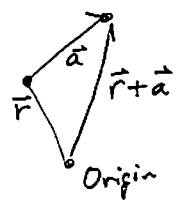
<u>Space-time symmetries</u> = translation, rotation, parity, charge conjugation, time-reversal

Only first 2 are exact in Nature.

Translation symmetry $\Longleftrightarrow$ all positions are intrinsically the same.

In QM, if we move everything by a vector $\vec{a}$:



$\vec{r}_i \to \vec{r}_i + \vec{a}$ for each particle labelled by i, then Hamiltonian H is unchanged.

For $\vec{a} = \delta\vec{r}$ infinitesimal, $H(\vec{r}_1', \vec{r}_2', \ldots) = H(\vec{r}_1 + \delta\vec{r}, \vec{r}_2 + \delta\vec{r}, \ldots)$
$= H(\vec{r}_1, \vec{r}_2, \ldots)$.

[For a single non-relativistic particle of mass m,

$$H = -\frac{p^2}{2m} = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \quad \checkmark \quad]$$

More generally, translation invariance is true for closed systems.

Consider a translation operator $\hat{D}$ for a single particle:

$$\hat{D}\underbrace{\psi(\vec{r})}_{\text{wavefunction}} = \psi(\vec{r} + \delta\vec{r}) = \psi(\vec{r}) + \delta\vec{r}\cdot\vec{\nabla}\psi \qquad \text{(expansion in small } \delta\vec{r}.)$$

So $\hat{D} = 1 + i\,\delta\vec{r}\cdot\hat{p}$ $\qquad (\hat{p} = -i\vec{\nabla}$ with $\hbar = 1)$

So $$\hat{D}H(\vec{r})\psi(\vec{r}) = \underbrace{H(\vec{r}+\delta\vec{r})}_{=H(\vec{r})}\psi(\vec{r}+\delta\vec{r}) = H(\vec{r})\psi(\vec{r}+\delta\vec{r}) = H(\vec{r})\hat{D}\psi(\vec{r})$$

Therefore, $(\hat{D}H - H\hat{D})\psi(\vec{r}) = 0$, so $[\hat{D}, H] = 0 \Rightarrow$

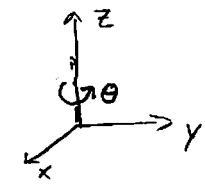
$$\boxed{[\hat{p}, H] = 0}$$

Invariance $\Rightarrow$ Conservation Law = Quantity commutes with H.

More generally, if there are N particles, the same argument works, with $\hat{p} = \sum_{i=1}^{N} \hat{p}_i$.

<u>Rotational Invariance</u> all directions in space are intrinsically same.

Do a rotation about z-axis



$$x_i \to x_i' = x_i \cos\theta - y_i \sin\theta$$
$$y_i \to y_i' = x_i \sin\theta + y_i \cos\theta$$
$$z_i \to z_i' = z_i$$

for all $i = 1, \ldots, N$ particles

Then $H(\vec{r}_1, \vec{r}_2, \ldots) = H(\vec{r}_1', \vec{r}_2', \ldots)$

Example: particle in a central potential:

$$H = -\frac{1}{2m}\nabla^2 + V(r)$$

Define a "rotation about z-axis operator by angle θ" operator $\hat{R}_z(\theta)$

Then $\hat{R}_z(\delta\theta)\,\psi(\vec{r}) \equiv \psi(\vec{r}') = \psi(x - \delta\theta\, y,\ y + \delta\theta\, x,\ z)$

expand in small $\delta\theta$

$$\psi(\vec{r}) + (\vec{r}' - \vec{r})\cdot\vec{\nabla}\psi(\vec{r}) = \psi(\vec{r}) - \delta\theta\left(y\frac{\partial\psi}{\partial x}\right) + \delta\theta\, x\frac{\partial\psi}{\partial y} = (1 + i\,\delta\theta\,\hat{L}_z)\,\psi(\vec{r})$$

if we define $\hat{L}_z = -i\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$ = z-component of orbital angular momentum operator.

