Novel ways to determine the Neutrino Mass Hierarchy

Stephen Parke
Fermilab

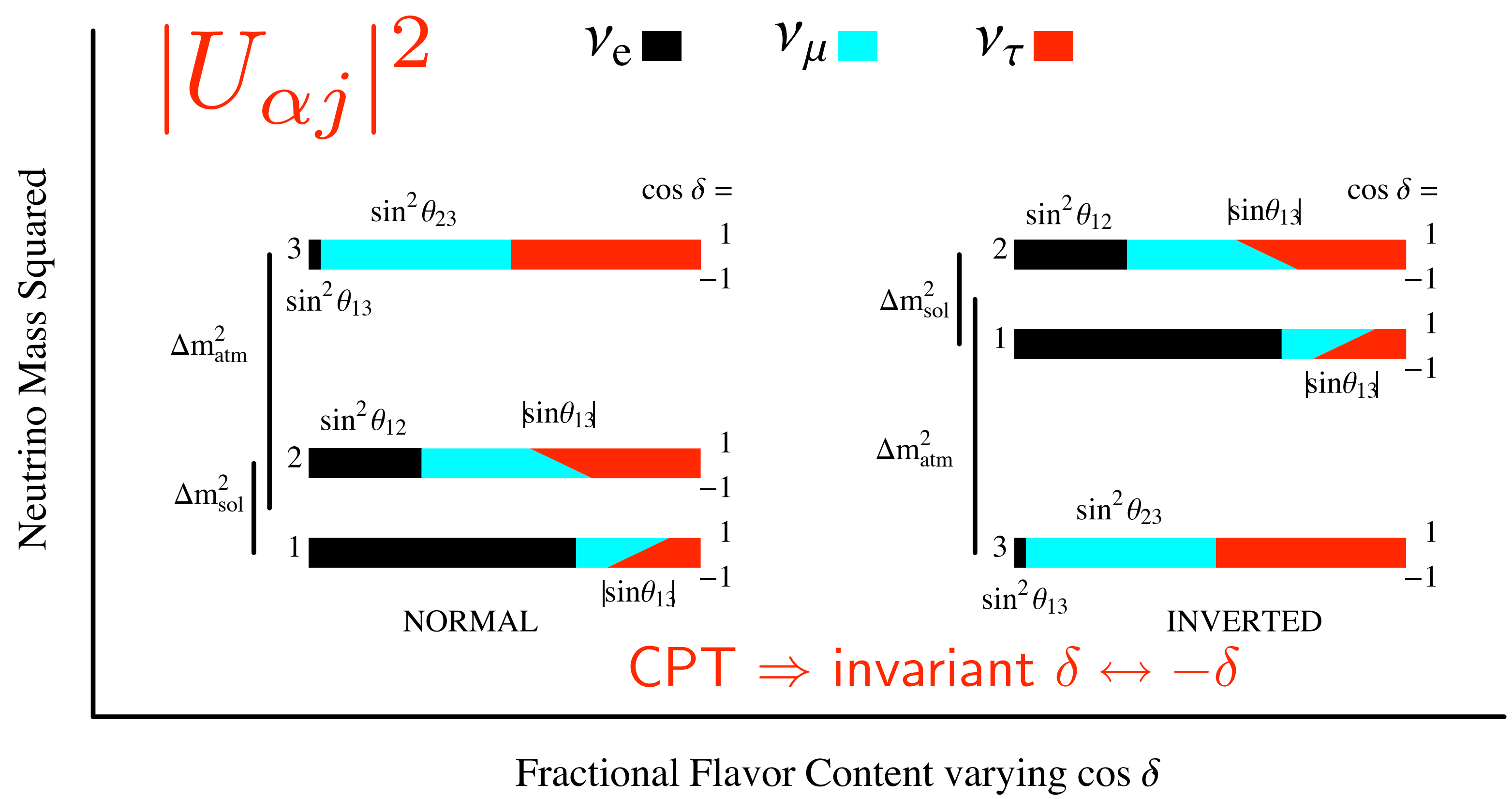
- Quick review
- CPT conjugate pairs

  Jansson, Mena, SP & Saoulidou: **Phys. Rev. D78:053002,2008**
- Mossbauer Neutrinos

  Minakata, Nunokawa, SP & Zukanovich Funchal: **Phys. Rev. D76:053004**

Atmospheric Neutrino Mass Hierarchy:

Mena + SP
hep-ph/0312131



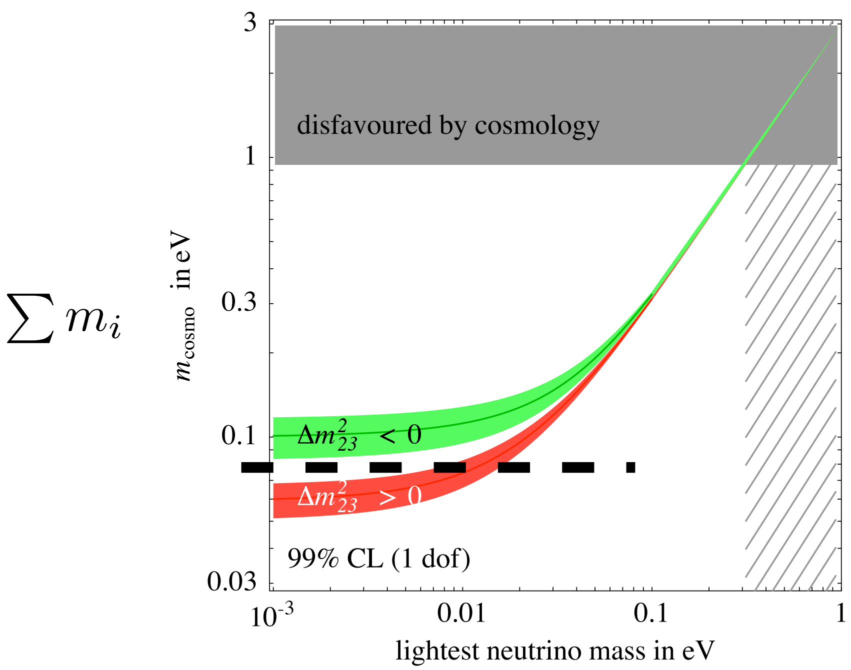
defn: $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$

Solar neutrino mass hierarchy (mass ordering of nu1 & nu2) was determined by SNO !

$$\{\frac{CC}{NC} = 0.34 < 0.5\}$$

Atmospheric neutrino mass hierarchy ????
(mass ordering of (nu1,nu2) and nu3)

Cosmology:



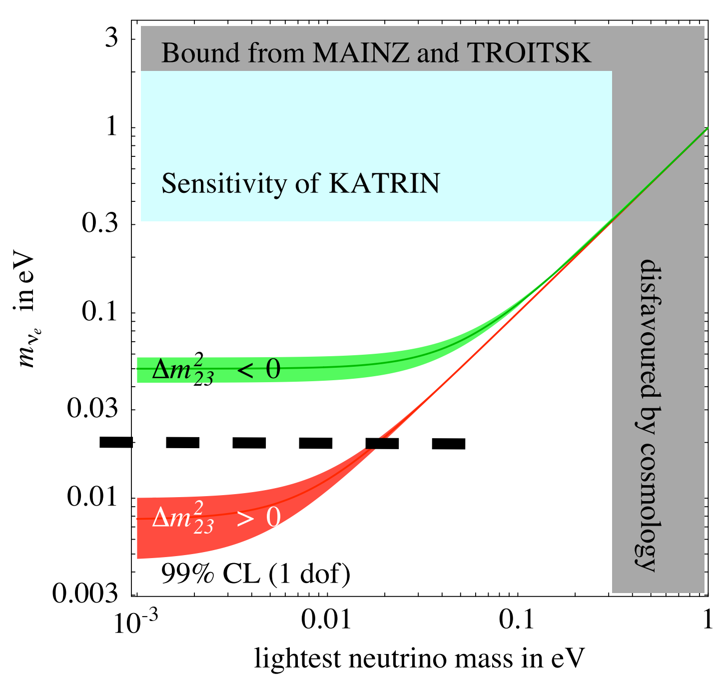
For NH need to show

$\sum m_i < 2\sqrt{\delta m^2_{atm}} \approx 100$ meV at say 90 or 95 % C.L.

However $\sum m_i \geq \sqrt{\delta m^2_{atm}} + \sqrt{\delta m^2_{sol}} \approx 59$ meV

systematics?

Tritium Beta Decay:

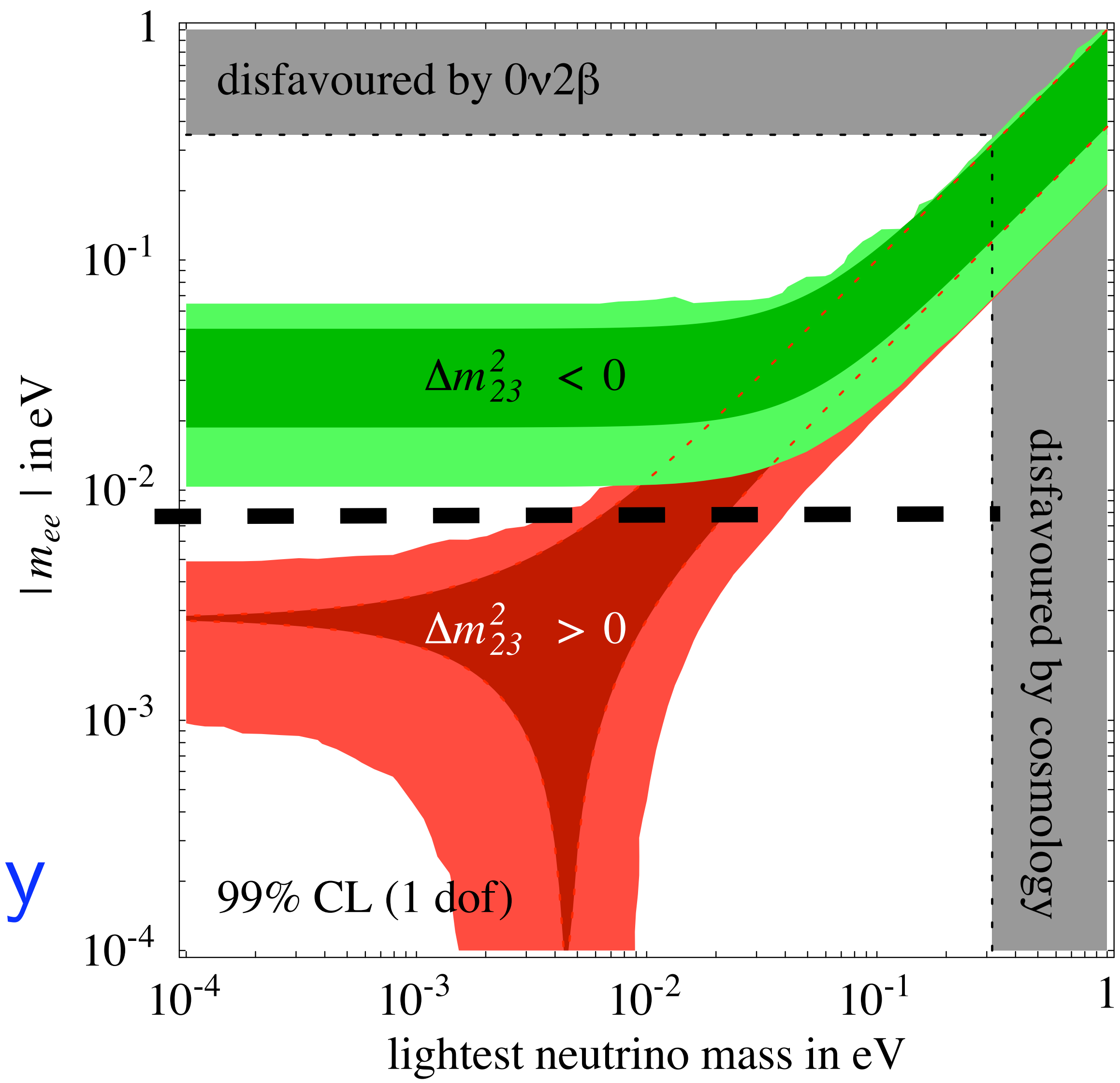


Similarly, if Tritium decay exp. (Hyper-Katrin) could exclude $m_{\nu_e} > \frac{1}{30}\ eV$, then Normal Hierarchy.

Neutrinoless double beta decay

$$< m >_{\beta\beta} \equiv \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right|$$
$$= \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\beta} + m_3 s_{13}^2 e^{2i(\gamma-\delta)} \right|$$

dividing point $m_{\beta\beta} \approx 10meV$ $\Rightarrow\Rightarrow$

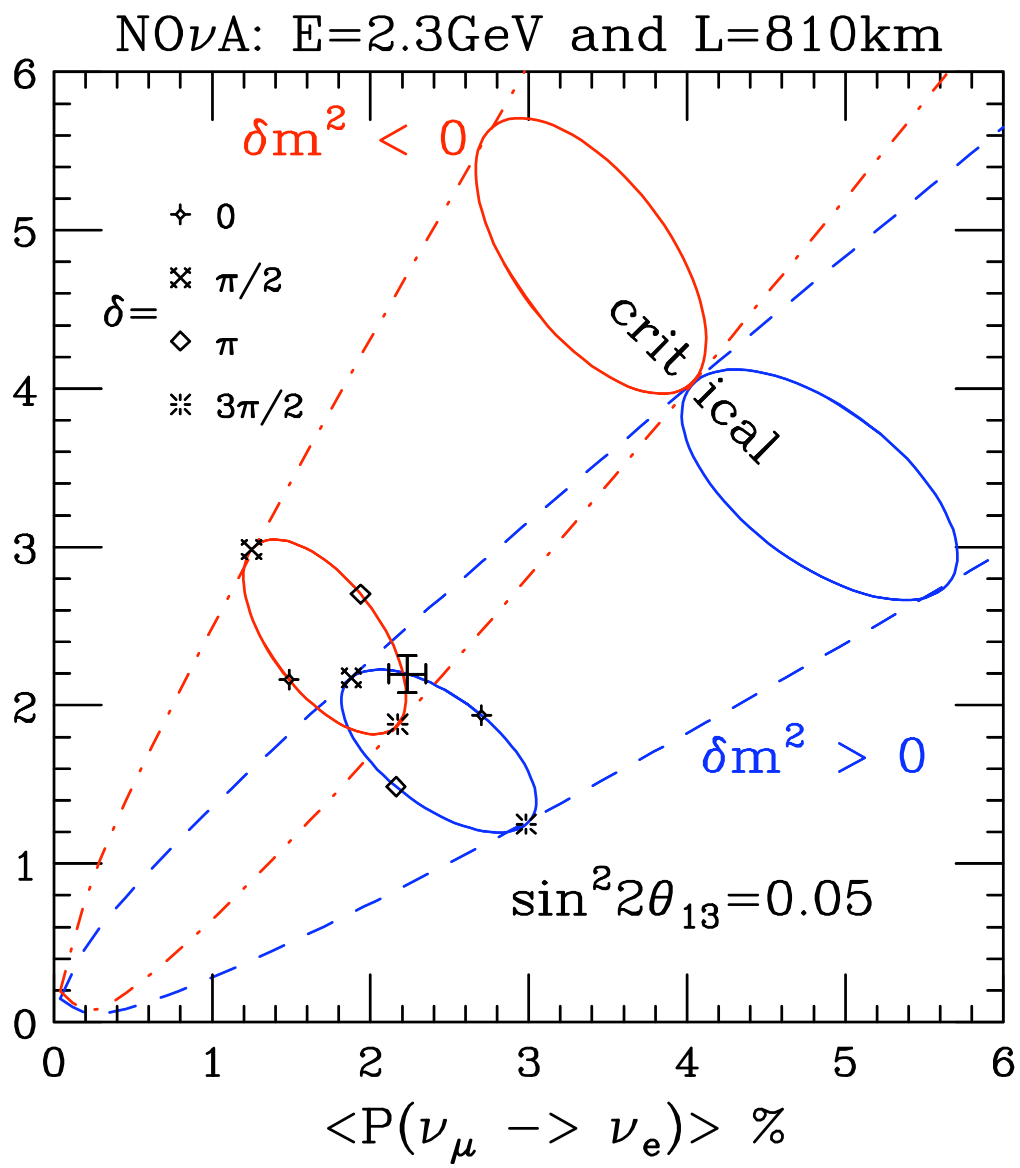


Signal below $\sim$ 10 meV would imply Majorana and Normal Hierarchy!

these 3 figs from Strummia and Vissani hep-ph/0503246

Neutrino v AntiNeutrino Channels in LBL:

NOvA:



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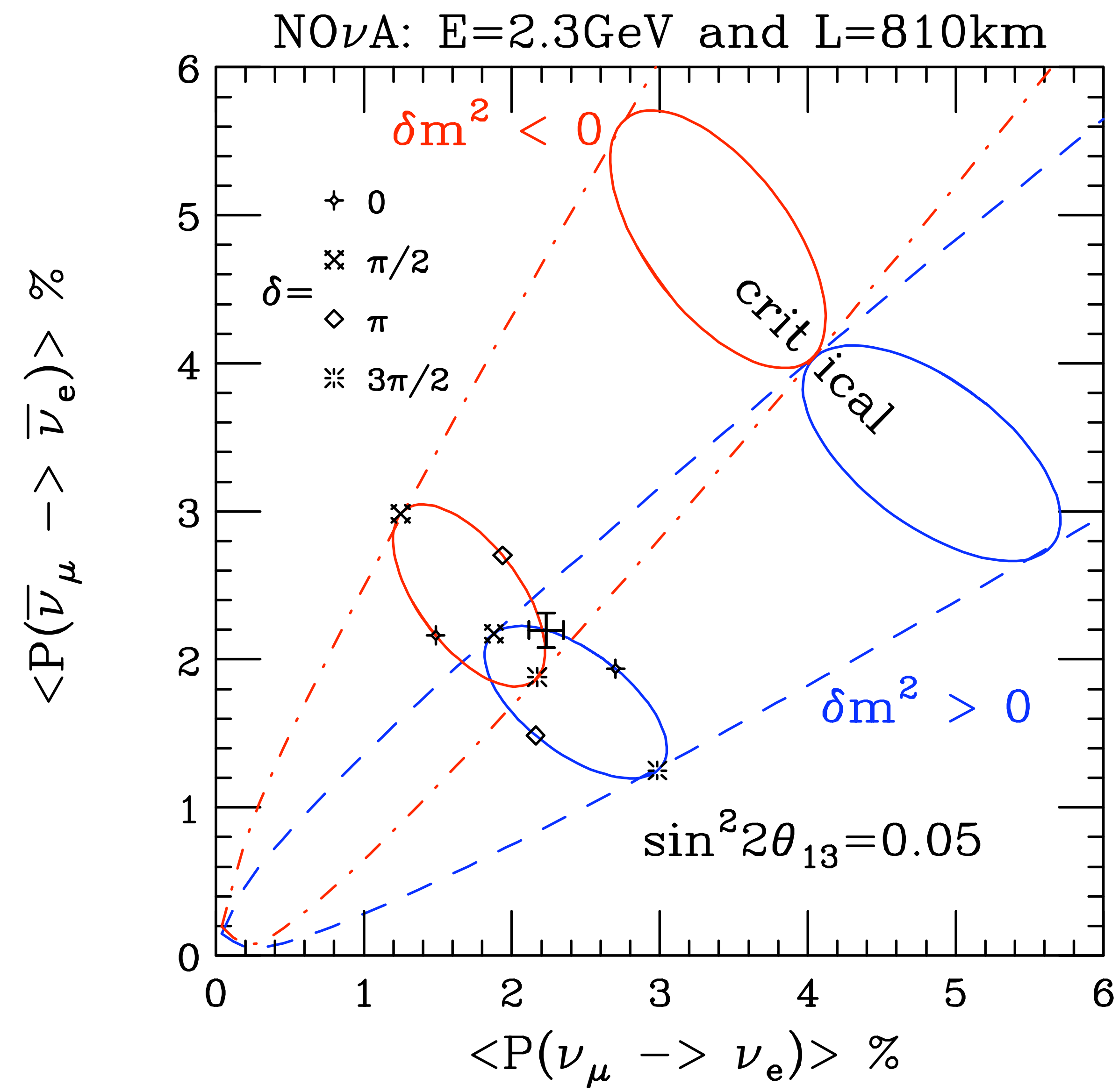

$$\theta_{crit} = \frac{\pi^2}{8}\,\frac{\sin 2\theta_{12}}{\tan\theta_{23}}\,\frac{\delta m^2_{21}}{\delta m^2_{31}}\left(\frac{4\Delta^2/\pi^2}{1-\Delta\cot\Delta}\right)/(aL) \sim 1/6$$

i.e. $\sin^2 2\theta_{crit} = 0.10$

in the overlap region

$$\langle\sin\delta\rangle_+ - \langle\sin\delta\rangle_- \;=\; 2\langle\theta\rangle/\theta_{crit} \;\approx\; 1.4\sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

