Homework # 2 Solutions

(1)

1. $\frac{dE}{dx} \simeq 1.8$ MeV/g/m^2 for MIPs

I stated that you should assume protons as MIPs for simplicity

(Any value 1.5 – 2 MeV/g/m^2 is acceptable)

$\rho(\text{air}) = 1.297$ kg/m^3 (ρ at STP)

$\simeq 1.3 \times 10^{-3}$ g/cc

$$\therefore \frac{dE}{dx} = 1.8 \times 1.3 \times 10^{-3} \text{ MeV/cm}$$

$$= 2.34 \times 10^{-3} \text{ MeV/cm}$$

So, for one turn in the Tevatron (6 km circumference $= 6 \times 10^5$ cm

$$\Delta E = 2.34 \times 10^{-3} \times 6 \times 10^5 \text{ MeV}$$

$$\simeq 1400 \text{ MeV} = 1.4 \text{ GeV/turn}$$

To lose the total energy of 1 TeV (ignoring the fact that dE/dx depends on $\beta\gamma$ and will be different when energy falls below that of MIP)

$$\#\text{ of turns} = \frac{1000 \text{ GeV}}{1.4 \text{ GeV}} \simeq 714 \text{ turns}$$

Time taken by a $\beta = 1$ particle/turn $= \frac{6 \times 10^5 \text{ cm}}{30 \text{ cm/ns}} = 20\,\mu s$

$$\therefore 714 * 20\,\mu s = 14.3 \text{ ms}$$

This is an estimate assuming loss of energy being at a constant level as a MIP.

2. From the plot of R/M vs $\beta\gamma$,

$$\frac{R}{M} = 50{,}000 \ g/cm^2/GeV \text{ for 10 GeV muons}$$

$$\therefore R = \underset{g/cm^2/GeV}{5\times10^4} \times \frac{M(GeV)}{\rho(g/cc)}$$

$$= 5\times10^4 \times \frac{(.106)}{7.87} \ cm = 673 \ cm$$

$$= \underline{\underline{6.73 \ m}}$$

3. $$E_c^{\mu} = E_c^{e} \cdot \left(\frac{m_\mu}{m_e}\right)^2$$

Given $E_c^e = 8$ MeV (in Lead)

$$\therefore E_c^{\mu} = 8 * \left(\frac{106}{.511}\right)^2 \simeq 344239 \text{ MeV}$$

$$\simeq \underline{\underline{344 \text{ GeV}}}$$

4. Cerenkov angle for 3 GeV pion, kaon and proton

(Assume a medium of refractive index n and above threshold for $\hat{c}$ production)

$$\cos\theta_c = \frac{1}{\beta n}$$

$$\theta_c = \cos^{-1}\frac{1}{\beta n}$$

$$\beta = \frac{p}{E} = \frac{p}{\sqrt{p^2+m^2}} \; ; \; \theta_c = \cos^{-1}\left[\frac{1}{n}\cdot\frac{\sqrt{p^2+m^2}}{p}\right]$$

$$\beta_\pi = \frac{3}{\sqrt{9+(.14)^2}} = 0.9989$$

$$\beta_K = \frac{3}{\sqrt{9+(.494)^2}} = 0.9867$$

$$\beta_p = \frac{3}{\sqrt{9+(.938)^2}} = 0.9544$$

5. $v_D = \mu \cdot E \cdot \frac{\rho_0}{\rho}$

$\mu = 2500 \ cm^2/V\text{-}s \quad ; \ E = 2\times10^3 \ V/cm$

$\frac{\rho_0}{\rho} = 1$

$\therefore v_D = 2500 * 2\times10^3 \ cm/s$

$= 5\times10^6 \ cm/s$ or $\underline{\underline{5 \ cm/\mu s}}$

$\sigma_x = \sqrt{\frac{2kT}{e} \cdot \frac{x}{E}} = \sqrt{\frac{2 * .025 \ eV}{e * 2.0\times10^3 V/cm} \cdot 1 \ cm}$

$= \sqrt{25\times10^{-6} \ cm^2}$

$= \underline{\underline{50 \mu m}}$

6. (i) $t_{max} = \dfrac{\ln(E/E_c)}{\ln 2}$; $E_c = \dfrac{800\,MeV}{Z}$

$= 9.75\,MeV$ (for Pb)

10 GeV: $t_{max} = \ln\left[10/(.8 * 82)\right] / \ln 2$

$= 10\,X_0$

$X_0 = 180\,A/Z^2 = 180 \times \dfrac{207}{(82)^2} = 5.54\,g/cm^2$

$(t_{max})_{10\,GeV} = \underline{\underline{55.4\,g/cm^2}}$

100 GeV: $t_{max} = 13.32\,X_0$

$= \underline{\underline{73.81\,g/cm^2}}$

(ii) $N_{max} = \dfrac{E_0}{E_c}$

10 GeV: $N_{max} = 1025$

100 GeV: $N_{max} = 10250$

6. (iii) $t_{95\%} = t_{max} + .082 + 9.6$

For 100 GeV e^-, $t_{95\%} = 13.3 + .082 + 9.6$

$= 29.46\ X_0$

$= \underline{\underline{163.21\ g/cm^2}}$

(iv) The "Stochastic Term" in the energy resolution

$$\frac{\sigma_E}{E} \sim \frac{1}{\sqrt{N_{ch}}} \sim \frac{1}{\sqrt{N_{max}}} \qquad N_{ch}^{max} \sim 2/3\, N_{max}$$

$\sim 1\%$ for 100 GeV e^-

But, I have given full credit of 1 point to everyone for this.