So $\hat{R}_z(\delta\theta) = 1 + i\,\delta\theta\,\hat{L}_z$ for infinitesimal $\delta\theta$.

For a general axis of rotation $\hat{z} \to \hat{n}$,

$$\hat{R}_n(\delta\theta) = (1 + i\,\delta\theta\,\hat{\vec{L}}\cdot\hat{n})$$

Just as for translations, $\boxed{[\hat{\vec{L}}, H] = 0.}$

That was assuming no spin = intrinsic angular momentum.

More generally, $\Psi = \psi(\vec{r})\,\chi$

total wavefunction; position wavefunction; spin wavefunction

and $[\hat{\vec{J}}, H] = 0$ where $\hat{\vec{J}} = \hat{\vec{L}} + \hat{\vec{S}}$ acts on χ.

For spin-½, $\hat{\vec{S}} = \frac{1}{2}\vec{\sigma}$, where $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ = spin up state

$\chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ = spin down state

$$\hat{\vec{J}}\,\Psi = (\hat{\vec{L}} + \hat{\vec{S}})\,(\psi(\vec{r})\chi) = \left(\hat{\vec{L}}\,\psi(\vec{r})\right)\chi + \psi(\vec{r})\left(\hat{\vec{S}}\,\chi\right).$$

Note that in general, $[\hat{L}, H] = -[\hat{S}, H] \neq 0$ when there are spin-dependent forces present.

(Example: spin-orbit coupling $H_{so} = \sim \vec{L}\cdot\vec{S}$ in atomic physics.)

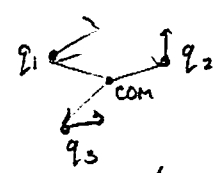
Classification of composite particles

(Atoms = nucleus + electrons, Nucleus = neutrons + protons, mesons = $q\bar{q}$, baryons = qqq.)

Spin of the composite particle = $\vec{J}$ (total angular momentum of constituents)

Let $\vec{S}$ = total spin of constituents

$\vec{L}$ = total angular momentum of constituents about center of mass.



Describe the total state in spectroscopic notation by

$^{2S+1}L_J$ (with $L \to S$ for $L=0$
$L \to P$ for $L=1$
$L \to D$ for $L=2$
$L \to F$ for $L=3$ ...)

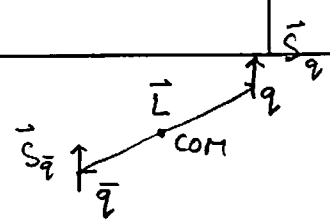
So a particle written as $^3S_1$ has $S=1$, $L=0$, $J=1$.

$^2D_{3/2}$ has $S=\frac{1}{2}$, $L=2$, $J=\frac{3}{2}$

$^3P_{5/2}$ is impossible!

The <u>deuteron</u> is a $^3S_1$ bound state of p, n.
(Usually, lowest lying bound states have $L=0$.)

Mesons in the quark model



$\vec{S} = \vec{S}_q + \vec{S}_{\bar{q}}$

Just one $\vec{L}$ = relative angular momentum

q and $\bar{q}$ have spin-½ $\Rightarrow$ $S = 0$ or 1.

For $L=0$ states (usually lowest energy = mass):

$$^{2S+1}L_J = {}^1S_0 \text{ or } {}^3S_1 \qquad (L=0)$$

For $L \geq 1$, $\begin{cases} J = L & (\text{if } S=0) \\ J = L+1, L, L-1 & (\text{if } S=1) \end{cases}$

So
$$^{2S+1}L_J = {}^1P_1 \text{ or } {}^3P_2, {}^3P_1, {}^3P_0 \qquad (L=1)$$
$$^1D_2 \text{ or } {}^3D_3, {}^3D_2, {}^3D_1 \qquad (L=2)$$
$$\vdots$$

Note: the spin of the meson is J, the lower subscript.

