Space-Time Symmetries (Quick Re-cap)

Symmetries play an important role in particle physics.

What is symmetry?

If an operation/transformation that you perform on a system (the system) leaves it invariant, the system possesses symmetry w.r.t. that transformation

→ Point to conservation laws

→ Allows us to make progress when complete dynamical theory is not yet available

◇ Noether's Theorem:

- Every symmetry of nature corresponds to a conservation law.
- Conversely, every conservation law reflects an underlying symmetry

Symmetries ⟷ Conservation laws

◇ Gauge Invariance: (Mathematical Implication)

Conservation laws lead to the invariance of the Lagrangian of the interaction under the transformations of a unitary group

Gauge Groups:

	$U(1)$	EM
$SU(2)\times U(1)$	$SU(2)$	Weak (Isospin)
← electroweak	$SU(3)$	Strong (color)

Types of Symmetries

Space-Time

Continuous : Translational invariance

→ Conservation of linear mom^{m}.

$[\hat{p}, H] = 0$

Rotational Invariance

→ Consv^{n} of angular momentum

$[\hat{L}, H] = 0$ for spin-0 particles

$[\hat{J}, H] = 0$ in general

Discrete Multiplicative : Spatial Reflection

→ Parity $[\hat{P}, H] = 0$

$P^2 = 1$, $P = \pm 1$

Conserved in Strong and EM interactions

Particle-Antiparticle Conjugation

→ $[\hat{C}, H] = 0$

$\hat{C}^2 = 1$, $\hat{C} = \pm 1$

Time Reversal

Discrete Additive : Electric charge, weak charge, color charge

Baryon number, Lepton number

Other Internal Symmetries : Isospin

(charge independence of strong forces)

Time Reversal Invariance
& Principle of Detailed Balance

$t \to t' = -t$

Conserved in strong and electromagnetic interactions

But no physical observable associated with it that is conserved as a consequence of T invariance

Although no conservation law, it leads to what is called "Principle of detailed balance"

i.e., given equivalent conditions of energy, momentum, spin, the reaction rates of "time-reversed" process would be the same as the "forward" process

Consider,

$$p + p \longrightarrow \pi^+ + d$$

Sum over all final state spins.

$$\frac{d\sigma}{d\cos\theta}(p+p \to \pi^+ + d) = \frac{(2S_\pi+1)(2S_d+1)}{2\pi} \cdot \frac{p_\pi^2}{v_i v_f} \overline{|M_{if}|^2}$$

$\overline{|M_{if}|^2}$ is the squared scattering amplitude averaged over initial spin states.

$$\text{|||ly} \quad \frac{d\sigma}{d\cos\theta}(\pi^+ + d \to p + p) = \frac{(2S_p+1)^2}{2\pi} \frac{p_p^2}{v_i v_f} \overline{|M_{fi}|^2}$$

Principle of detailed balance says,

$$\overline{|M_{if}|^2} = \overline{|M_{fi}|^2}$$

$$\therefore \frac{d\sigma(p+p \to \pi^+ + d)/d\cos\theta}{d\sigma(\pi^+ + d \to p+p)/d\cos\theta} = 3\left(\frac{P_\pi}{P_p}\right)^2 (2S_\pi + 1)$$

The measurement of this ratio established $S_\pi = 0$.

The Principle has been tested for several reactions and no T violation has been seen.

But weak interactions violate $\hat{C}$, P and CP.

CPT is thought to be a perfect symmetry and conserved. So there maybe a compensating T violation?

If CPT symmetry is perfect, particle, antiparticle masses should be exactly the same.

$K^0 - \bar{K}^0$ mass difference $< 10^{-18} \cdot m_{K^0}$

Maybe neutrino sector might reveal something surprising.

ISOSPIN SYMMETRY

In 1932, Heisenberg suggested that proton and neutron can be considered as charge sub-states of one particle, the "nucleon", and introducing a quantum number called the "Isospin" I.

$I = \frac{1}{2}$ for the nucleon

I_z or $I_3 = +\frac{1}{2}$ for proton ; $-\frac{1}{2}$ for neutron

Recall $Q = I_3 + \frac{1}{2}$

$p = uud$; $n = udd$; $m_p = 938\,MeV$; $m_n = 940\,MeV$

Isospin is conserved in strong interactions.

The earliest evidence for this came from the equivalence of n-p, p-p and n-n forces after Coulomb effects are subtracted.

