AXIONIC DARK MATTER

Daniel Grin

FermiLab Workshop: New Perspectives on Dark Matter

4/28/2014

Collaborators:
Marc Kamionkowski, Tristan Smith, Pedro Ferreira, Renee Hlozek, D. Marsh, K. Z. Khor, J.P. Kneib, Eric Jullo, Andrew Blain, Giovanni Covone

Outline

- Strong CP Problem, QCD Axion, couplings
- How to make axions in the expanding universe
- Experimental limits
- Cosmological limits [+BICEP2!]
- Ultra-light axions and ALFs from string theory

The strong CP problem

- Strong interaction violates CP through θ -vacuum term

$$\mathcal{L}_{\rm CPV} = \frac{\theta g^2}{32\pi^2} G\tilde{G}$$

- Limits on the neutron electric dipole moment are strong. Fine tuning?

$$d_n \simeq 10^{-16}\ \theta \ \mathrm{e\ cm}$$

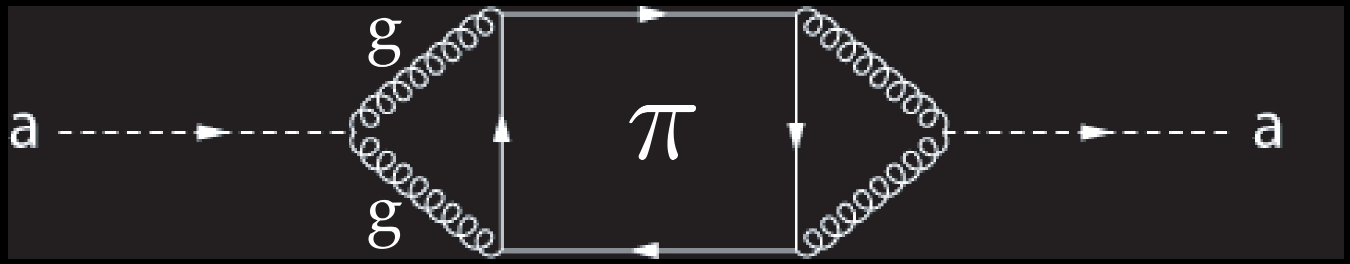
$$\theta \lesssim 10^{-10},$$

Axions solve the strong CP problem

- New field (axion) and U(1) symmetry dynamically drive net CP-violating term to 0

$$\mathcal{L}_{\mathrm{CPV}} = \frac{\theta g^2}{32\pi^2} G\tilde{G} - \frac{a}{f_{\mathrm{a}}} g^2 G\tilde{G}$$

- Through coupling to pions, axions pick up a mass

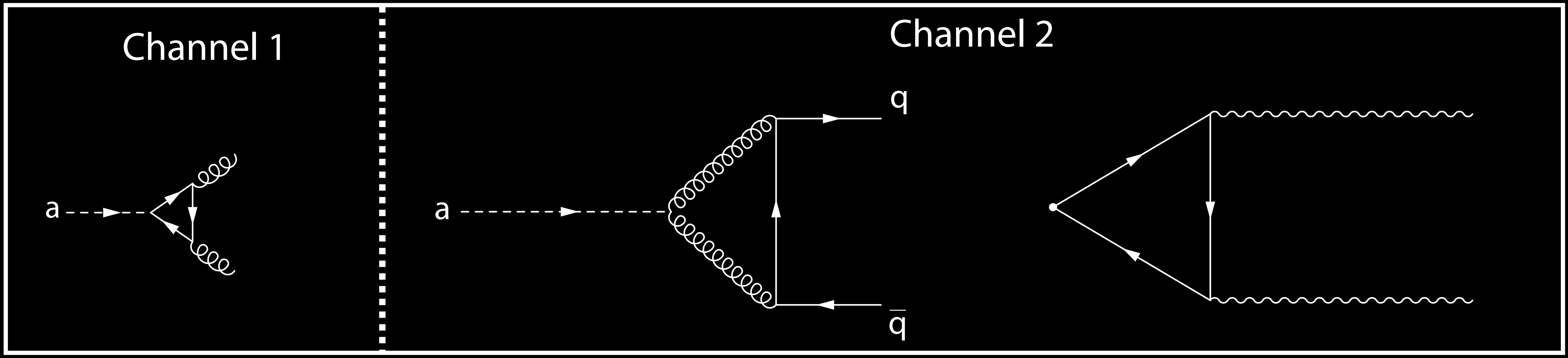


$$m_{\mathrm{a}} \simeq \frac{m_\pi f_\pi}{f_{\mathrm{a}}} \frac{\sqrt{r}}{1+r}$$

$$r \equiv m_{\mathrm{u}}/m_{\mathrm{d}}$$

$$m_a = 6.2\mu \text{ eV} \left(\frac{10^{12} \text{ GeV}}{f_{\mathrm{a}}} \right)$$

Two-photon coupling of axion

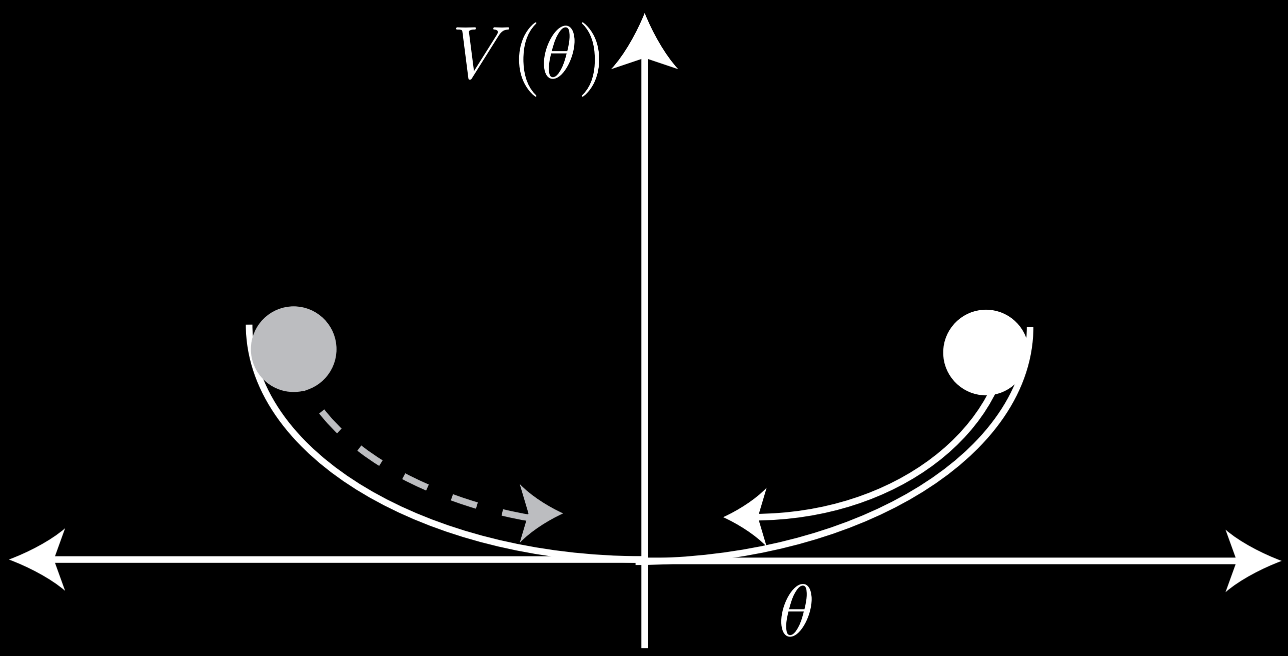


- Axions interact weakly with SM particles $\Gamma, \sigma \sim \alpha^2$
- Axions have a two-photon coupling

$$g_{a\gamma\gamma} = -\frac{3\alpha}{8\pi f_a}\xi$$

2 axion populations: Cold axions

$m_a < 10^{-2}$ eV



- Before PQ symmetry breaking, θ is generically displaced from vacuum value
- EOM: $\ddot{\overline{\theta}} + 3H\dot{\overline{\theta}} + m_{\mathrm{a}}^2(T)\,\overline{\theta} = 0$ $\quad m_{\mathrm{a}}(T) \simeq 0.1 m_{\mathrm{a}}(T=0)\left(\Lambda_{\mathrm{QCD}}/T\right)^{3.7}$
- After $m_{\mathrm{a}}(T) \gtrsim 3H(T)$, coherent oscillations begin, leading to $n_{\mathrm{a}} \propto a^{-3}$
- Axions are cold $p \ll m_a c$

Dark matter axion abundance

- QCD axion couples to quarks/pions, temp-dependent mass
  - High-temp regime

$$m_{\rm a} = 0.02 m_{\rm a}^{(T=0)} \left(\frac{\Lambda_{\rm QCD}}{T}\right)^4 \quad \text{if } T \gg \Lambda_{\rm QCD}$$

  - Low-temp regime $m_{\rm a} = m_{\rm a}^{(T=0)}$ if $T \lesssim \Lambda_{\rm QCD}$

$$\Omega_{\rm mis} h^2 = 0.236 \left\langle \theta_i^2 f(\theta_i) \right\rangle \left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{7/6} \quad \text{if } f_a \lesssim 10^{18}\ \text{GeV}$$

$$\Omega_{\rm mis} h^2 = 0.005 \left\langle \theta_i^2 f(\theta_i) \right\rangle \left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{3/2} \quad \text{if } f_a \gtrsim 10^{18}\ \text{GeV}$$

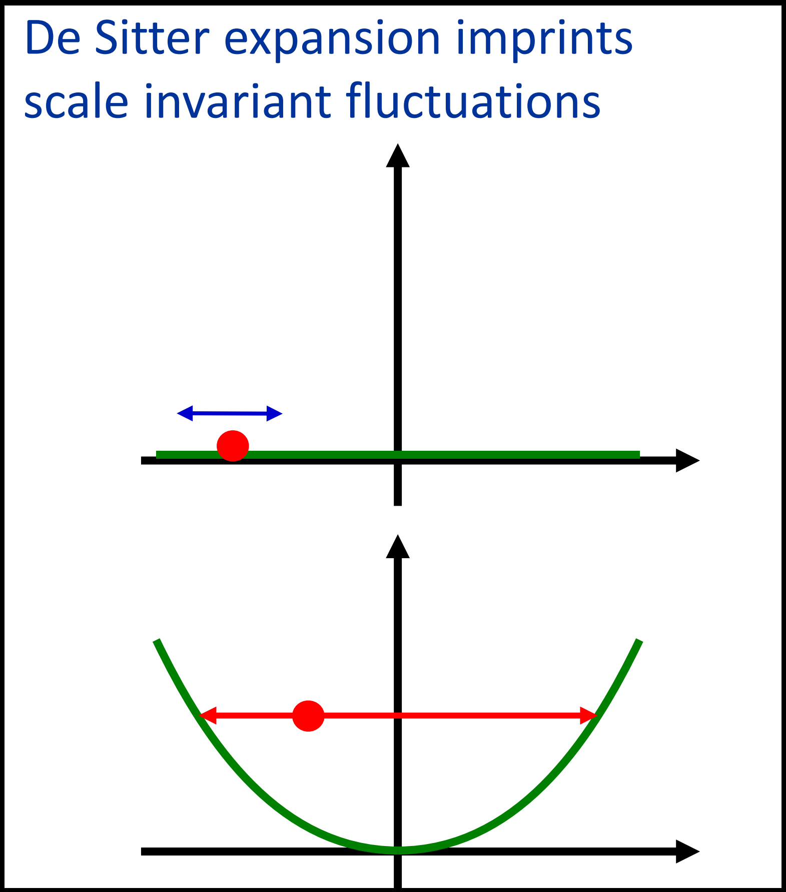
Anthropic axion window: $f_a > \max\{T_{\rm RH}, H_{\rm I}\}$

- Axion field is relatively homogeneous

$$\langle\theta^2\rangle = \bar{\theta}^2 + \left(\frac{H_I}{2\pi f_a}\right)^2$$

Vacuum fluctuations from inflation

Misalignment in our Hubble Patch



From Raffelt 2012

- Abundance

$$\Omega_a h^2 \simeq 0.43 \left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{7/6} \theta_i^2$$

$$\Omega_a h^2 \simeq 0.005 \left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{3/2} \theta_i^2$$

- $\bar{\theta}$ can be tuned to get DM abundance for many axion masses

Classic axion window: $f_a < \max\{T_{\rm RH}, H_{\rm I}\}$

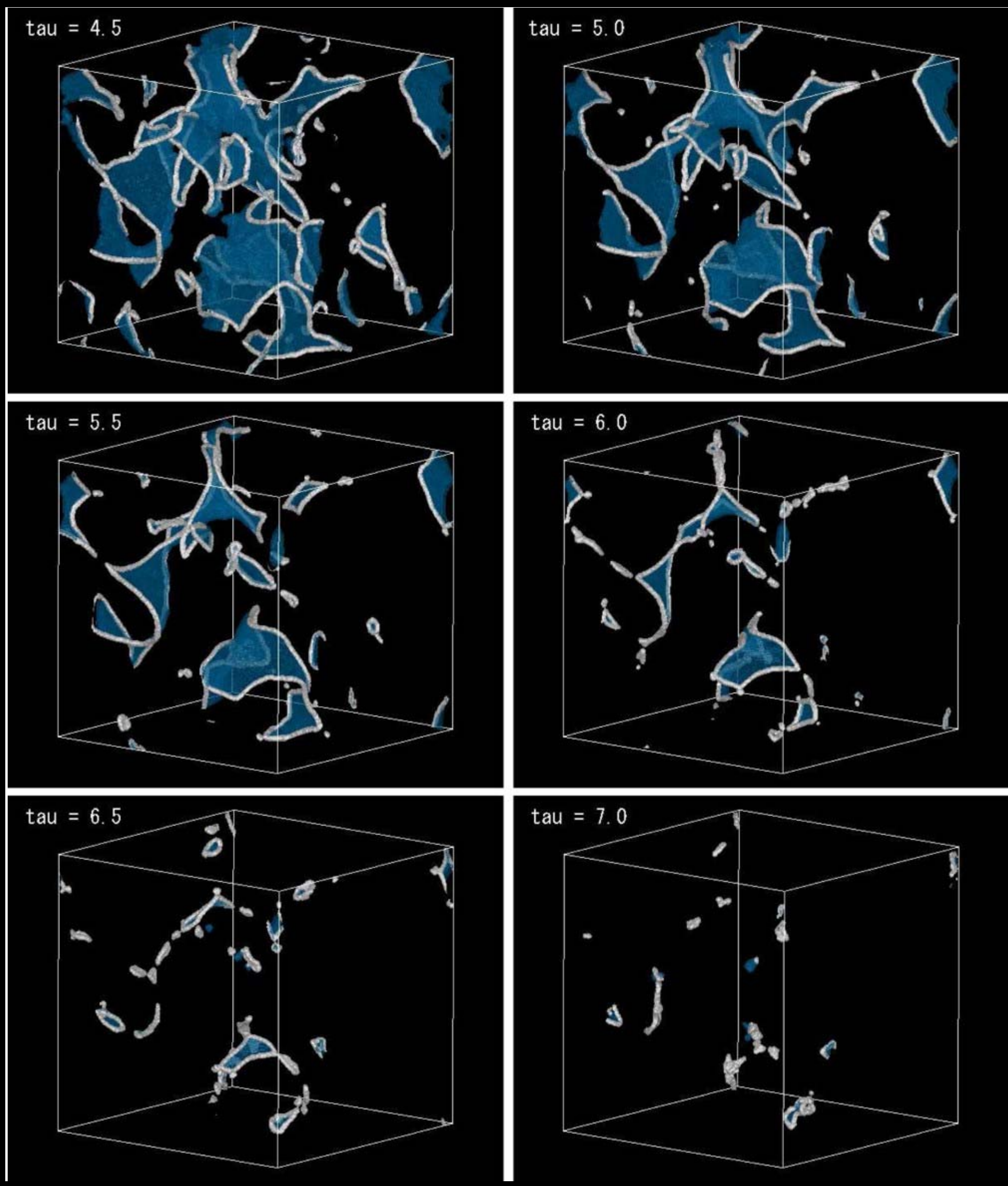
- ✴ Axion field is very inhomogeneous

$$\left\langle \overline{\theta}_i^2 \right\rangle = \frac{\pi^2}{6}$$

- ✴ Defects [domain walls, strings, etc..]

$$\mathcal{O}(1) \lesssim \alpha_{\rm defect} \lesssim \mathcal{O}(10^2)$$

CONTROVERSY!



From Hiramatsu 2012

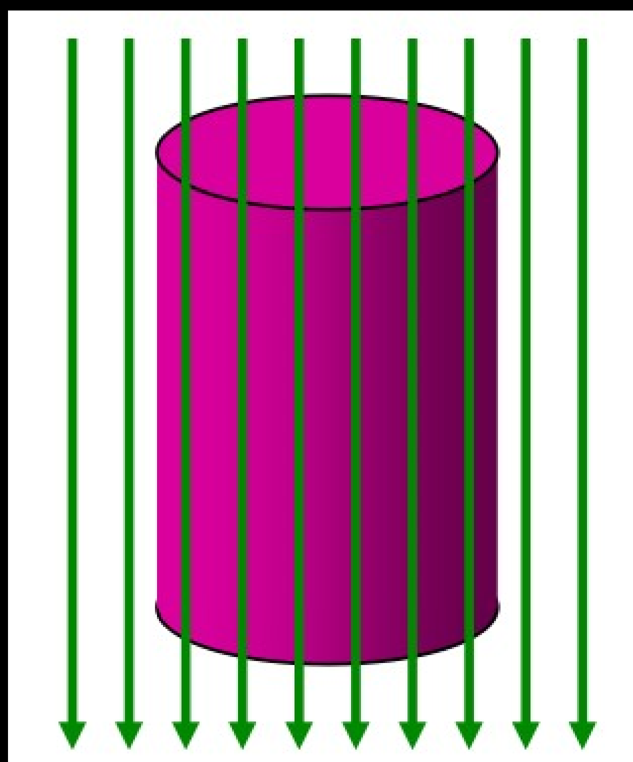
- ✴ Abundance

$$\Omega_a h^2 \simeq 2.0\left\{1+f_{\rm defect}\right\}\left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^{7/6}$$

Cavity searches [e.g. ADMX]

Images from Wong 2012

- Magnetized RF Cavity



- Axion excites cavity (TEM) modes [cavity must be tuned]

$$E_\gamma = m_a c^2$$

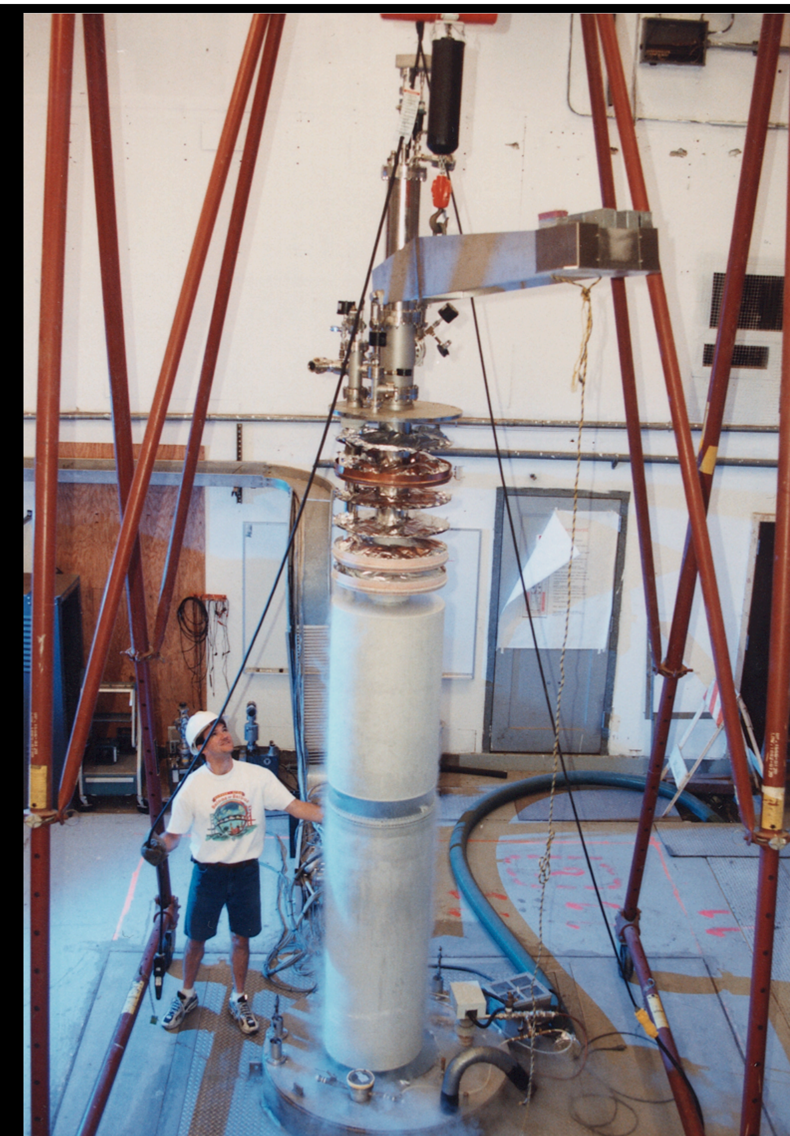
- Power

$$\text{Power} = g_{a\gamma}^2 \frac{V B^2 \rho_a Q}{m_a} \sim 10^{-21} \text{ Watts}$$