Lightest mesons are either spin-0 or spin-1.

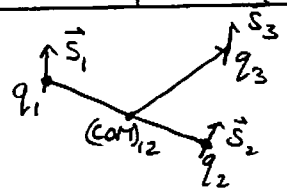
Examples:
$$\left.\begin{aligned} \pi^+ &= u\bar{d} \\ \pi^0 &= (u\bar{u} - d\bar{d})/\sqrt{2} \\ \pi^- &= d\bar{u} \end{aligned}\right\} \text{spin } 0, \quad {}^1S_0 \text{ states} \quad \sim 140 \text{ MeV}$$

and
$$\left.\begin{aligned} \rho^+ &= u\bar{d} \\ \rho^0 &= (u\bar{u} - d\bar{d})/\sqrt{2} \\ \rho^- &= d\bar{u} \end{aligned}\right\} \text{spin } 1 \quad {}^3S_1 \text{ states} \quad \sim 770 \text{ MeV}$$

Also,

strange mesons
$$\left.\begin{aligned} K^+ &= u\bar{s} \\ K^0 &= d\bar{s} \\ \bar{K}^0 &= s\bar{d} \\ K^- &= s\bar{u} \end{aligned}\right\} \text{spin } 0, \quad {}^1S_0 \text{ states} \quad \sim 500 \text{ MeV}$$
$$\left.\begin{aligned} K^{*+} &= u\bar{s} \\ K^{*0} &= d\bar{s} \\ \bar{K}^{*0} &= s\bar{d} \\ K^{*-} &= s\bar{u} \end{aligned}\right\} \text{spin } 1, \quad {}^3S_1 \text{ states} \quad \sim 900 \text{ MeV}$$

Baryons in the quark model



$\vec{L} = \vec{L}_{12} + \vec{L}_3$ (= integer ≥ 0)

$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$ (= 3/2 or 1/2)

So, for L=0, have states

$^2S_{1/2}$ or $^4S_{3/2}$ (L=0)

These should be the lowest lying baryons. But, can also have:

$^2P_{1/2}$, $^2P_{3/2}$, $^4P_{1/2}$, $^4P_{3/2}$, $^4P_{5/2}$ (L=1) and

$^2D_{3/2}$, $^2D_{5/2}$, $^4D_{1/2}$, $^4D_{3/2}$, $^4D_{5/2}$, $^4D_{7/2}$ (L=2) etc.

The most common $^2S_{1/2}$ baryon states are:

$p = uud$, $n = udd$ } 940 MeV

$\Lambda = uds$ $\Sigma^0 = uds$ $\Sigma^+ = uus$ $\Sigma^- = dds$ } 1116 ÷ 1197 MeV

$\Xi^0 = uss$ $\Xi^- = dss$ } ~ 1320 MeV

$\Lambda_c = udc$ ~ 2287 MeV (note: J = 1/2 not measured by experiment)

$\Lambda_b = udb$ ~ 5620 MeV (" ")

The most common $^4S_{3/2}$ baryon states are:

uuu Δ^{++}
uud Δ^+
udd Δ^0
ddd Δ^-
} ~ 1232 MeV

uus Σ^+
uds Σ^0
dds Σ^-
} ~ 1385 MeV

uus Ξ^0
dss Ξ^-
} ~ 1530 MeV

Note; the names are reused! One writes "Σ^-" for the $^2S_{1/2}$ state with mass 1197.45 MeV, and "$\Sigma(1385)^-$" for the $^4S_{3/2}$ state with mass 1387.2 MeV.

sss Ω^- ~ 1672 ← prediction of this state by Gell-Mann was big evidence for quark model.