This isospin symmetry must come from the equivalence of u and d quarks (except for the charge)

$$m_u \approx m_d$$

Isospin symmetry applies to all hadrons which transform one to another by interchange of u and d quarks.

$m_{\pi^\pm} = 140\,MeV$

$m_{\pi^0} = 135\,MeV$

$$\left.\begin{array}{ll} \pi^+ = u\bar{d} & (I_3 = +1) \\ \pi^- = \bar{u}d & (I_3 = -1) \\ \pi^0 = \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u}) & (I_3 = 0) \end{array}\right\} \begin{array}{l} I = 1 \text{ triplet.} \\ \text{Masses nearly} \\ \text{equal} \end{array}$$

Building Hadrons from Quarks

Baryons $\equiv q_1 q_2 q_3$

Mesons $\equiv q_1 \bar{q}_2$

Lightest hadrons are light mesons made from u, d, s

$I_3(u) = +\frac{1}{2}$

$I_3(d) = -\frac{1}{2}$

Hypercharge $Y = B + S + C + \tilde{B} + T$

Strangeness

Baryon number

Bottomness

Triplet of $I = 1$

$2 \otimes \bar{2} = 3 \oplus 1$

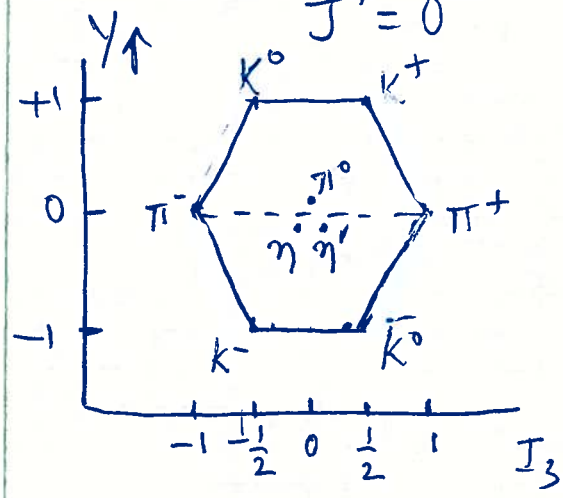
With only u, d, s

$u\bar{d}, d\bar{u}, \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}, \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$ $I = 0$

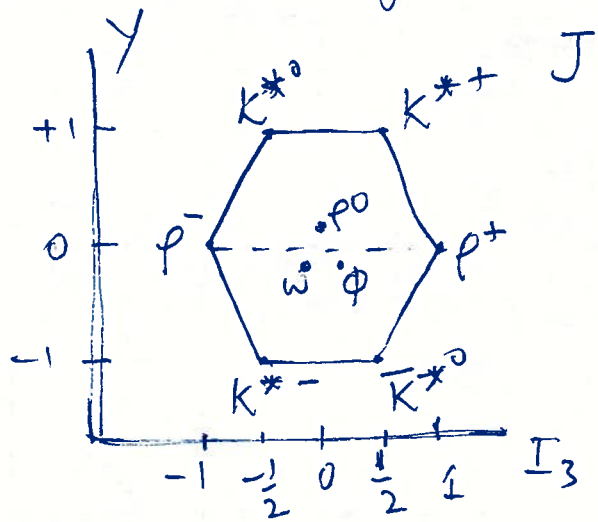
$u\bar{s}, \bar{u}s, s\bar{s}, d\bar{s}, s\bar{d}$

$Y = B + S = S \quad \because B = 0$ for Mesons

$J^P = 0^-$



$J^P = 1^-$



Meson Nonets with u, d, s

Parity of the meson

$$P = P_a P_b (-1)^L = (-1)^{L+1}$$

Flavorless singlet

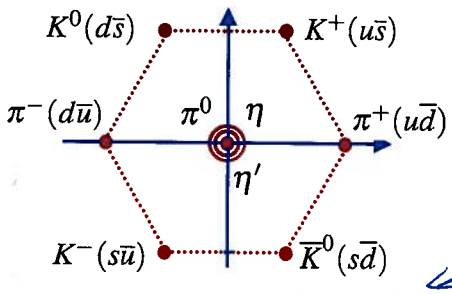
$\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$

$3 \times \bar{3} = 8 \oplus 1$

SU(3) flavor Meson Nonets

PSEUDOSCALAR MESONS (L=0, S=0, J=0, P= –1)

Spin



- Because SU(3) flavour is only approximate the physical states with $I_3 = 0,\ Y = 0$ can be mixtures of the octet and singlet states.

