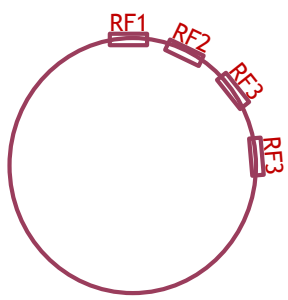
# Longitudinal Motion 2

## RF Manipulations

- As you'll show in homework, the synchrotron tune (longitudinal oscillations/turn) is generally <<1.
- That means that if there are multiple RF cavities around the ring, the orbiting particle will see the *vector sum* of the cavities.




$$\frac{\Delta E}{dn} = \sum_{i=1}^{N} V_i \sin(\phi_i) = V_{eff} \sin(\phi_{eff})$$

$\phi_i$ is the phase angle at the arrival of the particle at cavity *i*

- We will clearly get the maximum energy gain if all phases are the same, so (assuming all voltages are the same)

$$\frac{\Delta E}{dn} = N V_0 \sin(\phi_s)$$

## Do we always want the maximum acceleration?

- As we'll see, there are times when we want to change the amplitude of the RF quickly.
- Because cavities represent stored energy, changing their amplitude quickly can be difficult.
- Much quicker to change phase
- Standard technique is to divide RF cavities into two groups and adjust the relative phase. In the simplest case, we put half the RF cavities into group "A" and half into group "B". We can adjust the phases of these cavities relative to our nominal synchronous phase as

$$V_{eff}\sin(\phi_{eff}) = \frac{N}{2}V_0\sin(\phi_s+\delta)+\frac{N}{2}V_0\sin(\phi_s-\delta)$$
$$= \frac{N}{2}(\sin\phi_s\cos\delta+\cos\phi_s\sin\delta+\sin\phi_s\cos\delta-\cos\phi_s\sin\delta)$$
$$= NV_0\cos\delta\sin\phi_s$$
$$V_{eff} = NV_0\cos\delta;\phi_{eff}=\phi_s$$

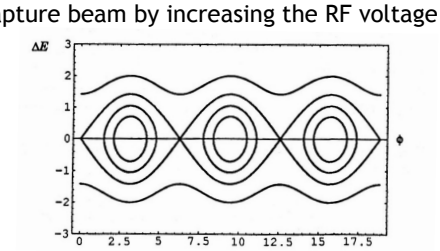
- So $\delta = 0 \Rightarrow V_{eff} = NV_0$

  $\delta = \frac{\pi}{2} \Rightarrow V_{eff} = 0$ ← Like "turning RF off"



## RF capture

- We can capture beam by increasing the RF voltage with no accelerating phase




- As we accelerate beam, Δt decreases. Recall $\frac{1}{\beta_L}(\Delta t)^2+\beta_L(\Delta E)^2 \equiv \varepsilon_L = \text{constant}$

$$\Delta E_{RMS} = \sqrt{\frac{\varepsilon_L}{\beta_L}} = \left(-\frac{\varepsilon_L^2 eV_0\omega_{rf}E_S\beta^2\cos\phi_s}{\tau\eta}\right)^{\frac{1}{4}}$$

$$\Delta t_{RMS} = \sqrt{\varepsilon_L\beta_L} = \left(-\frac{\tau\eta}{\varepsilon_L^2 eV_0\omega_{rf}E_S\beta^2\cos\phi_s}\right)^{\frac{1}{4}}$$

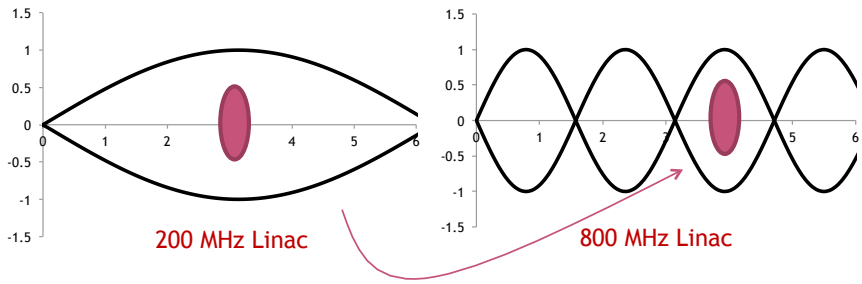
- So as beam accelerates, bunches get narrower

## Bucket to Bucket Transfer

- In general, the accelerating gradient of an RF structure is

$$\frac{V}{L} \propto \frac{V_{breakdown}}{\lambda_{RF}} \propto \omega_{RF} V_{breakdown}$$

- So when bunches get short enough, it's advantageous to transfer to a higher frequency. For example, in the Fermilab Linac



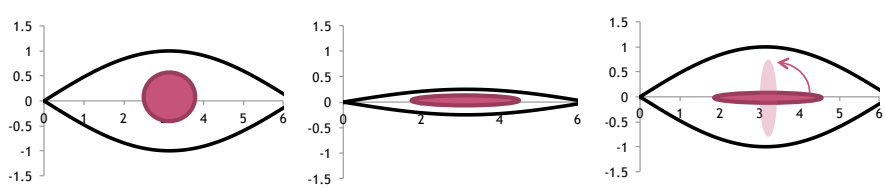

Transfer beam to every 4th bucket



## Bunch Rotation

- If we *slowly* change the RF voltage (or effective voltage by phasing), we can adiabatically change the bunch shape

$$\Delta E_{RMS} = \sqrt{\frac{\varepsilon_L}{\beta_L}} = \left( -\frac{\varepsilon_L^2 e V_0 \omega_{rf} E_S \beta^2 \cos\phi_s}{\tau\eta} \right)^{\frac{1}{4}} \propto V_0^{\frac{1}{4}}$$

$$\Delta t_{RMS} = \sqrt{\varepsilon_L \beta_L} = \left( -\frac{\tau\eta}{\varepsilon_L^2 e V_0 \omega_{rf} E_S \beta^2 \cos\phi_s} \right)^{\frac{1}{4}} \propto V_0^{-\frac{1}{4}}$$

- If we suddenly change the voltage, then the bunch will be mismatched and will rotate in longitudinal phase space