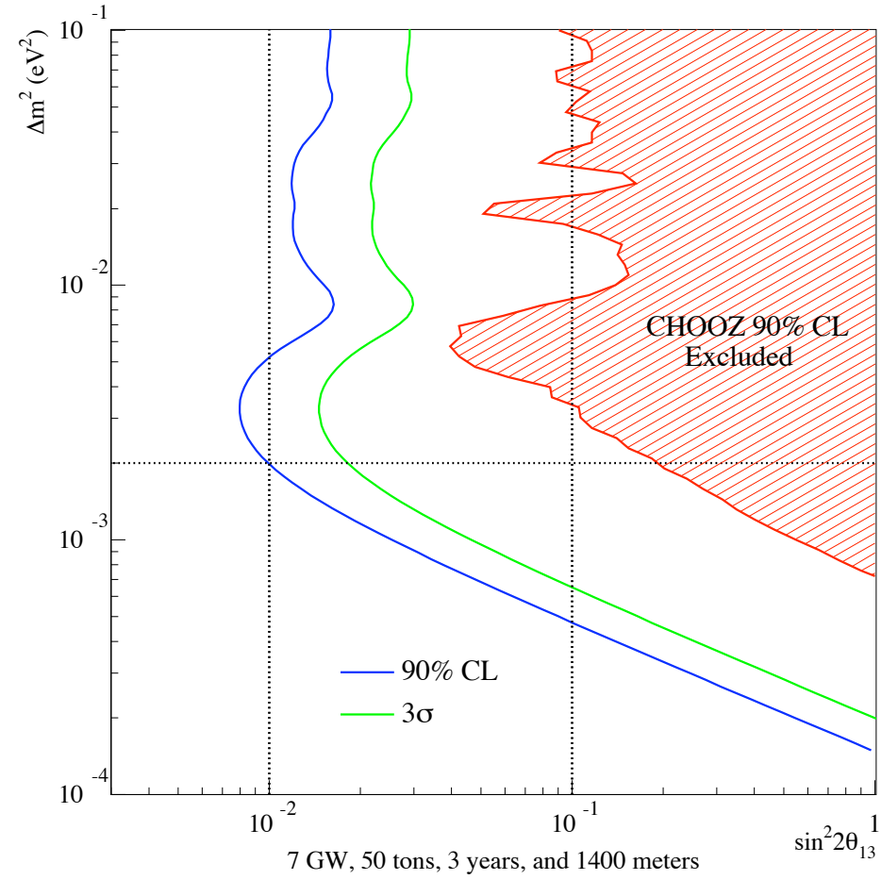
$$\sin^2\theta_{13} < 0.03 - 0.05$$

limit $|\delta m^2_{32}|$ dependent

$$0 \leq \delta_{CP} < 2\pi$$

Unknown!





J. Link, Columbia

- Pure measurement of $\sin^2\theta_{13}$ — no contamination from $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy.

With Off-axis measurements of $\nu_\mu \to \nu_e$:

- of $\sin^2\theta_{23}\sin^2\theta_{13}$ can help resolve $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy for $\sin^2 2\theta_{23} \neq 1$.
- Help resolve hierarchy and $\sin\delta \neq 0$, maybe.

Vacuum LBL:

$$\nu_\mu \longrightarrow \nu_e$$

Vacuum LBL: $\nu_\mu \longrightarrow \nu_e$

$$P_{\mu\to e} \approx |\sqrt{P_{atm}} e^{-i(\Delta_{32}\pm\delta)} + \sqrt{P_{sol}}|^2$$

CP violation !!!