Volume

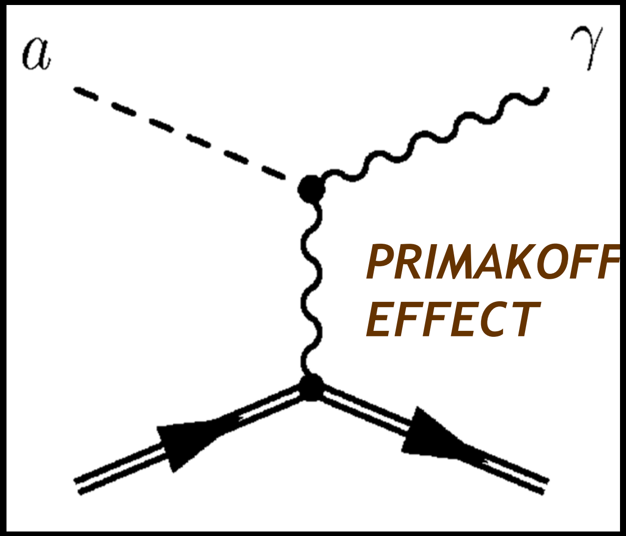
Quality factor

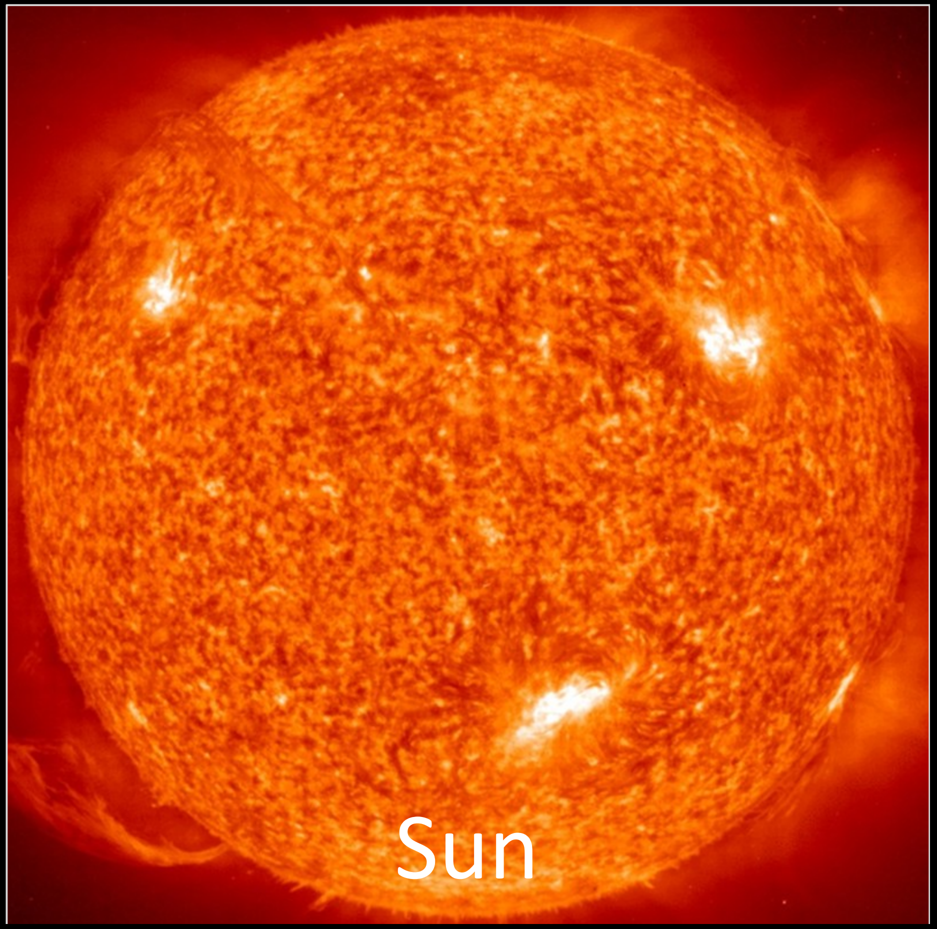
Axion energy density



Making axions in stars

- Primakoff process



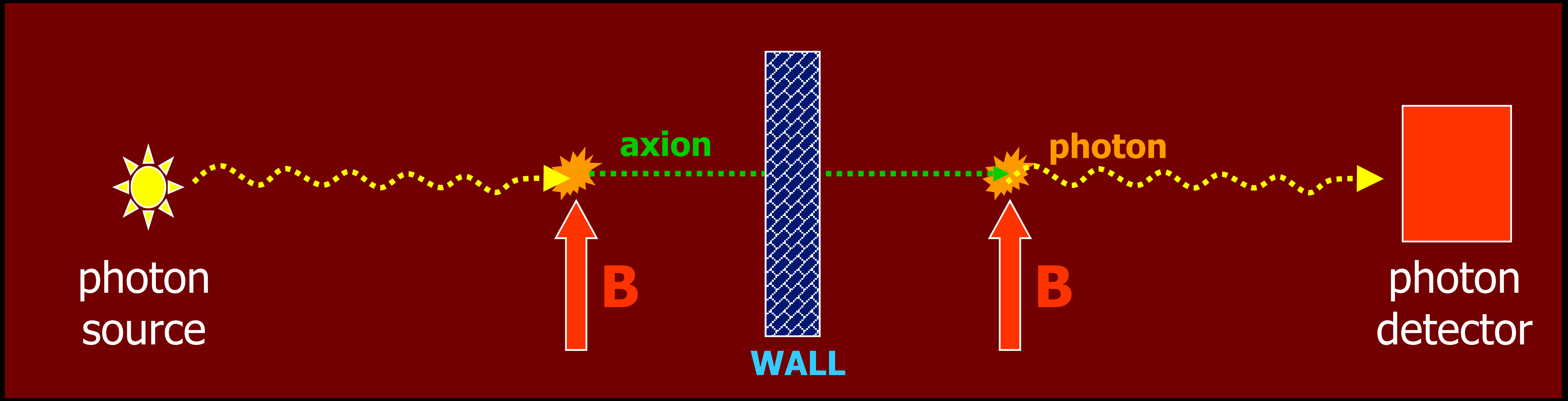


- Lifetime of our own sun/Solar luminosity/helioseismology impose constraint

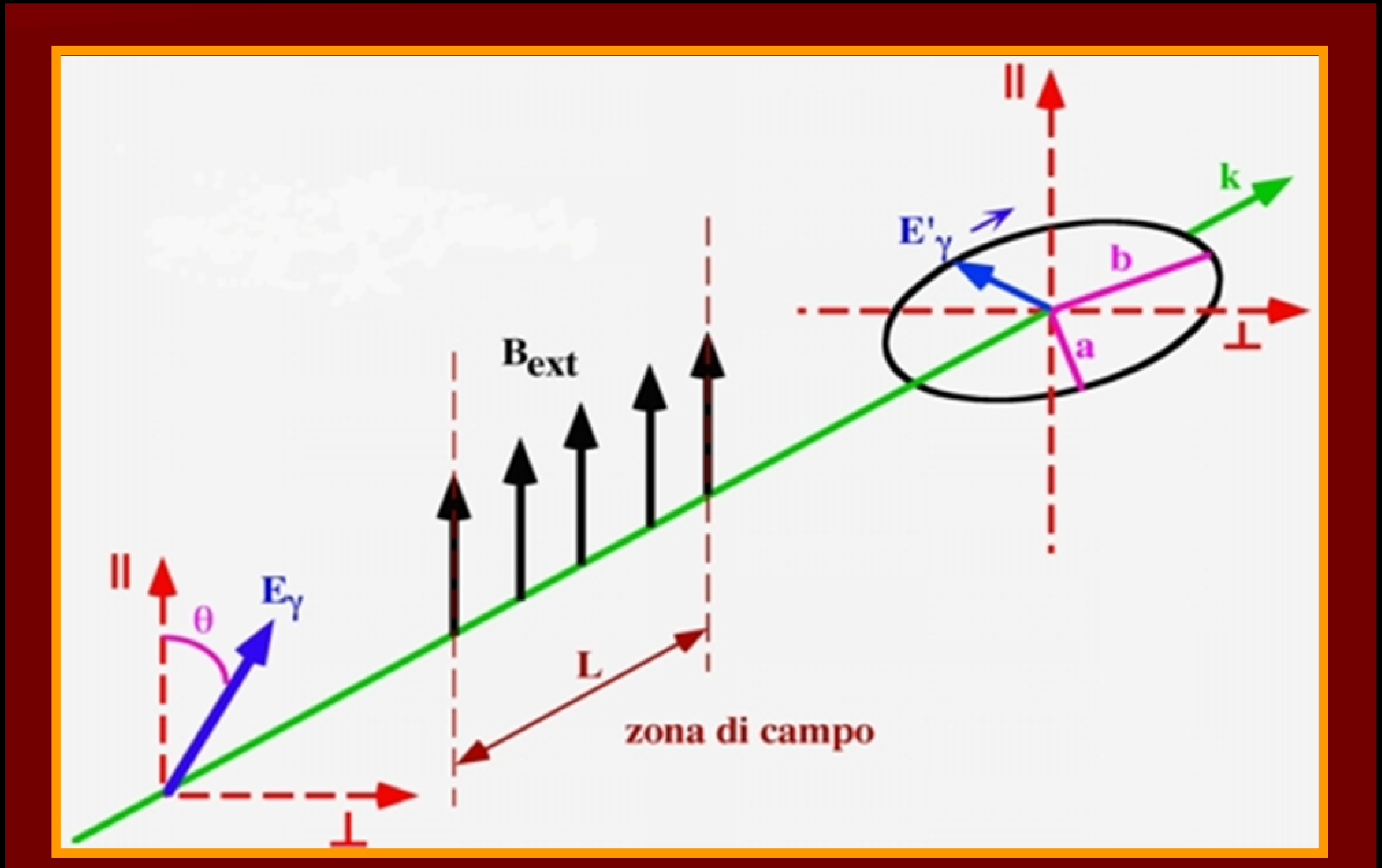
$$g_{a\gamma\gamma} \lesssim 1 - 3 \times 10^{-9}\ \mathrm{GeV}^{-1}$$

Laser experiments

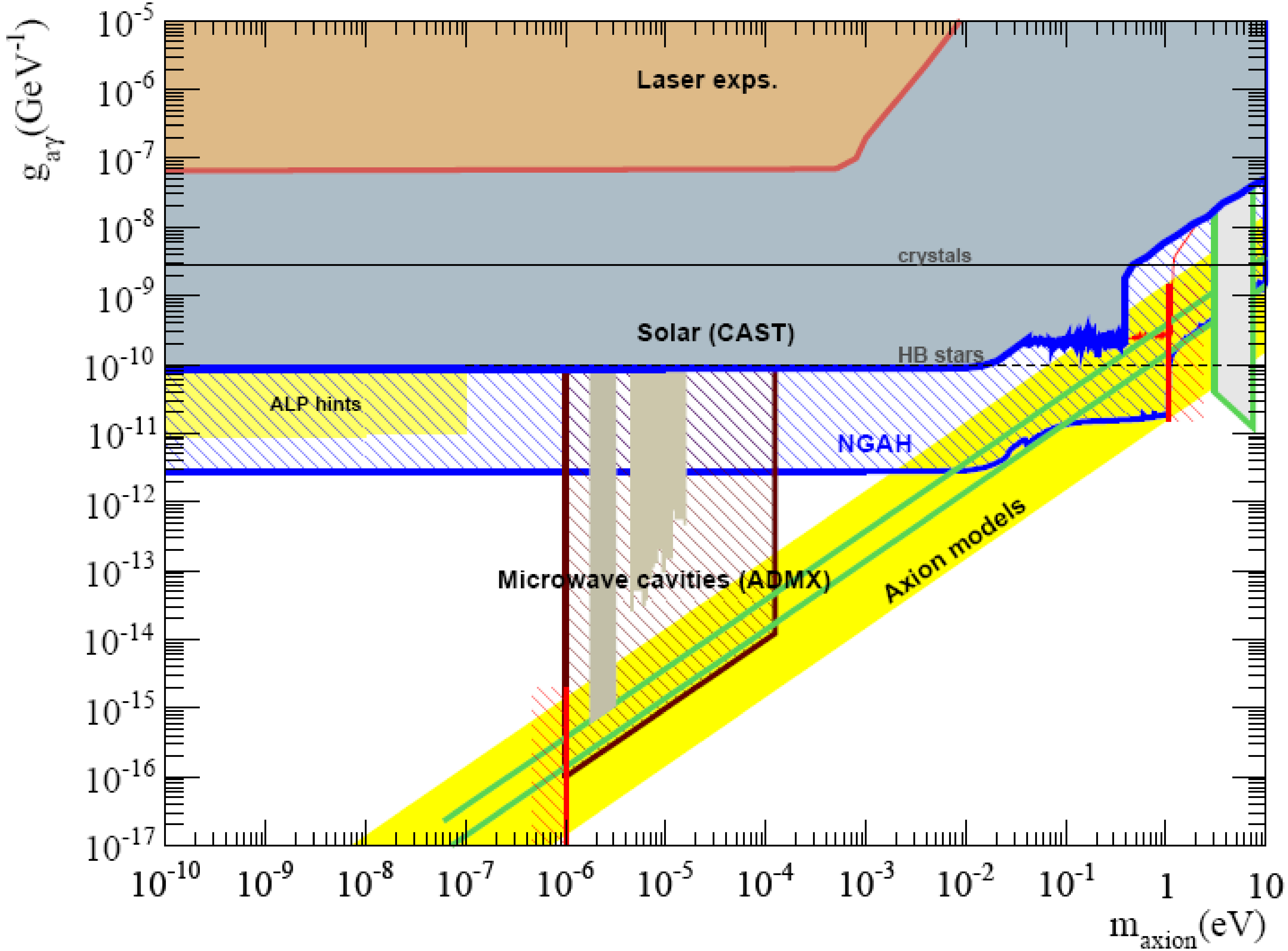
Light shining through walls (e.g. GammeV)



Polarization experiments (e.g. PVLAS)



Limits and horizon



Other methods

- Spectra of magnetic white dwarves **[New]**
- Extragalactic background light
- Pulsating white dwarf seismology [**New**]
- Dimming of gamma-ray blazars **[New]**
- Two-photon decays in galaxy clusters
- Light degrees of freedom at BBN [**New**]
- Helioscope in space [**New**]
- Supernovae 1987a
- White dwarf luminosity function
- Oscillating electric dipole moments of nucleons [**NEW**]

Axions carry isocurvature

- If PQ symmetry broken during/before inflation

$$\sqrt{\langle a^2 \rangle} = \frac{H_{\rm I}}{2\pi}$$

Quantum zero-point fluctuations!

- Subdominant species seed isocurvature fluctuations

$$\zeta \propto \frac{\rho_a}{\rho_{\rm tot}} \frac{\delta\rho_a}{\rho_a} \ll 10^{-5}$$

$$S_{a\gamma} = \frac{\delta n_a}{n_a} - \frac{\delta n_\gamma}{n_\gamma} = \frac{\delta\rho_a}{\rho_a} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} \sim 10^{-5}$$

The axion and the scale of inflation

... story laid out by Fox, Mack, Steinhart, Hertzberg, Wilczek, Gondolo [and others]

- ✳ Tensor mode amplitude set by inflationary energy scale

$$\frac{k^3 P_h}{2\pi^2} = 8\left(\frac{H_{\rm I}/M_{\rm pl}}{2\pi}\right)^2 \qquad \frac{k^3 P_{\rm R}}{2\pi^2} = \frac{1}{2\epsilon}\left(\frac{H_{\rm I}/M_{\rm pl}}{2\pi}\right)^2 \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$\frac{k^3 P_S}{2\pi^2} = 4\left(\frac{H_{\rm I}}{2\pi\phi}\right)^2$$

- ✳ Isocurvature probes quantity

$$\bar{\theta} \sim \Omega_a^{1/2} f_{\rm a}^{-7/12}$$

$$\left(\frac{H_I}{f_{\rm a}\bar{\theta}}\right)\left(\frac{\Omega_a}{\Omega_d}\right) \rightarrow f_a^{5/12} \gtrsim \left(\frac{H_{\rm I}}{10^{12}\ {\rm GeV}}\right)$$

The axion and the scale of inflation

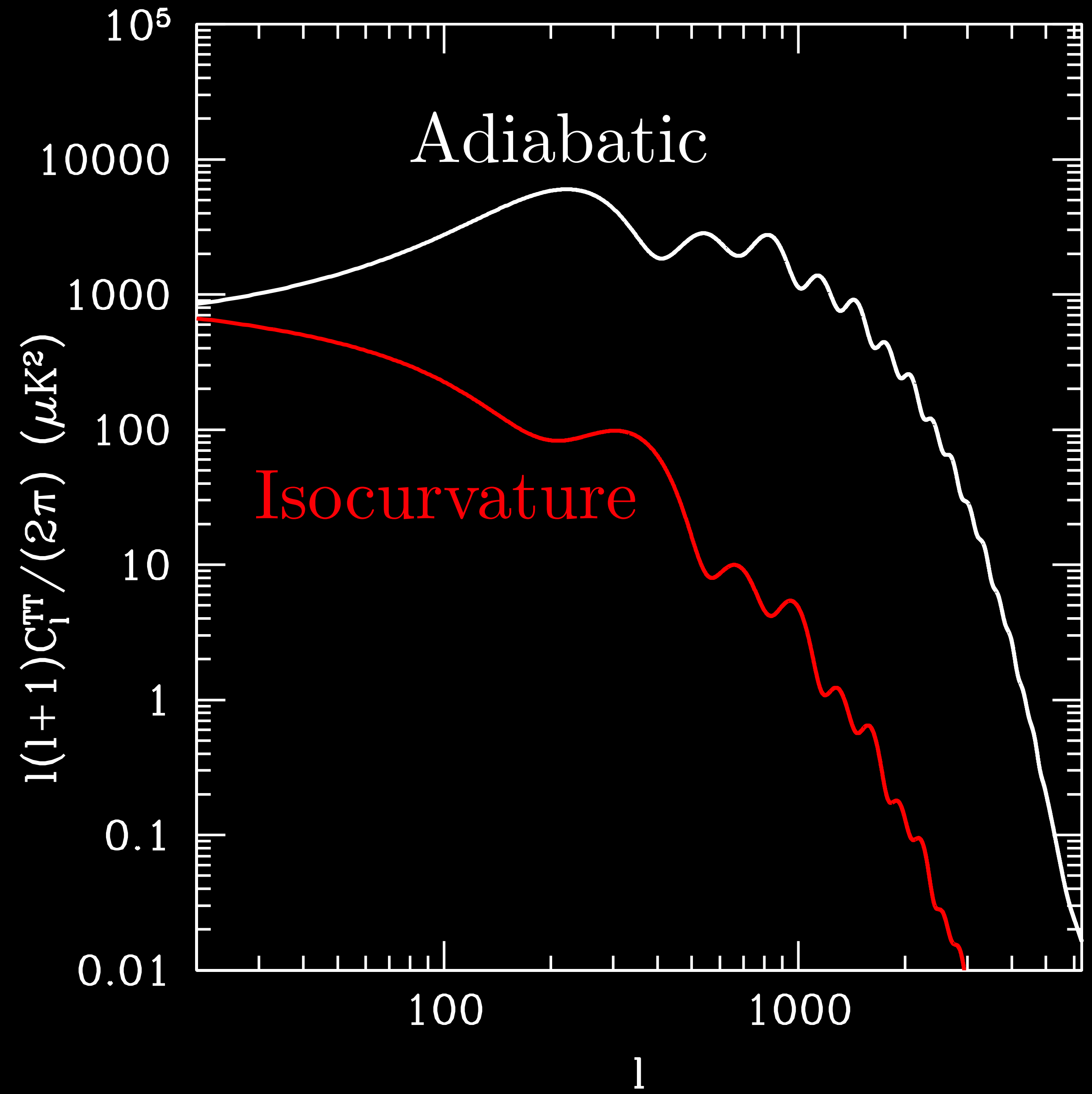
Komatsu al. 2008/2011 find

$$\alpha_{\rm ax} \lesssim 0.1$$

$$r \sim 5 \times 10^{-12} \left(\frac{\Omega_{\rm c}}{\Omega_{\rm a}}\right)^{2/7}$$

Komatsu al. 2008/2011 (WMAP)