<u>Parity</u> = space inversion is the transformation

$$\vec{r}_i \to \vec{r}_i{}' = -\vec{r}_i \quad \text{for every particle labelled by } i = 1, \ldots, N.$$

A system is invariant under parity if:

$$H(-\vec{r}_1, -\vec{r}_2, \ldots) = H(\vec{r}_1, \vec{r}_2, \ldots) \iff [\underbrace{\hat{P}}_{\text{parity operator}}, H] = 0$$

This is true for gravity, strong interactions, EM interactions but <u>not</u> for weak interactions (1957.)

Can often neglect weak interactions.

For a single particle,

$$\hat{P}\,\psi(\vec{r},t) \equiv \underbrace{P_a}_{\text{a constant}}\,\psi(-\vec{r},t) \qquad a \text{ labels type of particle}$$

Now $\hat{P}^2\psi(\vec{r},t) = \psi(\vec{r},t)$

$$\hat{P}^2\psi(\vec{r},t) = P_a^2\,\psi(\vec{r},t), \quad \text{so} \quad P_a^2 = 1 \Rightarrow P_a = \pm 1$$

For momentum eigenstates, $\psi_{\vec{p}}(\vec{r},t) = e^{i(\vec{p}\cdot\vec{r} - Et)}$, so

$$\hat{P}\,\psi_{\vec{p}}(\vec{r},t) = P_a\,\psi_{\vec{p}}(-\vec{r},t) = P_a\,\psi_{-\vec{p}}(\vec{r},t).$$

So if $\vec{p} = 0$ (particle at rest), P_a is the eigenvalue of $\hat{P}$, ($\equiv$ intrinsic parity).

Also, particles with definite angular momentum are eigenstates of $\hat{P}$:

$$\psi_{nlm}(\vec{r}) = \underbrace{R_{nl}(r)}_{\text{radial wavefunctions}}\,\underbrace{Y_{lm}(\theta,\phi)}_{\text{spherical harmonics}}$$

Under $\vec{r} \to -\vec{r}$, $\quad (r \to r,\ \theta \to \pi - \theta,\ \phi \to \phi + \pi)$.

So $\hat{P}\,\psi_{nlm}(\vec{r}) = P_a\,\psi_{nlm}(-\vec{r}) = P_a\,R_{nl}(r)\,\underbrace{Y_{lm}(\pi-\theta, \phi+\pi)}_{= (-1)^l\,Y_{lm}(\theta,\phi)}$

$$= P_a(-1)^l\,\psi_{nlm}(\vec{r})$$

So parity eigenvalue is: $\underbrace{P_a}_{\text{intrinsic parity for particle at rest.}}(-1)^l$

Parity for fermions and antifermions

Dirac equation shows: $P_{e^+} P_{e^-} = -1$

intrinsic parities

and more generally $P_f = -P_{\bar{f}}$

Useful for predictions. For example

$e^+ e^- \to$ (S-wave positronium) $\to \gamma\gamma$

Since $L=0$, total parity of e^+e^- system is $P = P_{e^+} P_{e^-} (-1)^{\overset{L}{0}} = -1$

For final state, $P = \underbrace{P_\gamma P_\gamma}_{=1} (-1)^{l_\gamma} = (-1)^{l_\gamma}$ where l_γ = relative angular momentum of photons.

So, predict l_γ = odd. ✓

Since electrons can't be created or destroyed without creating or destroying a positron, no process can determine P_{e^-} or P_{e^+} alone; only their product is observable.

Convention: $P_{e^-} = P_{\mu^-} = P_{\tau^-} = +1$. Then,

$P_{e^+} = P_{\mu^+} = P_{\tau^+} = -1$ follows.

Similarly, for quarks, $P_u = P_d = P_s = P_c = P_b = P_t = 1$

$P_{\bar{u}} = P_{\bar{d}} = P_{\bar{s}} = P_{\bar{c}} = P_{\bar{b}} = P_{\bar{t}} = -1$.