O. Mena + SP
hep-ph/0408070

Neutrino v AntiNeutrino Channels in LBL:

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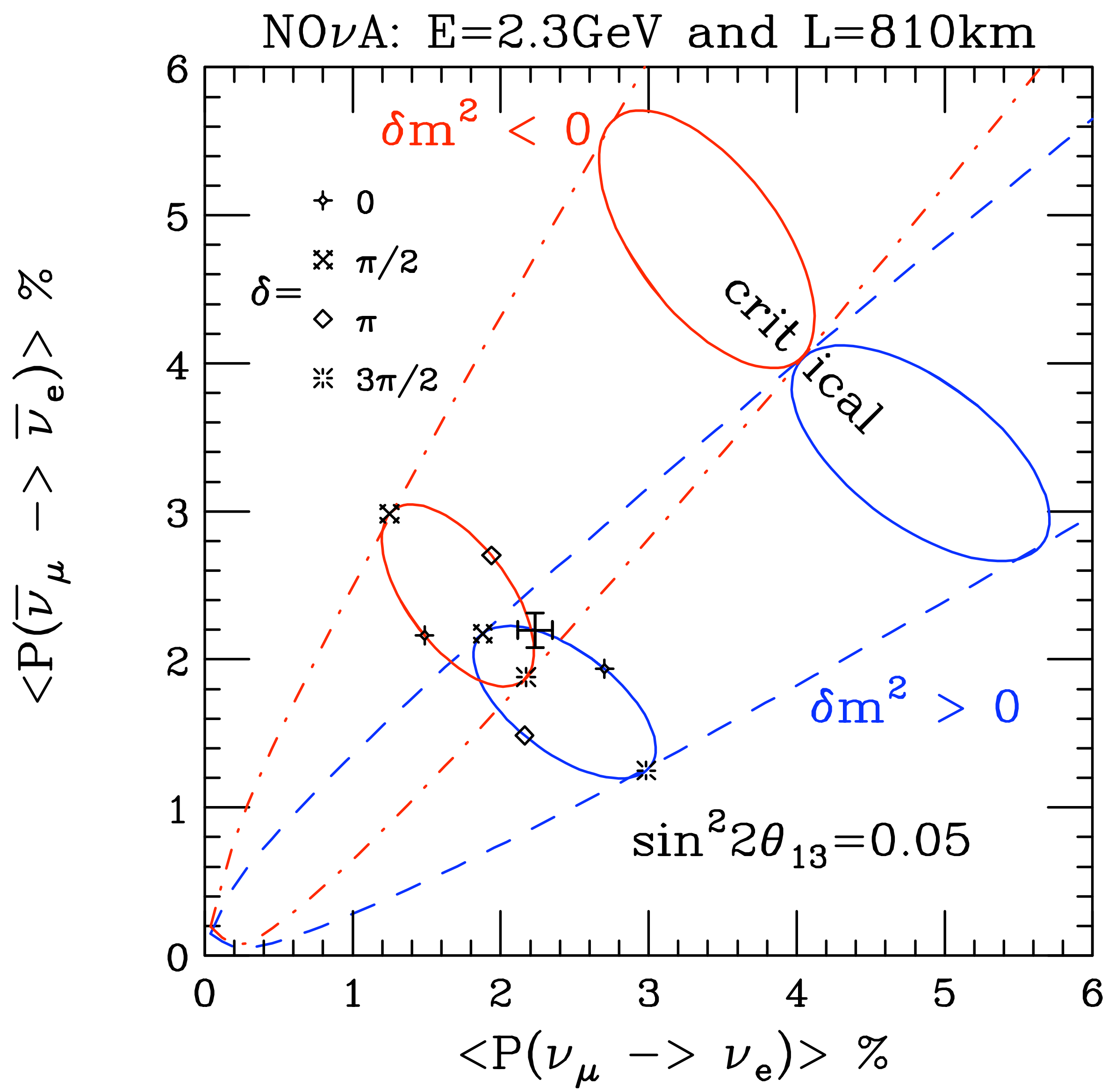
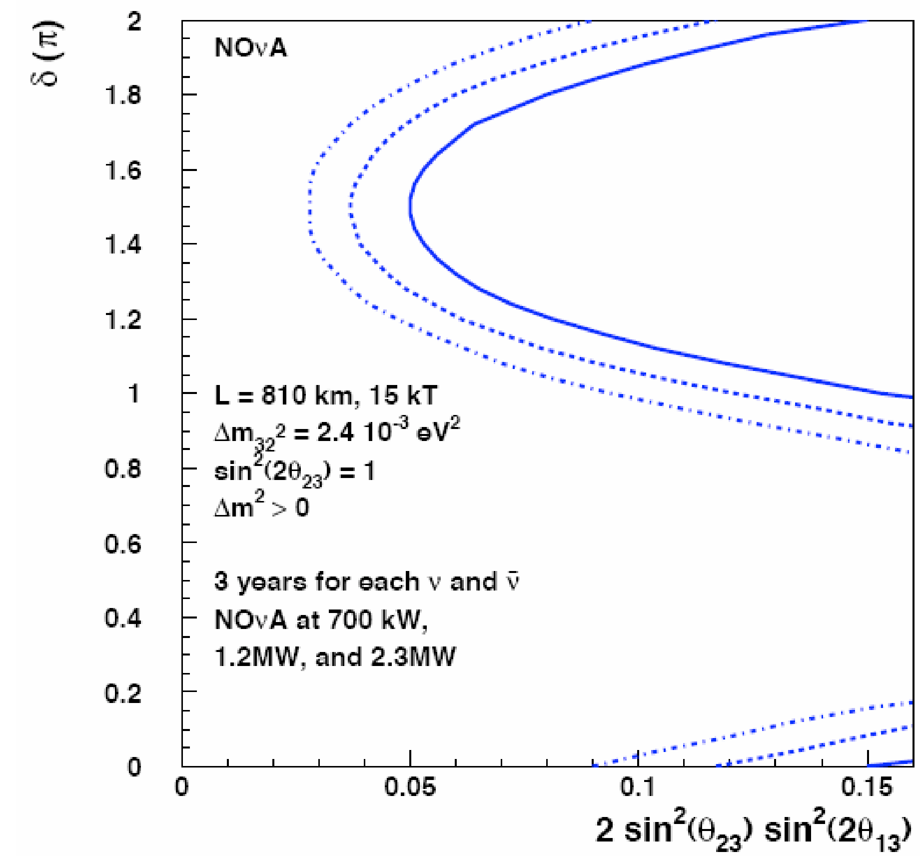


95% CL Resolution of the Mass Ordering
NOvA Alone



Normal Ordering

for Inverted Hierarchy $\delta \to \pi - \delta$

$$\theta_{crit} = \frac{\pi^2}{8}\,\frac{\sin 2\theta_{12}}{\tan\theta_{23}}\,\frac{\delta m^2_{21}}{\delta m^2_{31}}\left(\frac{4\Delta^2/\pi^2}{1-\Delta\cot\Delta}\right)/(aL) \sim 1/6$$

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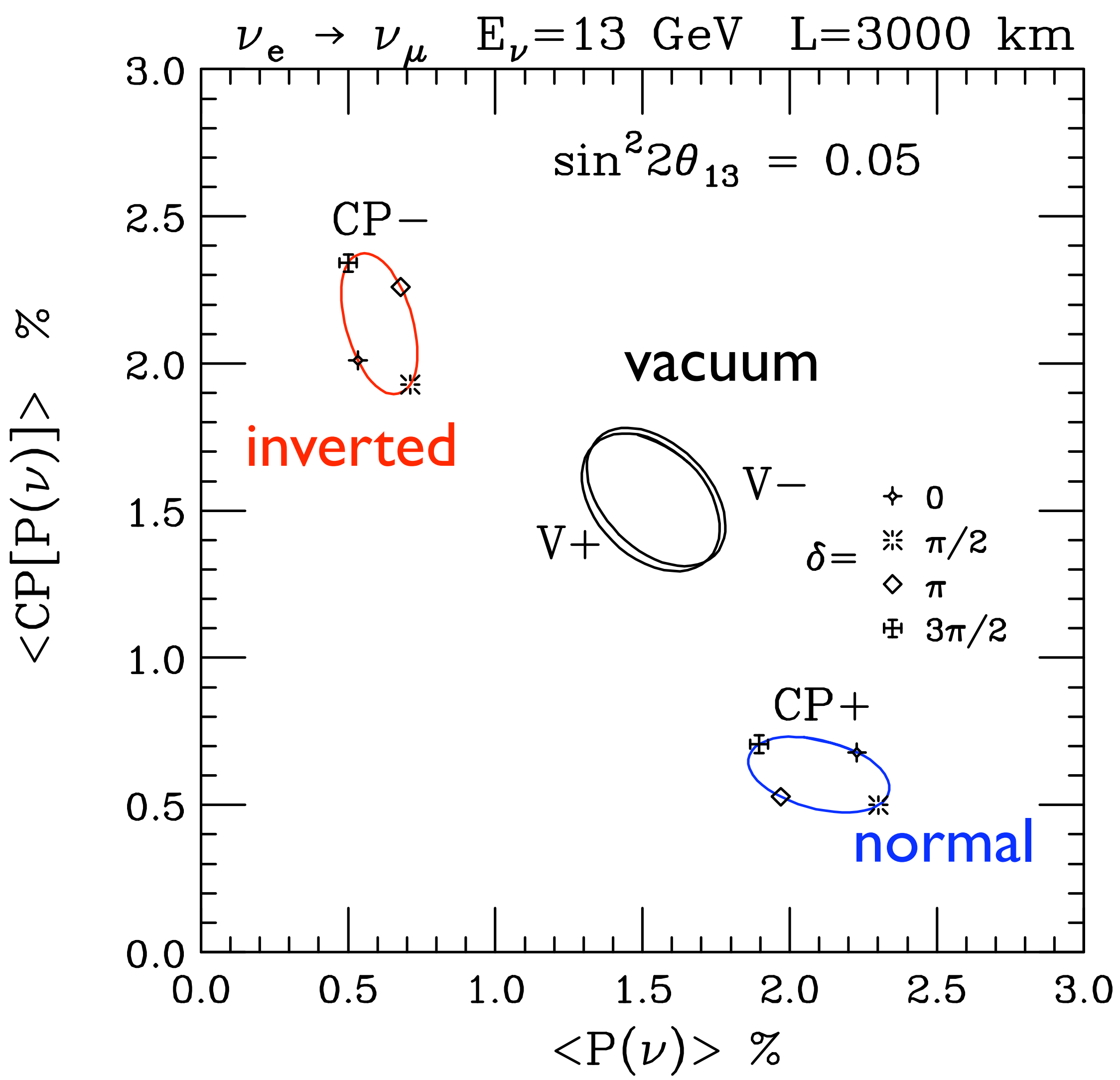
Neutrino Factory:

$\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$



$$\nu_e \leftrightarrow \nu_\mu$$

Suppression $\geq$ Enhancement

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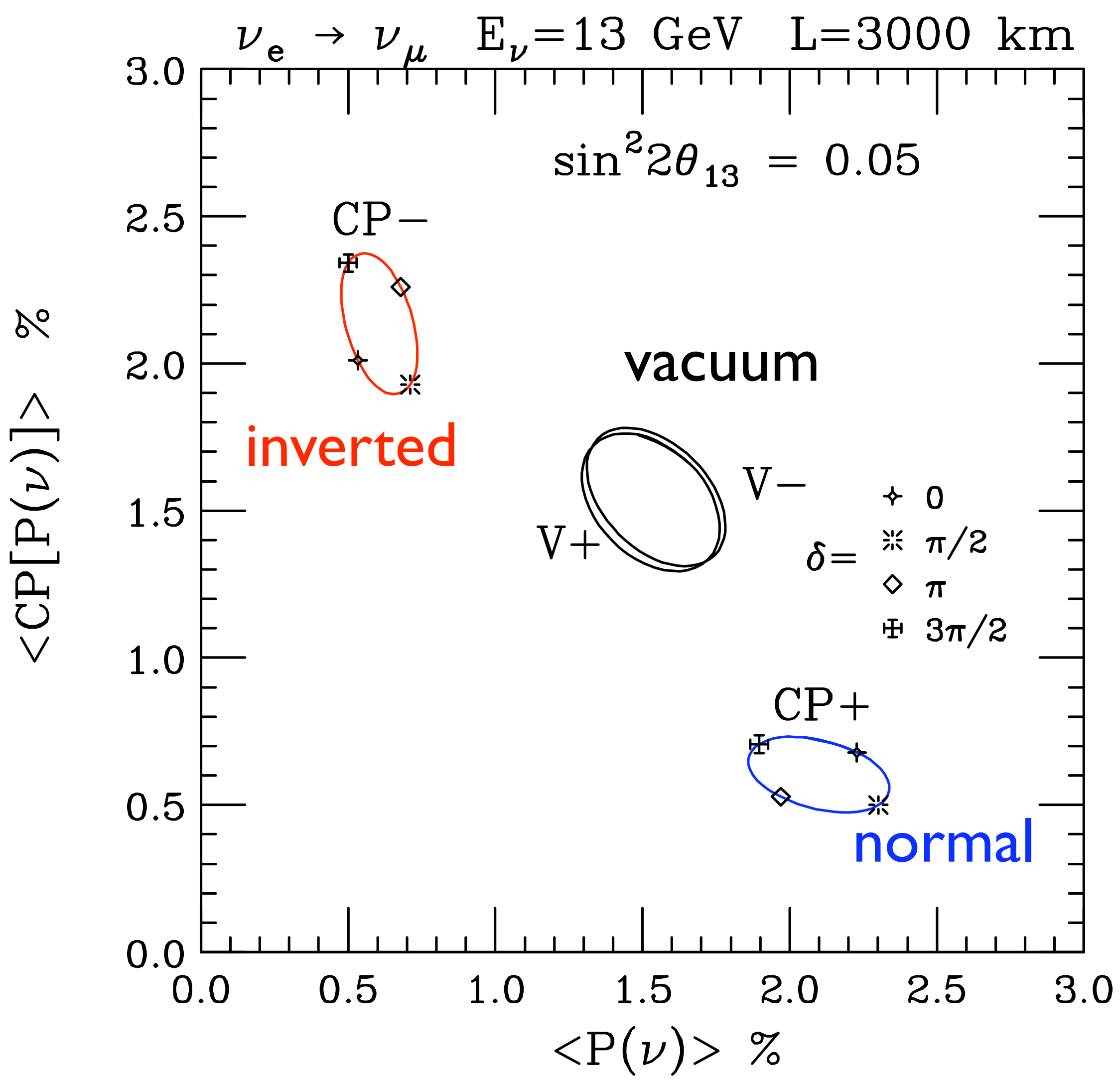
$$\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$$



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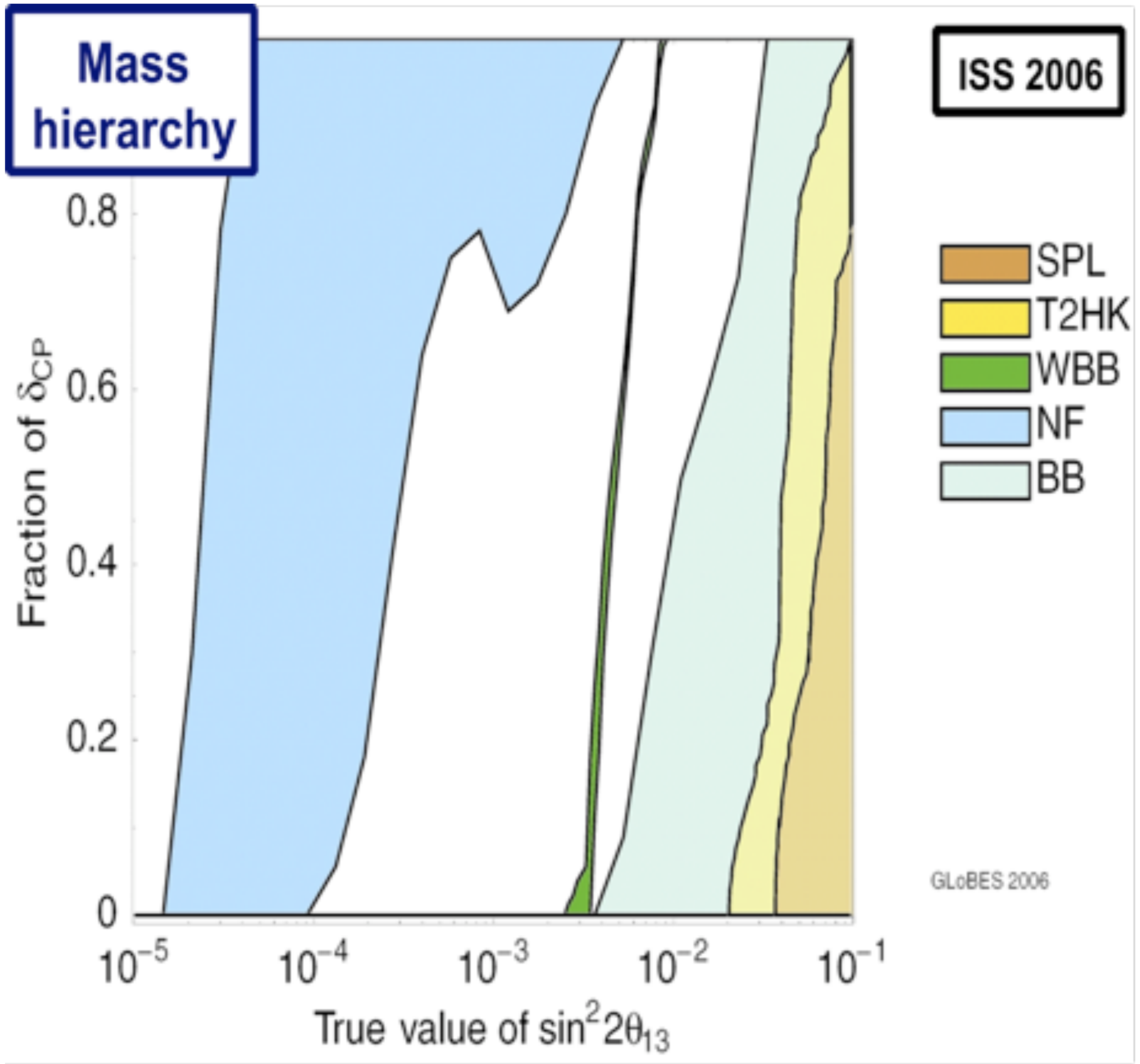
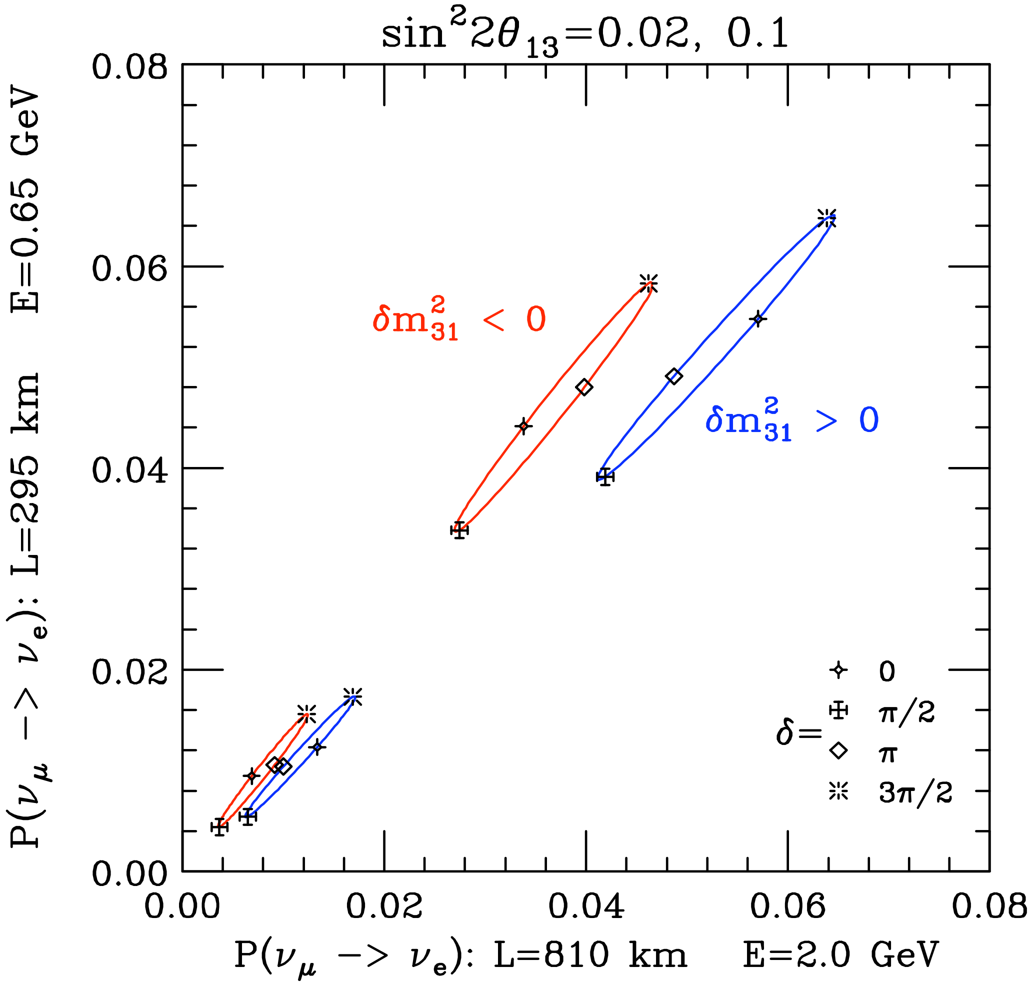
Suppression $\geq$ Enhancement