SACHS WOLFE-EFFECT & POWER SPECTRA

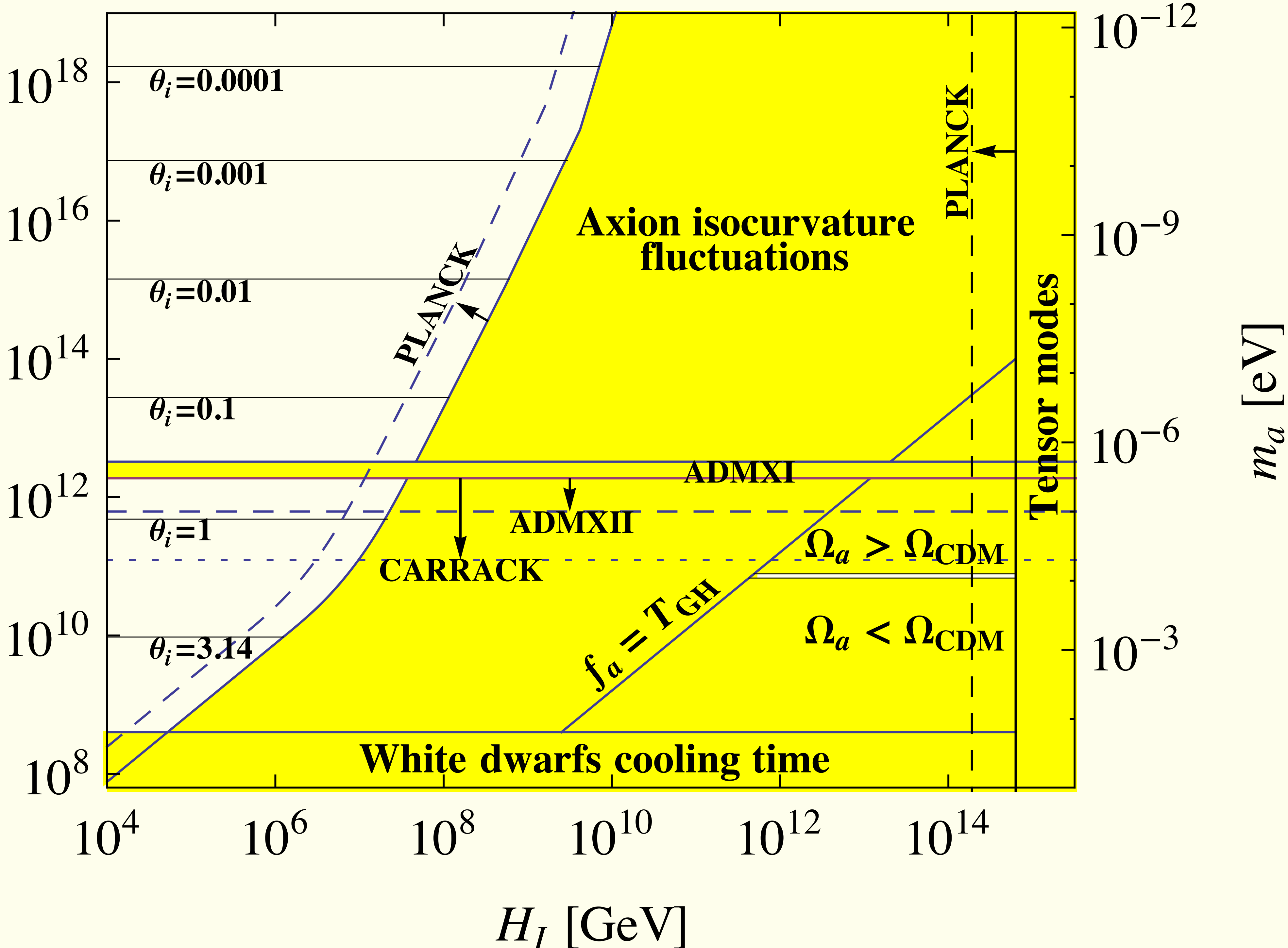


- Planck TT constraints

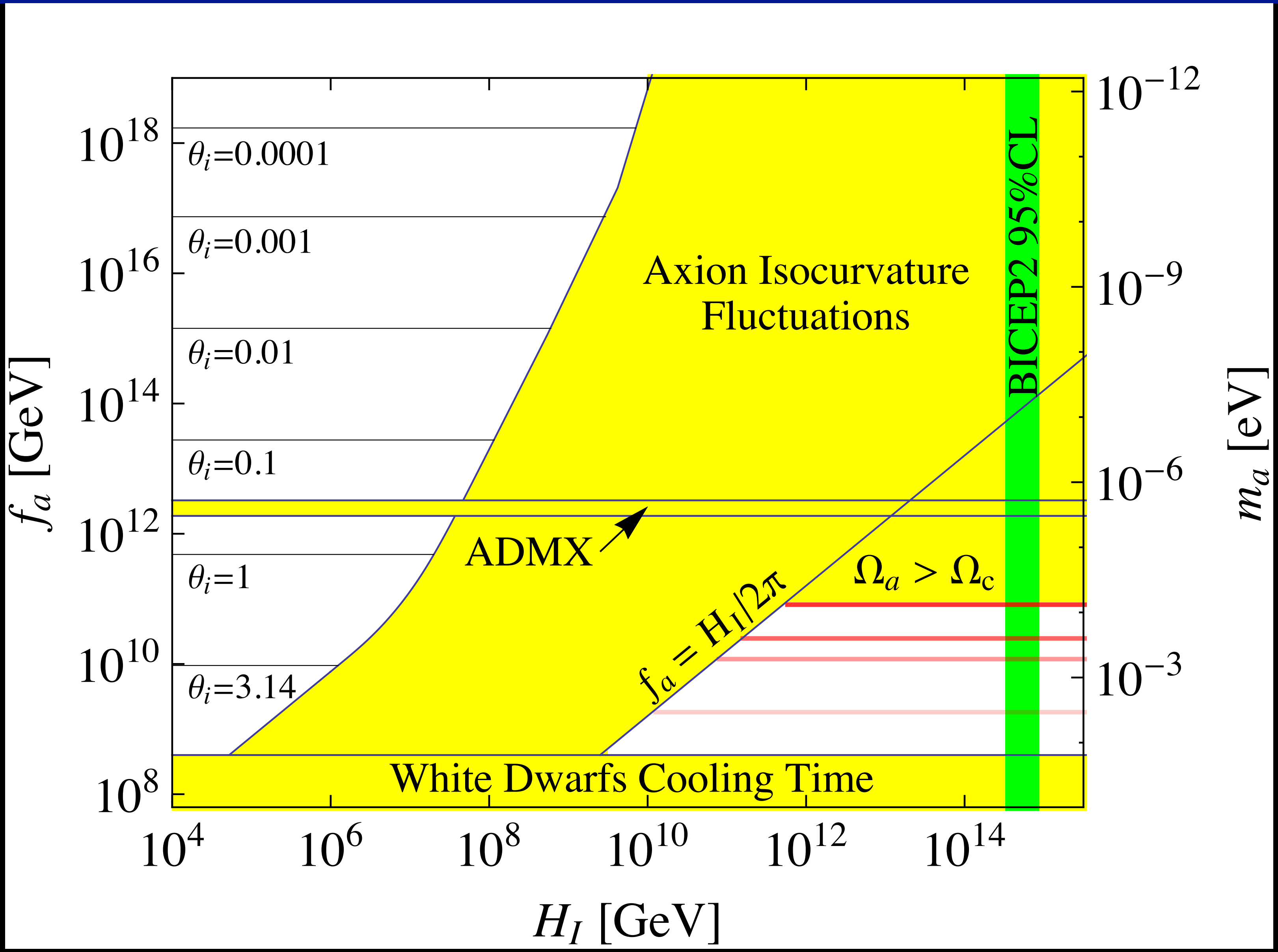
$$\frac{P_{\rm iso}}{P_{\rm tot}} \lesssim 1.6 \times 10^{-2}$$

$$\frac{H_I}{f_a \bar{\theta}} \frac{\Omega_a}{\Omega_{\rm d}} \lesssim 4 \times 10^{-5}$$

(Gondolo 2009):
ADMX axions still viable if low-scale inflation or in classical window



BICEP2 [inflationary energy scale detected?]



- Hard to accomodate QCD axion DM w/o defects! [Marsh+yours truly +others 1403.4216 (2014), Gondolo et al. 2014 1403.4594]

$$\frac{\Omega_a}{\Omega_d} \lesssim 5 \times 10^{-12} \left(\frac{f_a}{10^{16}\ \mathrm{GeV}} \right)^{5/6}$$

A new scale for perturbed scalars

- *Perturbations obey*

$$\delta\ddot{\phi} + 2\mathcal{H}\delta\dot{\phi} + \left(k^2 + m^2a^2\right)\delta\phi = -\dot{\phi}_0\dot{h}/2$$

- *Structure suppressed when*

$$k \gg k_{\rm J} \sim \sqrt{m\mathcal{H}}$$

- *Scales are very small for QCD axion*

$$\lambda \sim 10^{10}~{\rm cm}$$

What about lighter axions?

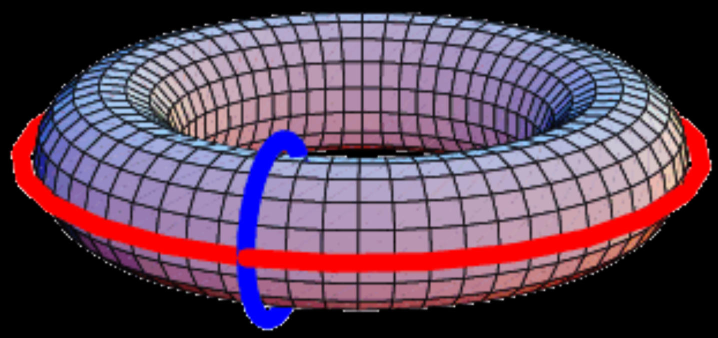
A cosmological search for ultra-light axions

with D. J.E. Marsh, R. Hlozek and P. Ferreira

arXiv:1303.3008, *Phys. Rev. D 87, 121701(R) (2013)*
(with MCMC results and methods paper in progress)

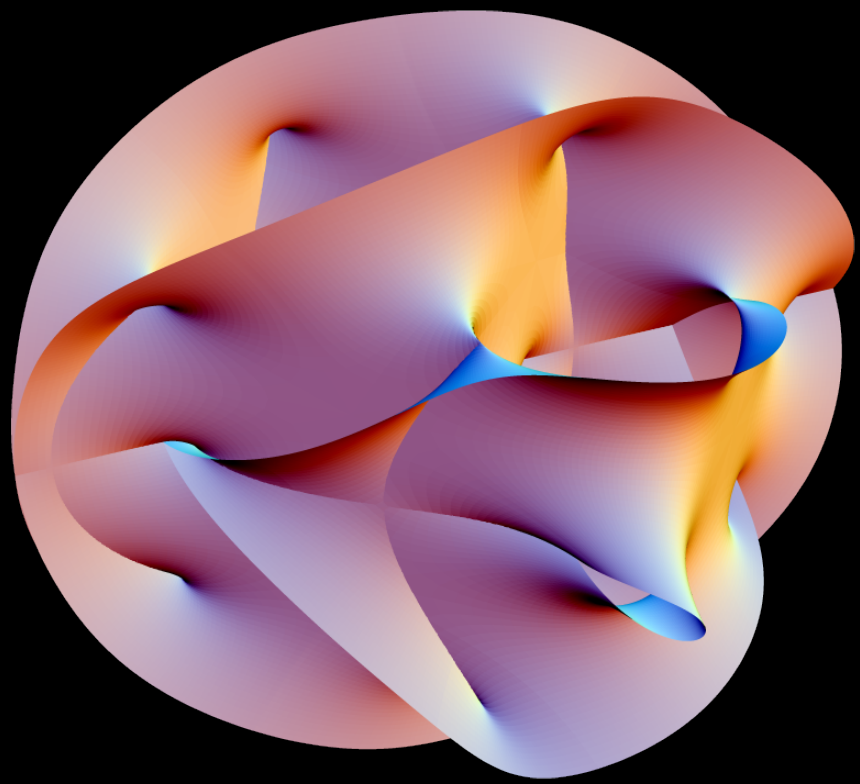
Light axions and string theory

- String theory has extra dimensions: *compactify (6)!*
- Form fields and gauge fields: `Axion' is KK zero-mode of form field



$$\mathcal{L} \propto \frac{aG\tilde{G}}{f_{\rm a}}$$

Axiverse! (Arvanitaki et al. 2009)



- Calabi-Yau manifolds

  Many 2-cycles ⟶ Many axions

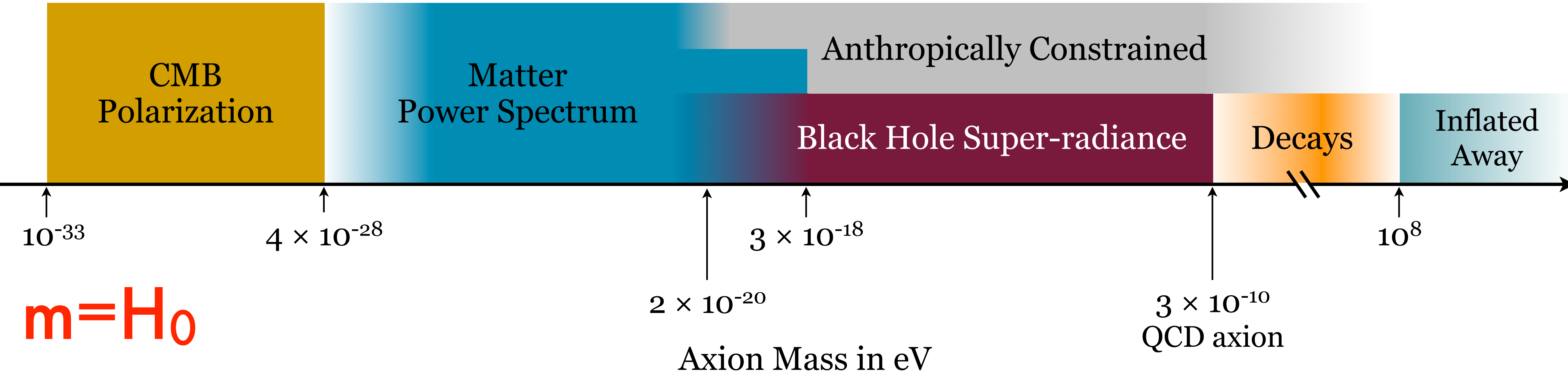
  Hundreds!

- Mass from non-perturbative physics (instantons, D-branes)

$$m_a^2 = \frac{\mu^4}{f_a^2} e^{-S} \quad f_a \propto \frac{M_{\rm pl}}{S}$$

Many decades in mass covered!

Axiverse! (Phenomena)



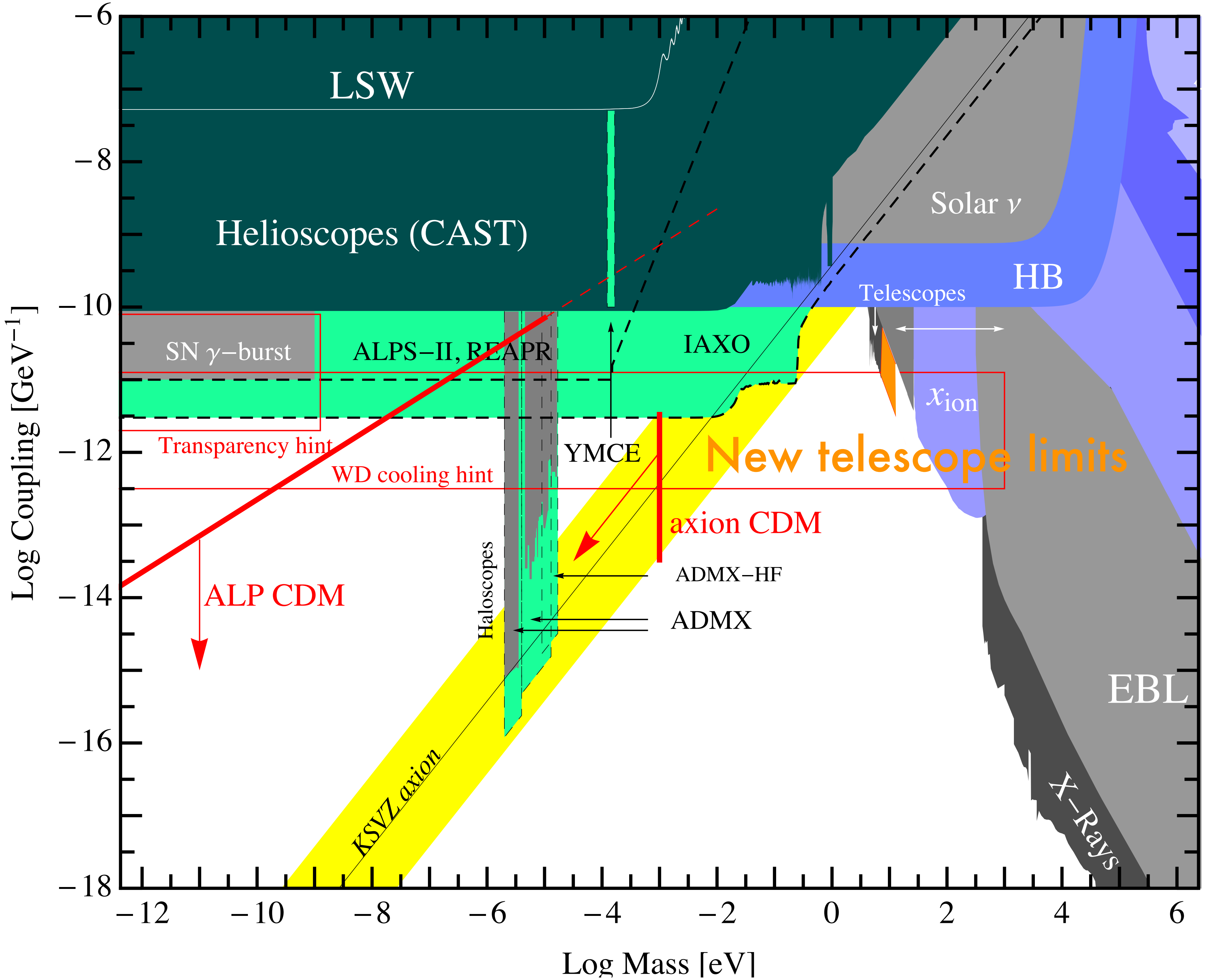
- ***Birefringence (Faraday rotation), model dependent:***

$$\mathcal{L} \propto \frac{a\vec{E}\cdot\vec{B}}{f_a}$$

- ***Decrement in matter power spectrum for***

$$k \gg k_{\rm J} \sim \sqrt{m\mathcal{H}}$$

Parameter space in context



Effective fluid approximation

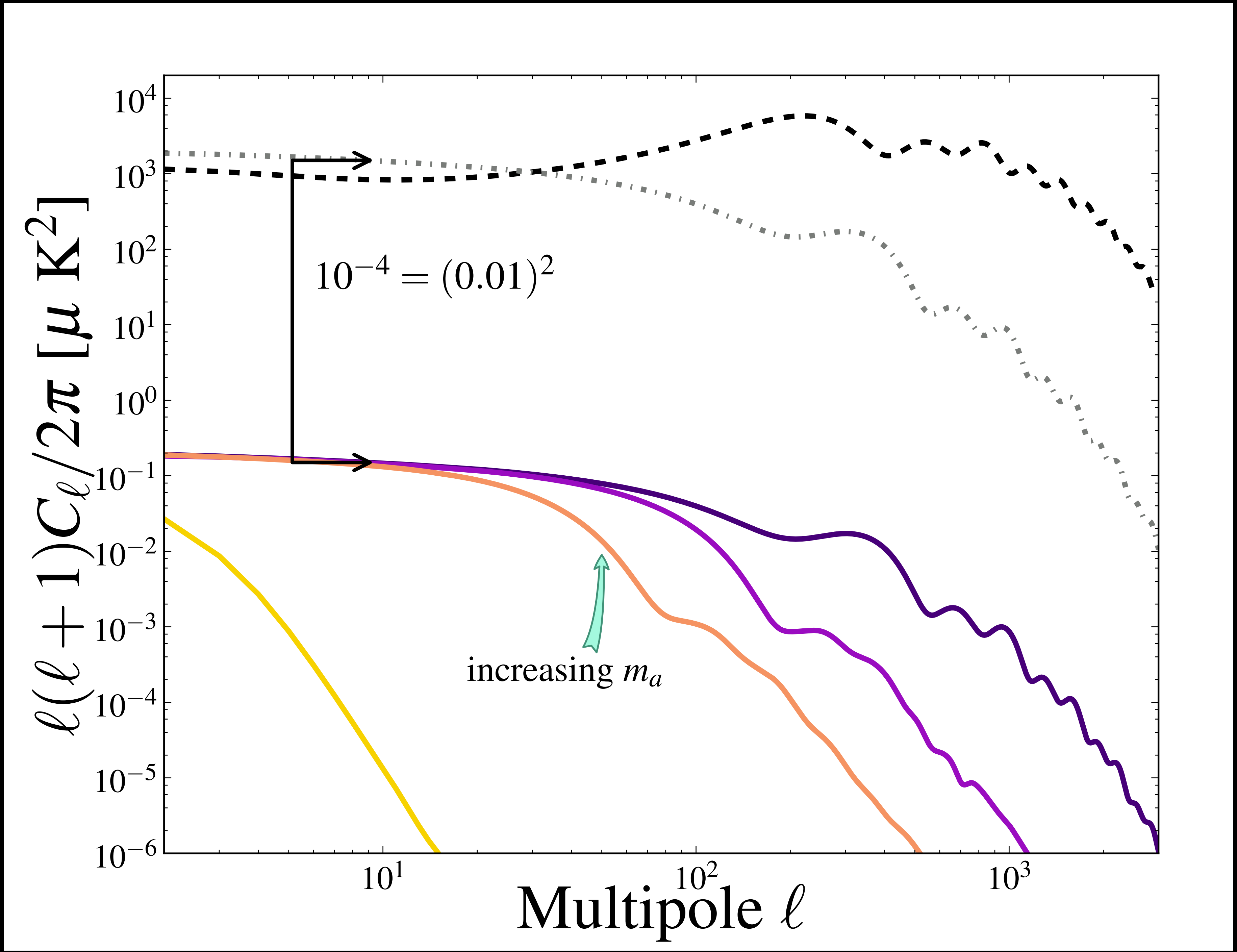
- Computing observables is expensive for $m \gg H_0$:
  - Coherent oscillation time scale

$$\Delta\eta \sim (ma)^{-1} \ll \Delta\eta_{\mathrm{CAMB}}$$

  - Ansatz $\delta\phi = A_c \Delta_c(k,\eta)\cos(m\eta) + A_s \Delta(k,\eta)\sin(m\eta)$

