Your Name:________________________________

USPAS Fundamentals of Accelerator Physics Final Exam

June 24, 2016

General Guidelines - Please Read!

- REMEMBER TO WRITE YOUR NAME above!
- This is an "open book" exam. You may use the texts, lectures, homeworks, or any of the online resources found at the course website (`http://home.fnal.gov/~prebys/misc/uspas_2016/`), including previous finals. You may use your computer (Excel, MathCad, etc) to do calculations; however, you are expected to work independently and to not seek out other online sources for the solutions.
- This exam consists of 12 questions, some of which have multiple parts. Not all parts are of equal difficulty and not all have equal weight.
- You may use anything that appeared in the lectures, textbook or assigned homework, without re-deriving it.
- You will need to work out some results on your computer or scratch paper, but **your final answer should be written on the exam itself!**
- You are not required to show your work, but if you include the key equations for any derivation, you may receive partial credit even if the final answer is incorrect. There is no need to show your work or explain your reasoning for multiple choice and yes/no answers.
- All problems are straightforward applications of what you have learned. There are no trick questions or complex calculations. If you find yourself working hard, it's a good sign you're not doing the problem correctly. Stop and think!

1. Please answer the following questions for a proton ($m_p c^2 = .938$GeV) traveling at a velocity of .9c.

   (a) What is its momentum [GeV/c]?

   (b) What is its kinetic Energy [GeV]?

   (c) What is its rigidity [T-m]?

   (d) If this proton travels through a 1cm long magnet with a 1T field, by what angle will it be deflected [rad]?

   (e) If a beam of these protons travels through a 0.5m long quadrupole, what gradient will give a focal length of 10m [T/m]?

2. We have frequently used the following parameterization for the transfer matrix

$$M = \begin{pmatrix} \cos\mu + \alpha \sin\mu & \beta \sin\mu \\ -\gamma \sin\mu & \cos\mu - \alpha \sin\mu \end{pmatrix}$$

   in which all the parameters (α, β, γ, and μ) are real numbers. Circle *all* of the following statement (a-e) that are TRUE for this representation.

   (a) It can be used to represent any combination of quadrupoles and drifts in a synchrotron or section of a beam line.

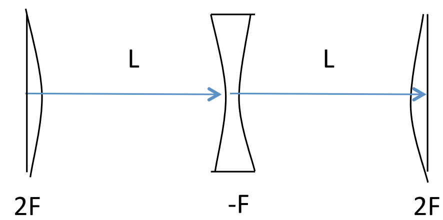
   (b) It can used to represent one cell in a periodic system for which stable solutions exist.

   (c) The parameters α, β, γ are the lattice functions at the ends of the cell.

   (d) The parameters α, β, γ are constant throughout the cell.

   (e) If I solve for the lattice functions of this cell in a periodic system, they are guaranteed to be the same for a *physically identical* cell in an extraction beam line.

3. Consider our standard, symmetric FODO cell.



(a) If the quadrupoles are separated by $L = 10$m, What focal length F is required to have a phase advance of $\mu = 60°$ over the cell [m]?

(b) What will be maximum betatron function β_{max} with this phase advance [m]?

4. A fundamental periodic cell of a synchrotron is described numerically (with units) by the matrix

$$\begin{pmatrix} 0 & 50\text{m} \\ ? & 0 \end{pmatrix}$$

(a) If this is a representation of a stable, periodic cell, what must the value (including units!) be for the lower left element?

(b) What are the lattice parameters α and β [m] at the ends of the cell?

(c) What is the minimum phase advance μ of this cell [degrees]?

5. A properly matched proton beam with an *unnormalized* RMS emittance $\epsilon = 2\mu$m is circulating in a synchrotron with a maximum betatron function in the x plane of $\beta_{max} = 30$m.

   (a) What is the maximum physical RMS size of the beam σ_x in the x plane [mm]?

   (b) If dispersion function at the location of β_{max} is $D_x = 3$m, what is the maximum fractional RMS momentum spread $\delta \equiv \sigma_p/p_0$ we can tolerate if we want the motion due to the momentum spread to be no larger than the beam spread due to betatron motion that you found in part (a)?