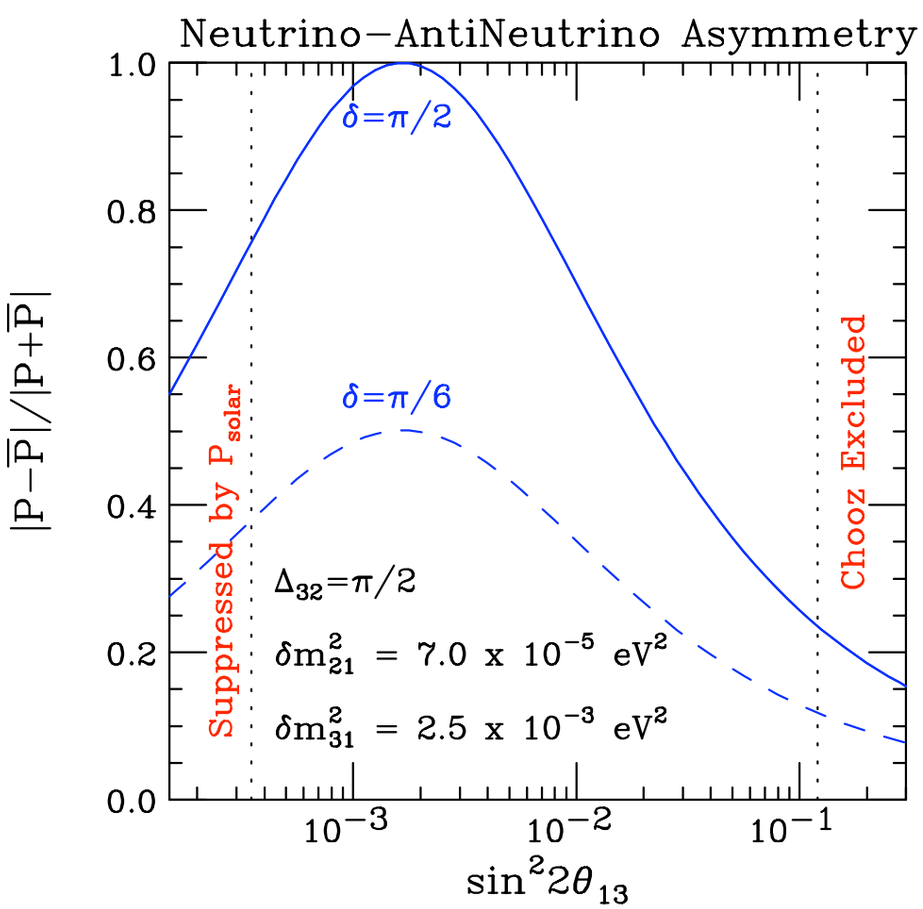
$$\Delta_{ij} = |\delta m^2_{ij}| L/4E$$