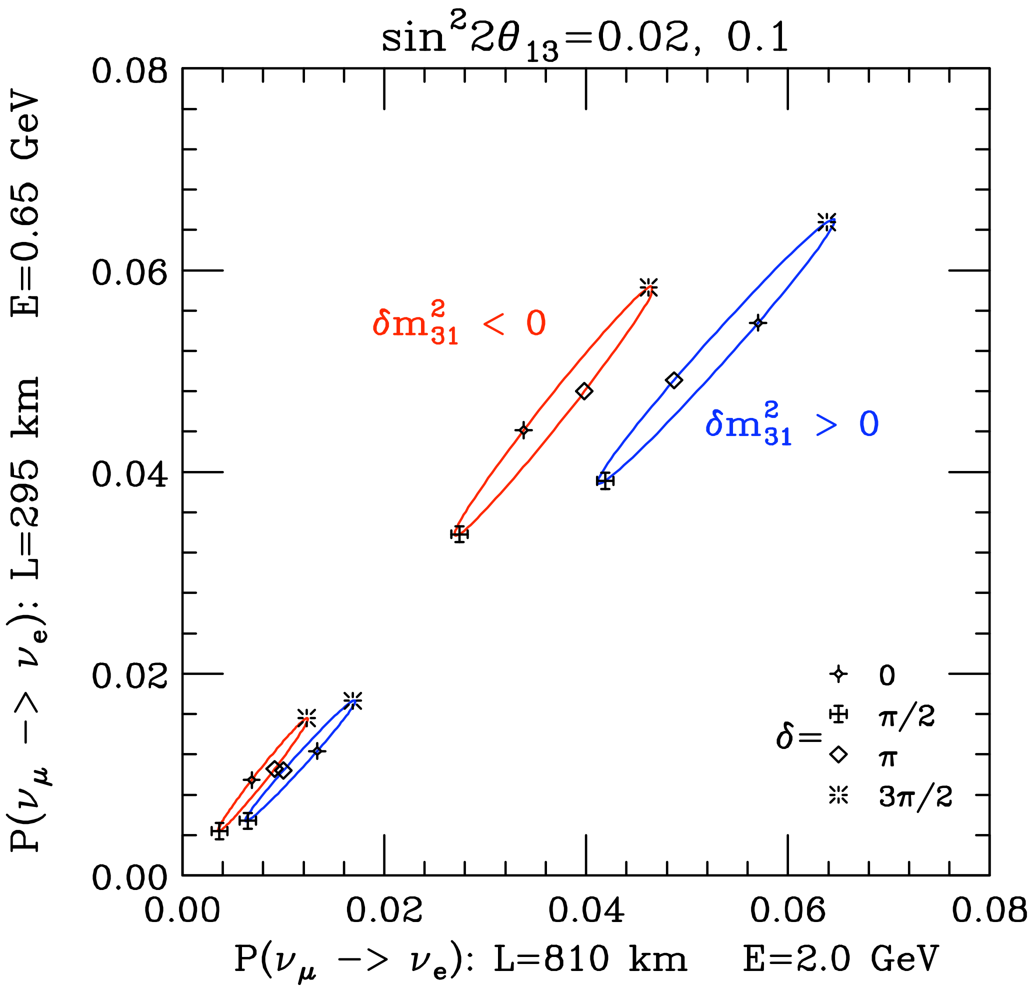


Two Neutrino Experiments different L same E/L :

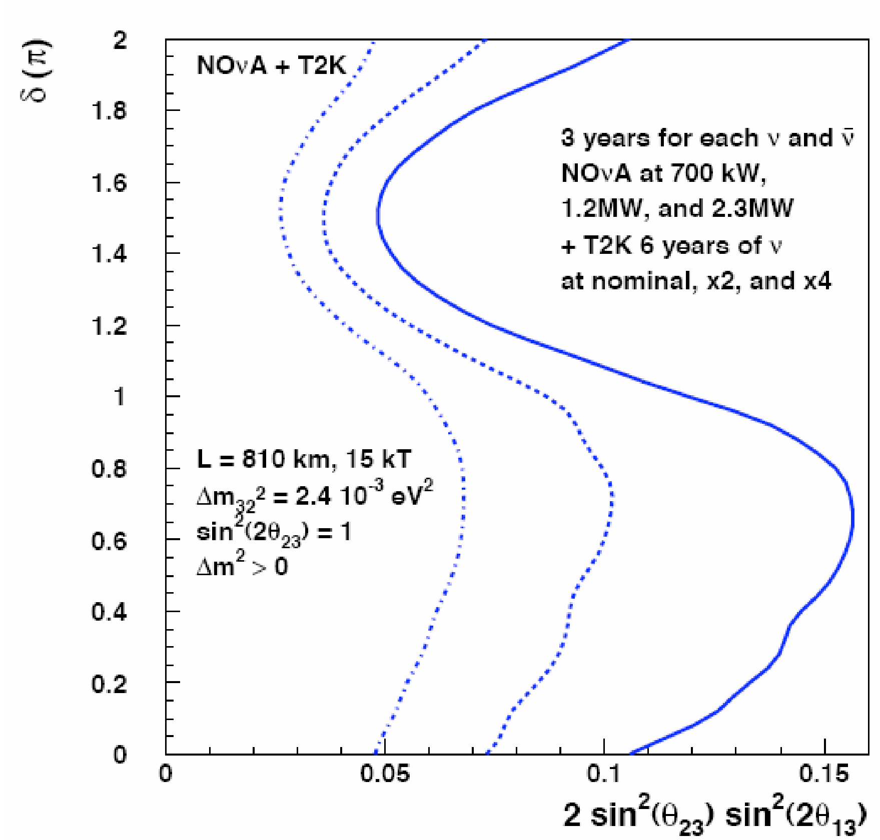


Neutrino (NOvA)
v
Neutrino (T2K)

Two Neutrino Experiments different L same E/L :



95% CL Resolution of the Mass Ordering
NOνA Plus T2K



Normal Ordering

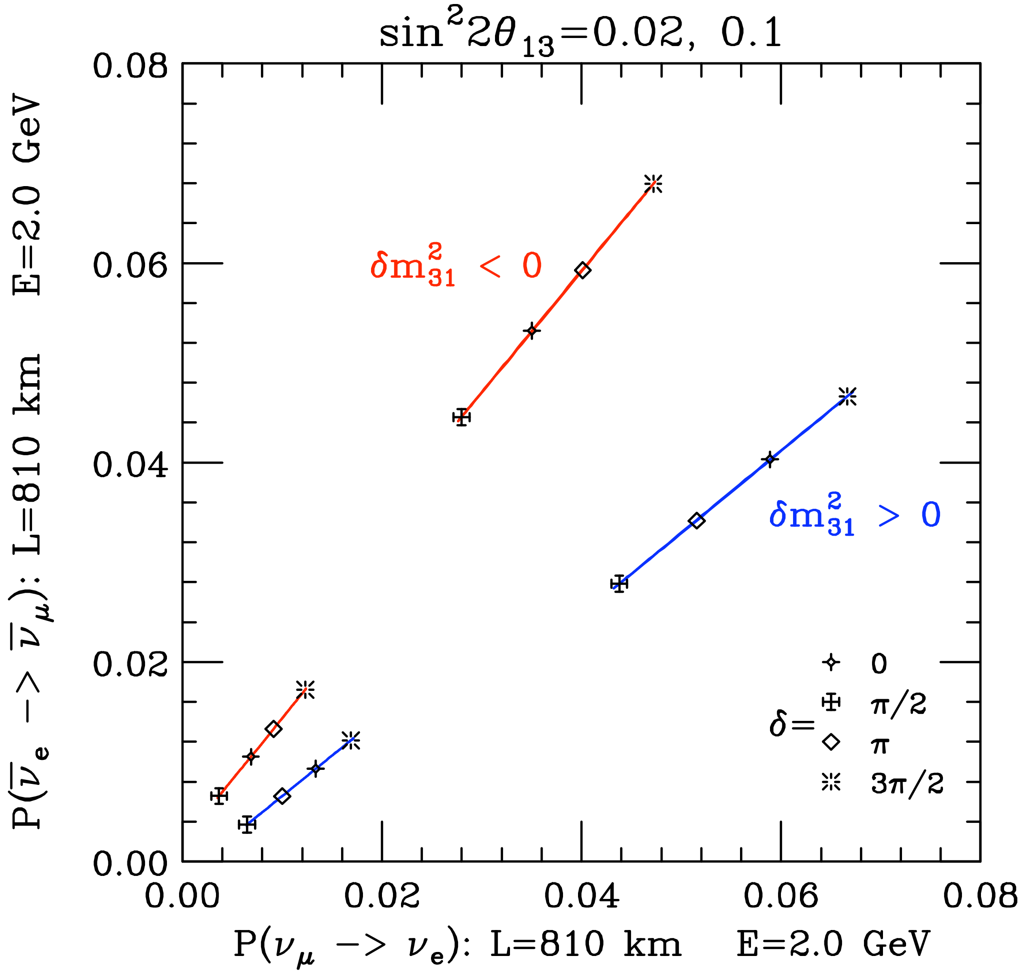
for Inverted Hierarchy $\delta \to \pi - \delta$

Neutrino (NOvA)
v
Neutrino (T2K)

- CPT conjugate pairs:

Jansson, Mena, SP & Saoulidou: **Phys. Rev. D78:053002,2008**

Hierarchy via CPT conjugate pairs:



$$P(\nu_\mu \to \nu_e) \; > \; \overline{P}(\bar{\nu}_e \to \bar{\nu}_\mu) \quad \text{for NH}$$

and $$P(\nu_\mu \to \nu_e) \; < \; \overline{P}(\bar{\nu}_e \to \bar{\nu}_\mu) \quad \text{for IH}$$

at the same E/L

Neutrino (NOvA)
v
CPT Antineutrino (NOvA)

The full amplitude for $\nu_\mu \to \nu_e$ is $(\pm\sqrt{X_\pm}\theta\ e^{-i(\pm\Delta_{31}+\delta)} + \sqrt{P_\odot})$.

$$\sqrt{P_\odot} \ = \ \cos\theta_{23}\sin 2\theta_{12}\frac{\sin(aL)}{(aL)}\ \Delta_{21},$$

where $\Delta_{ij} = |\delta m^2_{ij}|L/4E$ and $a = G_F N_e/\sqrt{2} \approx (4000\ km)^{-1}$

$$\sqrt{X_\pm} \ = \ 2\sin\theta_{23}\frac{\sin(\pm\Delta_{31} - aL)}{(\pm\Delta_{31} - aL)}\ \Delta_{31},$$

$$P(\nu_\mu \to \nu_e) \ = \ X_\pm\theta^2 \pm 2\sqrt{X_\pm}\sqrt{P_\odot}\ \theta\cos(\pm\Delta_{31} + \delta) + P_\odot$$

$$\overline{P}(\bar\nu_e \to \bar\nu_\mu) \ = \ X_\mp\theta^2 \pm 2\sqrt{X_\mp}\sqrt{P_\odot}\ \theta\cos(\pm\Delta_{31} + \delta) + P_\odot.$$

In vacuum these two probabilities must be identical !!!

$$X_+ = X_- \equiv X_0$$

$$P(\nu_\mu \to \nu_e) - \overline{P}(\bar{\nu}_e \to \bar{\nu}_\mu) \;=\; \pm\theta\;(\sqrt{X_+} - \sqrt{X_-})\;\left[(\sqrt{X_+} + \sqrt{X_-})\theta \pm 2\sqrt{P_\odot}\cos(\pm\Delta_{13} + \delta)\right].$$

$$P(\nu_\mu \to \nu_e) \;>\; \overline{P}(\bar{\nu}_e \to \bar{\nu}_\mu) \quad \text{for NH}$$

$$P(\nu_\mu \to \nu_e) \;<\; \overline{P}(\bar{\nu}_e \to \bar{\nu}_\mu) \quad \text{for IH}$$

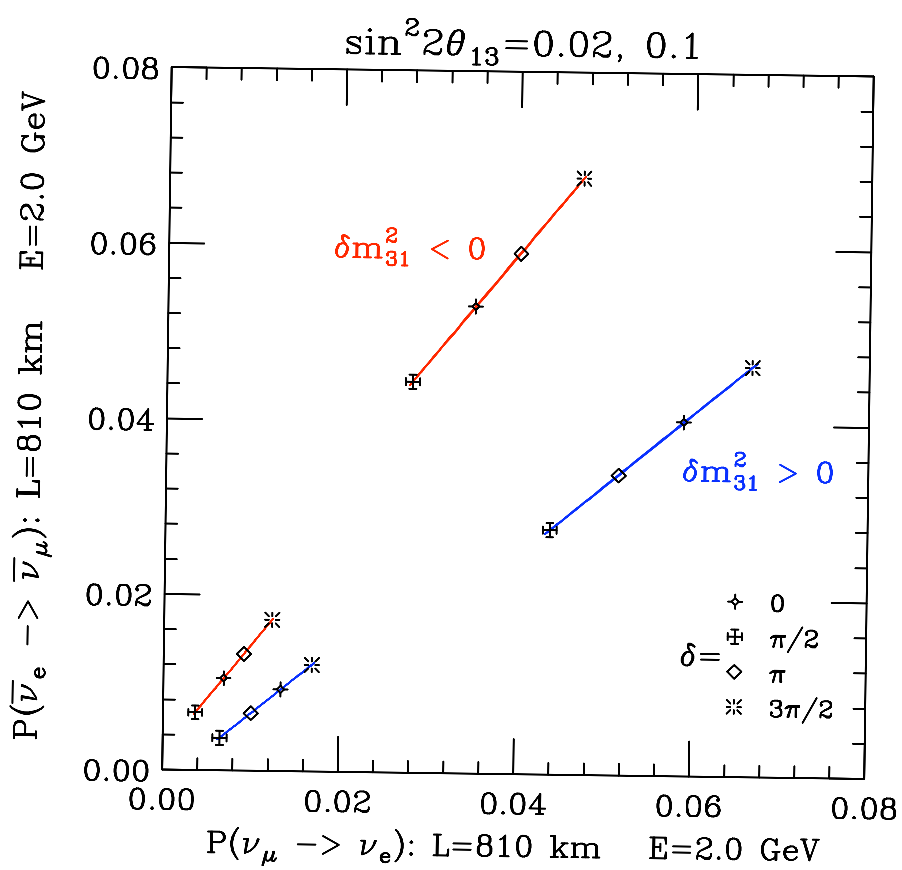
if

$$\sqrt{X_+} \;>\; \sqrt{X_-}$$

$$\theta > 2\sqrt{P_\odot}/(\sqrt{X_+} + \sqrt{X_-}) \approx \sqrt{P_\odot}/\sqrt{X_0},$$

$$\sin^2 2\theta_{13} > \frac{\sin^2 2\theta_{12}\Delta_{21}^2}{\tan^2\theta_{23}\sin^2\Delta_{31}} \sim 0.001 - 0.002,$$

$$\sqrt{X_\pm}\theta \;>\; \sqrt{P_\odot}$$



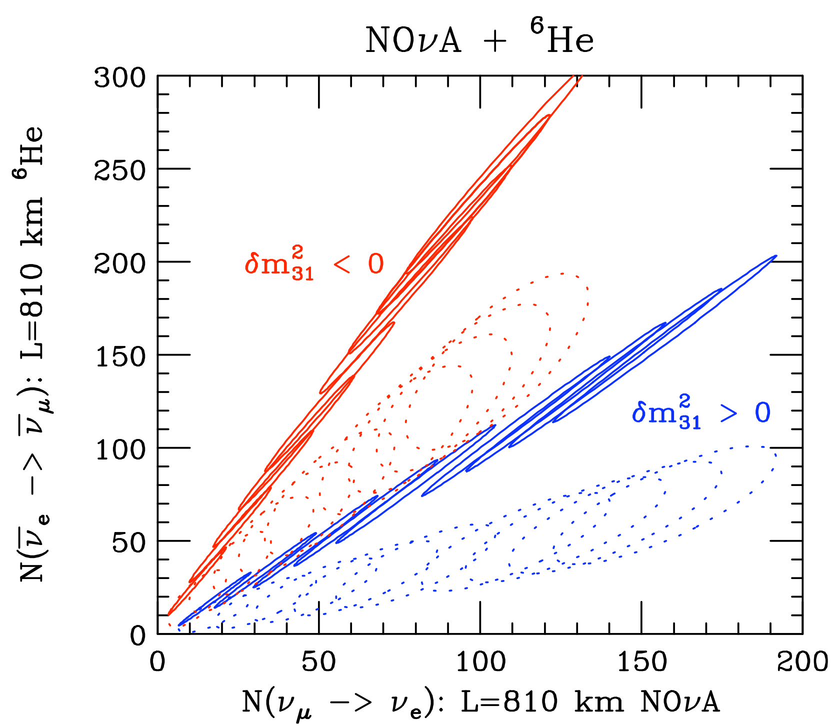
NOνA + ^{6}He

- Tevatron: $\gamma = 350$ and $\langle E_\nu \rangle \sim$ 1.2 GeV with 10^{18} useful decays/yr
- Detector: NOνA site, L=810 km:
  10 ktons of iron calorimeter or LAr TPC
- to suppress atmospheric backgrounds beam duty cycle $< 10^{-4}$!!!