6. Assuming a beam distribution is described by a Gaussian emittance and is properly matched to the lattice, circle *all* of the things in the following list (a-e) that will *change* in the x plane immediately after the beam has passed through a thin lens.

   (a) β_x

   (b) α_x

   (c) γ_x

   (d) The RMS size of the spatial distribution σ_x

   (e) The RMS size of the angular distribution $\sigma_{x'}$

7. With regard to sextupoles, octopoles, and other magnetic elements of higher order than a quadrupole, please circle *all* of the following statements (a-e) that are TRUE.

   (a) Such elements may be represented by linear transfer matrices.

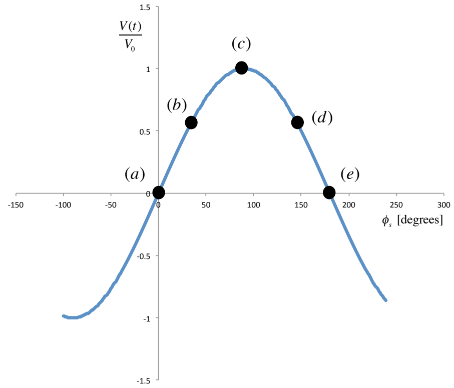
   (b) There are general, explicit solutions for particle trajectories in the presence of such magnets.

   (c) When analyzing particle dynamics, such elements must generally be treated perturbatively.

   (d) Such elements will generally result in chaotic motion and beam loss for sufficiently high amplitude particles.

   (e) The *only* only reason to include nonlinear elements in a lattice design is to cancel out the anomalous (unwanted) multipole moments of the dipole and quadrupole magnets.

8. The plot below shows the accelerating voltage $V(t)$ of the RF system of a proton synchrotron. Several possible values for the synchronous phase angle ϕ_s are labeled (a) through (e).



If the particles in the synchrotron have energies *above* the transition energy, please describe each point with the correct *two words* from the following two sets of choices: ["Accelerating" or "Not Accelerating"] and ["Stable" or "Unstable"] (e.g. "Accelerating, Unstable"):

(a)

(b)

(c)

(d)

(e)

9. A proton synchrotron has the following parameters:
   - Circumference: 3 km
   - RF Harmonic number: 500
   - Maximum RF voltage: 1 MV
   - Transition gamma (γ_T): 25
   - RMS Longitudinal emittance (ϵ_L): 0.1 eV-s

   Please calculate the following at the point when the beam is accelerating though $E_s = 100$GeV with a synchronous phase angle of $\phi_s = 120°$ You may assume the beam is traveling very close to the speed of light ($\beta \approx 1$).

   (a) What is the period of the machine [μs]?

   (b) What is the frequency of the RF [MHz]?

   (c) What is the ramp rate of the acceleration dE_s/dt [GeV/s]?

   (d) What is the slip factor η?

   (e) What is the the longitudinal betatron function β_L [s/eV]?

   (f) What is the RMS time distribution σ_t [ns]?

10. A collider collides two proton beams, each with n_b bunches of N_b particles each. If nothing else changes, by what factor will the luminosity increase if I double the bunch size N_b in both beams?

11. The maximum energy reached by the LEP e^-e^- collider was 104 GeV/beam. By what factor would they have had to increase the RF power to increase the energy per beam to 175 GeV - the energy needed to study the Higgs particle - in order to compensate for the increased synchrotron radiation loss? You may assume that the beam current and the RF efficiency stay the same.

12. A synchrotron is designed to operate with a tune in the x plane of $\nu_x = 9.82$. Trim quadrupoles are used to gradually lower the tune in the x plane only. The machine continues to operate smoothly until the tune drops below 9.69, at which point beam loss begins to increase. Of the following list (a-e), which would be the most likely explanation for this behavior (circle one):

    (a) A misaligned quadrupole

    (b) Anomalous quadrupole moments caused by imperfect magnets.

    (c) A rotated quadrupole.

    (d) Anomalous sextupole moments caused by imperfect magnets.

    (e) Anomalous octopole moments caused by imperfect magnets.

END OF EXAM