where $\sqrt{P_{atm}} = \sin\theta_{23}\sin 2\theta_{13}\ \sin\Delta_{31}$

and $\sqrt{P_{sol}} = \cos\theta_{23}\sin 2\theta_{12}\ \sin\Delta_{21}$

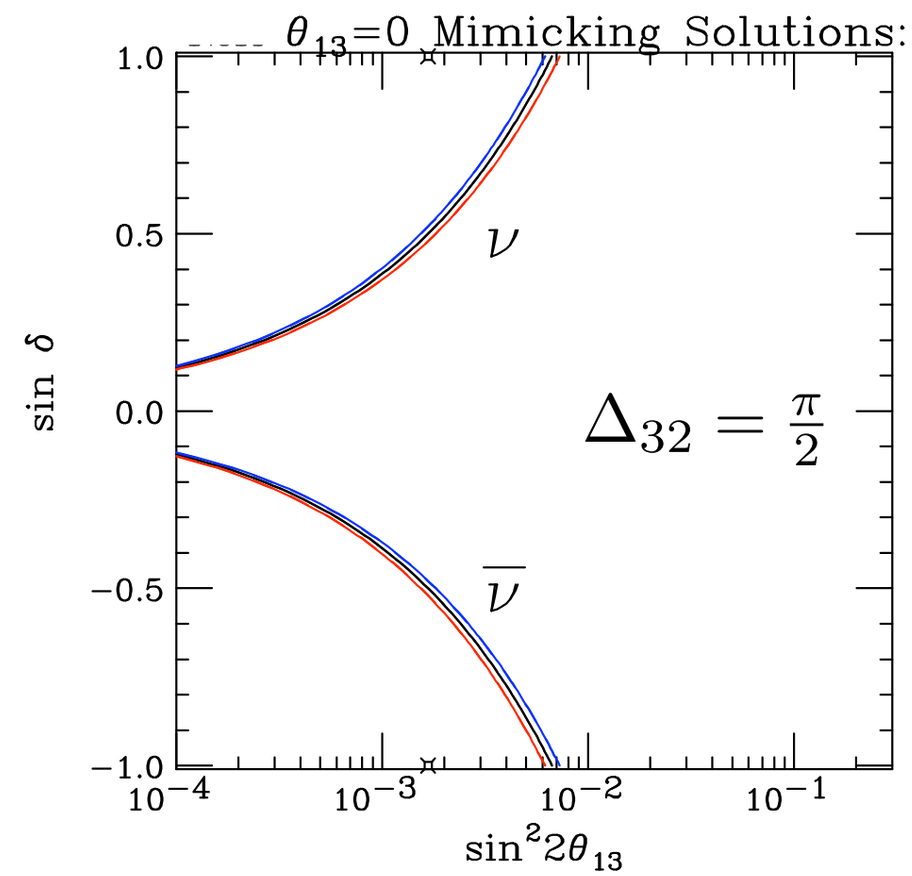
$$P_{\mu\to e} \approx \left| \sqrt{P_{atm}} e^{-i(\Delta_{32}\pm\delta)} + \sqrt{P_{sol}} \right|^2$$

Asymmetry Peaks:



$$\sqrt{P_{atm}} = \sqrt{P_{sol}}$$

Zero Mimicking Solutions:



$$\sqrt{P_{atm}} = -2\sqrt{P_{sol}}\cos(\Delta_{32} \pm \delta)$$

$\nu_\mu \to \nu_e$ with MATTER

$$P_{\mu\to e} \approx |\sqrt{P_{atm}} e^{-i(\Delta_{32}\pm\delta)} + \sqrt{P_{sol}}\,|^2$$

where $\sqrt{P_{atm}} = \sin\theta_{23}\sin 2\theta_{13}\,\frac{\sin(\Delta_{31}\mp aL)}{(\Delta_{31}\mp aL)}\,\Delta_{31}$