Parity for hadrons (mesons, baryons)

For a meson $M = a\bar{b}$, $(a, b = u, d, s, c, b)$

$$P_M = P_a P_{\bar{b}} (-1)^L = (-1)^{L+1}$$

For the lightest mesons, $L=0$, so $P_M = -1$.

Agrees with observed parities of π, K, D mesons.

For baryons, $B = abc$, $P_B = \underbrace{P_a P_b P_c}_{1 \cdot 1 \cdot 1} (-1)^{L_{12}} (-1)^{L_3} = (-1)^{L_{12} + L_3}$

For the <u>lightest</u> baryons, $L_{12} = L_3 = 0$, so $P_B = +1$.

This assignment is valid for $B = p, n, \Lambda, \Sigma^0, \Sigma^-, \Sigma^+, \Xi^-, \Xi^0$ ($^2S_{1/2}$)

as well as the $^4S_{3/2}$ states $B = \Delta^-, \Delta^0, \Delta^+, \Delta^{++}, \Sigma^{*-}, \Sigma^{*0}, \Sigma^{*+}, \Xi^{*-}, \Xi^{*+}$ and Ω^-.

What about antibaryons? $\bar{B} = \bar{a}\bar{b}\bar{c}$, so

$$P_{\bar{B}} = \underbrace{P_{\bar{a}} P_{\bar{b}} P_{\bar{c}}}_{(-1)^3} (-1)^{L_{12}} (-1)^{L_3} = (-1)^{L_{12} + L_3 + 1}.$$

For the lightest baryons, $P_{\bar{B}} = -1$ (for $\bar{p}, \bar{n}, \ldots$).

Note that assigning definite parity to these states is only valid if you neglect the weak interactions in the binding energy, but that's a <u>very</u> good approximation.

Martin + Shaw say (bottom page 131):

"...hadrons containing the t quark $\underbrace{\text{have not yet been discovered}}$."

Never will be; don't exist!

Top quark lifetime $\sim 10^{-25}$ sec

Time needed to form bound states much larger.

<u>Test of parity of the π^-</u>

$\pi^- + D \to n + n$

For the deuteron $D = pn$ in $L=0$ state, $P_D = P_p P_n (-1)^{L=0} = 1 \cdot 1 \cdot 1 = +1$.

$\pi^- + D$ reacts in an S-wave ($L=0$) state (wavefunctions overlap maximally).

So $P_{\text{initial state}} = P_{\pi^-} P_D (-1)^{L=0} = P_{\pi^-}$.

Meanwhile, the total angular momentum is $J = 1$ (spin of D).

Also, $P_{\text{final state}} = P_n P_n (-1)^{L_{\text{final}}} = (-1)^{L_{\text{final}}}$.

For total $n+n$ spin $S=1$, wavefunction is symmetric in spin ($|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$ or $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$), so spatial wavefunction must be

antisymmetric by Fermi statistics. $\Rightarrow L_{final}$ odd.

So $J_{final} = 1$, $S_{final} = 1 \Rightarrow L_{final} = 1$.

So $P_{initial} = P_{final} \Rightarrow P_{\pi^-} = -1$. ✓

What about $S_{final} = 0$? That is an antisymmetric spin state $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$, so the spatial wavefunction is symmetric, so L_{final} = even. But $J_{final} = 1$, $S_{final} = 0 \Rightarrow L_{final} = 1$. ✗

Contradiction, so $P_{\pi^-} = -1$, and the final state neutrons have $S = 1$.

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<u>Parity of the Photon</u>

In QM, the vector potential corresponds to photon wavefunction (in Coulomb gauge, with $\vec{\nabla}\cdot\vec{A} = 0$ and $\Phi = 0$ (electric potential).

$$\begin{cases} \vec{A}(\vec{r},t) = \hat{n}\, e^{i(\vec{k}\cdot\vec{r} - Et)} \\ \Phi(\vec{r},t) = 0 \end{cases}$$

$\hat{n}$ ← polarization vector

$\hat{n}\cdot\vec{k} = 0$ in Coulomb gauge.