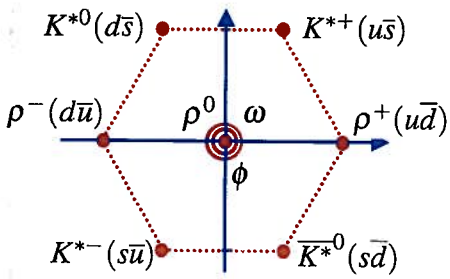
Empirically find:

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$
$$\eta \approx \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
$$\eta' \approx \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

singlet

Spin

VECTOR MESONS (L=0, S=1, J=1, P= –1)



- For the vector mesons the physical states are found to be approximately "ideally mixed":

$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$
$$\omega \approx \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
$$\phi \approx s\bar{s}$$

MASSES

$\pi^\pm : 140\,\text{MeV}$	$\pi^0 : 135\,\text{MeV}$
$K^\pm : 494\,\text{MeV}$	$K^0/\bar{K}^0 : 498\,\text{MeV}$
$\eta : 549\,\text{MeV}$	$\eta' : 958\,\text{MeV}$

$\rho^\pm : 770\,\text{MeV}$	$\rho^0 : 770\,\text{MeV}$
$K^{*\pm} : 892\,\text{MeV}$	$K^{*0}/\overline{K^{*0}} : 896\,\text{MeV}$
$\omega : 782\,\text{MeV}$	$\phi : 1020\,\text{MeV}$

7

Table 14.2: Suggested $q\bar{q}$ quark-model assignments for some of the observed light mesons. Mesons in bold face are included in the Meson Summary Table. The wave functions f and f' are given in the text. The singlet-octet mixing angles from the quadratic and linear mass formulae are also given for the well established nonets. The classification of the 0^{++} mesons is tentative and the mixing angle uncertain due to large uncertainties in some of the masses. Also, the $f_0(1710)$ and $f_0(1370)$ are expected to mix with the $f_0(1500)$. The latter is not in this table as it is hard to accommodate in the scalar nonet. The light scalars $a_0(980)$, $f_0(980)$, and $f_0(600)$ are often considered as meson-meson resonances or four-quark states, and are therefore not included in the table. See the "Note on Scalar Mesons" in the Meson Listings for details and alternative schemes.

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	−11.5	−24.6
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	38.7	36.0
$1\,^1P_1$	1^{+-}	$b_1(1235)$	K_{1B}†	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	K_{1A}†	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)$†	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	32.0	31.0
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$					
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		

† The $1^{+\pm}$ and $2^{-\pm}$ isospin $\frac{1}{2}$ states mix. In particular, the K_{1A} and K_{1B} are nearly equal (45°) mixtures of the $K_1(1270)$ and $K_1(1400)$. The physical vector mesons listed under 1^3D_1 and 2^3S_1 may be mixtures of 1^3D_1 and 2^3S_1, or even have hybrid components.

Table 14.3: $q\bar{q}$ quark-model assignments for the observed heavy mesons. Mesons in bold face are included in the Meson Summary Table.

$n\,^{2s+1}\ell_J$	J^{PC}	I = 0 $c\bar{c}$	I = 0 $b\bar{b}$	I = $\frac{1}{2}$ $c\bar{u}$, $c\bar{d}$; $\bar{c}u$, $\bar{c}d$	I = 0 $c\bar{s}$; $\bar{c}s$	I = $\frac{1}{2}$ $b\bar{u}$, $b\bar{d}$; $\bar{b}u$, $\bar{b}d$	I = 0 $b\bar{s}$; $\bar{b}s$	I = 0 $b\bar{c}$; $\bar{b}c$
$1\,^1S_0$	0^{-+}	$\eta_c(1S)$	$\eta_b(1S)$	D	$D_s^{\pm}$	B	B_s^0	$B_c^{\pm}$
$1\,^3S_1$	1^{--}	$J/\psi(1S)$	$\Upsilon(1S)$	D^*	$D_s^{*\pm}$	B^*	B_s^*	
$1\,^1P_1$	1^{+-}	$h_c(1P)$		$D_1(2420)$	$D_{s1}(2536)^{\pm}$			
$1\,^3P_0$	0^{++}	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$D_0^*(2400)$	$D_{s0}^*(2317)^{\pm\dagger}$			
$1\,^3P_1$	1^{++}	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$		$D_{s1}(2460)^{\pm\dagger}$			
$1\,^3P_2$	2^{++}	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$	$D_2^*(2460)$	$D_{s2}(2573)^{\pm}$			
$1\,^3D_1$	1^{--}	$\psi(3770)$						
$2\,^1S_0$	0^{-+}	$\eta_c(2S)$						
$2\,^3S_1$	1^{--}	$\psi(2S)$	$\Upsilon(2S)$					
$2\,^3P_{0,1,2}$	$0^{++}, 1^{++}, 2^{++}$		$\chi_{b\,0,1,2}(2P)$					