in vac $\sin\Delta_{31}$

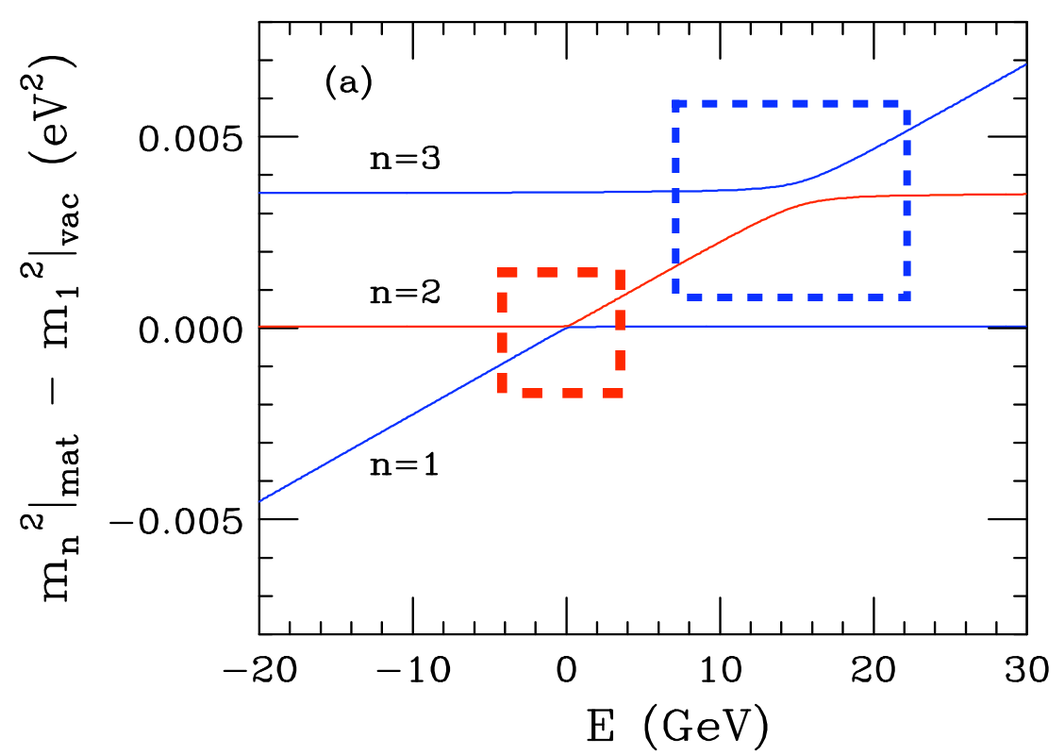
and $\sqrt{P_{sol}} = \cos\theta_{23}\sin 2\theta_{12}\,\frac{\sin(aL)}{(aL)}\,\Delta_{21}$

in vac $\sin\Delta_{21}$

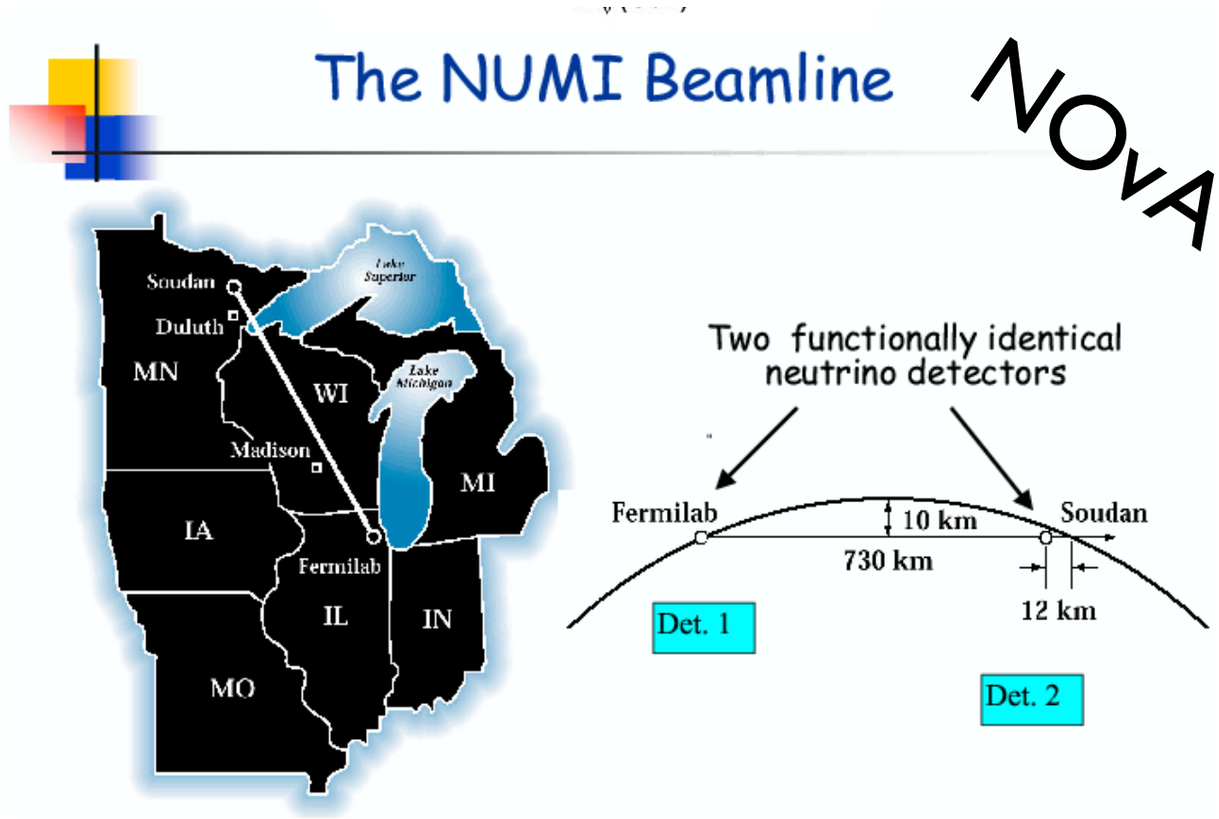
$$a = G_F N_e/\sqrt{2} = (4000\ km)^{-1},$$