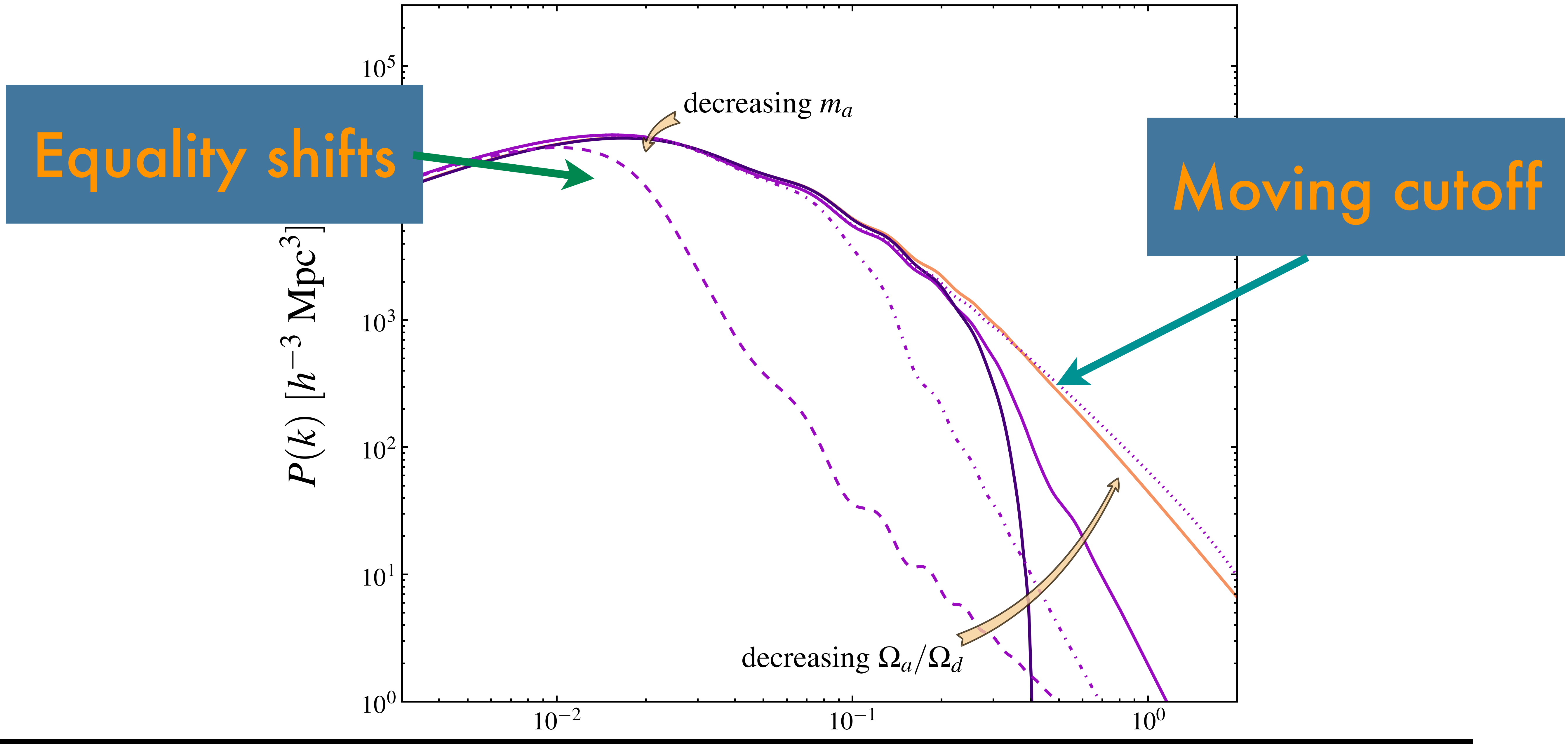
$$c_a^2 = \frac{\delta P}{\delta \rho} = \frac{k^2/(4m^2a^2)}{1+k^2/(4m^2a^2)}$$

CMB anisotropy power spectra



Power spectra may now be quickly computed for 15 orders of magnitude in axion mass!

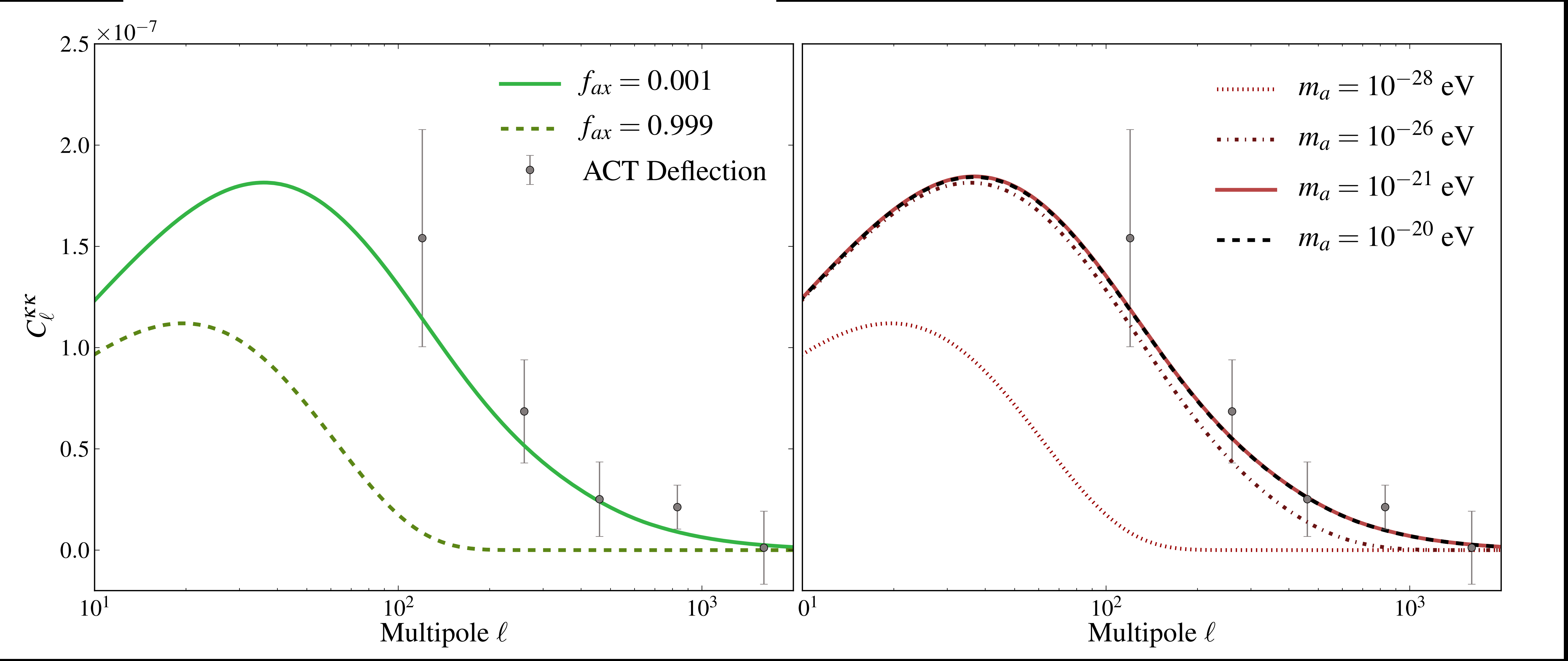
Matter power spectrum



We may now probe ultra-light axions and the axiverse with an MCMC covering 15 orders of magnitude in axion mass

CMB lensing [a probe of axions]

$$m_a \sim 10^{-28}\ \text{eV}$$



A new isocurvature signature [e.g. TE polarization]

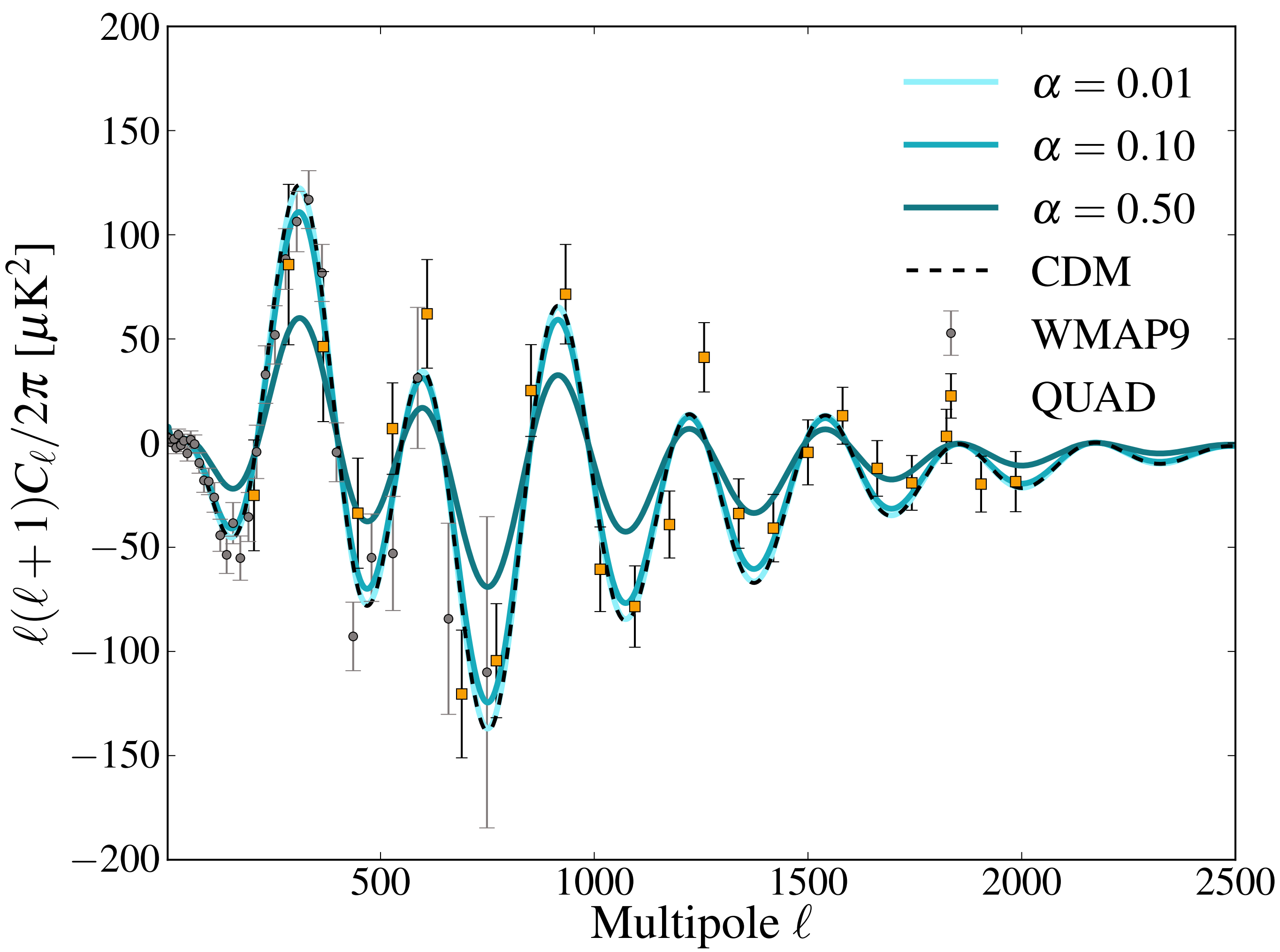


FIG. 5: CMB adiabatic and isocurvature TE polarization power spectra, varying the isocurvature amplitude from $\alpha = 0.01, 0.1, 0.5$ for fixed axion mass, with axions making up nearly all of the DM, $f_{\rm ax} = 0.9999$. The spectra are are a sum of $(1-\alpha)C_\ell^{ad} + \alpha C_\ell^{iso}$, hence adding in isocurvature removes adiabatic power, as can be seen by comparing the combined spectra to the adiabatic CDM-only spectrum, shown by the dashed curve.

The axiverse and the scale of inflation

- Tensor mode amplitude set by inflationary energy scale

Komatsu al. 2008/2011 find

$$\frac{k^3 P_h}{2\pi^2} = 8\left(\frac{H_{\rm I}/M_{\rm pl}}{2\pi}\right)^2 \qquad \frac{k^3 P_{\rm R}}{2\pi^2} = \frac{1}{2\epsilon}\left(\frac{H_{\rm I}/M_{\rm pl}}{2\pi}\right)^2\left(\frac{k}{k_0}\right)^{n_s-1}$$

$$\alpha_{\rm ax} \lesssim 0.1$$

$$\frac{k^3 P_S}{2\pi^2} = 4\left(\frac{H_{\rm I}}{2\pi\phi}\right)^2 \qquad \left(\frac{\phi}{M_{\rm pl}}\right)^2 = \frac{6H_0^2\Omega_a}{m_a^2 a_{\rm osc}^3}$$

$$r = 2.3\,\Omega_d h$$

$$r = 0.3\left(\frac{\Omega_d/\Omega_a}{100}\right)\left(\frac{10^{-33}\,{\rm eV}}{m_a}\right)^{1/2}$$

$$\left(\frac{\alpha}{1-\alpha}\right)$$

Stay tuned for MCMC constraints to the axiverse!

Motivation/anticipated contours

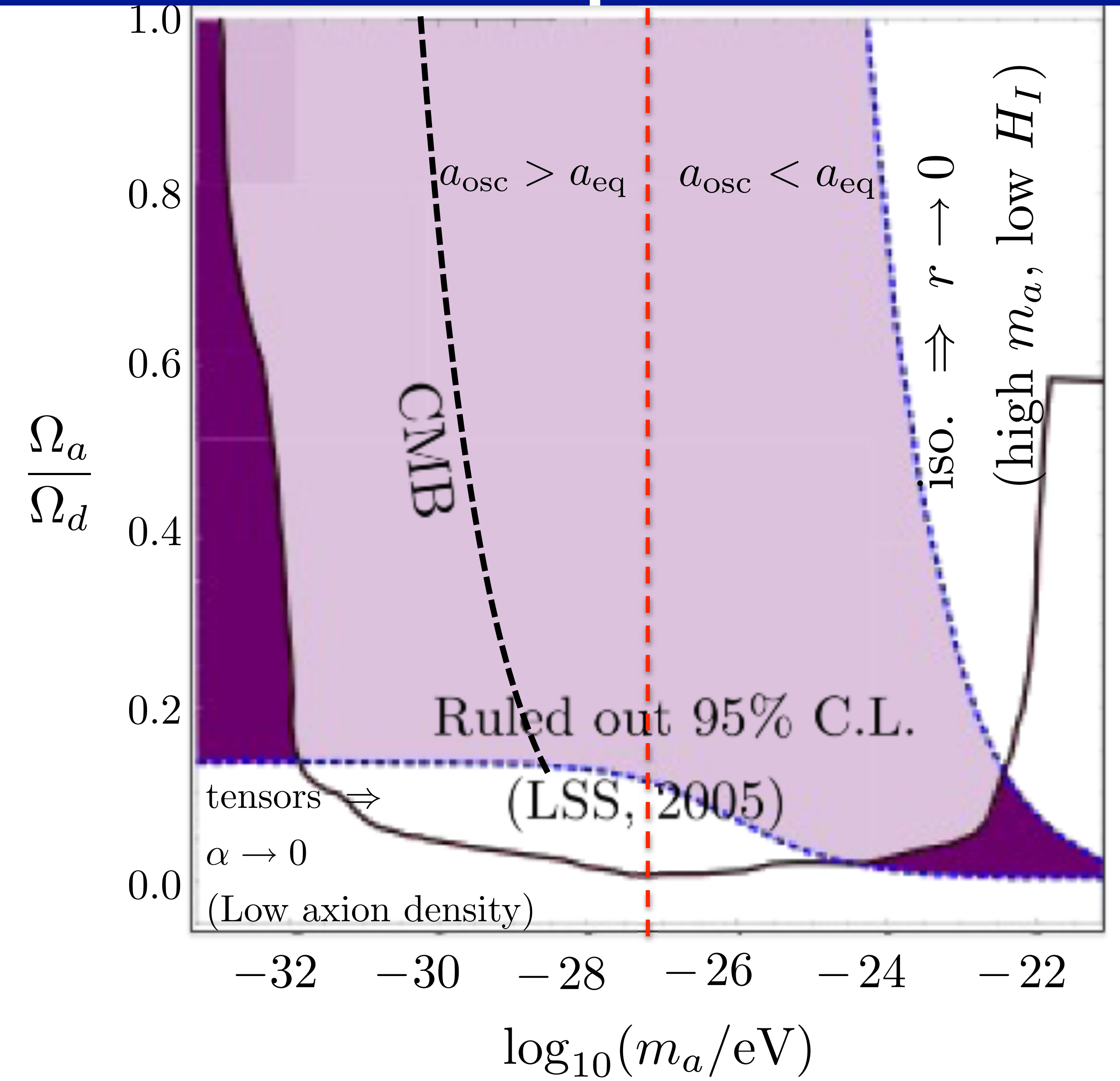
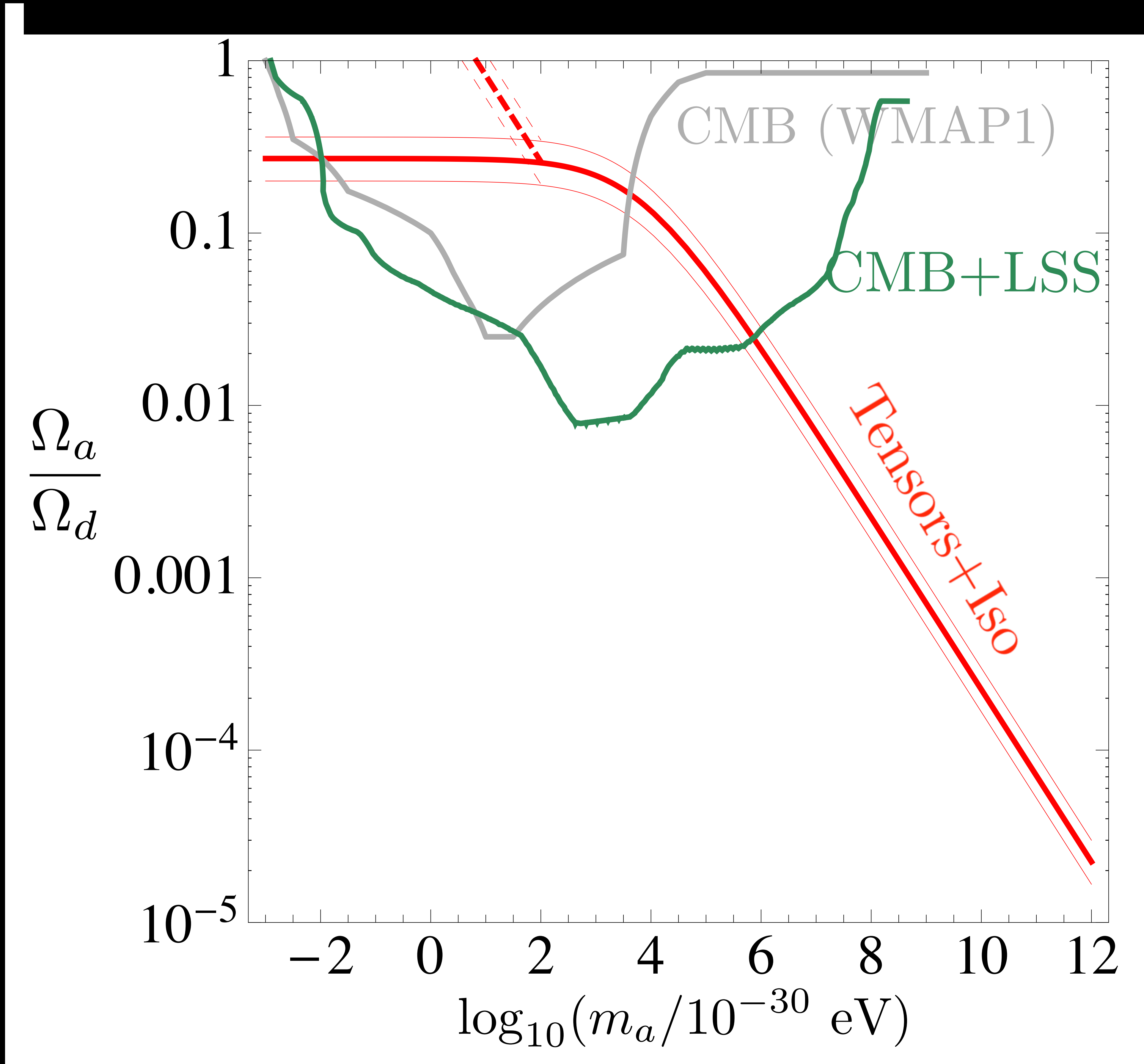