Under a parity transformation $\vec{r} \to -\vec{r}$, charge density transforms like $\rho(\vec{r},t) \to \rho(-\vec{r},t)$. Now

$\vec{\nabla}\cdot\vec{E} = \frac{\rho(\vec{r},t)}{\epsilon_0}$, and $\vec{\nabla} \to -\vec{\nabla}$ under parity, so

$\vec{E}(\vec{r},t) \to -\vec{E}(-\vec{r},t)$.

But also $\vec{E} = -\underbrace{\nabla\Phi}_{0} - \frac{\partial\vec{A}}{\partial t}$, so $\vec{A}(\vec{r},t) \to -\vec{A}(-\vec{r},t)$.

So define the intrinsic parity of the photon:

$\vec{A}(\vec{r},t) \to P_\gamma A(-\vec{r},t)$, where $P_\gamma = -1$.

<u>Charge Conjugation</u> Define an operator $\hat{C}$ that changes every particle into its antiparticle. Notes:

* Momenta, positions, angular momenta, spins unchanged
* Some particles are their own antiparticle (photon, π^0, ...)

* Charge and magnetic moment are reversed by $\hat{C}$.
* All charged, and <u>some</u> neutral particles are not their own antiparticle. p, π^+, n have antiparticles $\bar{p}, \pi^-, \bar{n}$.
* Like parity, $\hat{C}$ is a symmetry ($[\hat{C}, H] = 0$) of gravity, strong, and EM interactions, but <u>not</u> weak interactions.

If $a = \pi^+, K^+, p, n, \ldots$ and $\alpha = \pi^0, \gamma, \ldots$, then

$$\hat{C}|a\Psi\rangle = |\bar{a}\Psi\rangle, \quad \text{and} \quad \hat{C}|\alpha\Psi\rangle = C_\alpha|\alpha\Psi\rangle$$

phase factor here chosen $= +1$, by convention. possible phase factor.

Since $\hat{C}^2 = 1$, must have $C_\alpha = \pm 1$.

For multi-particle states;

$$\hat{C}\,|\alpha_1 \alpha_2 \ldots; a_1 a_2 \ldots\rangle = C_{\alpha_1} C_{\alpha_2} \ldots |\alpha_1 \alpha_2 \ldots; \bar{a}_1, \bar{a}_2 \ldots\rangle$$

Eigenstates of $\hat{C}$ must have particle-antiparticle pairs.

Example: $\pi^+\pi^-$ pair with relative orbital angular momentum L.

$$\hat{C}\,|\pi^+\pi^-; L\rangle = |\pi^-\pi^+; L\rangle = (-1)^L |\pi^+\pi^-; L\rangle.$$

For fermions, there is an additional factor (-1) for odd total spin, and an additional factor (-1) for exchanging fermions.

So for any particle-antiparticle state:

$$\hat{C}\,|f\bar{f}; J, L, S\rangle = (-1)^{L+S}|f\bar{f}; J, L, S\rangle$$

So $^1S_0$ neutral mesons (for example π^0) have $C = 1$.
$^3S_1$ neutral mesons (for example ρ^0) have $C = -1$.

In the Review of Particle Properties, for example, you'll see

$$J^{PC} = 0^{-+} \text{ for } \pi^0 \quad \text{and} \quad J^{PC} = 1^{--} \text{ for } \rho^0.$$

Note that C_{π^+} is not defined; $\hat{C}\pi^+ = \pi^-$, so <u>not</u> an eigenstate.

In the RPP entry for π^+, it just says $J^P = 0^-$.