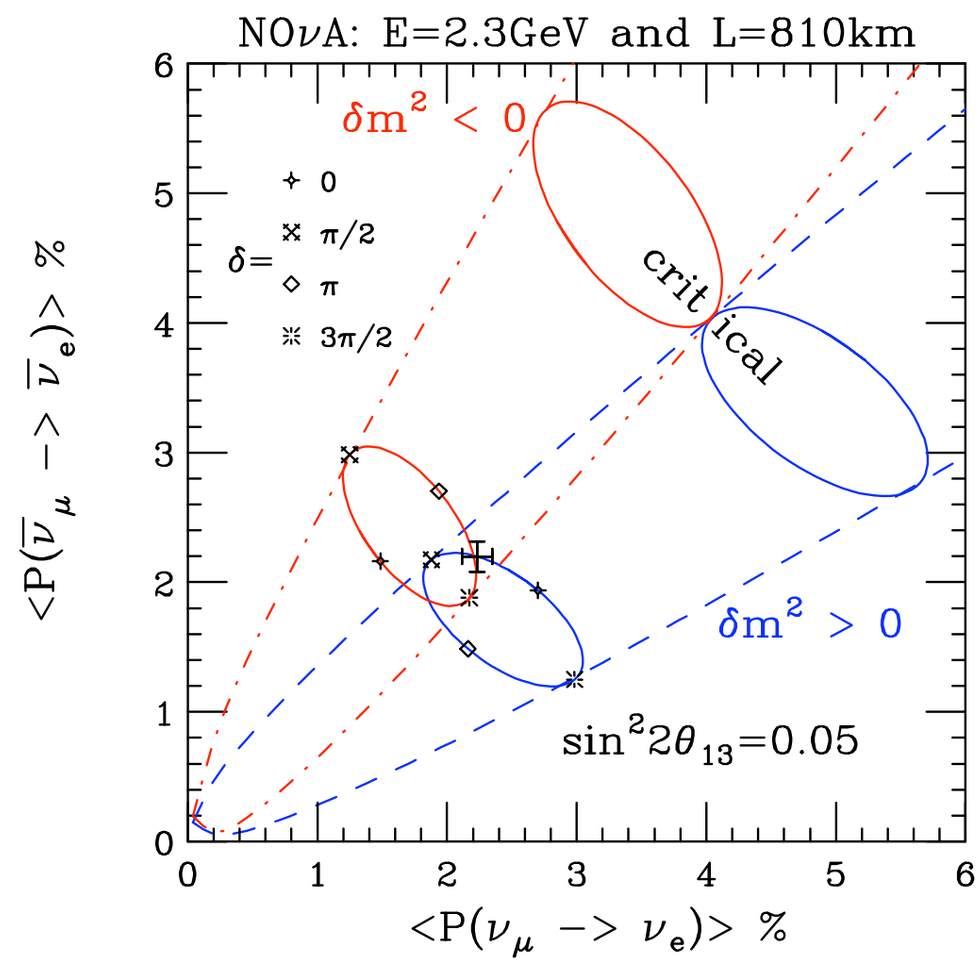
$$\pm = sign(\delta m^2_{31}) \qquad \Delta_{ij} = |\delta m^2_{ij}|L/4E$$