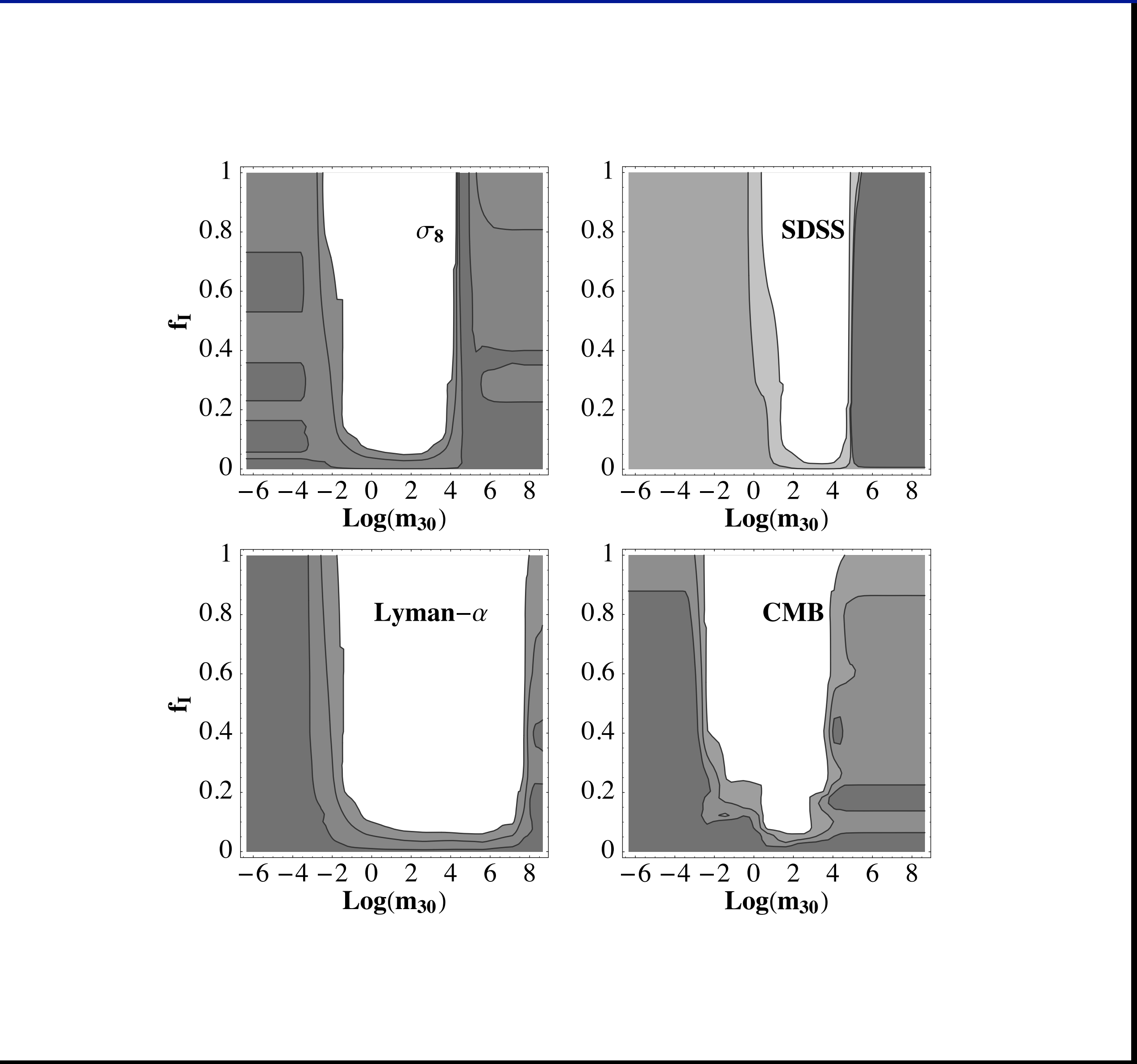
FIG. 2 (color online). Phenomenology in the $\{m_a, \Omega_a/\Omega_d\}$ plane. The shaded regions lie between the dashed contours and satisfy $\{0.01 < r < 0.1, 0.01 < \alpha_{\rm CDM} < 0.047\}$, evading current constraints, while being potentially observable with future data.

TAKE-AWAY message: for ULAs, tensors and isocurvature are simultaneously observable

BICEP makes this more than an anticipatory game



Amendola and Barbieri



Old power spectrum constraints from Amendola and Barbieri, arXiv:hep-ph/0509257

1) Grid search

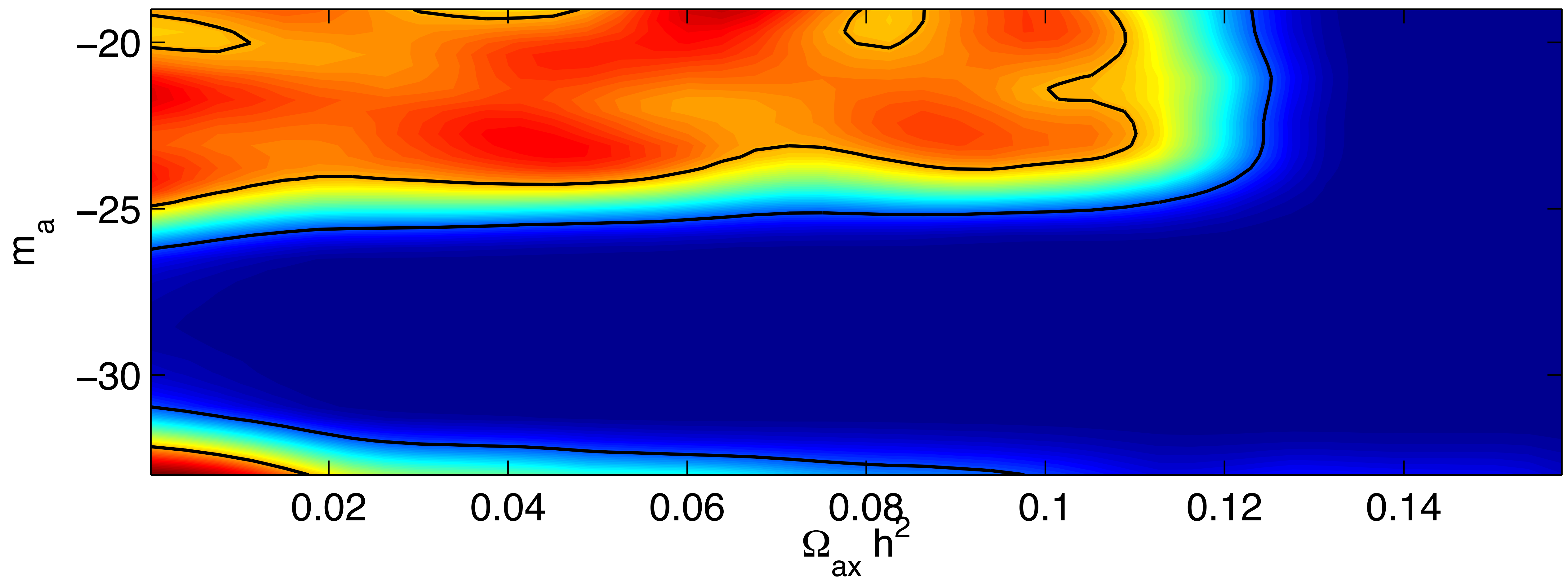
2) No isocurvature

3) No marginalization over foregrounds

4) No lensing, no polarization

5) No real Boltzmann code [step in power spectrum, or unclustered DE at low m]

Preliminary adiabatic results

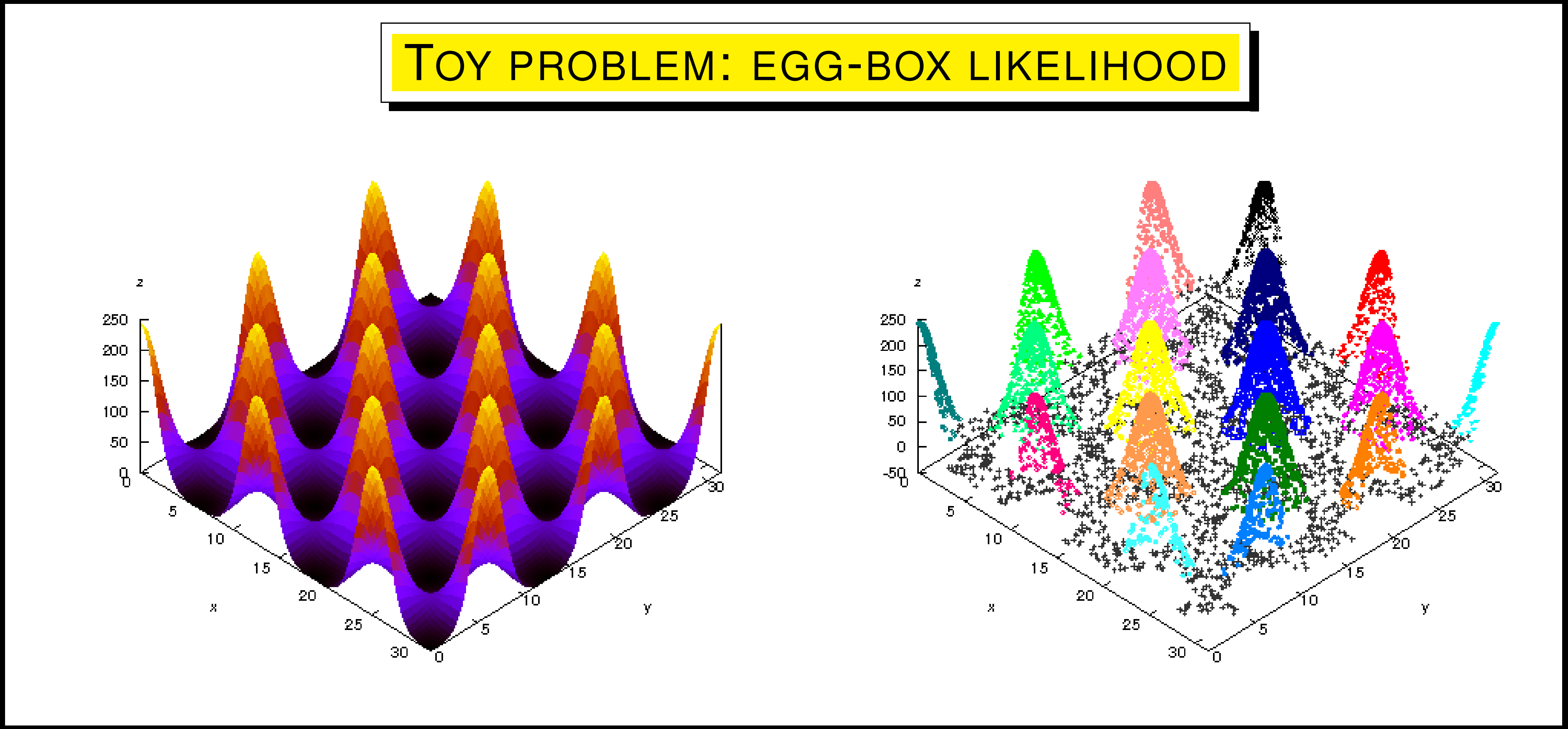


Convergence and climbing contours required MCMC with nested sampling instead of Metropolis-Hastings!

Old power spectrum constraints from Amendola and Barbieri, arXiv:hep-ph/0509257

We use nested sampling instead

From Hobson 2012



Conclusions

- ✴ QCD axion DM is under some tension as a result of BICEP detection [if confirmed], but possibilities still exist
- ✴ Ultra-light axion dark matter will soon be strongly probed

The strong CP problem

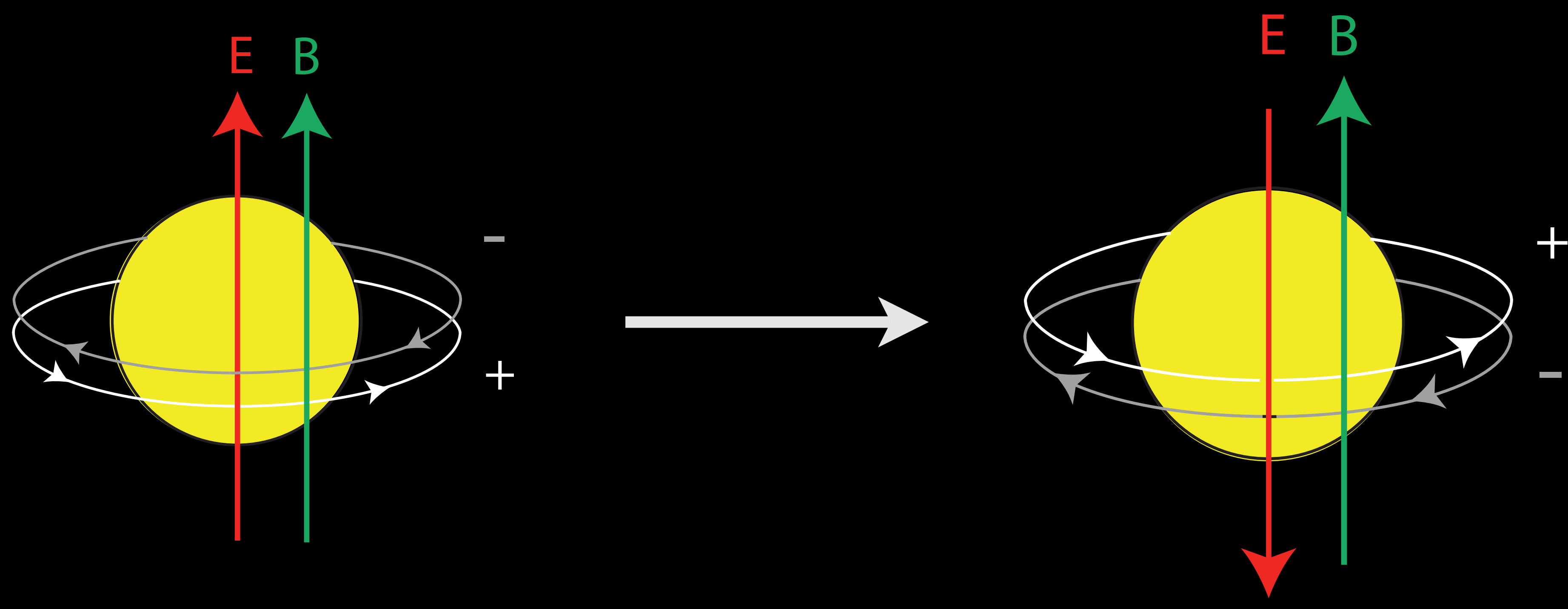
- Strong interaction violates CP through θ-vacuum term

$$\mathcal{L}_{\rm CPV} = \frac{\theta g^2}{32\pi^2} G\tilde{G}$$

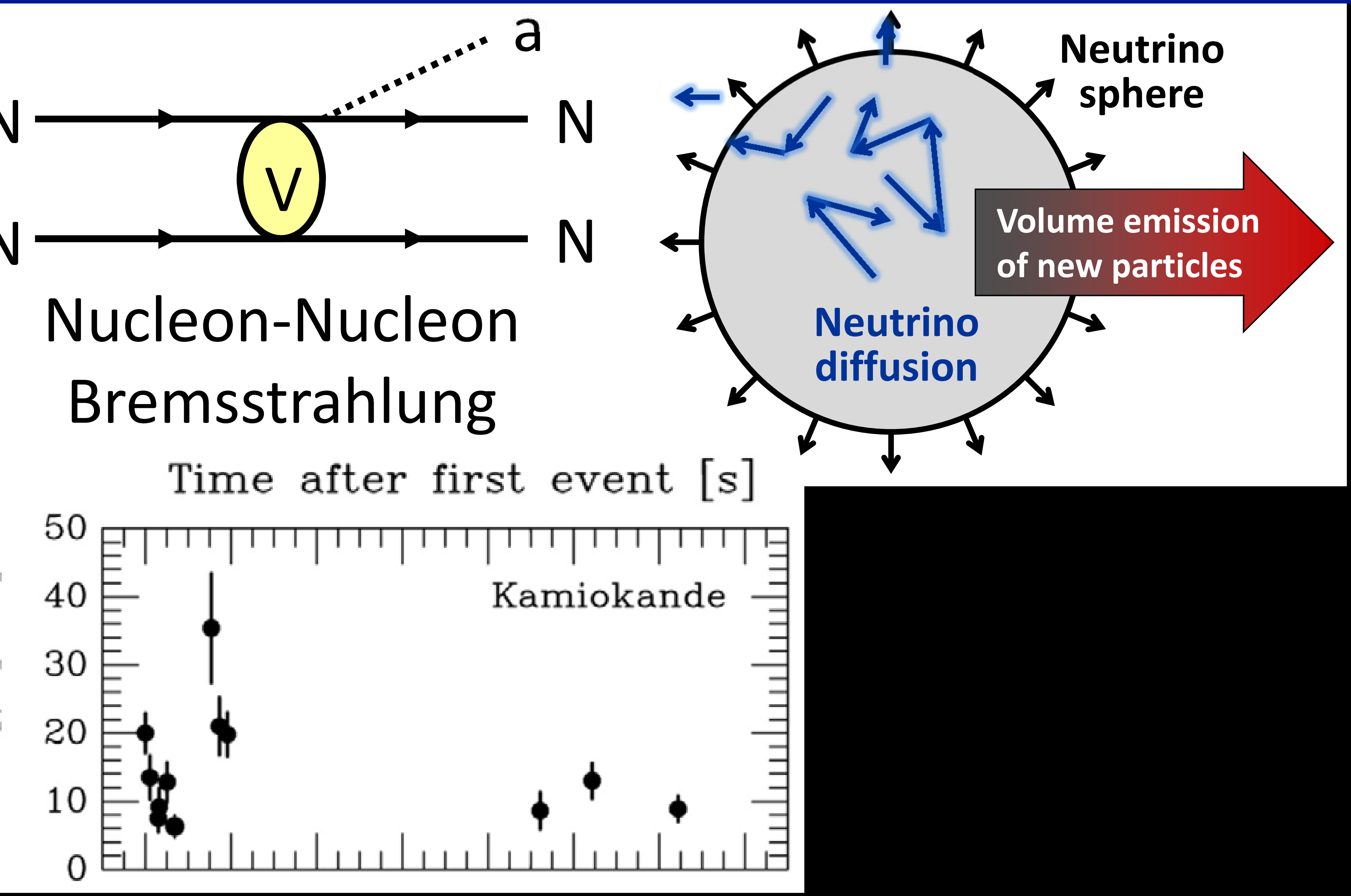
- Limits on the neutron electric dipole moment are strong. Fine tuning?