<u>Charge conjugation for the photon</u>

Under $\hat{C}$, charge $\to$ −charge, so from $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$, we have $\vec{E}(\vec{r},t) \to -\vec{E}(\vec{r},t)$. So, as for parity,

$\vec{A}(\vec{r},t) \to C_\gamma A(\vec{r},t)$, where $\boxed{C_\gamma = -1}$.

Test this in π^0 decays:

<u>$\pi^0 \to \gamma\gamma$</u> $\quad \hat{C}|\pi^0\rangle = C_{\pi^0}|\pi^0\rangle = |\pi^0\rangle$ (initial state has $C=+1$)

For the final state, $\hat{C}|\gamma\gamma\rangle = \underbrace{C_\gamma C_\gamma}_{(-1)^2}|\gamma\gamma\rangle = |\gamma\gamma\rangle$ (final state has $C=+1$). ✓

Now consider the alternate final state:

<u>$\pi^0 \to \gamma\gamma\gamma$</u> Here $\hat{C}|\gamma\gamma\gamma\rangle = (C_\gamma)^3|\gamma\gamma\gamma\rangle = -|\gamma\gamma\gamma\rangle$.

This final state has the wrong C, so predict it can't happen (except by weak interactions $\ll$ EM interactions)

Experimental result is $\dfrac{\Gamma(\pi^0\to\gamma\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)} < 3\times10^{-8}$ (never been seen).

Decays of the η meson (neutral, spin 0, $J^{PC} = 0^{-+}$)

$\eta \to \gamma\gamma$ $\quad$ BR = 0.39 $\quad$ ← clearly electromagnetic, involves γ

$\eta \to \pi^0\pi^0\pi^0$ $\quad$ BR = 0.33

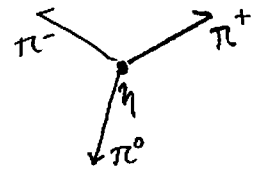
$\eta \to \pi^+\pi^-\pi^0$ $\quad$ BR = 0.23

← must be electromagnetic also, because comparable.

The $\eta\to\gamma\gamma$ proves that $C_\eta = +1$, just like π^0.

Check: $\hat{C}|\pi^0\pi^0\pi^0\rangle = (C_{\pi^0})^3|\pi^0\pi^0\pi^0\rangle = +1\,|\pi^0\pi^0\pi^0\rangle$ ✓

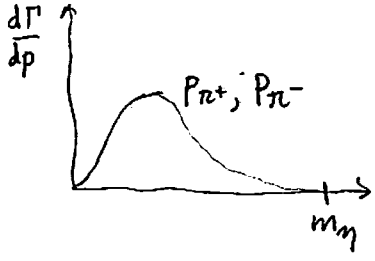
Now consider the last decay: ____



$\eta \to \pi^+(p_1) + \pi^-(p_2) + \pi^0(p_3)$ Act on this with $\hat{C}$.

$\eta \to \pi^+(p_2) + \pi^-(p_1) + \pi^0(p_3)$ must be same state.

So the π^+ and π^- must have <u>same</u> momentum distributions.



Individual events will have different p_{π^+}, p_{π^-}, but their distributions are same.

<u>Positronium</u> = e^+e^- bound state (like H atom)

$V(r) = -\frac{\alpha}{r}$ ($\alpha = e^2$ in natural units)