$\{\delta m^2 \sin 2\theta\}$ is invariant



Neutrino v Anti-Neutrino One Expt.





in the overlap region

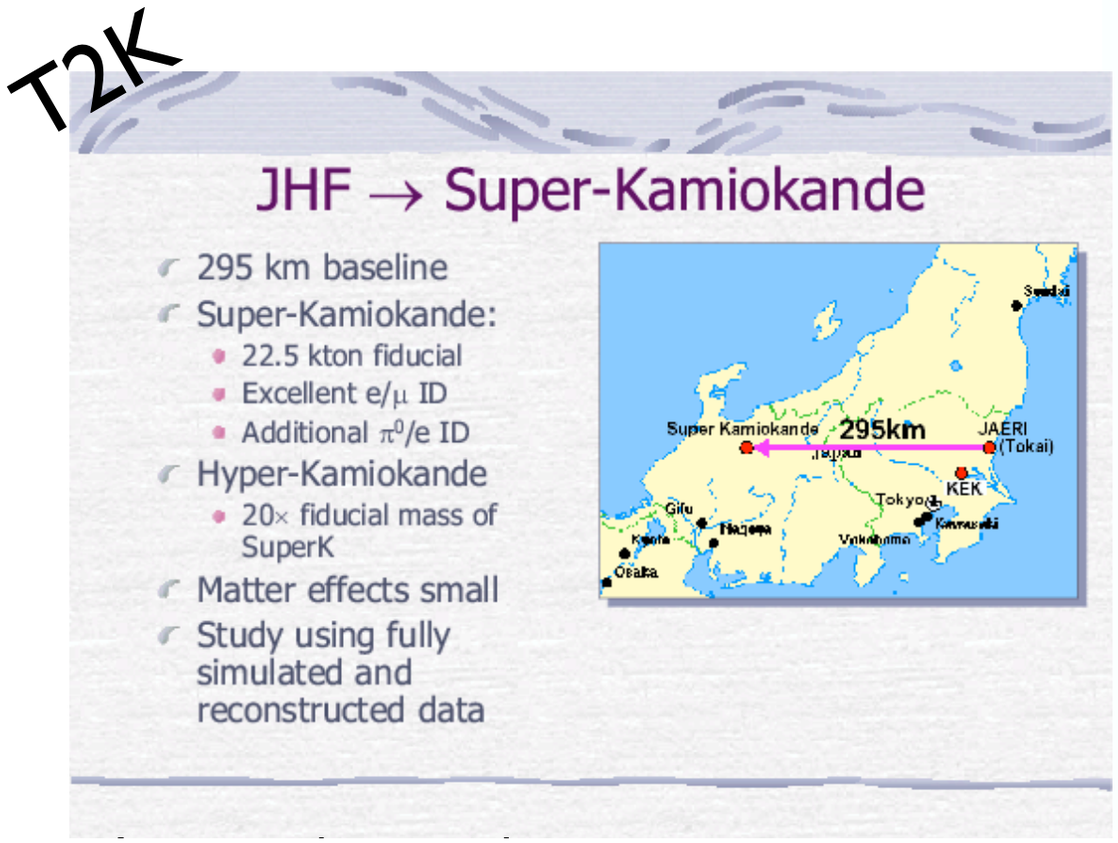
$$\langle \sin\delta \rangle_+ - \langle \sin\delta \rangle_- \quad = \quad 2\langle\theta\rangle/\theta_{crit} \quad \approx \quad 1.4\sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

