

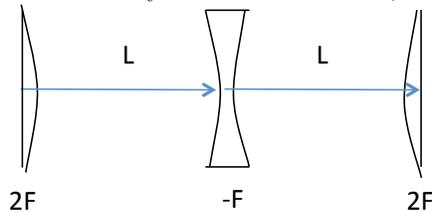
# USPAS Accelerator Physics Final Exam

January 29, 2015

## General Guidelines

- This is an “open book” exam. You may use the texts, lectures, homeworks, or any of the online resources, including previous finals. You are expected to work independently and to not seek out other sources for the solutions.
- There are a total of three problems, which do not have equal weight.
- You may use anything that appeared in the lectures, textbook or assigned homework, without re-deriving it.
- Full or partial credit will only be given if your reasoning can be followed, so show your work. Please give answers in the requested [units] when specified.
- The exam is due at 9AM tomorrow. Late exams will have their score reduced by 10%, with an additional 10% deducted for each additional hour.
- All problems are straightforward applications of what you have learned. There are no trick questions or complex calculations. If you find yourself working hard, it’s a good sign you’re not doing the problem correctly.
- If you think there’s a problem with the test, contact Eric at 630-336-1893 or [prebys@fnal.gov](mailto:prebys@fnal.gov). Any necessary corrections or clarifications will be sent to the email list and posted on the web page, so check both frequently!

All three problems will be based on our standard symmetric FODO cell, which we have seen many times:



where each cell contains focusing and defocusing quadrupoles of focal lengths (in the horizontal plane) of  $F$  and  $-F$ , respectively, spaced  $L$  apart. Positions  $s$  within the cell are measured from the center of the first focusing quadrupole. You may use the usual thin lens approximation for the quadrupoles. In all cases, “focusing quadrupole” and “defocusing quadrupole” will refer to the quadrupoles which focus or defocus, respectively, in the bend ( $x$ ) plane.

## Problem 1

- Write expressions for all Twiss parameters in the bend plane ( $\alpha_x, \beta_x$ , and  $\gamma_x$ ) immediately before and immediately after each focusing and defocusing quadrupole (ie, four sets total). Express your answers in terms of (as appropriate)  $\beta_{max}$ ,  $\beta_{min}$ ,  $F$ , and  $L$ . (Hint: invoke appropriate symmetry arguments to simplify the problem.)
- The quadrupoles are placed  $L=10\text{m}$  apart, and the desired phase advance per cell is  $103^\circ$ . What is the required focal length  $F$ ? [m]
- What are the values of  $\beta_{max}$  and  $\beta_{min}$ ? [m]
- If I build a ring out of  $N=58$  of these cells, what are the circumference  $C$  [m] and the tune  $\nu$  [number]?
- As you learned when you studied coupling, having exactly equal tunes in both planes is not actually a good idea, If I modify each cell by uniformly increasing the magnetic gradient of *all 58* of the focusing quadrupoles by 0.1%, what will be the approximate total change in the tunes  $\nu_x$  and  $\nu_y$  in the  $x$  and the  $y$  planes, respectively? [numbers] (Hint: think of the change in gradient as the *addition* of a small quadrupole right next to the existing one, and be careful with your signs.)
- If my ring has anomalous magnetic errors up to and including sextupole terms, what values of the fractional tune could cause resonant instabilities?
- Based on your answers to the previous two questions, what would be the smallest magnitude change in the gradients of the 58 focusing magnets that could subject the beam to a resonant instability? [percentage] (be sure to look at both signs and both planes)

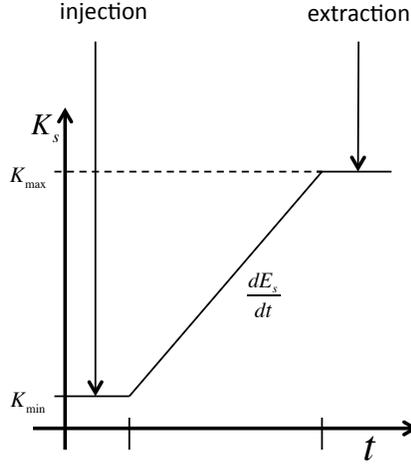
## Problem 2

Assume that the unperturbed lattice described above is designed to accelerate protons from a kinetic energy of  $K_{min}=5$  GeV to a kinetic energy of  $K_{max}=50$  GeV. The injected beam has a normalized RMS emittance of  $1 \mu\text{m}$  in both planes. The quadrupoles are each 2 m long.