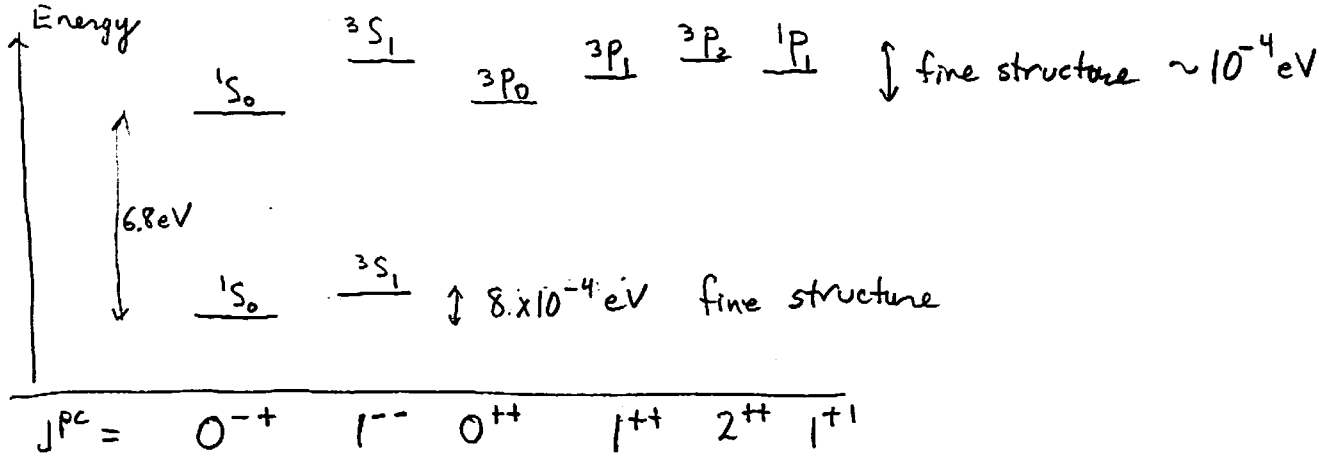
$E_n = -\frac{(13.6\,\text{eV})}{2n^2}$ only difference from H atom.

For each n, $L = 0, 1, \ldots, n-1$.
Also, $S = S_{e^+} + S_{e^-} = 0$ or 1. $\Big\}$ $J = 0, \ldots, n$

$P = \underbrace{P_{e^+} P_{e^-}}_{-1} (-1)^L = (-1)^{L+1}$

$C = (-1)^{L+S}$

The $n=1$ and $n=2$ energy levels:



Charmonium ($c\bar{c}$) and Bottomonium ($b\bar{b}$) have similar structure, but larger binding energy.

C-parity and positronium annihilation

Parapositronium $= {}^1S_0$, $J^{PC} = 0^{-+}$ state (typo in MS, 2 lines before 5.64)
Orthopositronium $= {}^3S_1$, $J^{PC} = 1^{--}$ state.

So, since a $\gamma\gamma$ state has $C=+1$, and $\gamma\gamma\gamma$ has $C=-1$,

$$\left.\begin{array}{ll} {}^1S_0 \to \gamma\gamma & \text{allowed} \\ {}^1S_0 \to \gamma\gamma\gamma & \text{forbidden} \end{array}\right\} \text{ \underline{if} } C \text{ is conserved}$$

Experimental result:

$$\frac{\Gamma({}^1S_0 \to \gamma\gamma\gamma)}{\Gamma({}^1S_0 \to \gamma\gamma)} < 1 \times 10^{-5} \quad \text{for parapositronium.} \checkmark$$

Also,

$$\left.\begin{array}{ll} {}^3S_1 \to \gamma\gamma & \text{forbidden} \\ {}^3S_1 \to \gamma\gamma\gamma & \text{allowed} \end{array}\right\} \text{ \underline{if} } C \text{ is conserved}$$

So, para and ortho are distinguished by how many γ's in final state:
${}^1S_0 \to \gamma\gamma$ and ${}^3S_1 \to \gamma\gamma\gamma$.

Since each extra γ implies an extra factor of $\alpha \simeq \frac{1}{137}$, we expect

$$R = \frac{\tau({}^1S_0)}{\tau({}^3S_1)} \sim \alpha \sim \frac{1}{137}.$$

In reality, $R = \dfrac{1.25 \times 10^{-10}\ \text{sec}}{1.42 \times 10^{-7}\ \text{sec}} \simeq 0.88 \times 10^{-3}\ \text{sec} \approx 0.12\,\alpha.$

O.K. for an order-of-magnitude