(Alternative one could use the MINOS detector $\times$ 2,
here the beam duty cycle $\sim$ 1 %)

solid [1.5,2.4] GeV

dashed [1.0,1.5] GeV



(a) NOνA–^{6}He βeta-beam

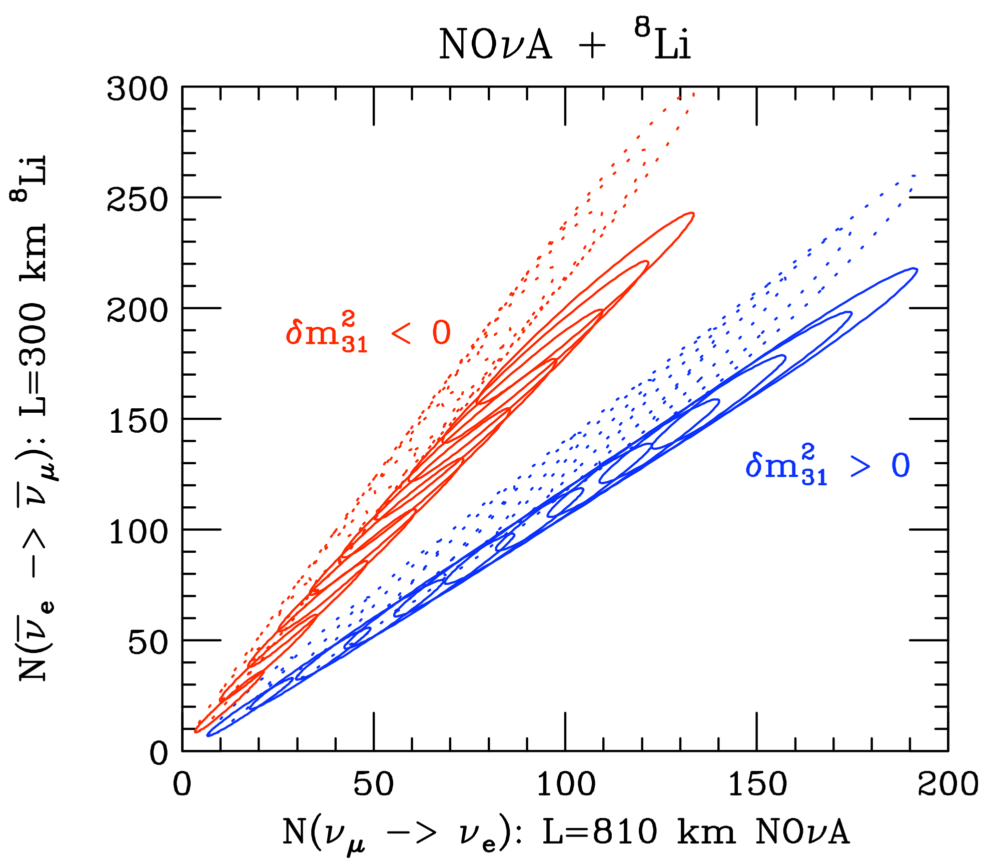
NOνA + ^{8}Li

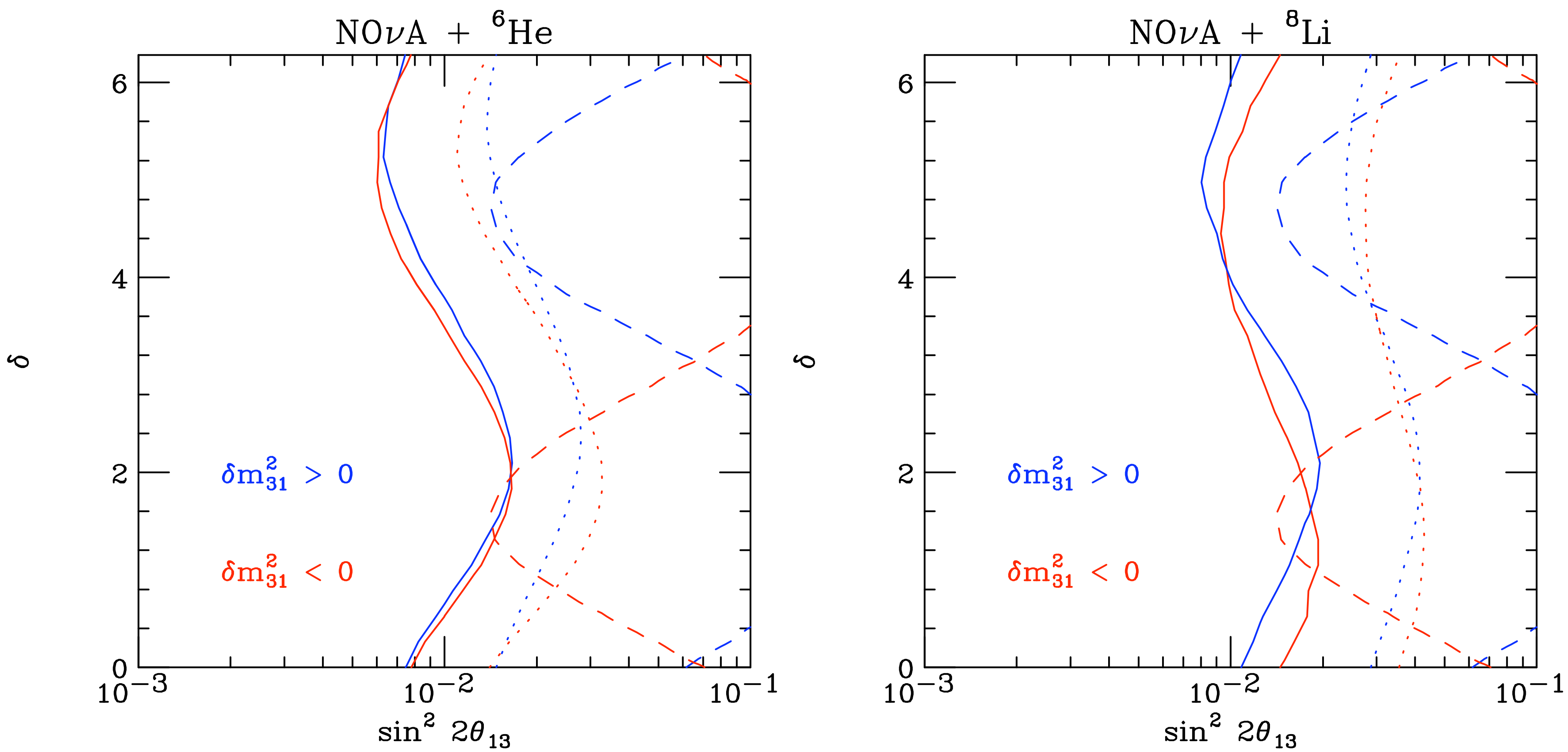
- Main Injector: $\gamma = 50$ and $\langle E_\nu \rangle \sim$ 0.9 GeV with $5(1) \times 10^{19}$ useful decays/yr

- Detector: L=300 km:
  10 (50) ktons of TASD, LAr TPC or Water Cerenkov

solid [1.0,1.5] GeV

dashed [0.75,1.0] GeV





	^6He	^8Li	
solid	2×10^{20}	5×10^{21}	useful ion decays times ktons
dotted	5×10^{19}	1×10^{21}	useful ion decays times ktons
dashed	$5 \times$ NOνA	$5 \times$ NOνA	5 yrs ν + 5 yrs $\bar{\nu}$

Hierarchy Determined for $\sin^2 2\theta_{13} > 0.01$ at 90 % C.

(large statistics!!!)

- Mossbauer Neutrinos

Minakata, Nunokawa, SP & Zukanovich Funchal: **Phys. Rev. D76:053004**

Mossbauer Neutrinos Review:

Mossbauer effect with Neutrinos in the ${}^3H -{}^3He$ system:

Source: ${}^3H \rightarrow ({}^3He + e^-_B) + \bar{\nu}_e$

Detector: $\bar{\nu}_e + ({}^3He + e^-_B) \rightarrow {}^3H$

count via
decay or
mass spectro.

$Q = 18.6$ keV and $\Gamma_{^3H} = 1.2 \times 10^{-24}$ eV

Various line broadening effects which significantly increase Γ_{eff}

Serious technical difficulties exist but it is not impossible (Raghaven, Potzel)

For $\Gamma_{eff} \sim 10^{-11}$ eV ($\Delta E/E \sim 10^{-15}$) then $\sigma \sim 10^{-33} cm^2$!!!

Do Mossbauer Neutrinos Oscillate? YES

(Akhmedov, Kopp, Lindner 0802.2513, 0803.1424)

(see also Bilenky, Feilitzsch, Potzel)

ν_e Disappearance

solar osc. (first min 270m) $P_\odot$

atm osc. (first min 9m)

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$
$$-\sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}\right]$$

$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$ (kinematic phase).

$\Delta_{21} = \Delta_{31} - \Delta_{32}$.

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$\Delta_{21} = \Delta_{31} - \Delta_{32}$.

$$\cos^2\theta_{12} > \sin^2\theta_{12}$$

- for Normal Hierarchy (NH): $|\Delta_{31}| > |\Delta_{32}|$

  phase of atmospheric oscillation ADVANCES by $2\pi\sin^2\theta_{12}$ for every solar osc.

- for Inverted Hierarchy (IH): $|\Delta_{31}| < |\Delta_{32}|$

  phase of atmospheric oscillation RETARDED by $2\pi\sin^2\theta_{12}$ for every solar osc.

ν_e Disappearance

solar osc. (first min 270m) $P_\odot$

atm osc. (first min 9m)

$$P(\bar{\nu}_e \to \bar{\nu}_e) \quad = \quad 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$
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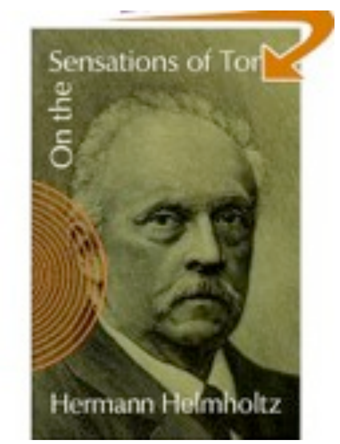

1875

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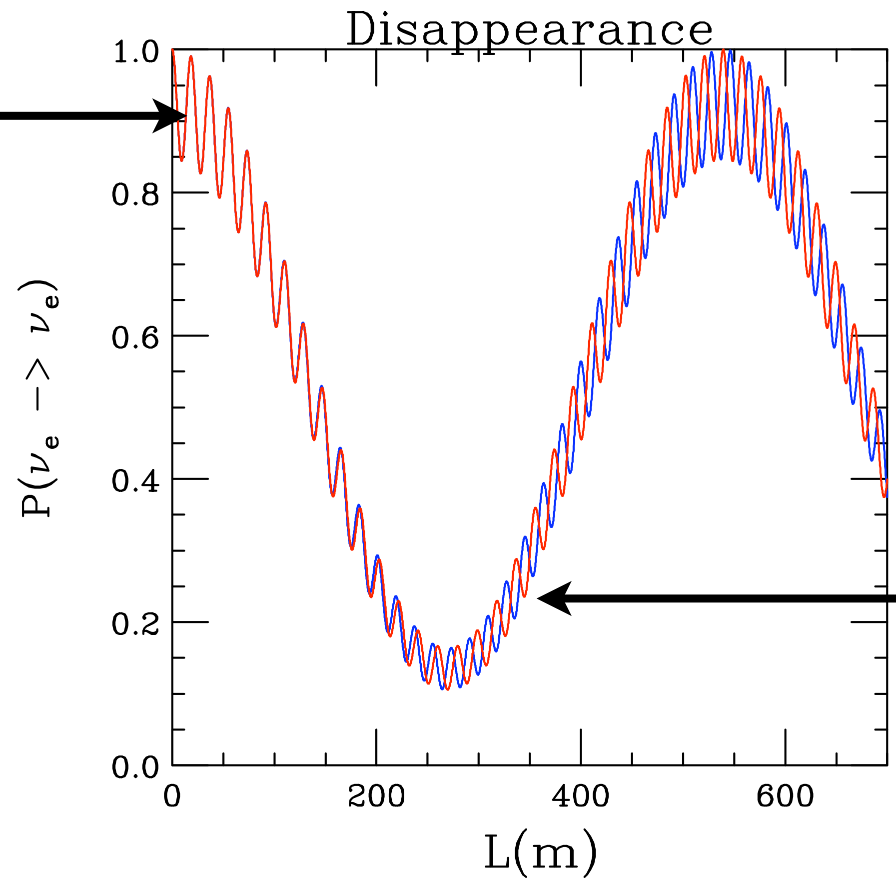
- for Inverted Hierarchy (IH): $|\Delta_{31}| < |\Delta_{32}|$

  phase of atmospheric oscillation RETARDED by $2\pi\sin^2\theta_{12}$ for every solar osc.

What about when $\sin^2\theta_{12} = \frac{1}{2}$?

in phase



out of phase by pi/2

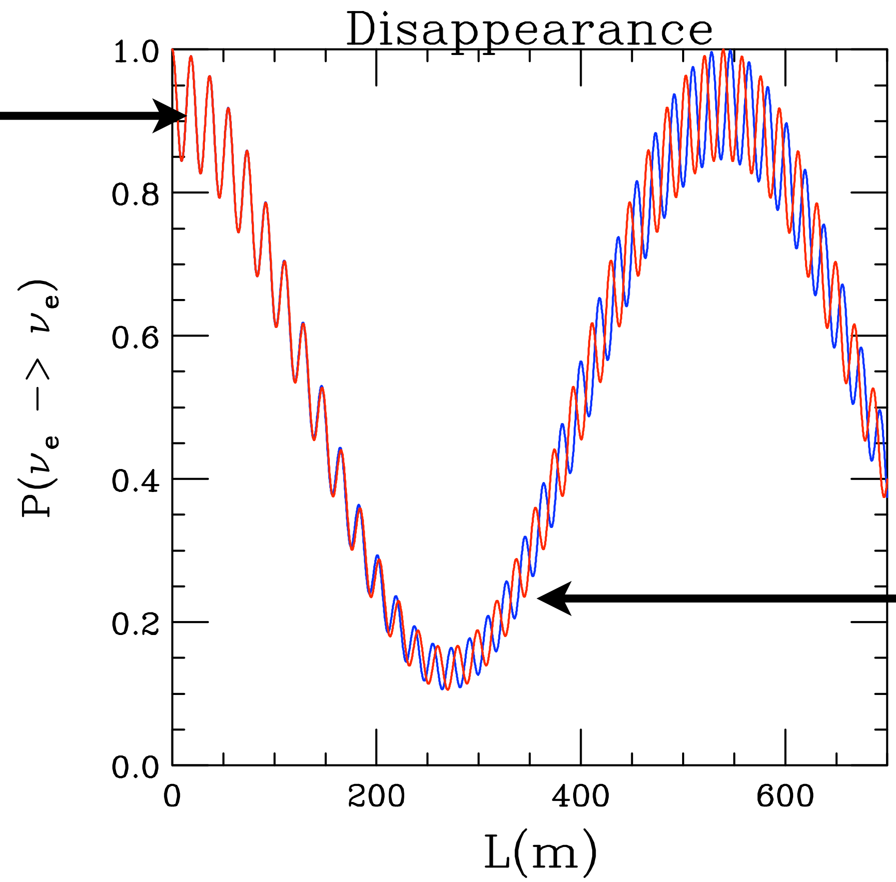
$$\delta m^2_{31} > 0$$

$$\delta m^2_{31} < 0$$

in phase

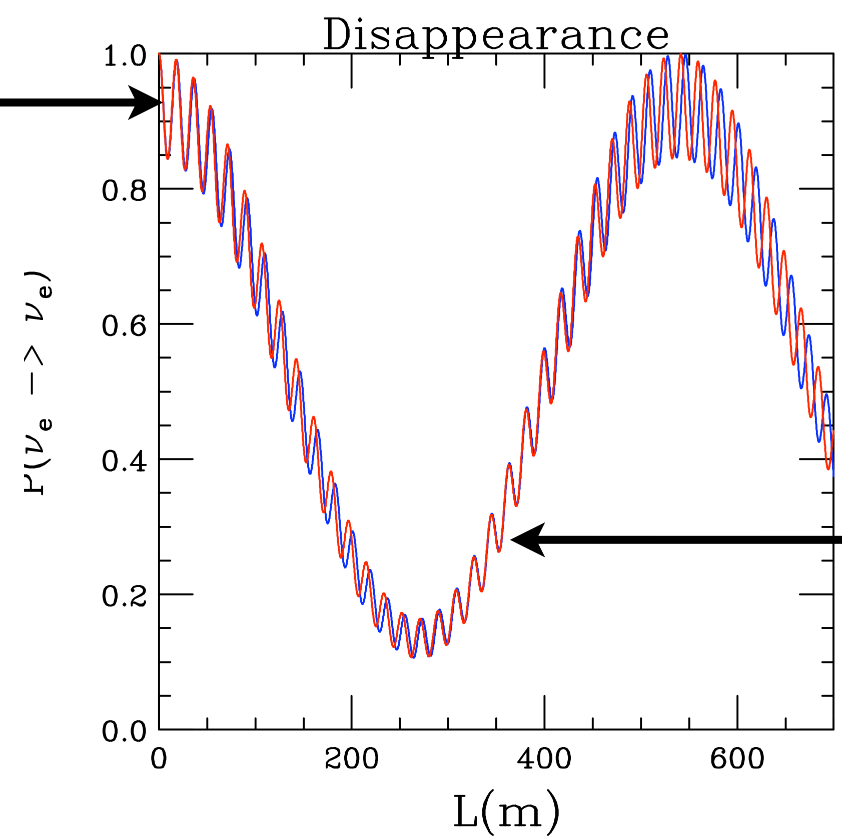
$\delta m^2_{31} > 0$

$\delta m^2_{31} < 0$



out of phase by pi/2

$\delta m^2_{IH} = 1.03 \times \delta m^2_{NH}$



in phase

Combining the Atm Osc:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \quad = \quad 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$-\frac{1}{2}\sin^2 2\theta_{13}\left\{1 - \sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}\cos(2\Delta_{ee}\pm\phi_\odot)\right\}$$

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- the $(2\Delta_{ee} \pm \phi_{\odot})$ part:
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- the $(2\Delta_{ee}\pm\phi_\odot)$ part:
    - $\pm$ Hierarchy: $+$ Normal and - Inverted.
    - linear term $2\Delta_{ee} \equiv \Delta m^2_{ee}L/2E$:

$$\begin{aligned} \Delta m^2_{ee} &= c^2_{12}|\Delta m^2_{31}| + s^2_{12}|\Delta m^2_{32}| > 0 \\ &= |m^2_3 - (c^2_{12}m^2_1 + s^2_{12}m^2_2)| \end{aligned}$$

ν_e weighted average of m^2_1 and m^2_2

Combining the Atm Osc:

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

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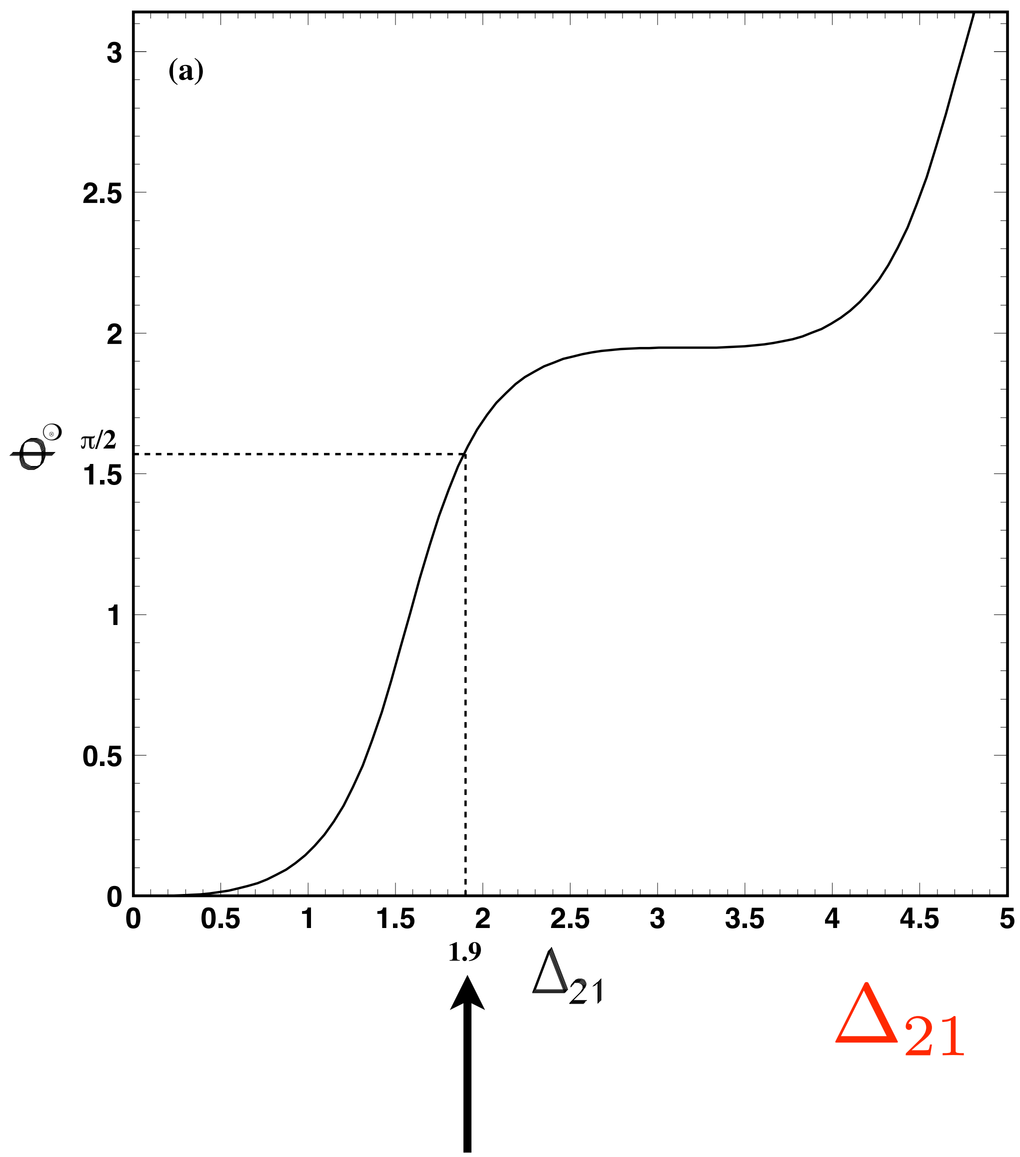
$$\begin{aligned}\Delta m^2_{ee} &= c^2_{12}|\Delta m^2_{31}| + s^2_{12}|\Delta m^2_{32}| > 0\\ &= |m^2_3 - (\underbrace{c^2_{12}m^2_1 + s^2_{12}m^2_2}_{\nu_e \text{ weighted average of } m_1^2 \text{ and } m_2^2})|\end{aligned}$$

  - everything else $\phi_\odot$: and only depends on Δ_{21} and θ_{12}.