$$d_n \simeq 10^{-16}\ \theta\ \mathrm{e\ cm}$$

$$\theta \lesssim 10^{-10},$$



Making axions in (exploding) stars, III



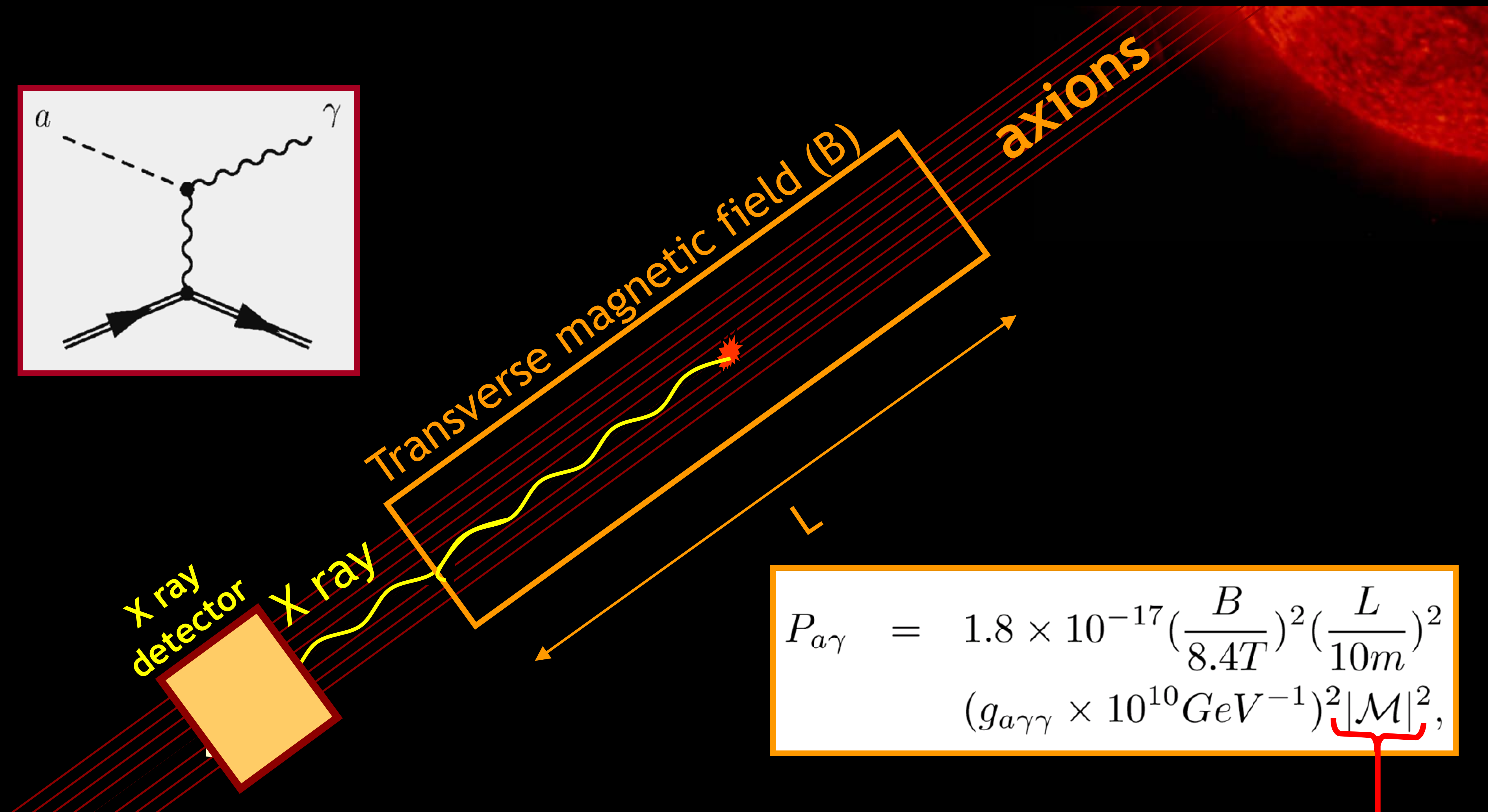
From Raffelt 2012

Raffelt, Seckel, and many more

Axion helioscopes

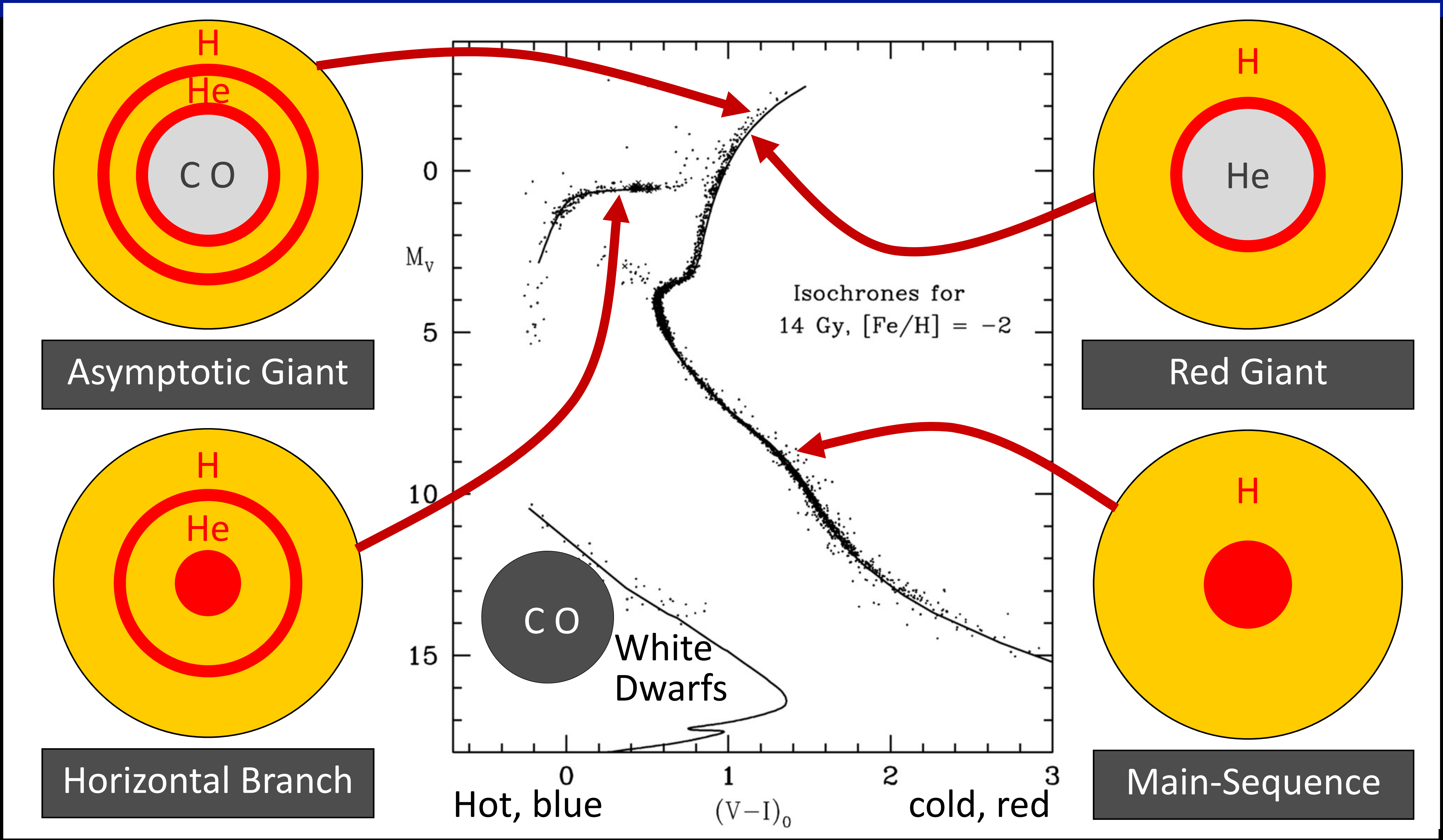
- Backwards Primakoff process (Sikivie, Zioutas, and many others)

From Irastorza 2013



$$P_{a\gamma} = 1.8 \times 10^{-17} \left(\frac{B}{8.4T}\right)^2 \left(\frac{L}{10m}\right)^2 (g_{a\gamma\gamma} \times 10^{10} GeV^{-1})^2 |\mathcal{M}|^2,$$

Making axions in stars, II



From Raffelt 2012

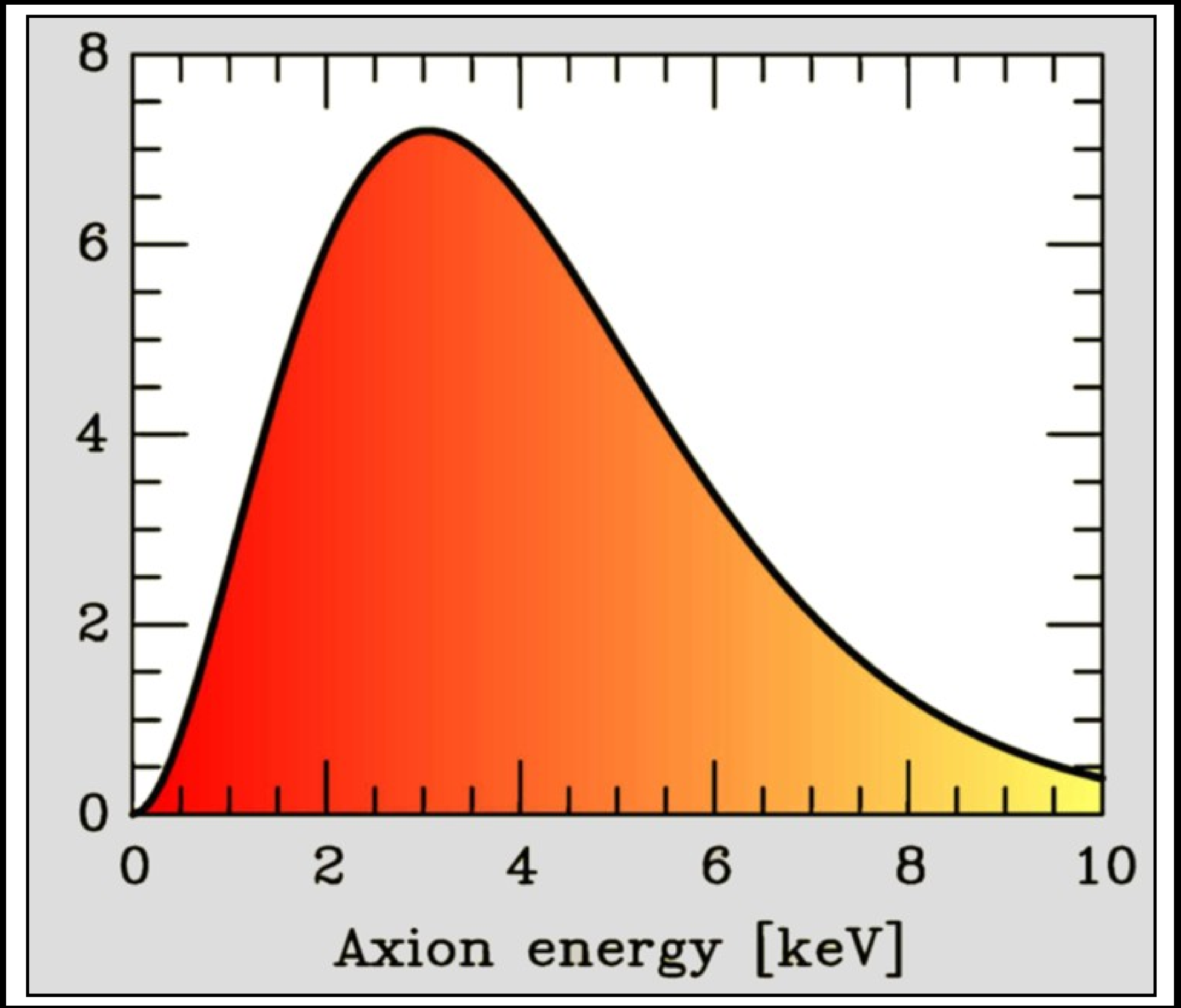
$$g_{a\gamma\gamma} \lesssim 10^{-10}\ \mathrm{GeV}^{-1}$$

Axion helioscopes

- ✳ Resonance condition

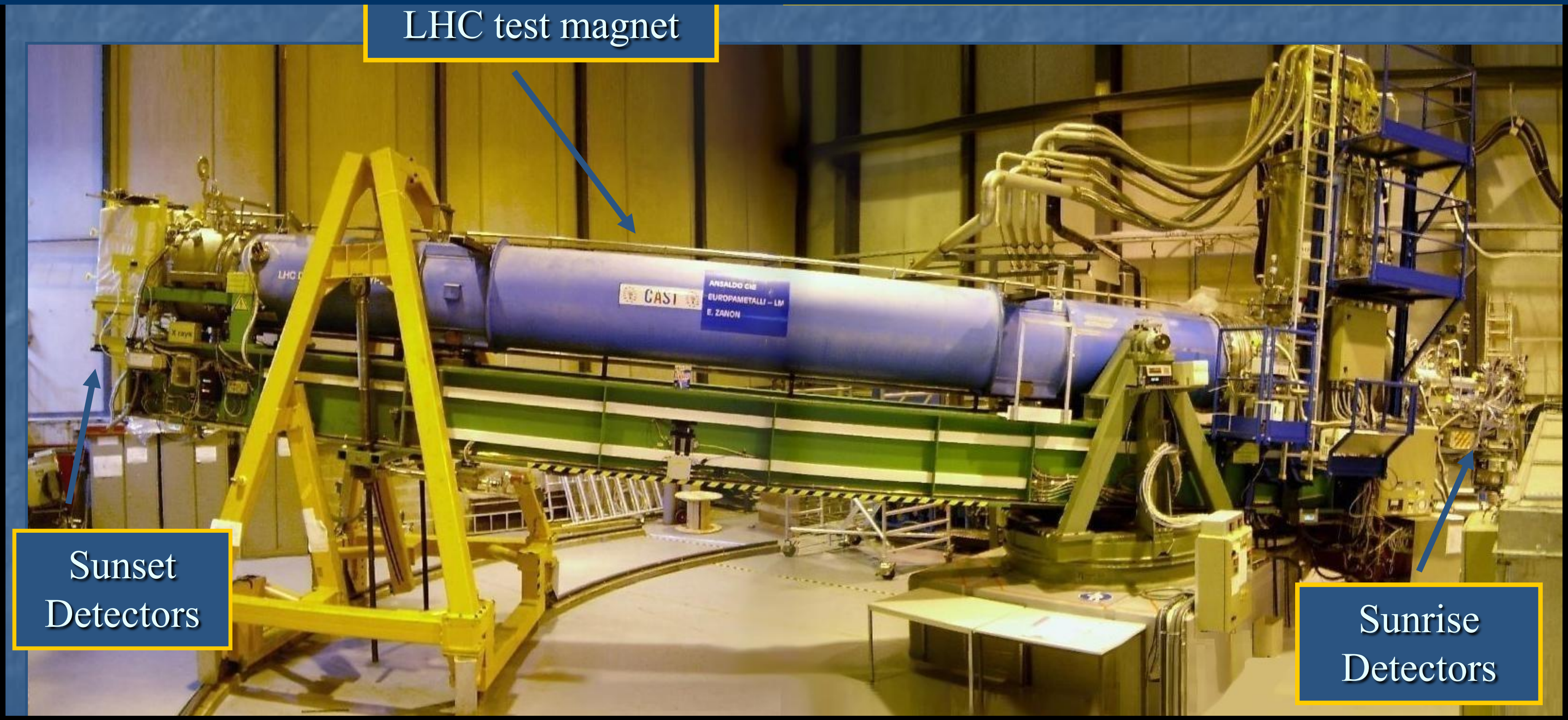
$$qL<\pi \quad\Rightarrow\quad \sqrt{m_\gamma^2-\frac{2\pi E_a}{L}}<m_a<\sqrt{m_\gamma^2+\frac{2\pi E_a}{L}}$$

- ✳ Broad axion energy spectrum



CAST/IAXO

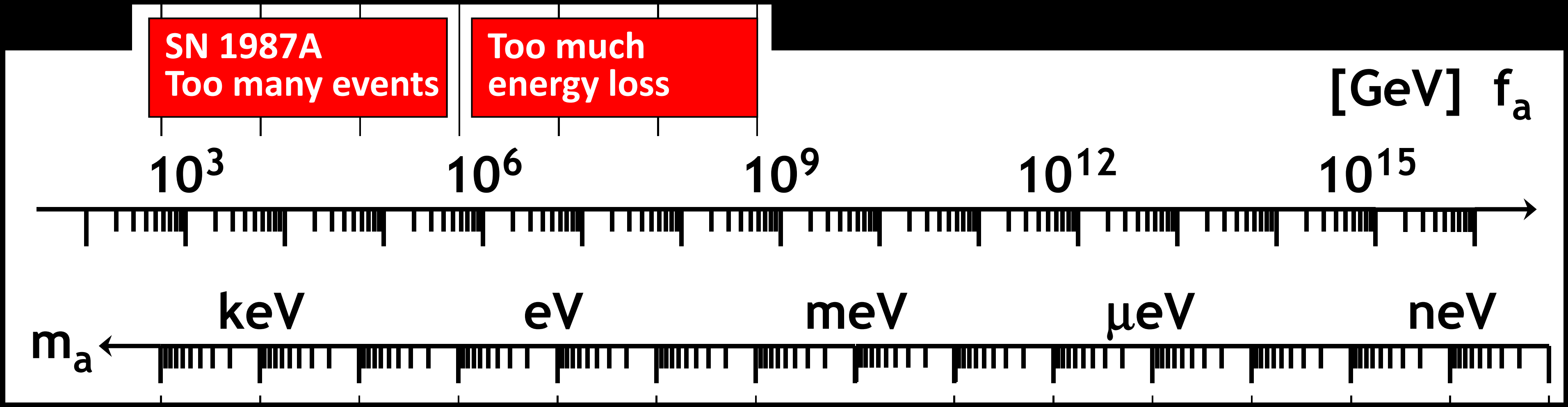
- ✳ CAST
  - ➢ LHC test magnet (B=9 T, L=9.26 m)



Lakic 2012

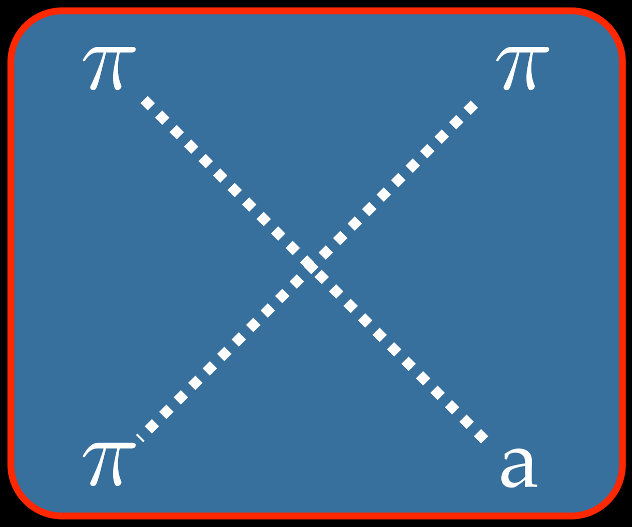
- ✳ IAXO proposal: 15-20m length magnet, optimized shape [not LHC DUD]

Making axions in (exploding) stars, III

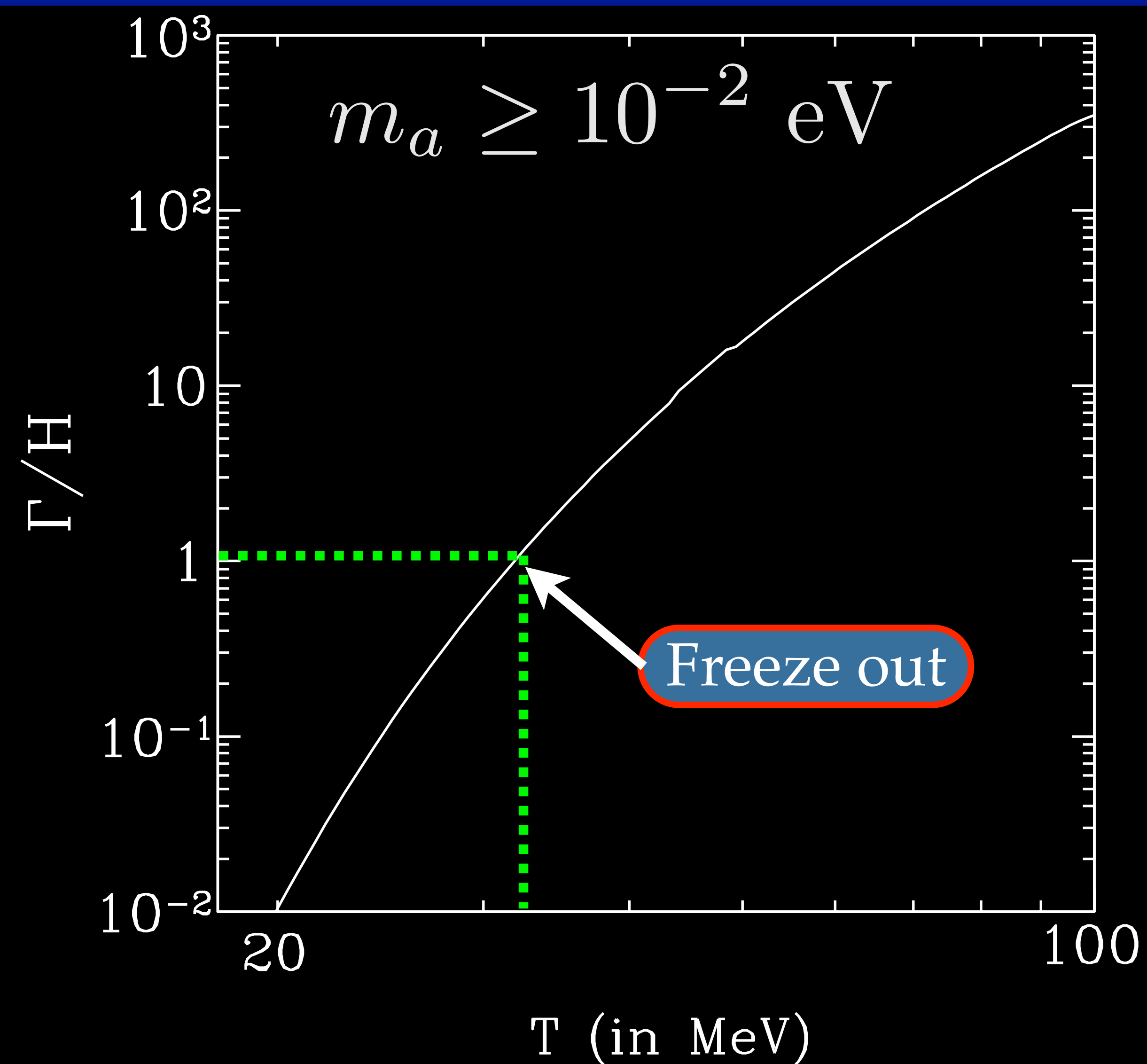


Hot axion production at early times

Axion Production:



$$\Omega_a h^2 = \frac{m_{a,\mathrm{eV}}}{130}\left(\frac{10}{g_{*,\mathrm{F}}}\right)$$