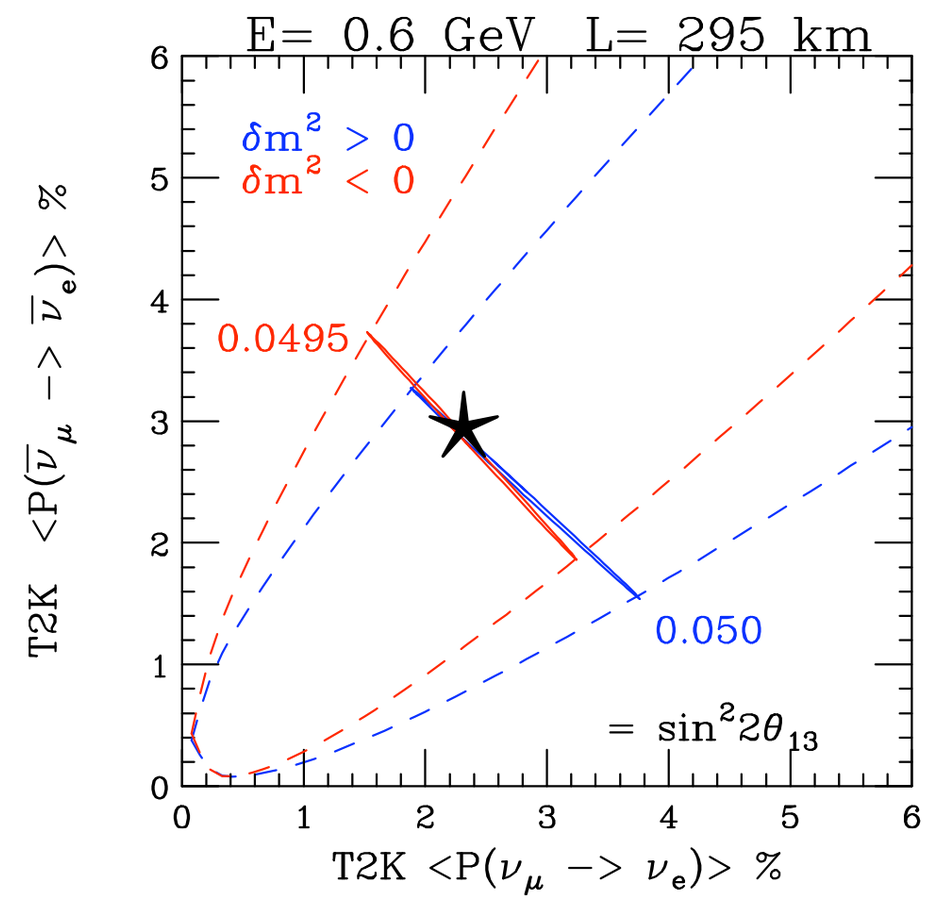
exact along diagonal --- approximately true throughout the overlap region!!!

$$\theta_{crit} = \frac{\pi^2}{8}\,\frac{\sin 2\theta_{12}}{\tan\theta_{23}}\,\frac{\delta m^2_{21}}{\delta m^2_{31}}\left(\frac{4\Delta^2/\pi^2}{1-\Delta\cot\Delta}\right)/(aL) \sim 1/6$$