- Calculate the momentum  $p$  [GeV/c], relativistic  $\beta$ , period  $\tau$  [ $\mu\text{sec}$ ], and beam rigidity ( $B\rho$ ) [T-m] at the minimum and maximum energies.
- What is the maximum magnetic gradient needed in the quadrupoles? [T/m]
- Assuming the injected beam is properly matched, what are the maximum and minimum beam sizes  $\sigma_{x,max}$  and  $\sigma_{x,min}$  at the minimum energy? [mm]
- Assuming the injected beam is properly matched, what are the maximum and minimum angular distributions  $\sigma_{x',max}$  and  $\sigma_{x',min}$  at the minimum energy? [radians]
- Based on your previous two answers, sketch the phase space distributions in the  $x$  plane at the center of the focusing and defocusing quads at injection (minimum energy). In each plot, clearly indicate the RMS extrema  $\sigma_{x,max}$ ,  $\sigma_{x,min}$ ,  $\sigma_{x',max}$  and  $\sigma_{x',min}$  that you calculated above. (The plots need not be terribly precise, but should be drawn to the same scale such that the relative sizes of the key features are qualitatively correct).
- Now assume that the injected beam has the correct emittance, but is mismatched, such that the phase space distribution at the center of the first *focusing* magnet it encounters looks instead like the distribution for the *defocusing* magnet in the previous question. In this case, what is the new effective (diluted) emittance of this mismatched beam? [ $\mu\text{m}$ ] (Hint: this question is really easy. If you're doing a lot of work, you're doing it wrong.).

### Problem 3

In this problem, we will consider the RF system for the synchrotron described above. The RF system operates at a harmonic  $h=188$  and the transition gamma is  $\gamma_t=15$ . When the beam is injected, the synchrotron is not accelerating. Once the protons have been injected, they are accelerated from the kinetic energy  $K_{min}=5$  GeV to  $K_{max}=50$  GeV and then extracted, as illustrated below



The injected beam is transferred from another accelerator, where it was bunched at the same RF frequency  $f_{inj}$ . Each bunch in the injected beam has an RMS energy spread of  $\sigma_E/E$  of 0.1% and an RMS time distribution of  $\sigma_t=2$  ns.

- What are the RF frequencies  $f_{inj}$  and  $f_{ext}$  at injection and extraction? [MHz].
- What are the slip factors  $\eta_{inj}$  and  $\eta_{ext}$  at injection and extraction?
- What is the RMS longitudinal emittance  $\epsilon_L$  of the injected beam? [eV-s]
- If I want to correctly match my RF to the longitudinal bunch shape of the injected beam, what value of peak RF voltage  $V_0$  do I need? [MV]
- I advance the synchronous phase angle to  $\phi_s=60^\circ$ . What is the initial acceleration of the beam  $dE_s/dt$ ? [GeV/s]
- I adiabatically accelerate the beam to  $K_{max}$ , and then adiabatically stop accelerating. If my peak voltage is still the  $V_0$  I calculated above, what are the values for the RMS values  $\sigma_E$  [MeV] and  $\sigma_t$  [ns] at this point?
- I wish to do a bunch rotation to decrease the time distribution slightly prior to extraction. I quickly (non-adiabatically) increase the RF voltage to  $V'_0 = 4 * V_0$ . How many revolution periods of the machine do I then need to wait before extracting the beam in order to achieve the minimum RMS time distribution  $\sigma_t$ ? [turns]
- If I extract the beam at this optimum time, what is the RMS of the time distribution  $\sigma_t$  of each extracted bunch? [ns] (Hint: this is not hard, but you have to think a bit. Consider the relative sizes of the dimensions of the bunch to what they would be for a properly matched bunch right after the voltage is increased, and then think about what this implies after the rotation.).