- Axions produced through interactions between non-relativistic pions in chemical equilibrium with rate

Axion hot dark matter

* Axion free-streaming length

$$\lambda_{\rm fs} \simeq \frac{196\ \rm Mpc}{m_{\rm a,eV}}$$

* Entropy generation, e.g. modulus decay

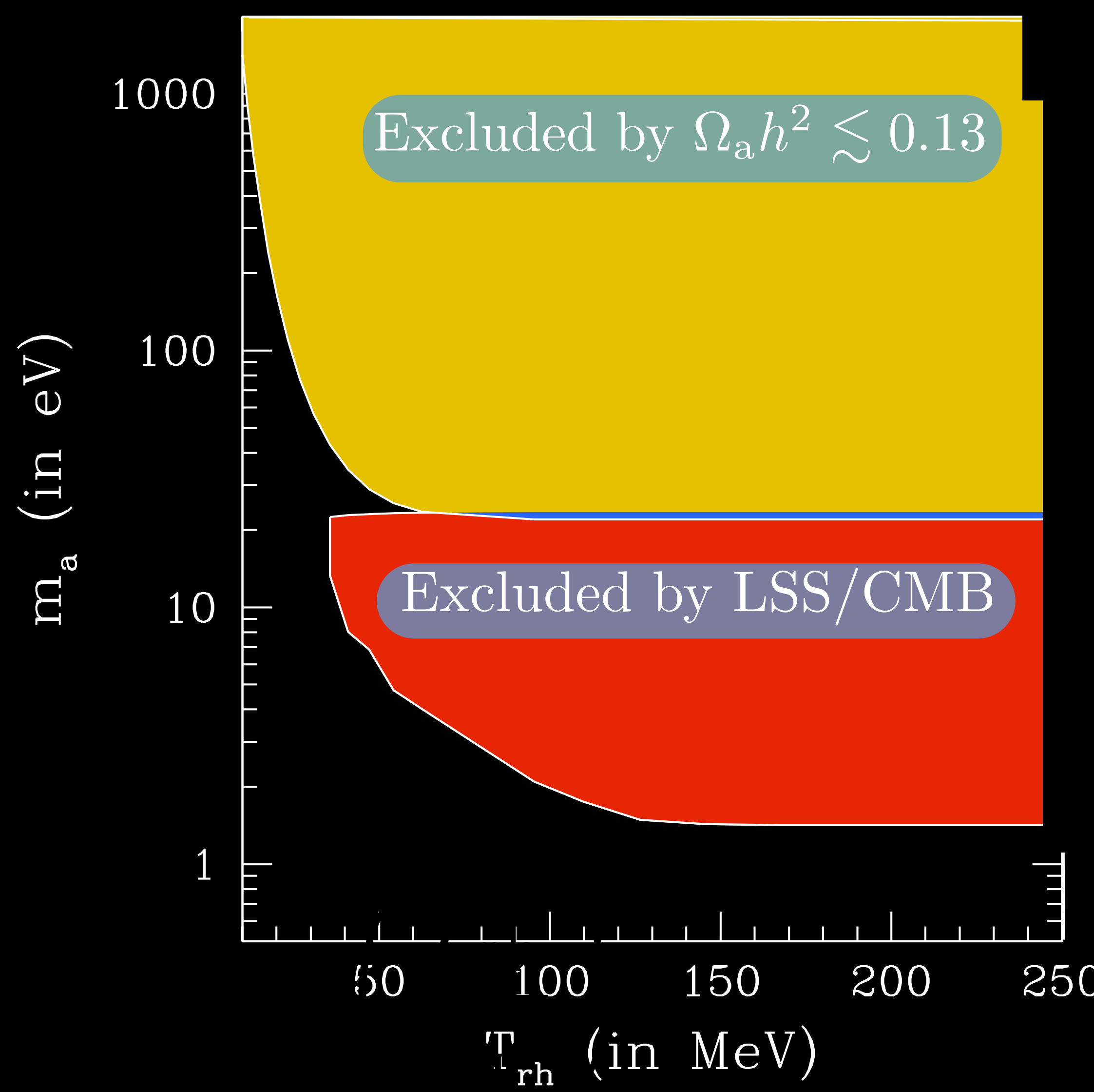
$$T_{\rm rh} \sim 10\ {\rm MeV} \left(\frac{m_\phi}{\rm TeV}\right)^{3/2}$$

* Axion temperature lowered

$$\frac{T_{\rm a}}{T_\nu} \propto \left(\frac{T_{\rm rh}}{T_{\rm F}}\right)^{5/3}$$

* Free streaming-length modified

$$\lambda_{\rm fs} \simeq \frac{196\ \rm Mpc}{m_{\rm a,eV}} \left(\frac{T_{\rm a}}{T_\nu}\right)$$



with T.L. Smith and M. Kamionkowski
Phys. Rev. D77 085020, 0711.1342

$$\Omega_a \to \Omega_a \left(\frac{T_{\rm rh}}{T_{\rm F}}\right)^5$$

A new telescope search for decaying relic axions

with K.Z. Khor, M. Kamionkowski, E.Jullo, G.Covone, J.P-Kneib

Axion HDM: Decay line

- Monochromatic emission line:

$$\lambda = \frac{c}{m_a c^2/2h} = 24800\mathring{A}\ \frac{(1+z_c)}{m_a/\ \mathrm{eV}}$$

Visible

- Axions decay:

$$\tau = 6.8 \times 10^{24} \xi^{-2} m_{a,\mathrm{eV}}^{-5}\ \mathrm{s}$$

Following in the footsteps of Ressell, Bershady, Turner 1991

Axion HDM: Galaxy clusters

- Galaxy clusters are huge axion reservoirs

$$N_{\rm ax} = 10^{80} m_{a,\rm eV}^{-1} \ !$$

- Reasonably wide line $\sigma_{1000} \sim 1$

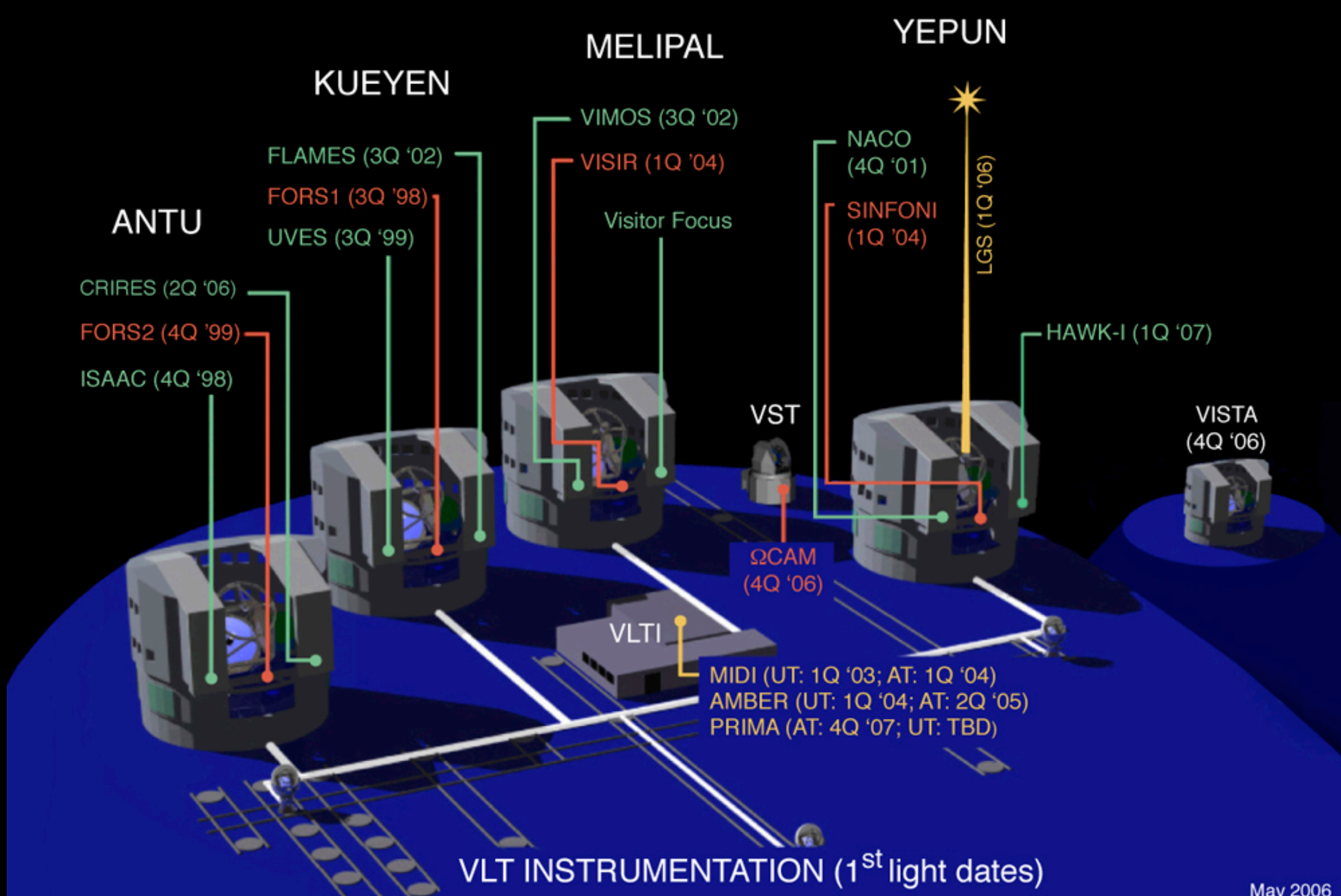
- Strong/weak gravitational lensing mass maps available

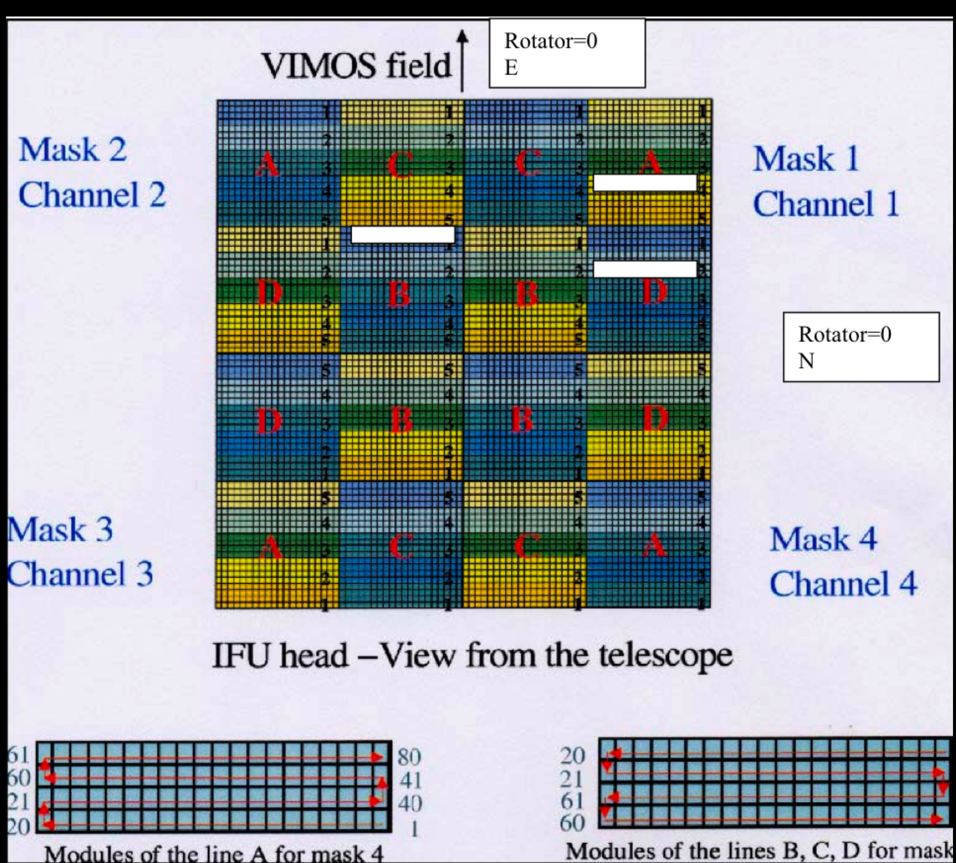
- Comparable to sky brightness

$$I_\lambda \simeq 10^{-18} \ {\rm cgs} \frac{{\rm m}_{\rm a,eV}^7 \xi^2}{(1+{\rm z_c})^4} \frac{\Sigma}{10^{12} {\rm M}_\odot {\rm pix}^{-2}}$$

Axion HDM: VIMOS IFU

- At VLT (Very Large Telescope) array of ~8 m instruments at Paranal, Chilé
- VIMOS IFU yields spatially resolved spectroscopy (6400 fibers in 1 arcmin^{2})



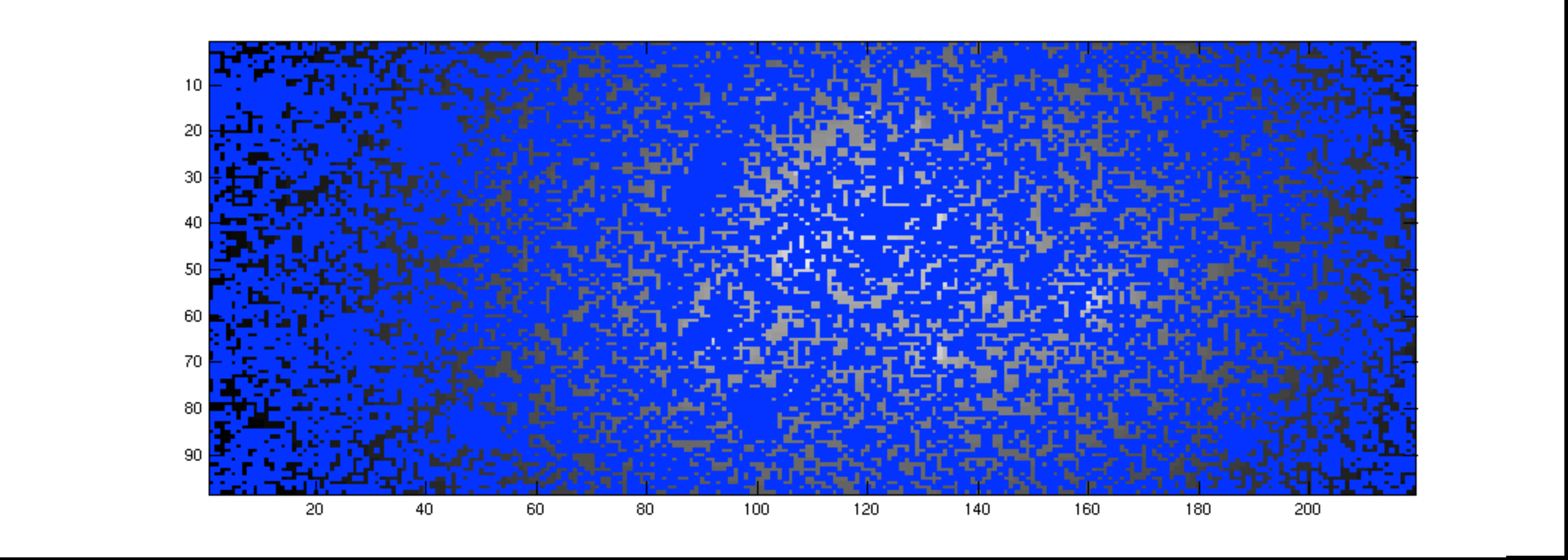


Axion HDM: Modern optical telescope searches

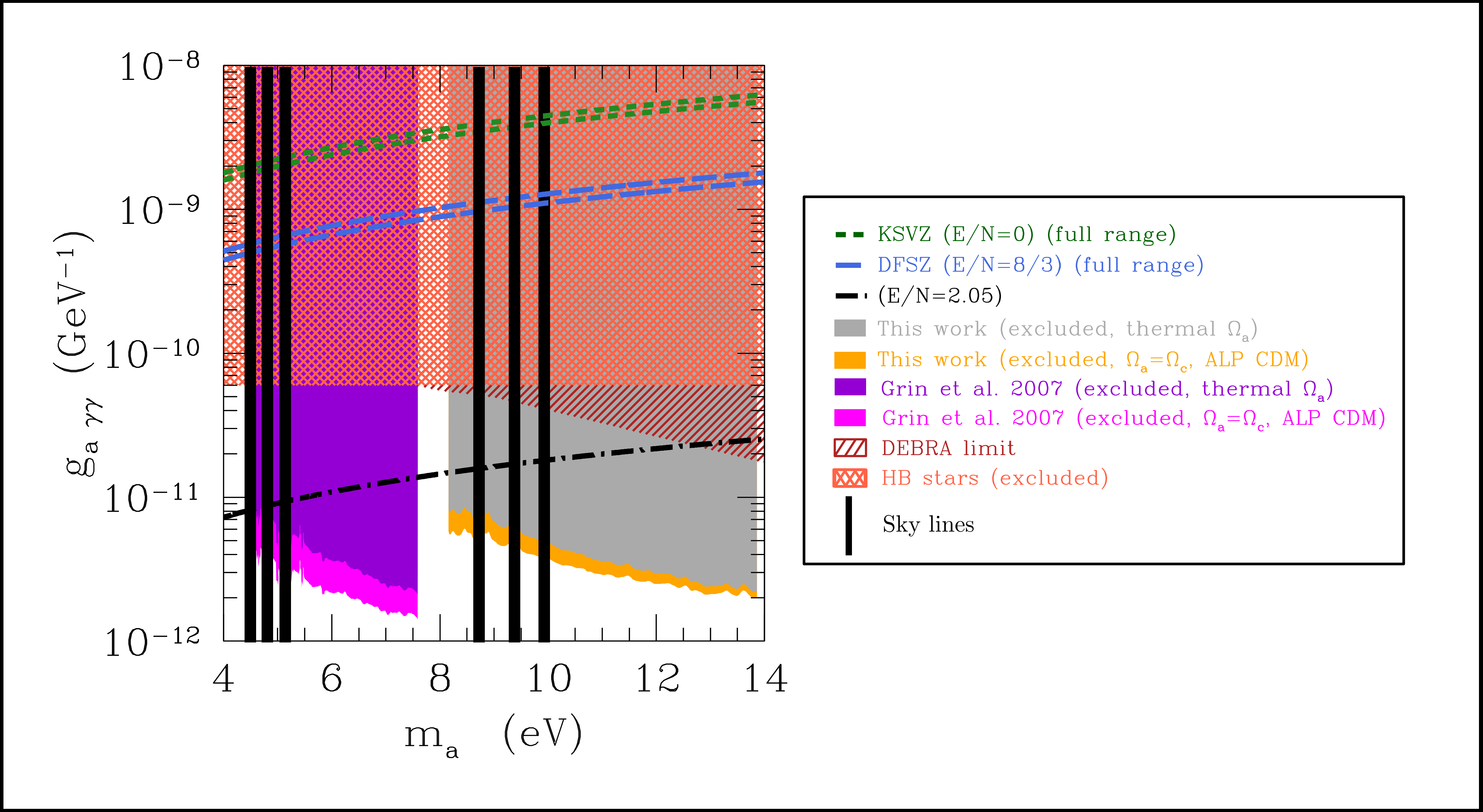
Grin et al. 2007: Abell 2667/2390 PRD, astro-ph/0611502

K.Z. Khor (Princeton Class of 2014)