The Phase:

- $\phi_\odot = \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \Delta_{21} \cos 2\theta_{12}$

$\phi_\odot$



Δ_{21}

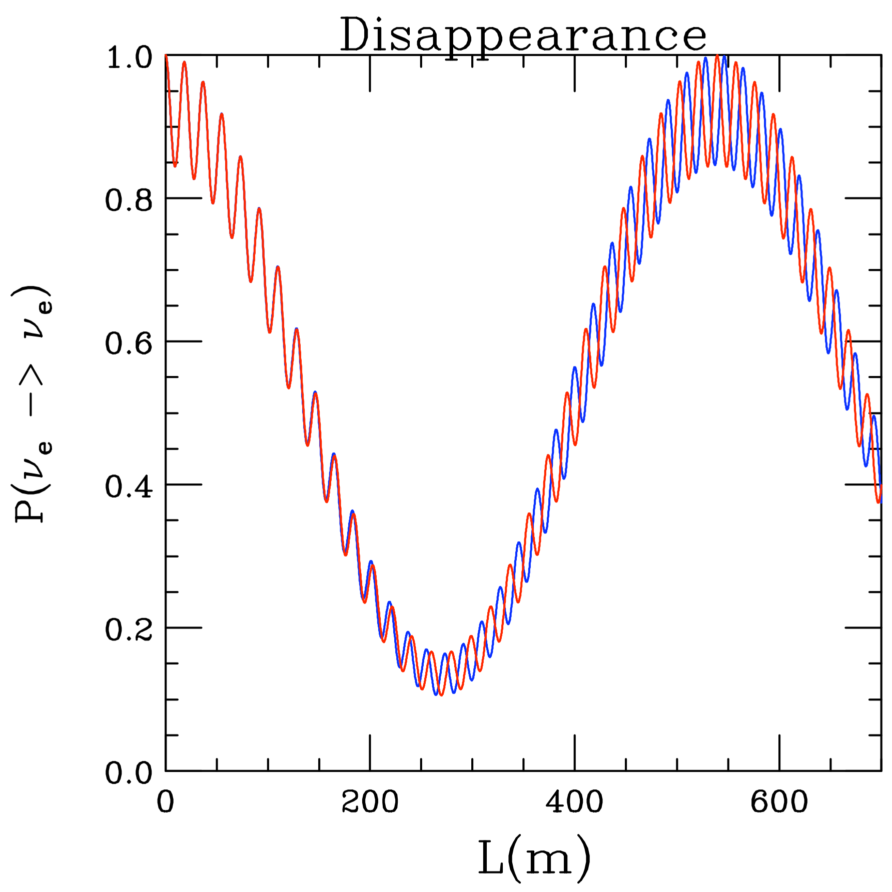
350 m

$$\phi_\odot(\Delta_{21} + \pi) = \phi_\odot(\Delta_{21}) + 2\pi \sin^2 \theta_{12},$$

$$\frac{d\phi_\odot}{d\Delta_{21}}\Big|_{n\pi} = 0$$

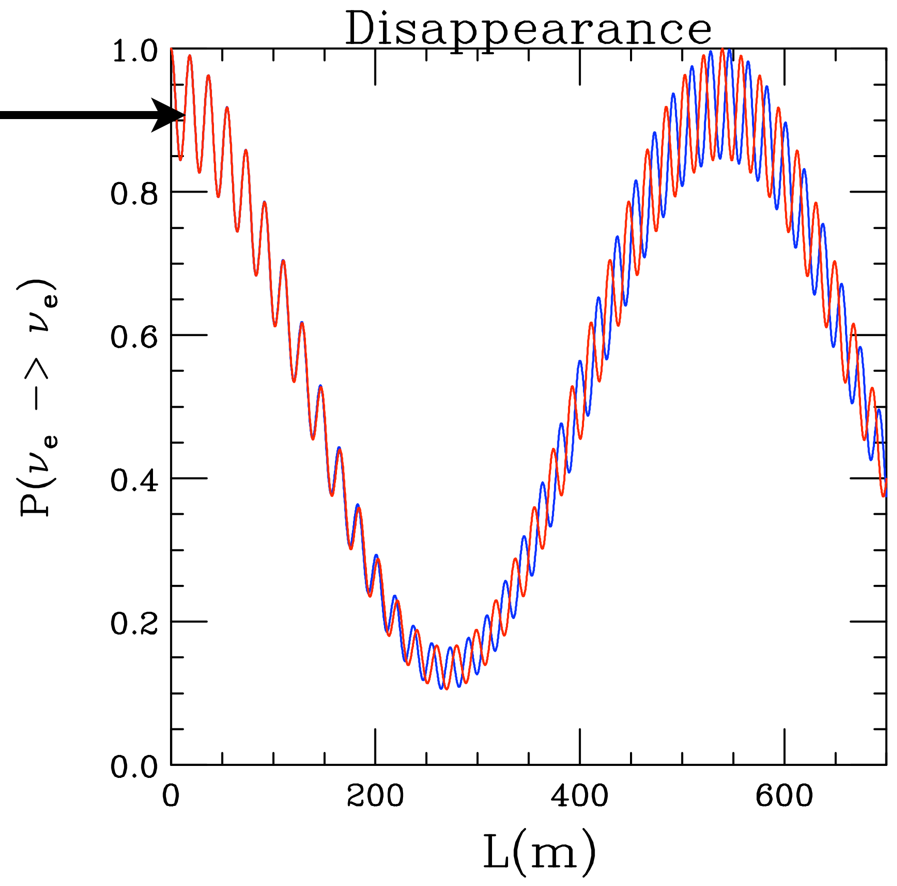
for $n = 0, 1, 2, \ldots$

Strategy:



Strategy:

(I) Precision (<1%) measurement of δm^2_{ee} at L around 10 m



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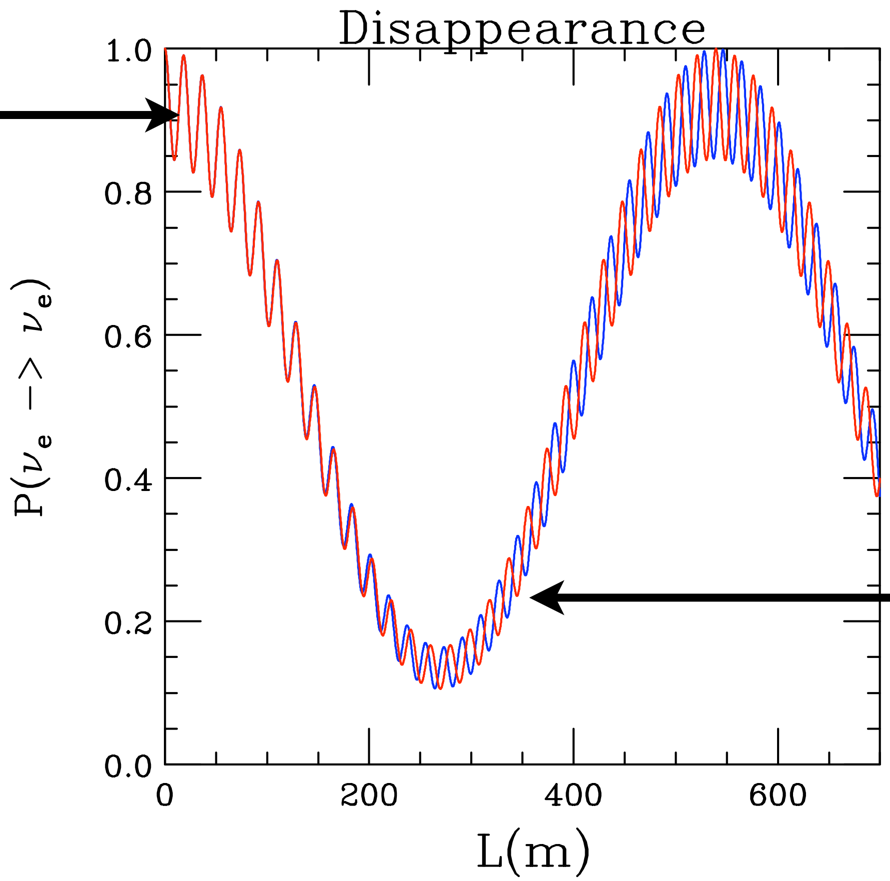

(II) determination of phase at L=350 m

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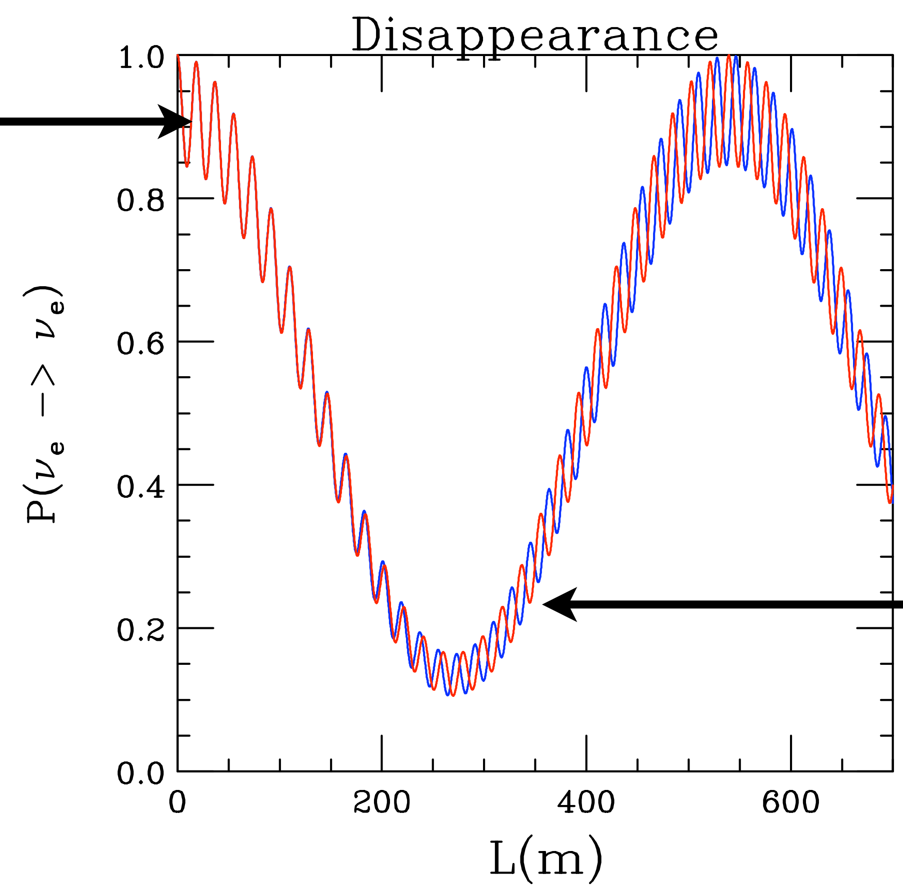


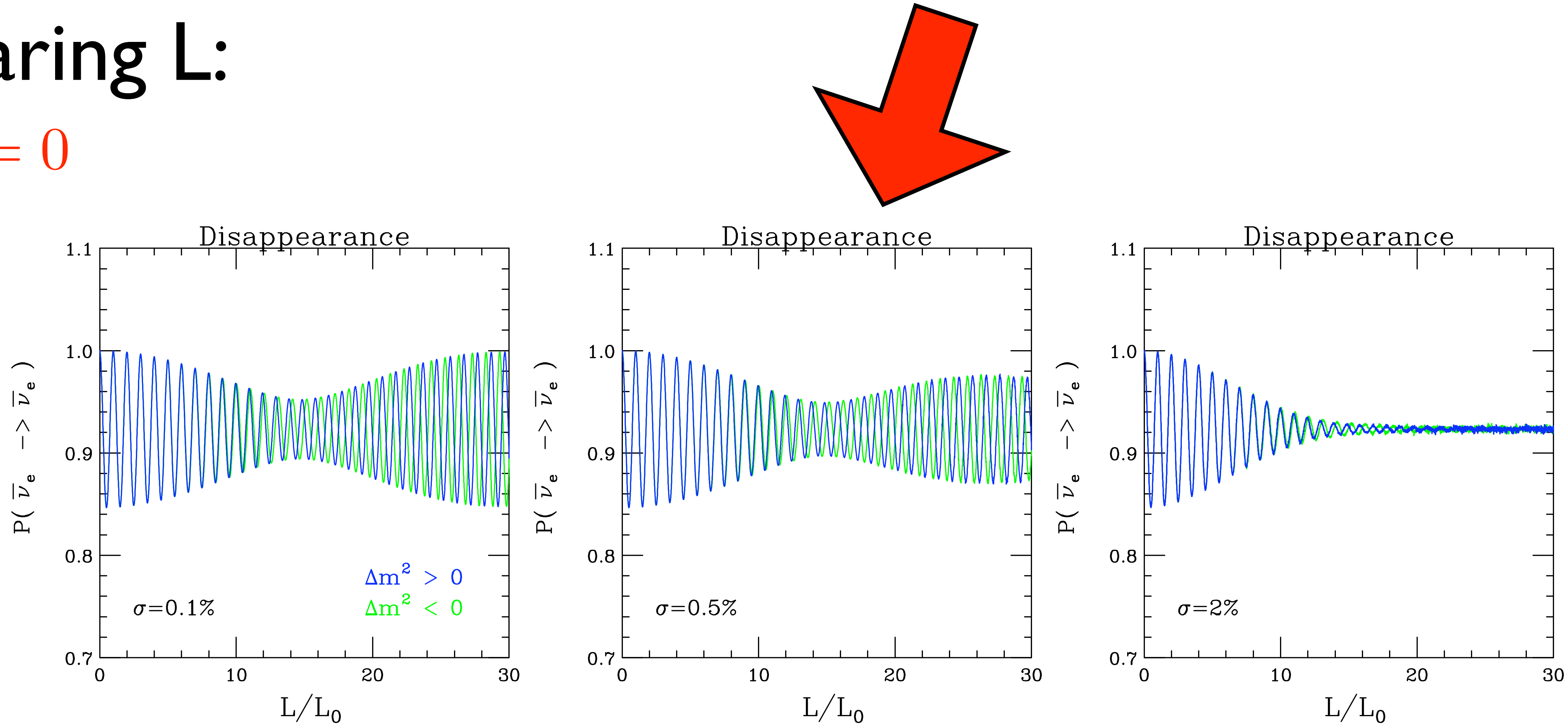
(II) determination of phase at L=350 m

But this is after 20 or so oscillation !!!
What about smearing in the L/E?

E ok, as $\Delta E/E \sim 10^{-15}$

Smearing L:

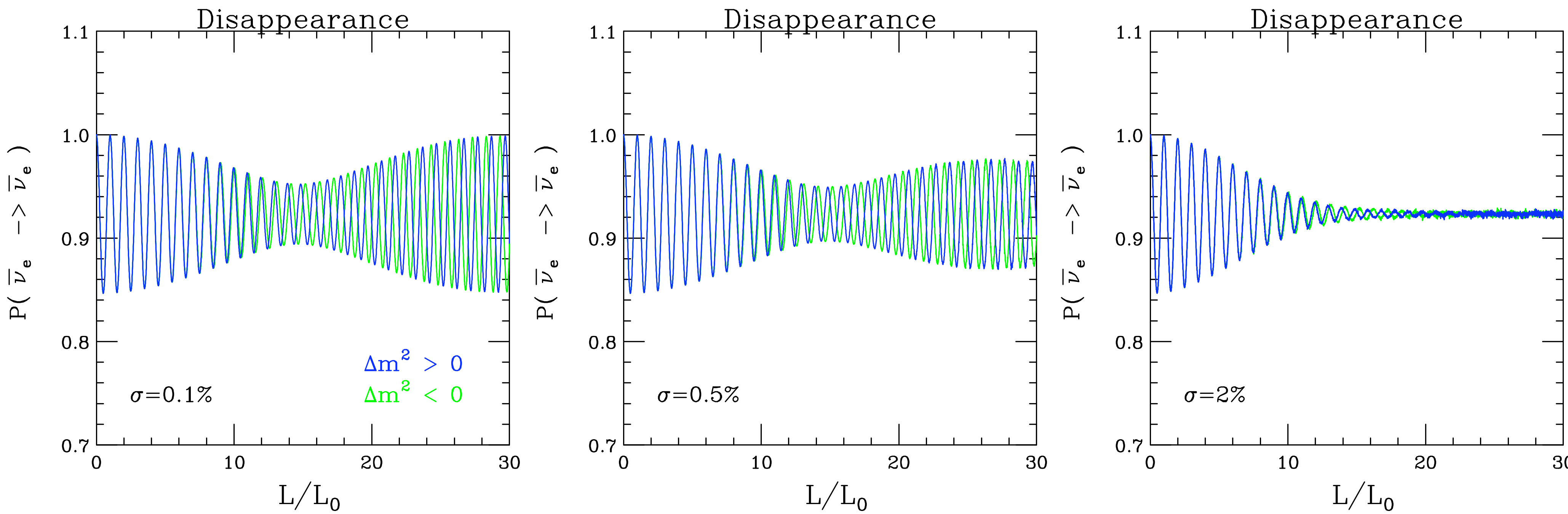
$P_{\odot} = 0$



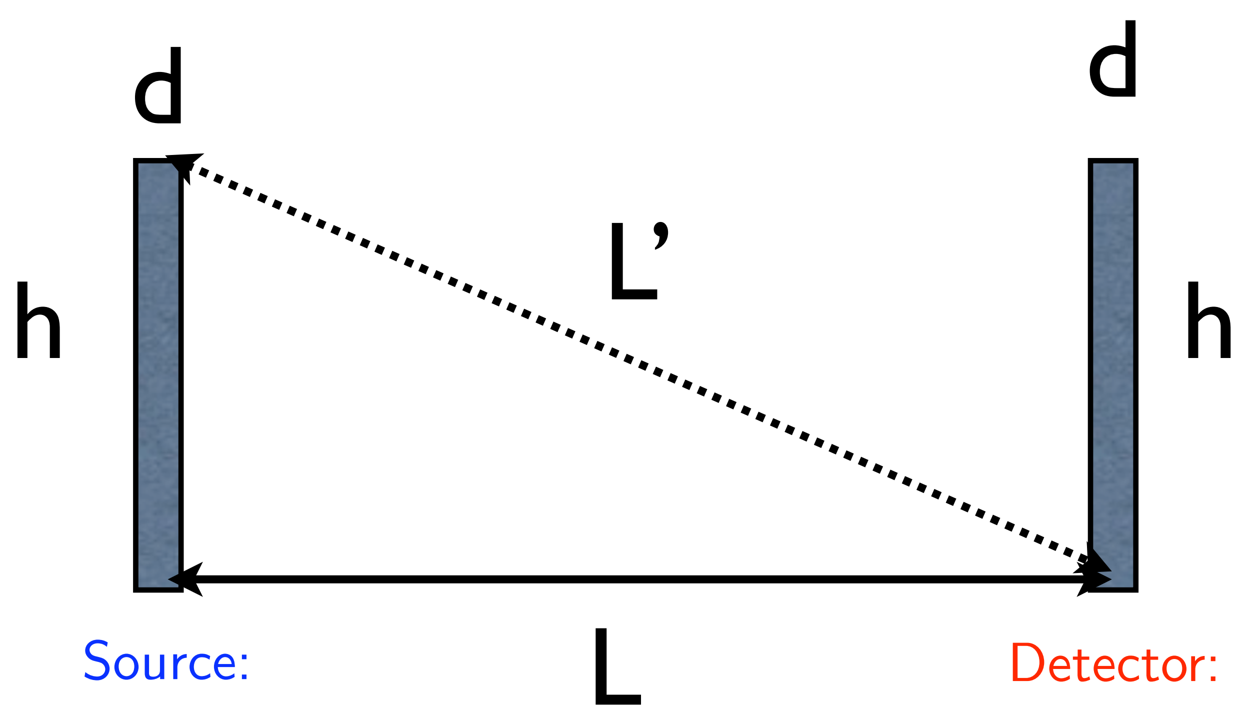
(note: amplitude modulation, 40% at solar minimum!)