i.e. $\sin^2 2\theta_{crit} = 0.10$

O. Mena + SP
hep-ph/0408070



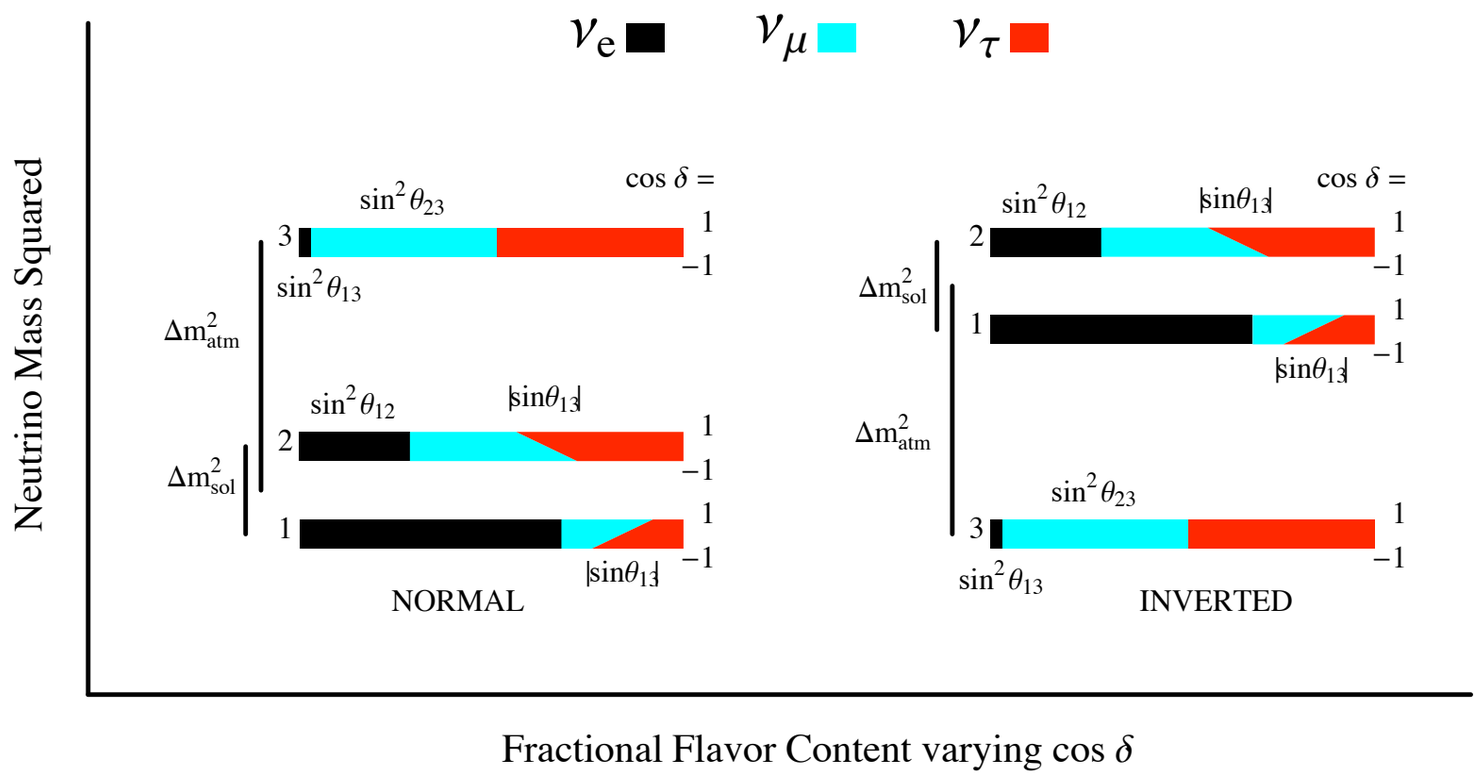


$$\langle \sin\delta \rangle_+ - \langle \sin\delta \rangle_- \;=\; 2\langle\theta\rangle/\theta_{crit} \;\approx\; 0.47\sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

(ρL) for NOvA <u>three</u> times larger than (ρL) than T2K.

- ν_e fraction of ν_3: – $\sin^2\theta_{13}$
- mass hierarchy: – sign of δm^2_{31}
- CP violation: – $\sin\delta \neq 0$

^ observable



CPT: $\delta \Leftrightarrow -\delta$ Invariant!

- Absolute Neutrino Mass
- Majorana ν Dirac — Seesaw
- Leptogenesis