Axion HDM: Cluster mass maps and



Axion HDM: RDCS 1252/A2667+A2390

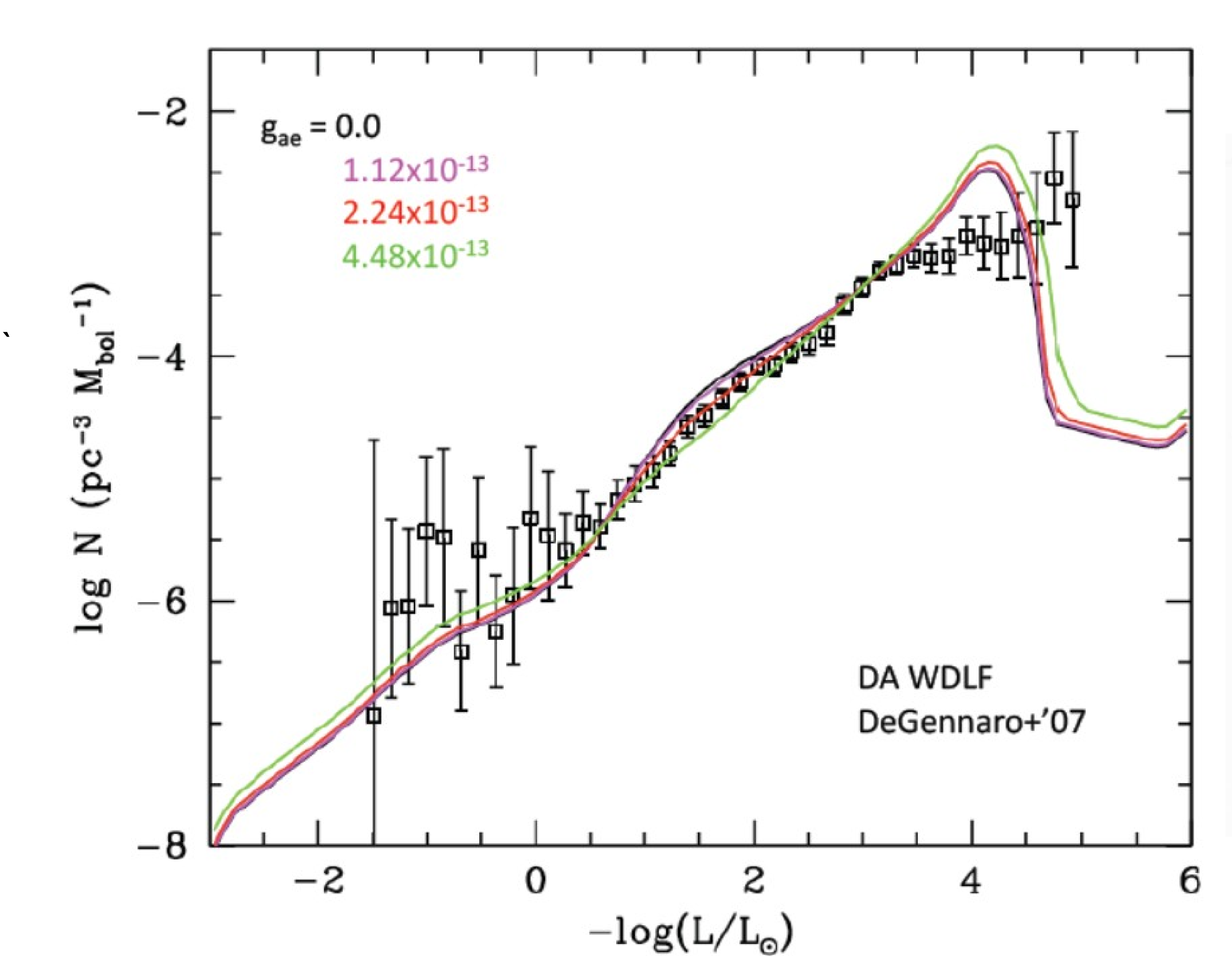


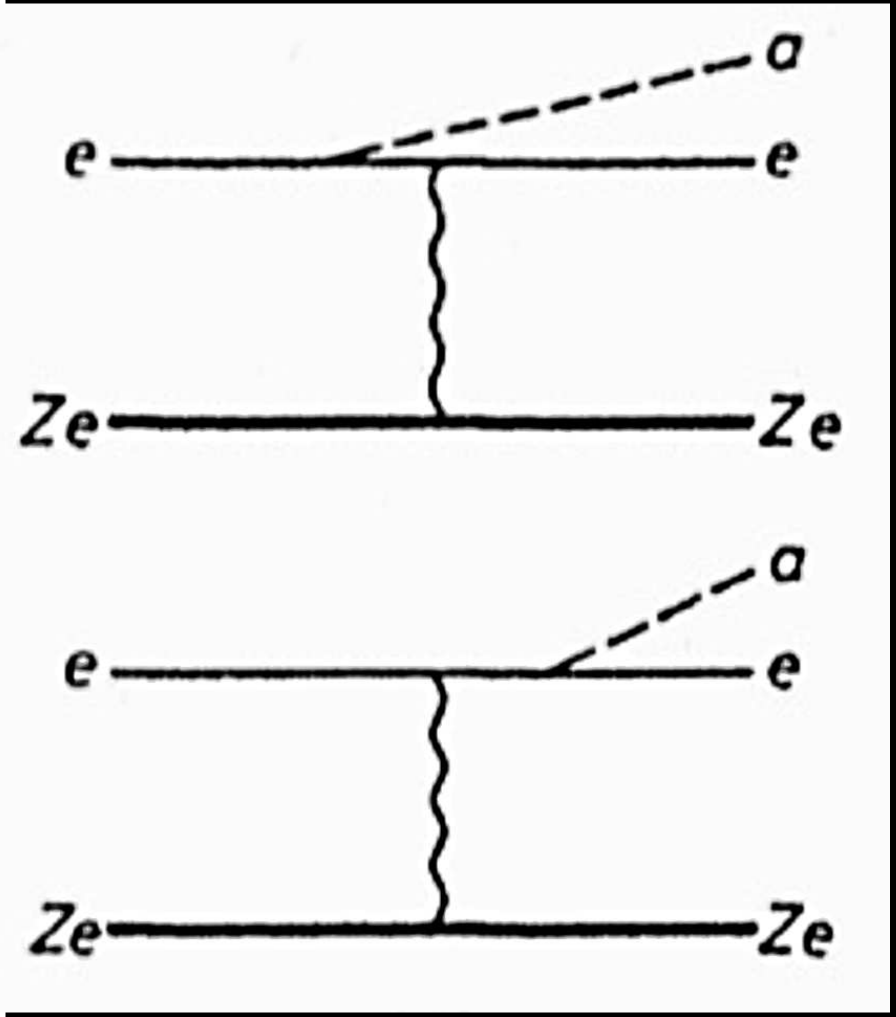
astro-ph/0611502, Phys.Rev.D75:105018,2007

+manuscript in progress

Making axions in degenerate stars, IV

- WDs are remnants of $1\ M_\odot$ main – sequence stars
- Axio-electric coupling provides additional cooling channel





Axion HDM: Decay line

✳ Monochromatic emission line:

Visible

$$\lambda = \frac{c}{m_a c^2/2h} = 24800 \mathring{A} \frac{(1+z_c)}{m_a/\text{ eV}}$$

✳ Resolvable $\delta\lambda = 195 \sigma_{1000} m_{a,\text{eV}}^{-1} \mathring{A}$

✳ Axions decay:

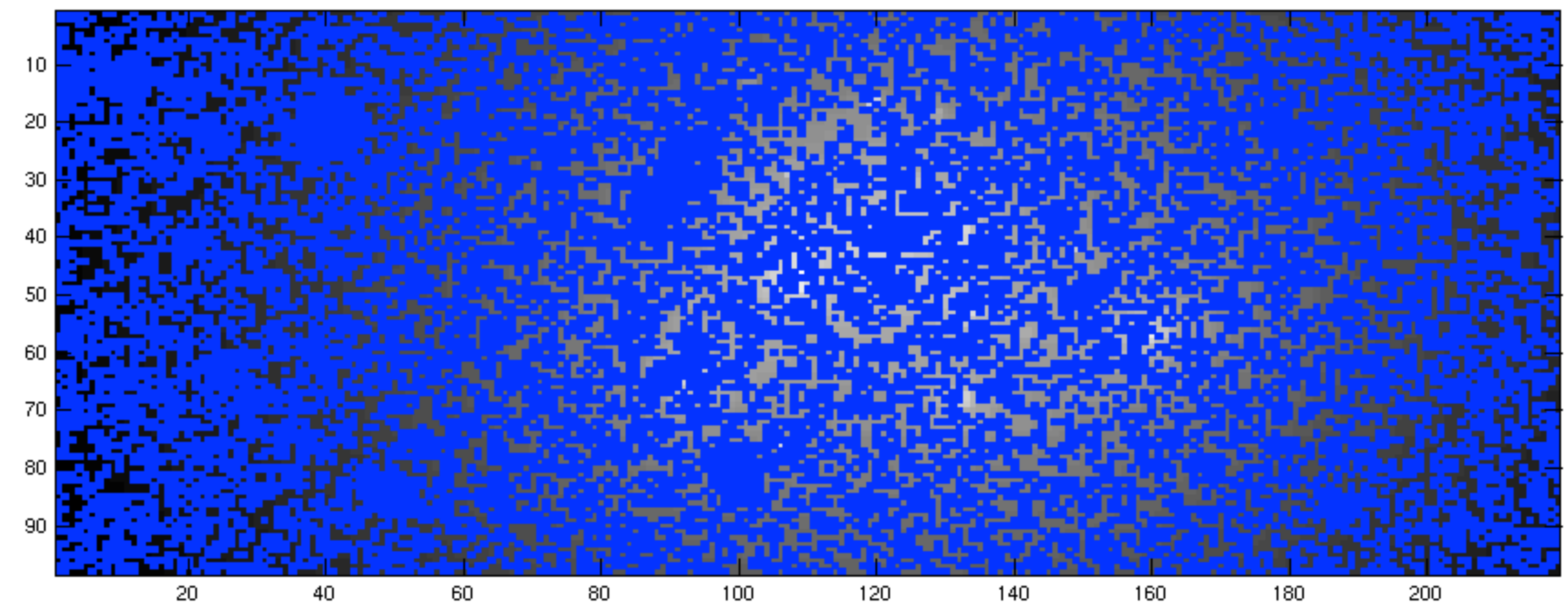
$$\tau = 6.8 \times 10^{24} \xi^{-2} m_{a,\text{eV}}^{-5} \text{ s}$$

✳ Axion thermal abundance

Following in the footsteps of Ressell, Bershady, Turner 1991

$$\Omega_{\text{ax}} h^2 \simeq \frac{m_a}{130 \text{ eV}}$$

Axion HDM: Cluster mass maps and



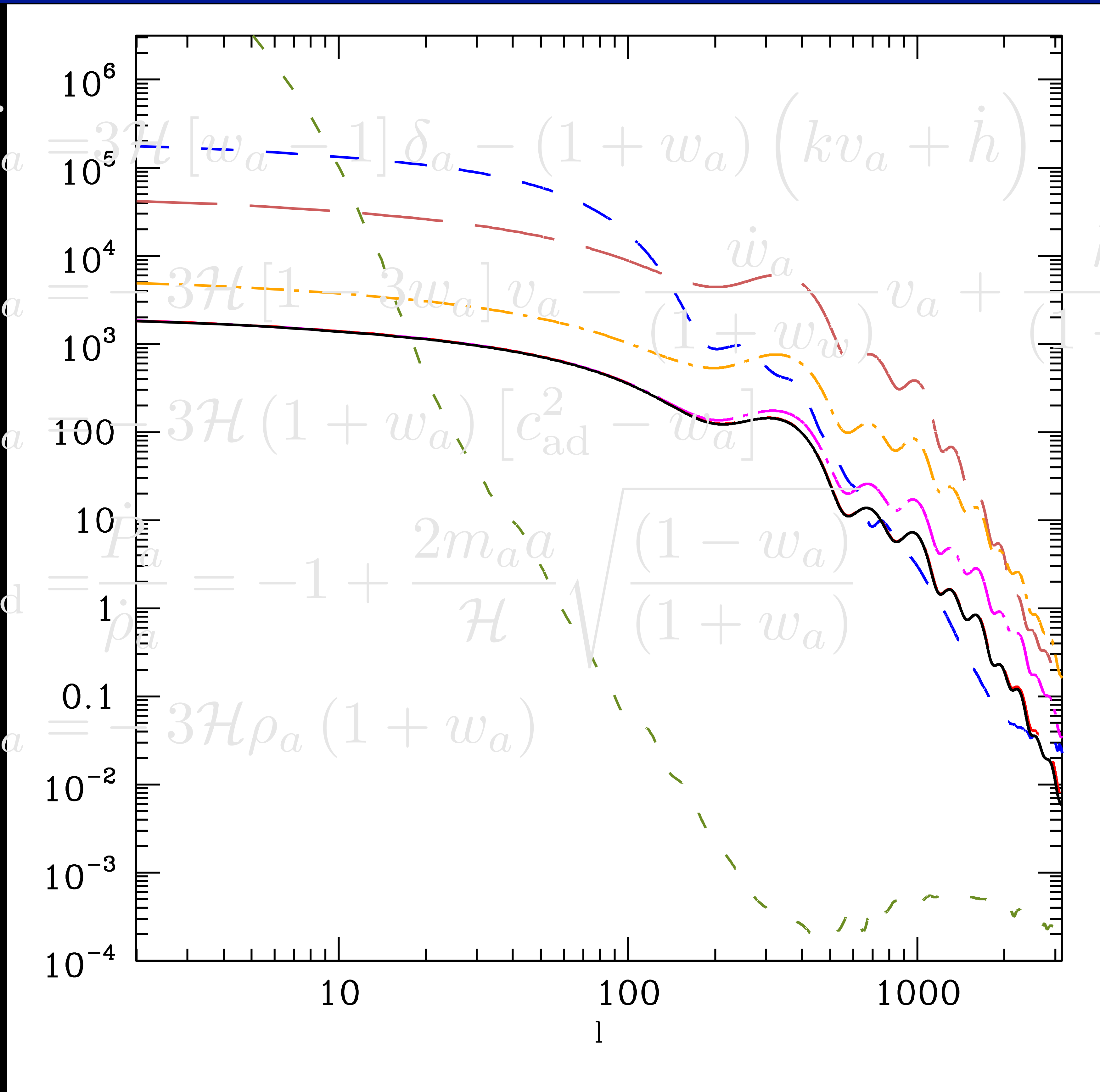
$\Sigma(10^{12} M_\odot \text{ pix}^{-2})$

- ✴ Cluster galaxies selected by redshift
- ✴ BCG, galaxies near arcs, cluster-scale mass component modeled individually

$$\Sigma(R) = \frac{\Sigma_0 r_0}{1 - r_0/r_{\mathrm{t}}} \left(\frac{1}{\sqrt{r_0^2 + R^2}} - \frac{1}{\sqrt{r_{\mathrm{t}}^2 + R^2}} \right)$$

✴HST Shear map (Rosati et al.) and arc locations fit

Getting under the hood: The need for numerical care



$$\dot{\delta}_a = 3\mathcal{H}\left[w_a - 1\right]\delta_a - (1+w_a)\left(kv_a + \dot{h}\right)$$

$$\dot{v}_a = -3\mathcal{H}\left[1 - 3w_a\right]v_a - \frac{\dot{w}_a}{(1+w_a)}v_a + \frac{k\delta_a}{(1+w_a)}$$

$$\dot{w}_a = -3\mathcal{H}\left(1+w_a\right)\left[c_{\rm ad}^2 - w_a\right]$$

$$c_{\rm ad}^2 = \frac{\dot{P}_a}{\dot{\rho}_a} = -1 + \frac{2m_a a}{\mathcal{H}}\sqrt{\frac{(1-w_a)}{(1+w_a)}}$$

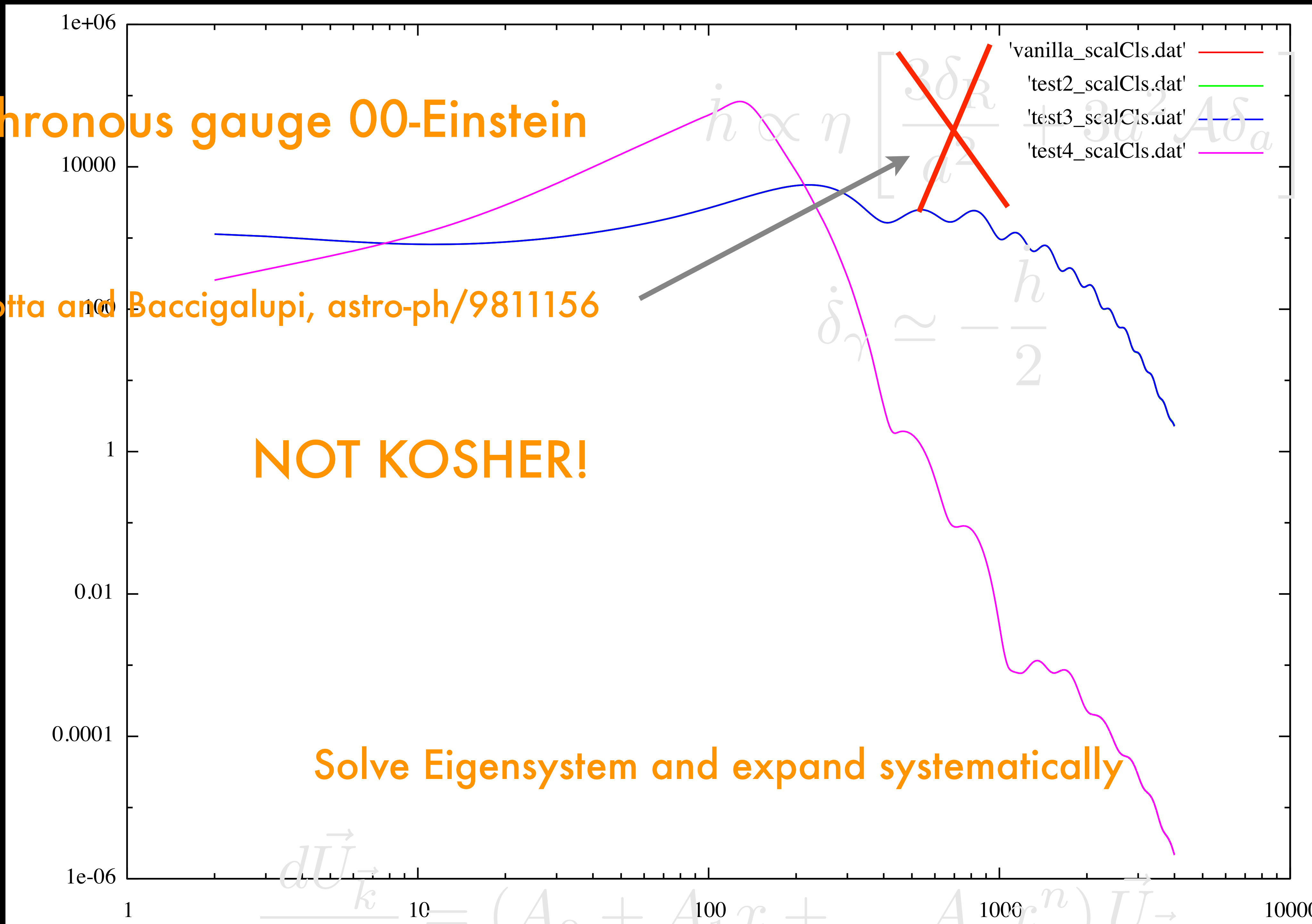
$$\dot{\rho}_a = -3\mathcal{H}\rho_a\left(1+w_a\right)$$

Getting under the hood: The need for correct (super-horizon) initial conditions

Synchronous gauge 00-Einstein

Perrotta and Baccigalupi, astro-ph/9811156

NOT KOSHER!



Solve Eigensystem and expand systematically

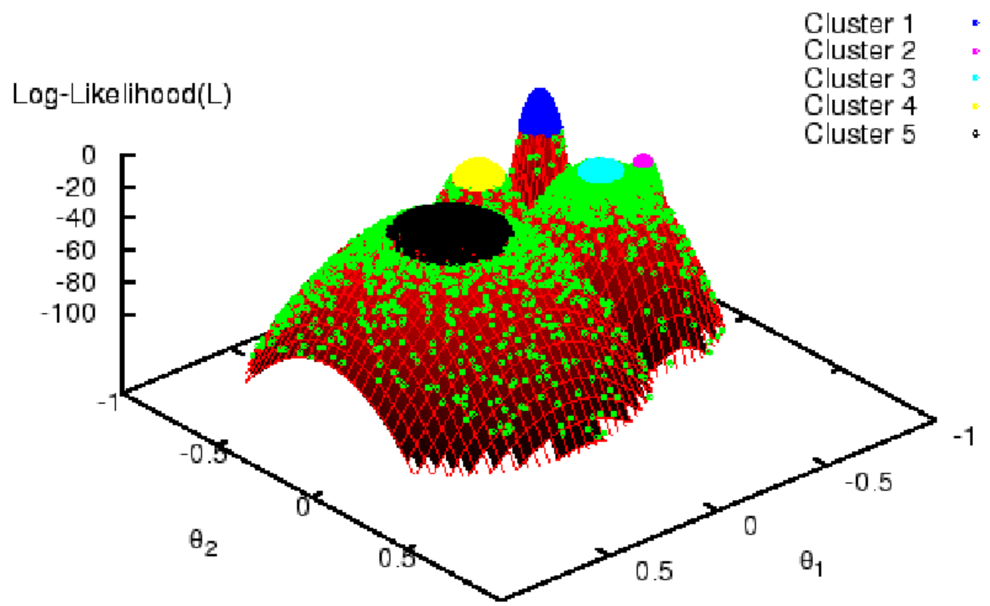
$$\frac{d\vec{U}_{\vec{k}}}{d\ln x} = (A_0 + A_1 x + \ldots A_n x^n)\,\vec{U}_{\vec{k}}$$

Bucher, Moodley, and Turok, PRD62, 083508, sol'ns can be obtained using this technique, outlined in Doran et al. , astro-ph/0304212

We use nested sampling instead

From Hobson 2012

TOY PROBLEM: MULTIPLE GAUSSIAN LIKELIHOOD



- Likelihood = five 2-D Gaussians of varying widths and amplitudes; prior = uniform
- Analytic evidence integral $\log E = -5.27$
- MULTINEST: $\log E = -5.33 \pm 0.11$, $N_{\text{like}} \approx 10^4$
- Thermodynamic integration (+ error): $\log E = -5.24 \pm 0.12$, $N_{\text{like}} \approx 4 \times 10^6$
- Typical of real applications (see later): $\sim 500\times$ efficiency of standard MCMC

We use nested sampling instead