Smearing L:

$P_{\odot} = 0$



(note: amplitude modulation, 40% at solar minimum!)



$d < L/200$

$L' \approx L(1 + \frac{1}{2}\frac{h^2}{L^2})$ so $h < L/10$

OK

Phase I: Measurement of δm^2_{ee}

(the atm δm^2 near the first osc. minima for a $\bar{\nu}_e$ disapp. exp.)

Event Rate:

$$R_{ench} = 3 \times 10^5 \left(\frac{S}{1MCi} \right) \left(\frac{M_T}{100g} \right) \left(\frac{L}{10m} \right)^{-2} day^{-1}$$

Minakata and Uchinami: hep/0602046

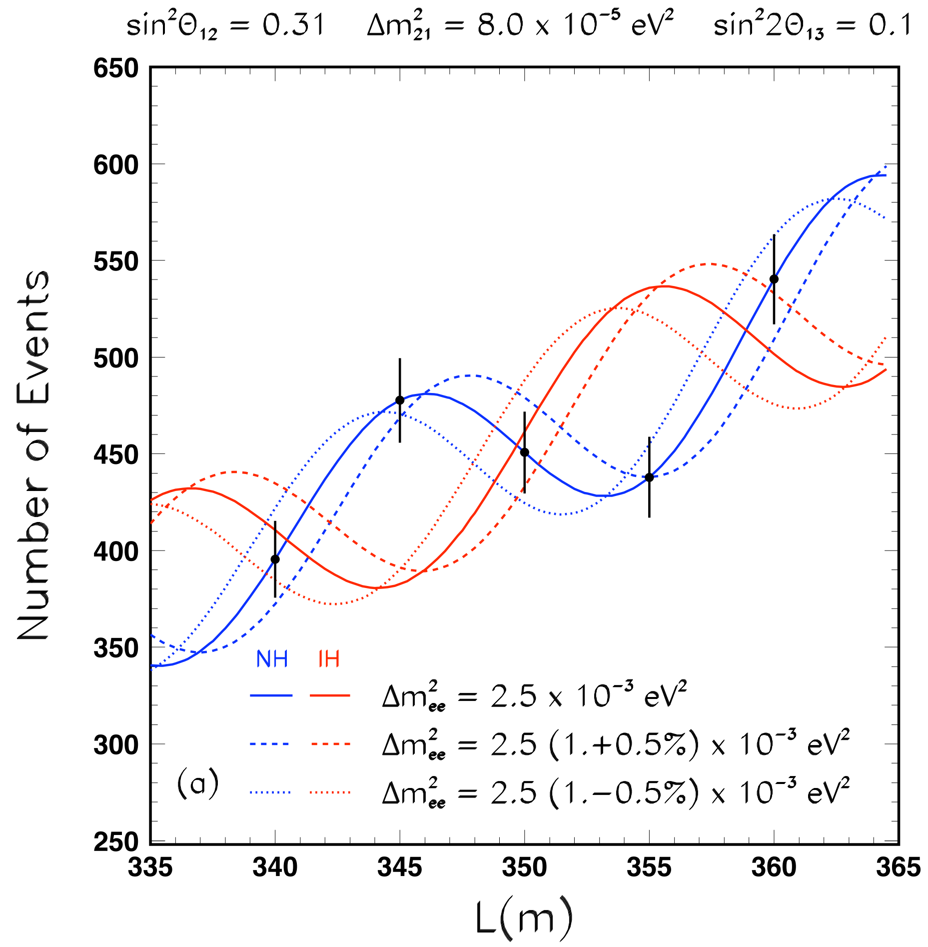
- Run IIB = 10 measurement points at (1/5, 3/5, $\cdots$ 19/5)L_{OM}
- 10^6 events each, $\sigma_{usys} = 0.2\%$, $\sigma_c = 10\%$
- Sensitivity in $\delta m^2_{ee} \approx 0.3 \left(\frac{\sin^2 2\theta_{13}}{0.1} \right)^{-1} \%$

Phase II: phase at 350 m

Event Rate:

$$R_{ench} = 2 \times 10^2 \left(\frac{S}{1MCi}\right)\left(\frac{M_T}{100g}\right)\left(\frac{L}{350m}\right)^{-2} day^{-1}$$

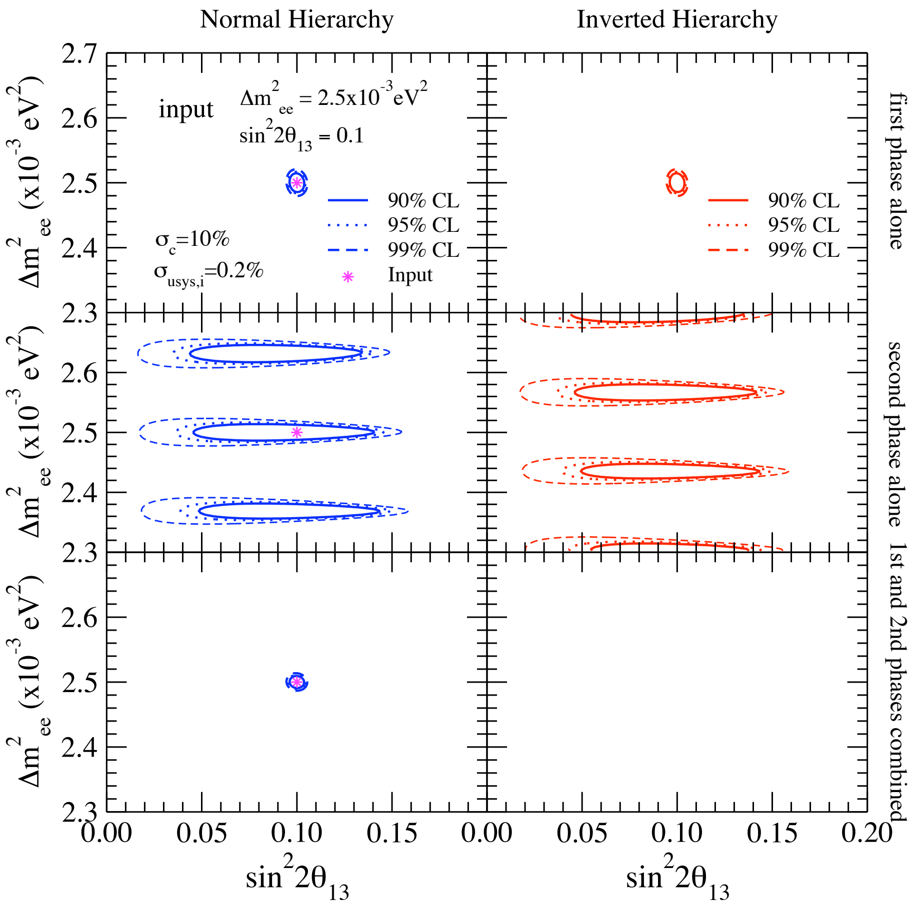
5 Baselines: $L = 350 \pm 5 \pm 10$ m



Phase I

Phase II

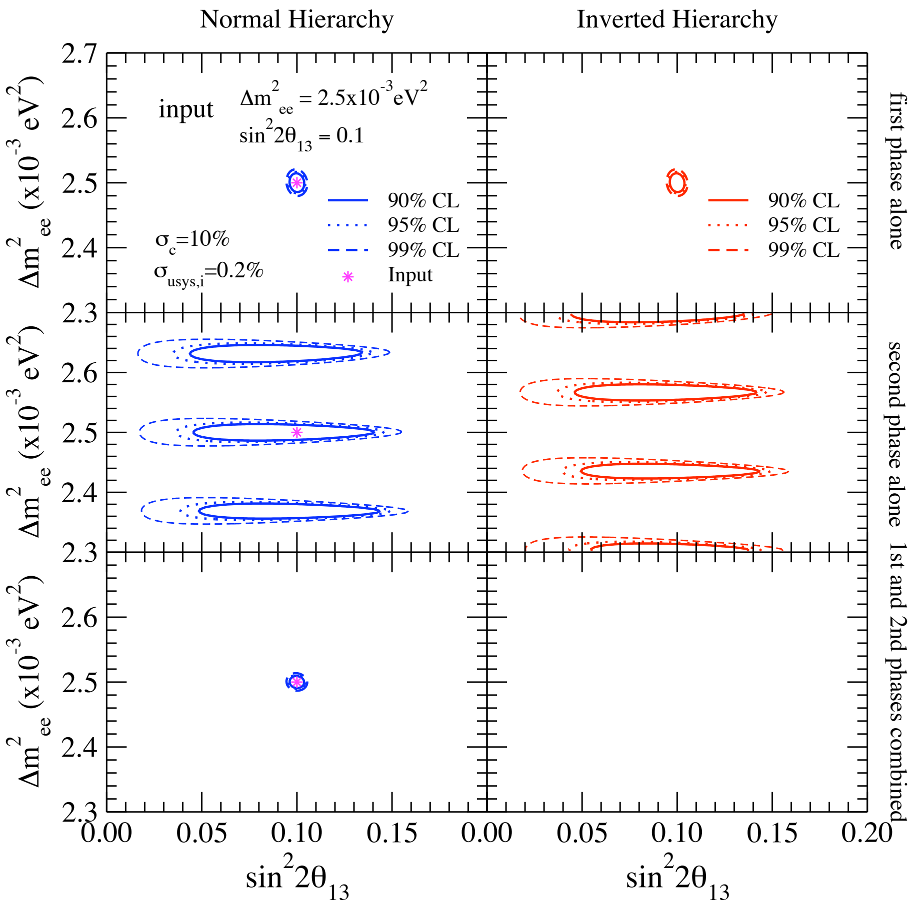
Phase I+II



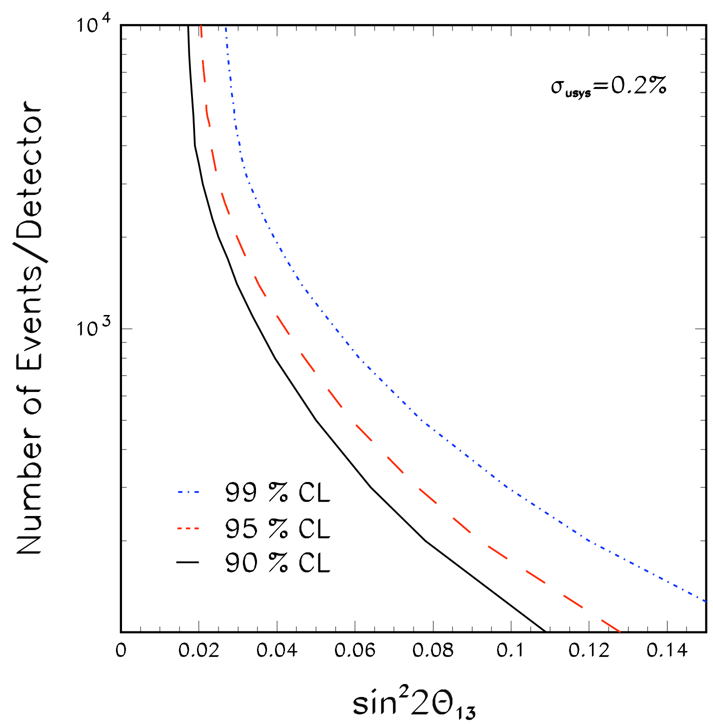
Phase I

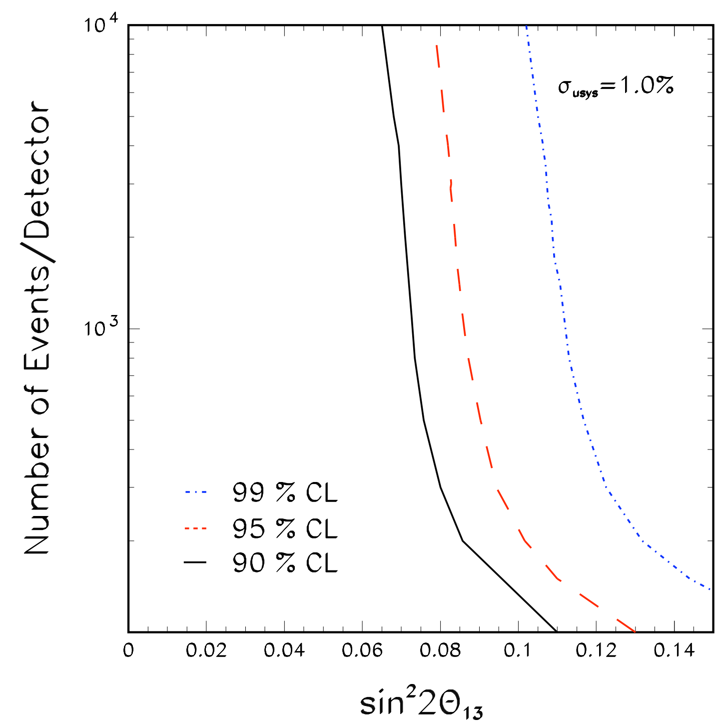
Phase II

Phase I+II



Sensitivity:



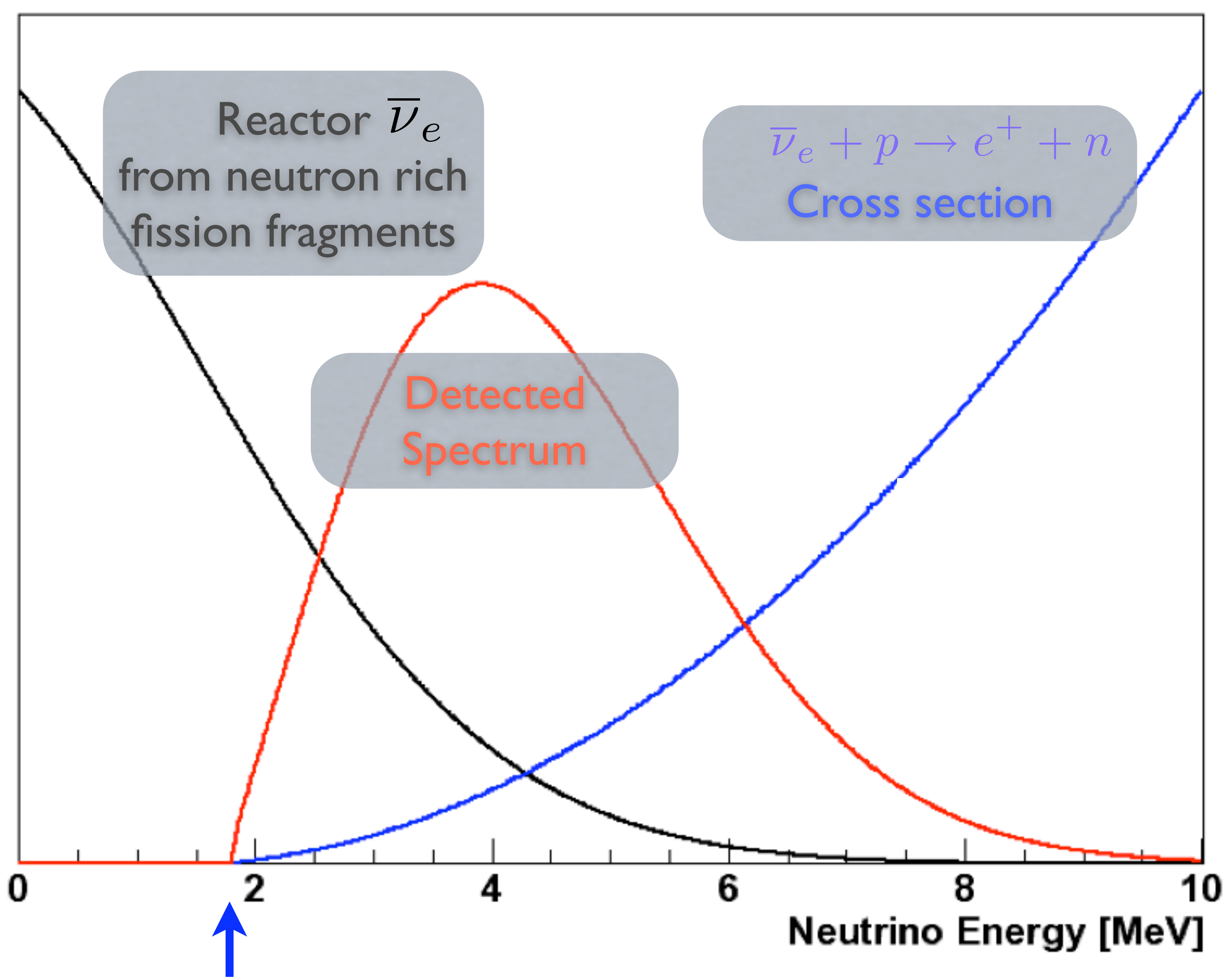


Reactor Neutrinos:

Reactor Neutrinos: Mass Hierarchy

Petcov, Piai hep-ph/0112074
Choubey, Petcov and Piai hep-ph/0306017

Detected Spectrum



Hawaii Antineutrino Observatory
Hanohano

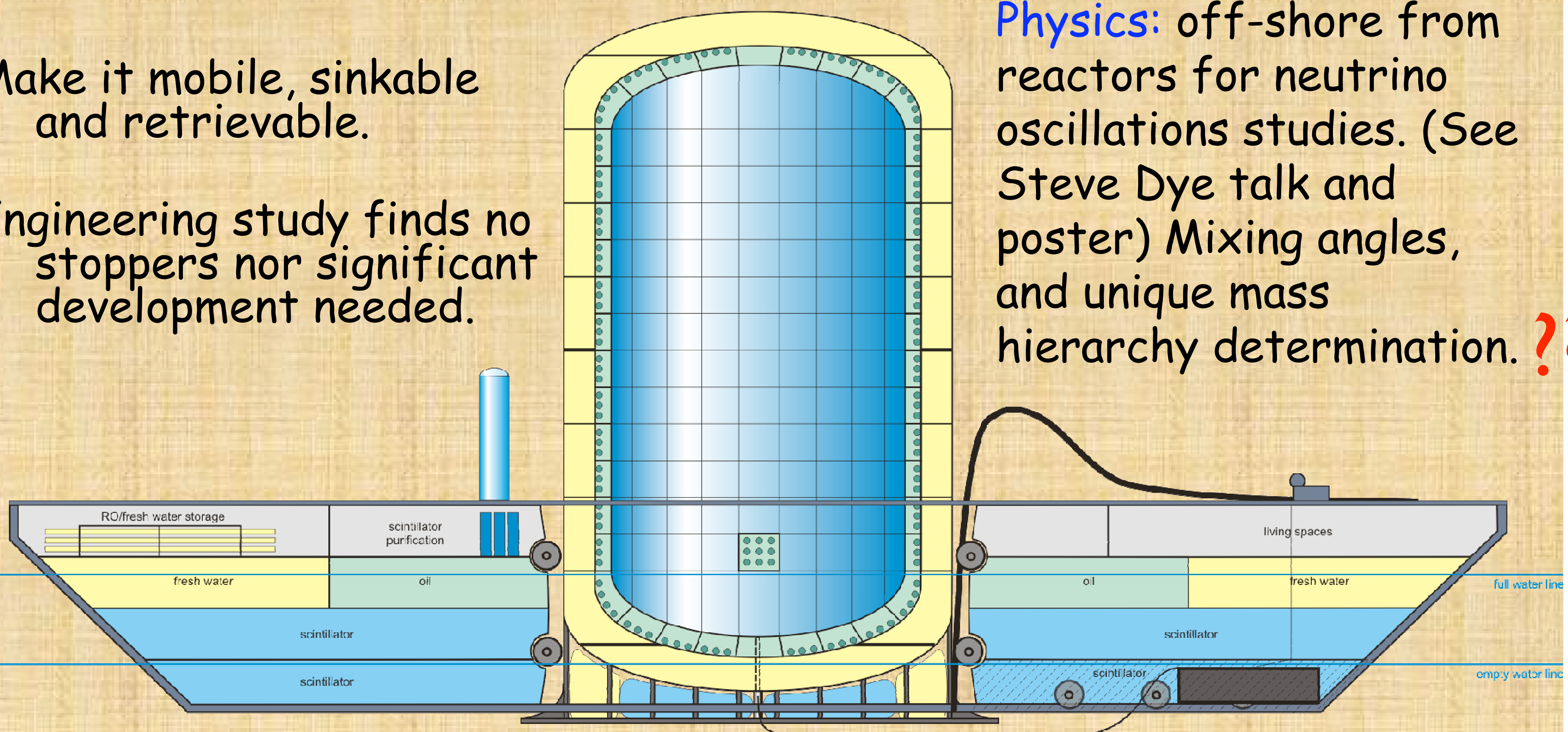
Idea: detector based on KamLAND technology adapted for deep ocean, but >10 x larger (for good counting rate)

Make it mobile, sinkable and retrievable.

Engineering study finds no stoppers nor significant development needed.

Geology: mid-Pacific and elsewhere for geo-neutrinos from mantle.

Physics: off-shore from reactors for neutrino oscillations studies. (See Steve Dye talk and poster) Mixing angles, and unique mass hierarchy determination. ???



Fourier Transforms: Hanohano

Learned, Dye, Pakvasa, and Svoboda, *hep-ex/0612022*

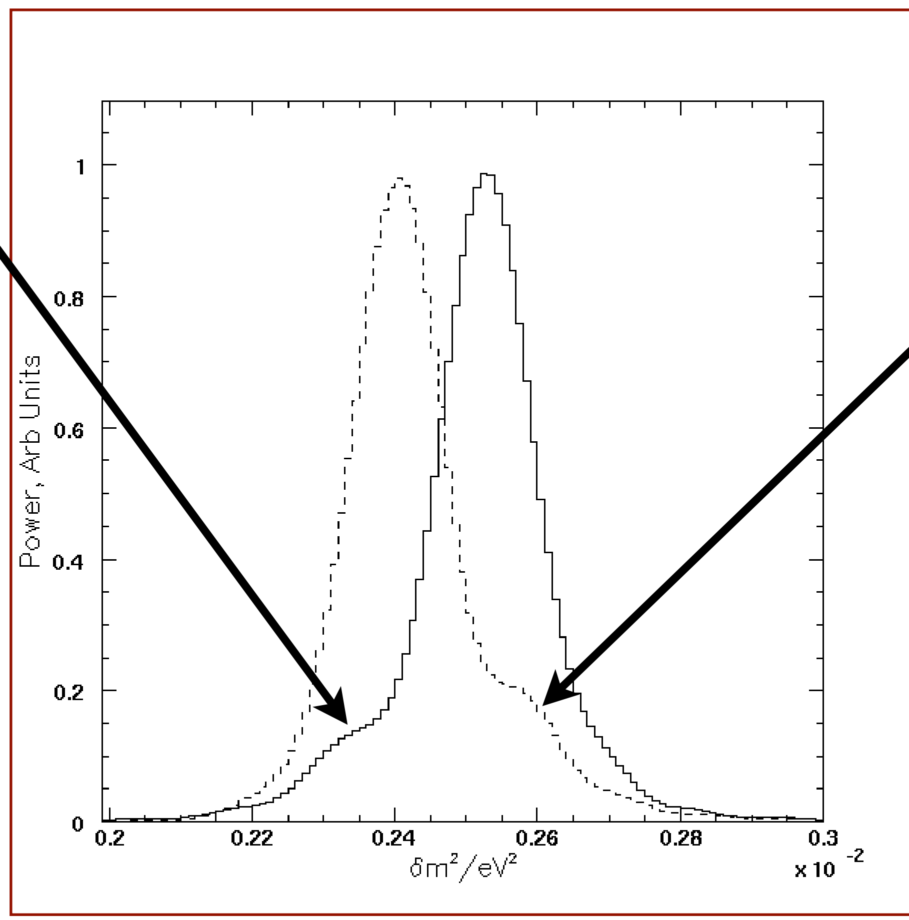
$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$-\sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}\right]$$

dominant frequency

sub-dominant frequency
(1/5 the power)

NH:
shoulder at
smaller freq.



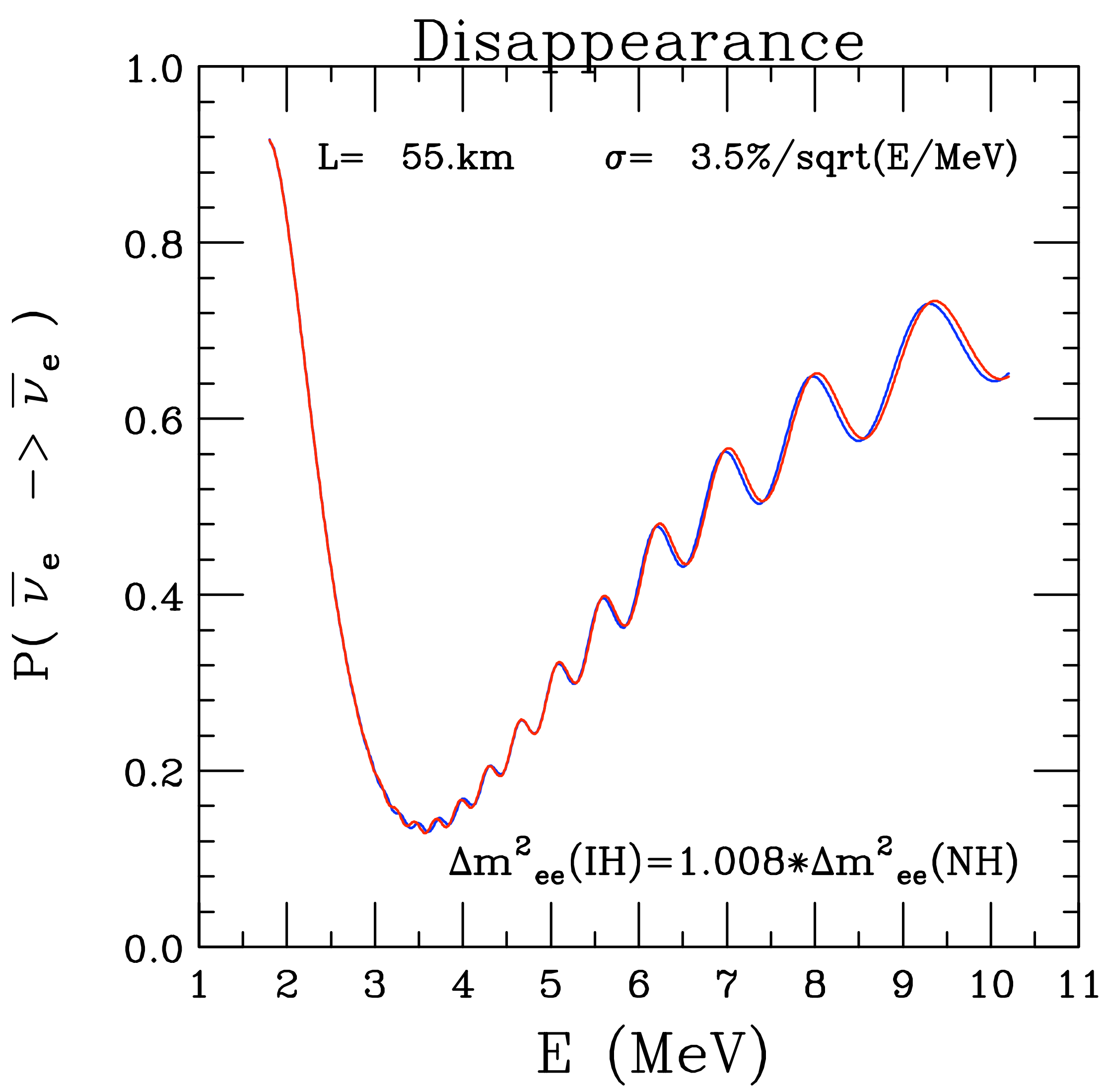
IH:
shoulder at
higher freq.

$\sin^2 2\theta_{13} > 0.05$ for 10 Kton-yr

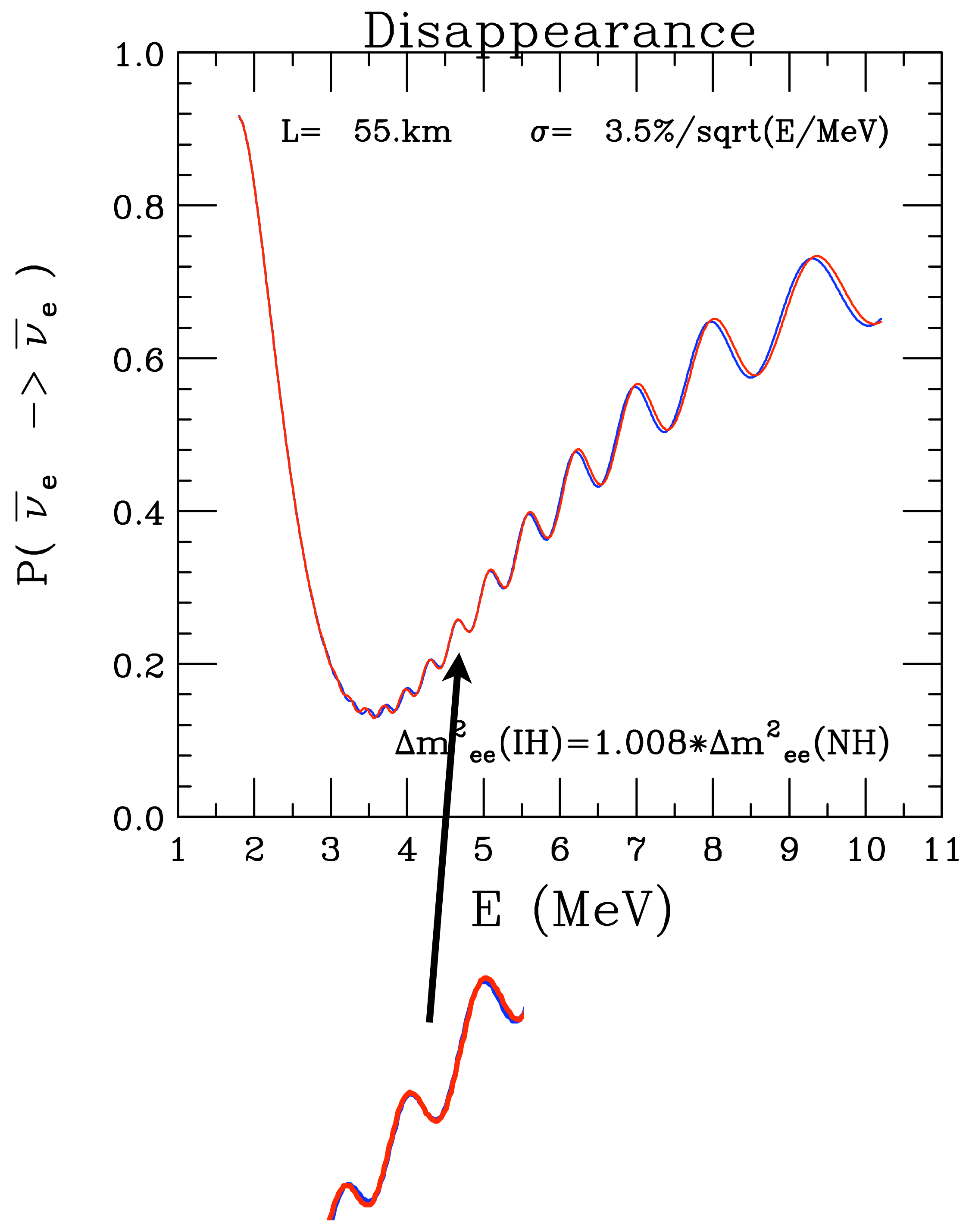
$\sin^2 2\theta_{13} > 0.02$ for 100 Kton-yr

also L. Zhan, Y. Wang, J. Cao and L. Wen, arXiv:0807.3203

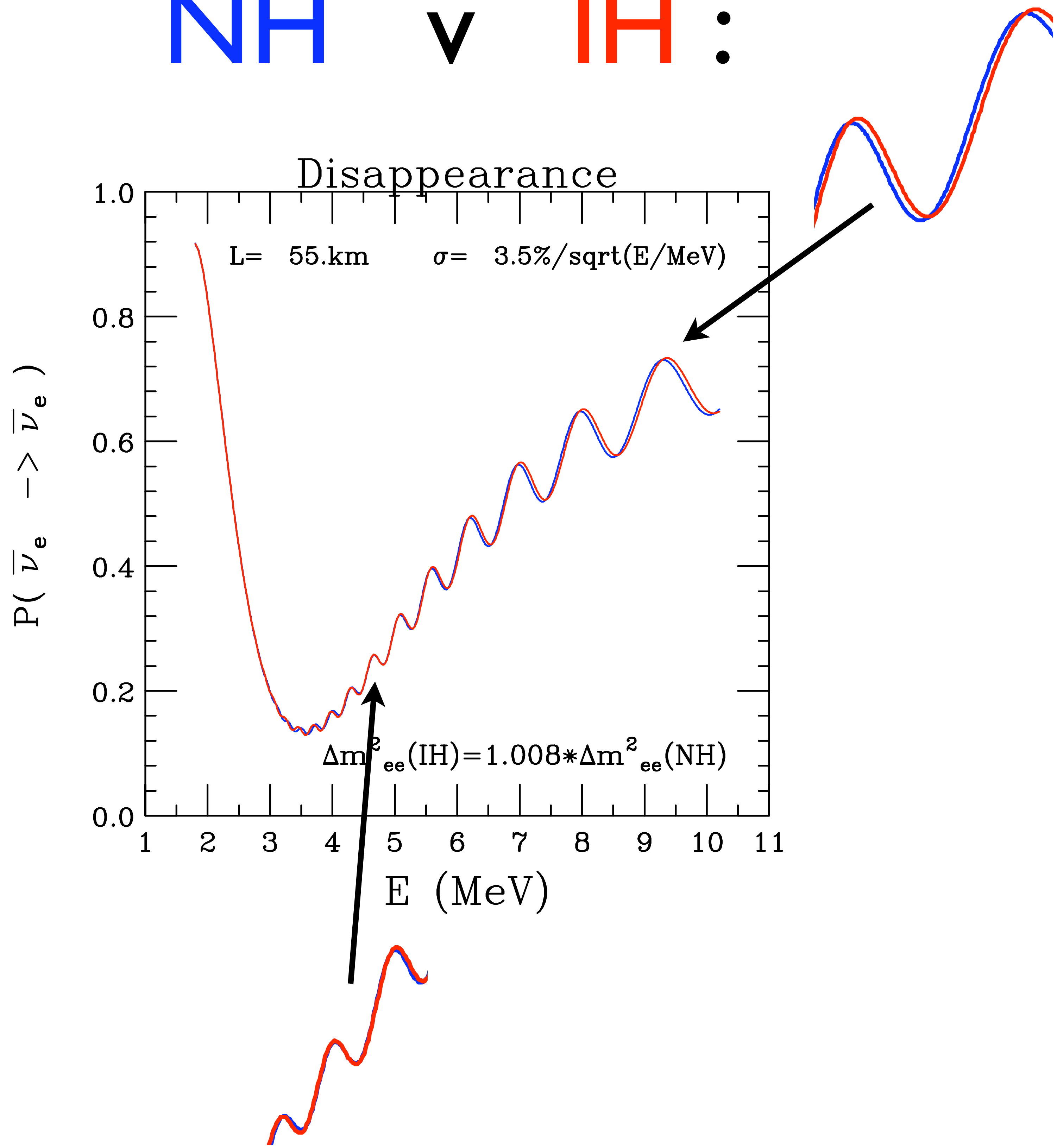
NH v IH :


Disappearance
L= 55.km σ= 3.5%/sqrt(E/MeV)
Δm²ee(IH)=1.008*Δm²ee(NH)
P(ν̄e -> ν̄e)
E (MeV)
1.0
0.8
0.6
0.4
0.2
0.0
1 2 3 4 5 6 7 8 9 10 11

NH v IH :

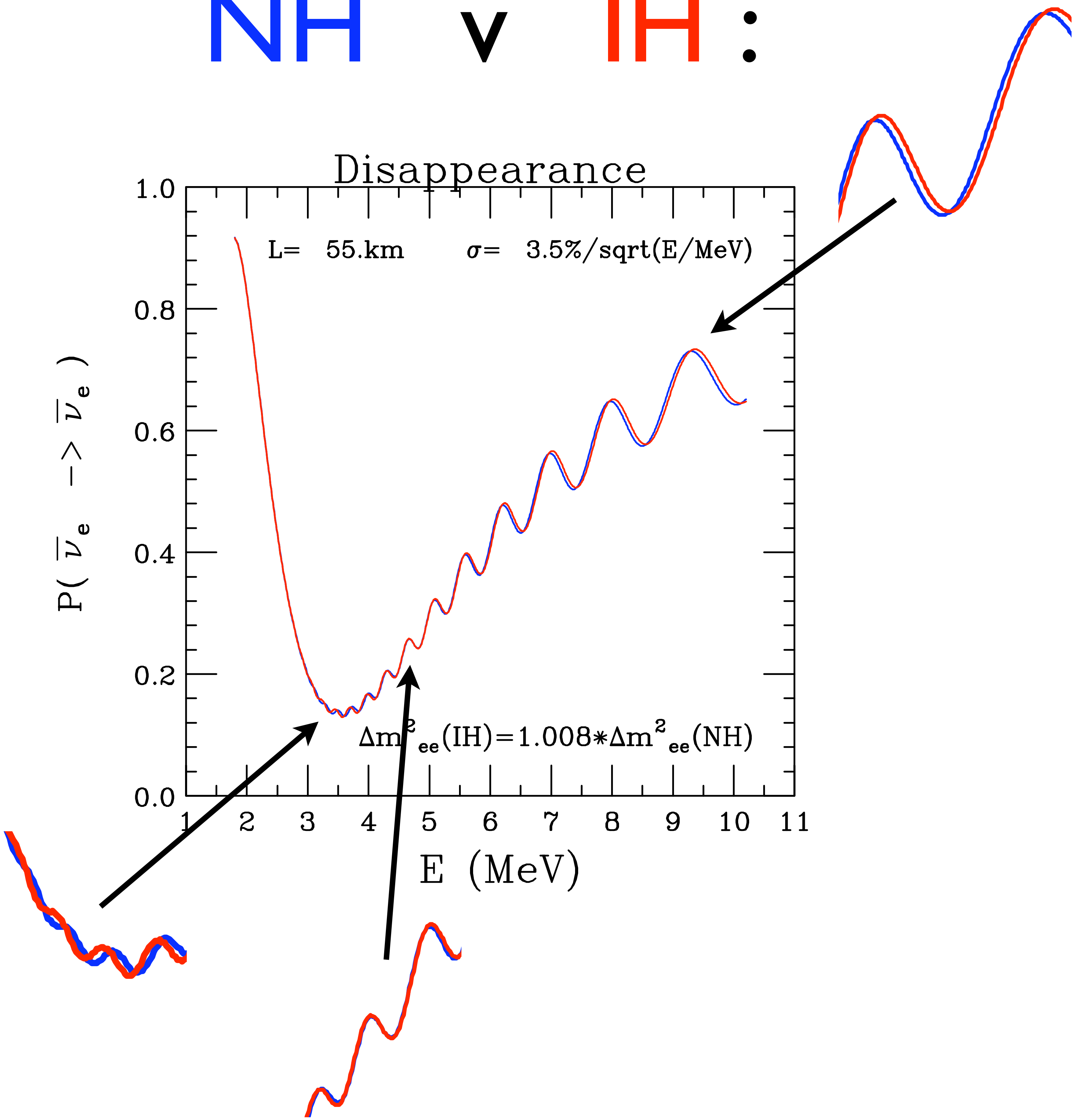

Disappearance
L= 55.km σ= 3.5%/sqrt(E/MeV)
Δm²ee(IH)=1.008*Δm²ee(NH)
P(ν̄e -> ν̄e)
E (MeV)
1.0
0.8
0.6
0.4
0.2
0.0
1
2
3
4
5
6
7
8
9
10
11

NH v IH :

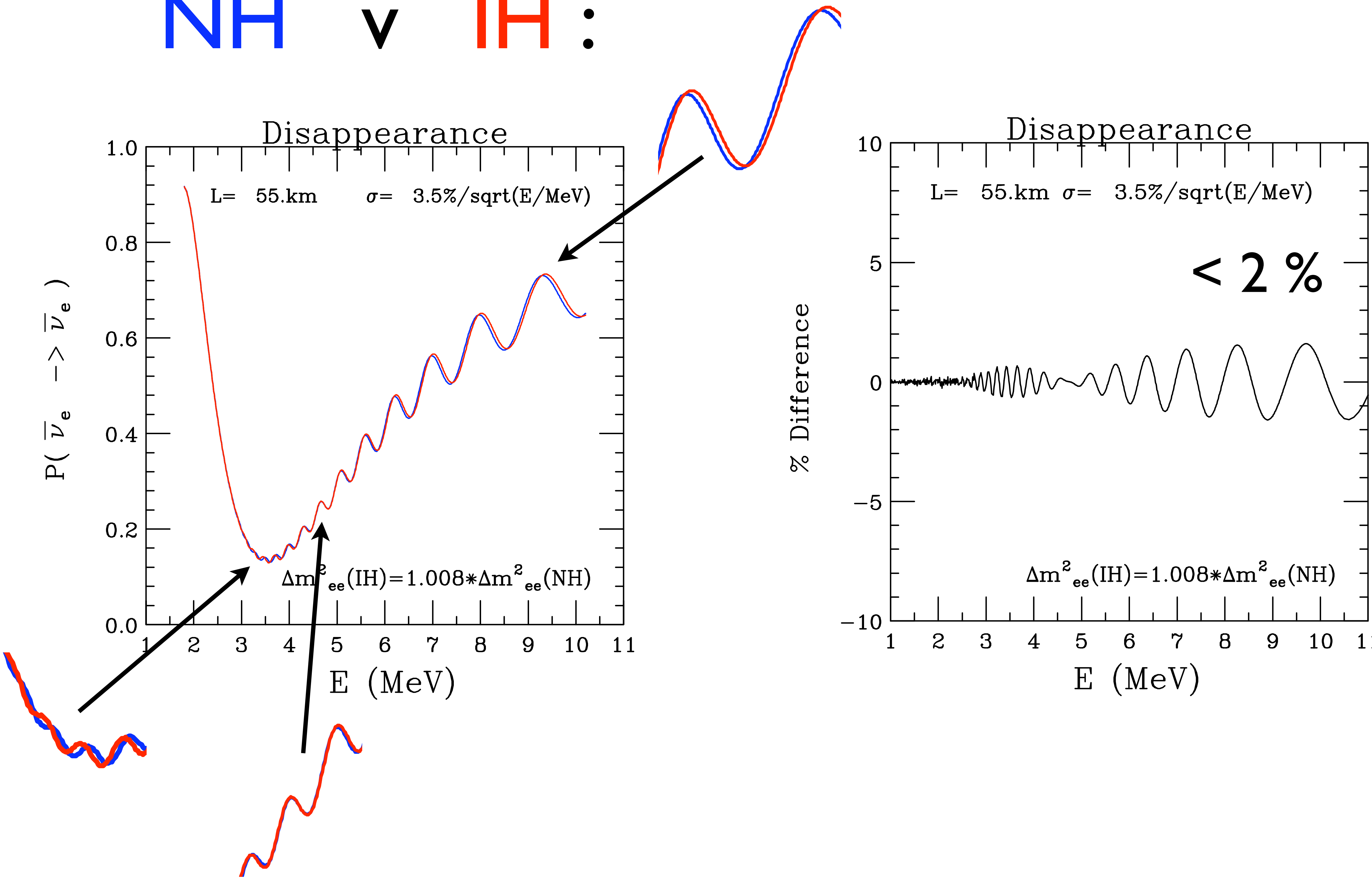


Disappearance
L= 55.km
σ= 3.5%/sqrt(E/MeV)
Δm²ee(IH)=1.008*Δm²ee(NH)
P(ν̄e -> ν̄e)
E (MeV)
0.0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6
7
8
9
10
11

NH v IH :


Disappearance
L= 55.km σ= 3.5%/sqrt(E/MeV)
Δm²ee(IH)=1.008*Δm²ee(NH)
P(ν̄e -> ν̄e)
E (MeV)
1.0
0.8
0.6
0.4
0.2
0.0
1
2
3
4
5
6
7
8
9
10
11

NH v IH :

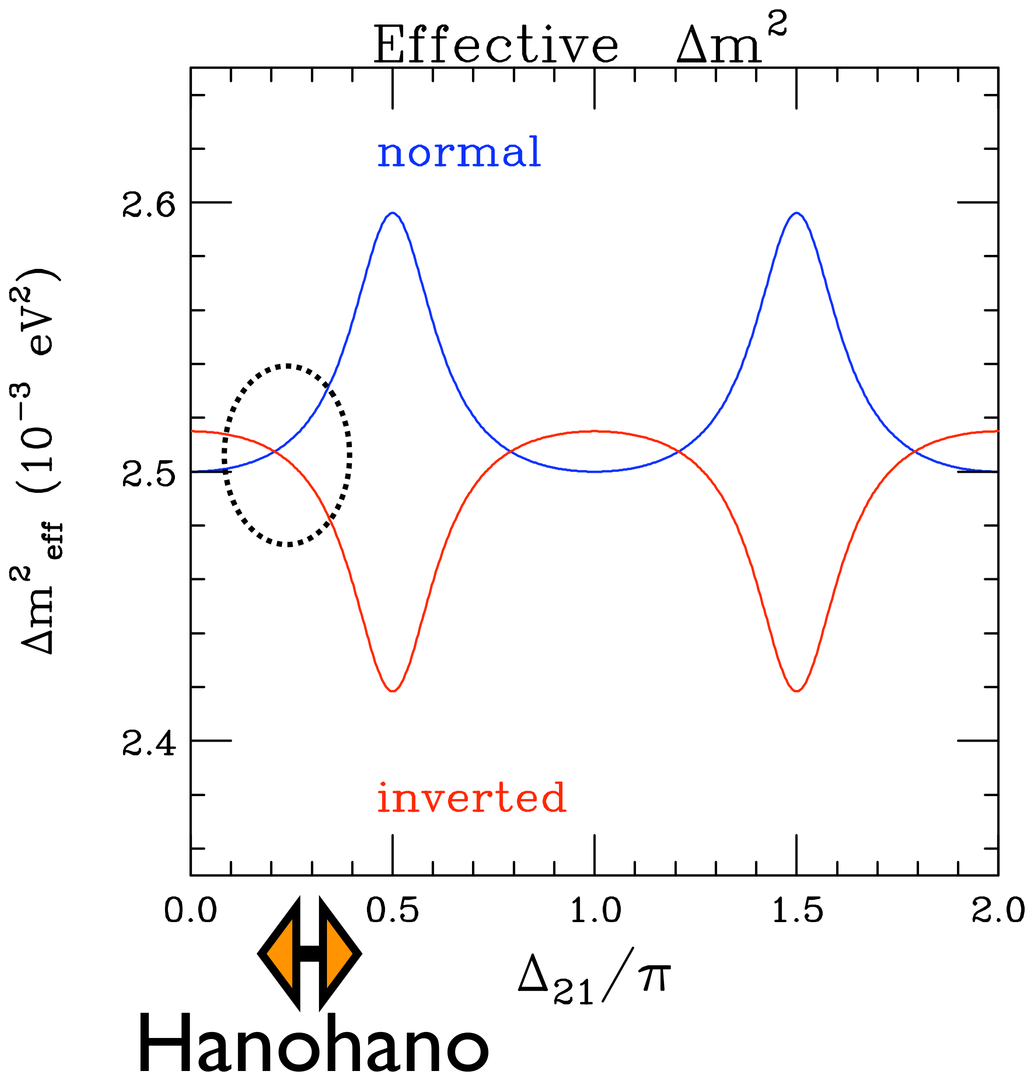

Disappearance
L= 55.km σ= 3.5%/sqrt(E/MeV)
Δm²ee(IH)=1.008*Δm²ee(NH)
P(ν̄e -> ν̄e)
E (MeV)
Disappearance
L= 55.km σ= 3.5%/sqrt(E/MeV)
< 2 %
Δm²ee(IH)=1.008*Δm²ee(NH)
% Difference
E (MeV)

the "instantaneous" δm^2

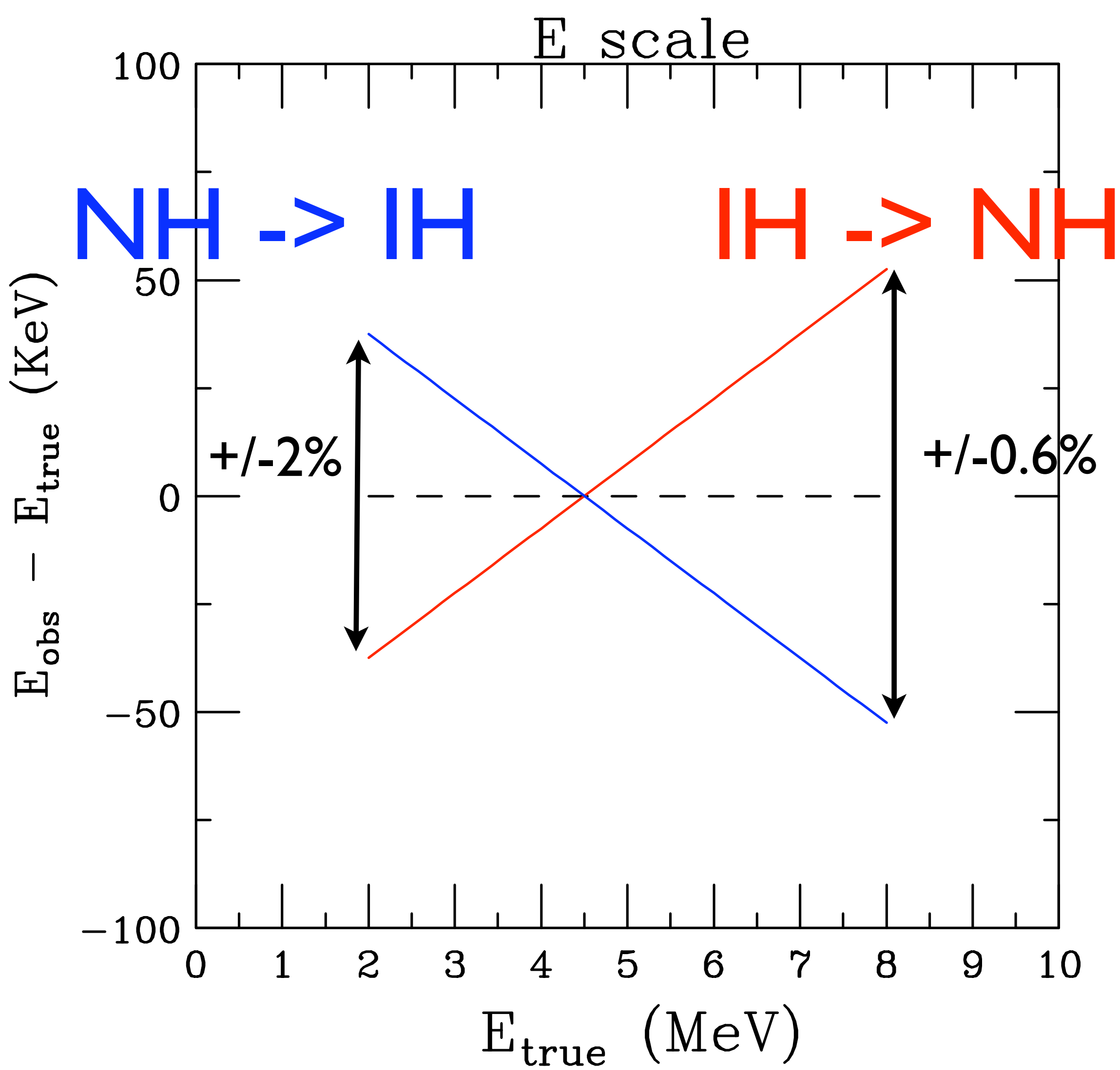
argument of the atm cosine term is $2\Delta_{ee} \pm \phi_\odot \equiv \frac{1}{2}\int_0^{L/E} d\rho\; \delta m^2_{eff}(\rho)$

$$\delta m^2_{eff}(L/E) = \delta m^2_{ee} \pm \frac{1}{2}\,\delta m^2_{21}\cos 2\theta_{12}\,\frac{\sin^2 2\theta_{12}\sin^2\Delta_{21}}{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}$$

derivative of $\phi_\odot$



Uncertainty in E scale ???
between 2 and 8 MeV !!!

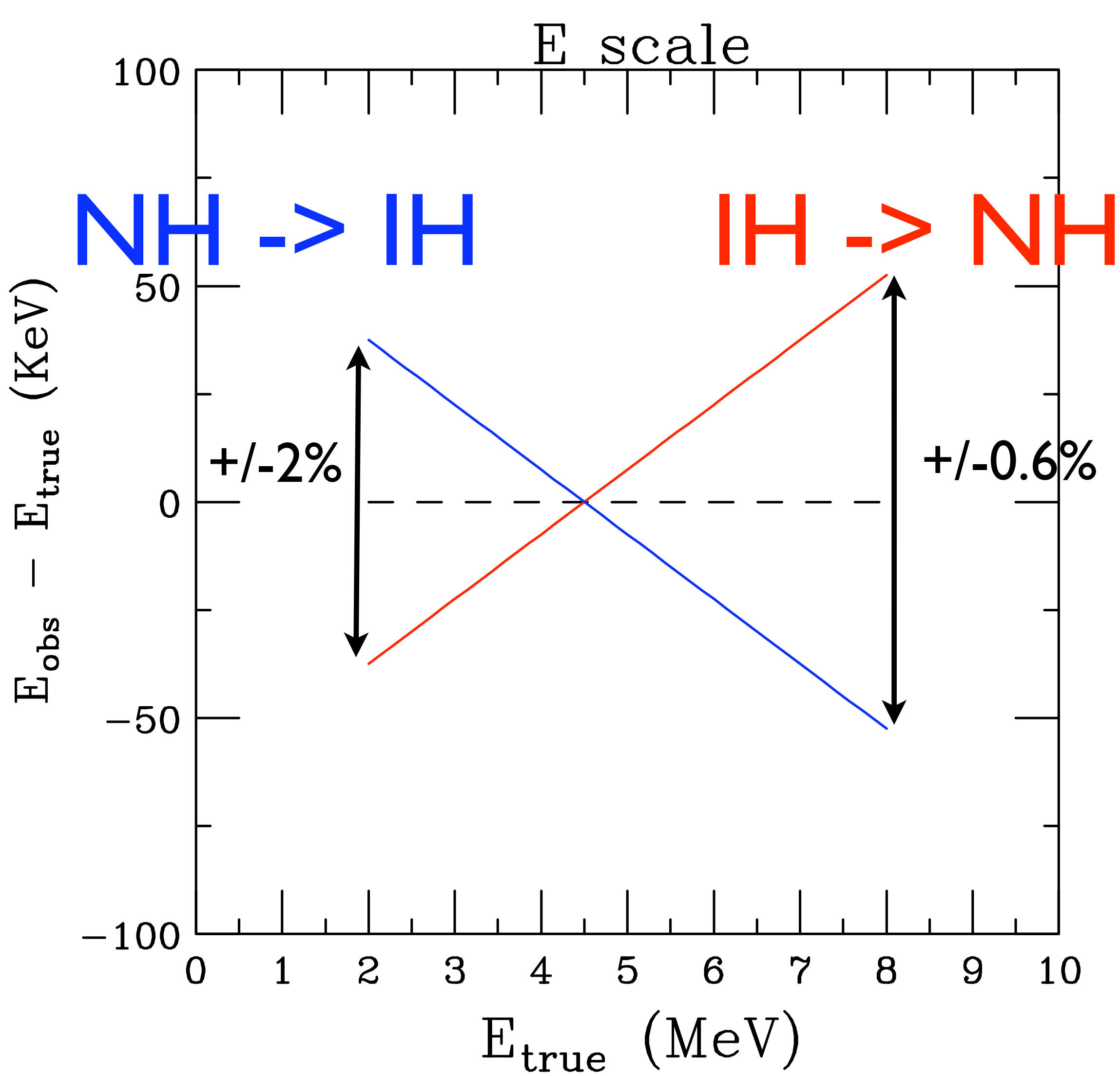


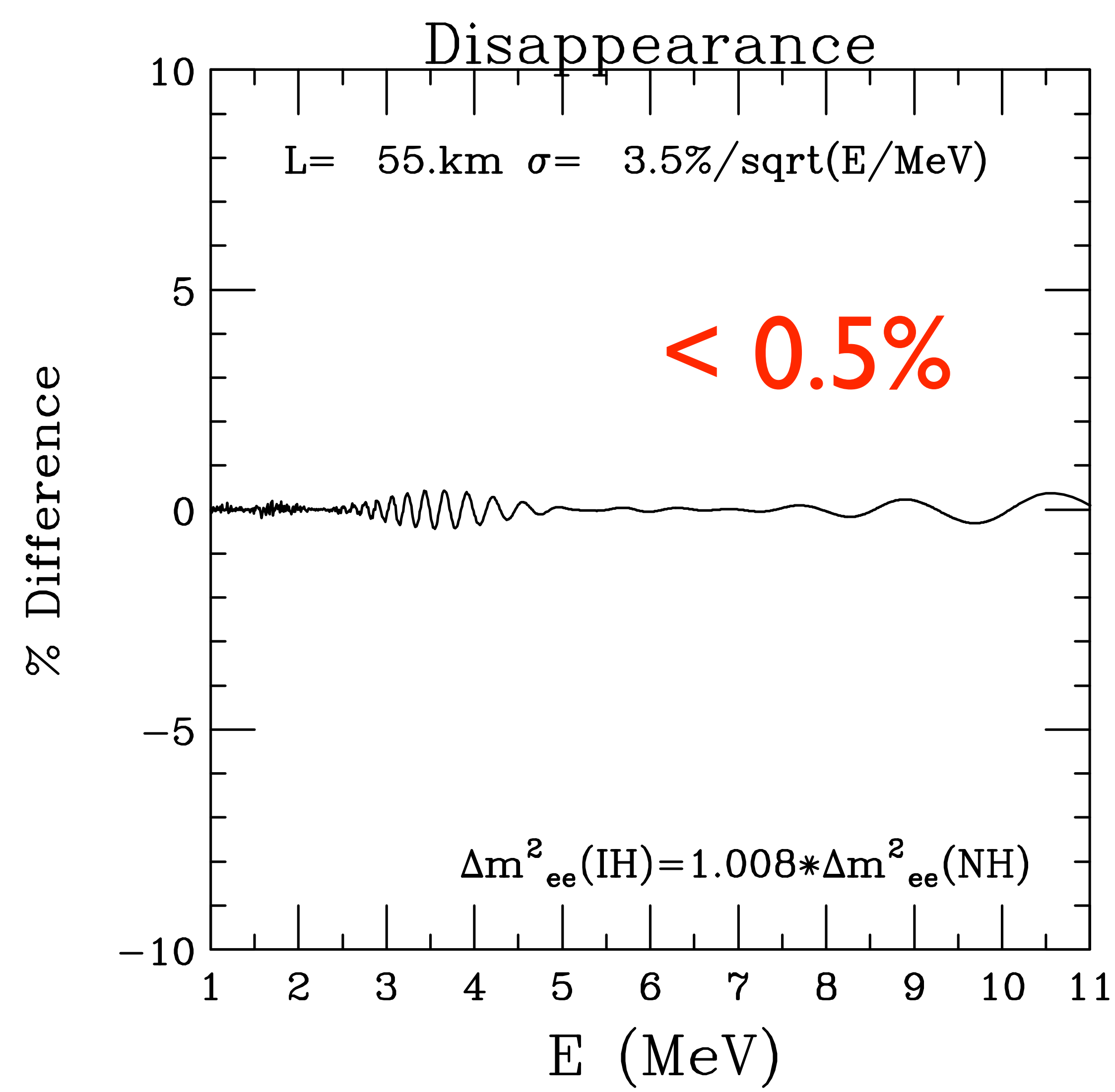
$$E_{obs} = E_{true} + 0.015 \times (E_{true} - 4.5)$$

$$E_{obs} = E_{true} - 0.015 \times (E_{true} - 4.5)$$

Uncertainty in E scale ???
between 2 and 8 MeV !!!

$$\frac{P_{IH}(E_{obs}) - P_{NH}(E_{true})}{P_{NH}(E_{true})}\%$$





$$E_{obs} = E_{true} + 0.015 \times (E_{true} - 4.5)$$

$$E_{obs} = E_{true} - 0.015 \times (E_{true} - 4.5)$$

Mossbauer Neutrinos:

Summary & Conclusions

The phase advancement or retardation of the atmospheric oscillation allows for the possibly determination of the neutrino mass hierarchy in $\bar{\nu}_e$ disappearance experiments: but it's quite a challenge:

- Even for monochromatic $\bar{\nu}_e$ beams (Mossbauer) this would require a high precision measurement of δm^2_{atm} around the first oscillation minimum as well as a determination of the phase 20 or so oscillations out !

  Challenging, but the high event rate that maybe possible with Mossbauer neutrinos could make this possible with modest size detectors.

- Reactor neutrinos using multi-cycle analyses (Fourier) requires high precision relative determination of the neutrino energy from 2 to 8 MeV.

  E.g. what you call a "6 MeV neutrino" must have twice the energy of what you call a "3 MeV neutrino" to about 1%, otherwise the hierarchies can be confused. This requirement is very challenging for reactor neutrinos.