† The masses of these states are considerably smaller than most theoretical predictions. They have also been considered as four-quark states (See the "Note on Non-$q\bar{q}$ Mesons" at the end of the Meson Listings). The $D_{s1}(2460)^{\pm}$ and $D_{s1}(2536)^{\pm}$ are mixtures of the $1^{+\pm}$ states.

Adding charm,

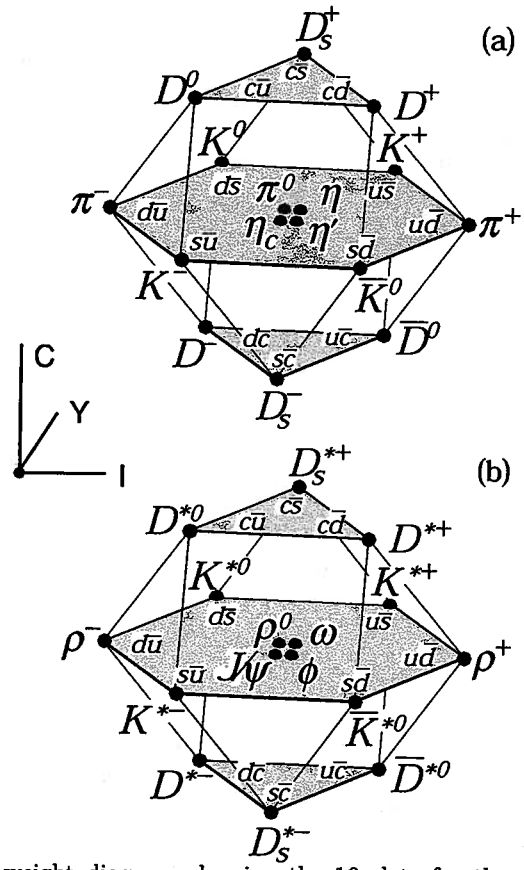


Figure 14.1: SU(4) weight diagram showing the 16-plets for the pseudoscalar (a) and vector mesons (b) made of the u, d, s, and c quarks as a function of isospin I, charm C, and hypercharge $Y = S + \mathcal{B} - \frac{C}{3}$. The nonets of light mesons occupy the central planes to which the $c\bar{c}$ states have been added.

Table 14.2: Suggested $q\bar{q}$ quark-model assignments for some of the observed light mesons. Mesons in bold face are included in the Meson Summary Table. The wave functions f and f' are given in the text. The singlet-octet mixing angles from the quadratic and linear mass formulae are also given for the well established nonets. The classification of the 0^{++} mesons is tentative and the mixing angle uncertain due to large uncertainties in some of the masses. Also, the $f_0(1710)$ and $f_0(1370)$ are expected to mix with the $f_0(1500)$. The latter is not in this table as it is hard to accommodate in the scalar nonet. The light scalars $a_0(980)$, $f_0(980)$, and $f_0(600)$ are often considered as meson-meson resonances or four-quark states, and are therefore not included in the table. See the "Note on Scalar Mesons" in the Meson Listings for details and alternative schemes.

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	−11.5	−24.6
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	38.7	36.0
$1\,^1P_1$	1^{+-}	$b_1(1235)$	K_{1B}†	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	K_{1A}†	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)$†	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	32.0	31.0
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$					
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		

† The $1^{+\pm}$ and $2^{-\pm}$ isospin $\frac{1}{2}$ states mix. In particular, the K_{1A} and K_{1B} are nearly equal (45°) mixtures of the $K_1(1270)$ and $K_1(1400)$. The physical vector mesons listed under 1^3D_1 and 2^3S_1 may be mixtures of 1^3D_1 and 2^3S_1, or even have hybrid components.

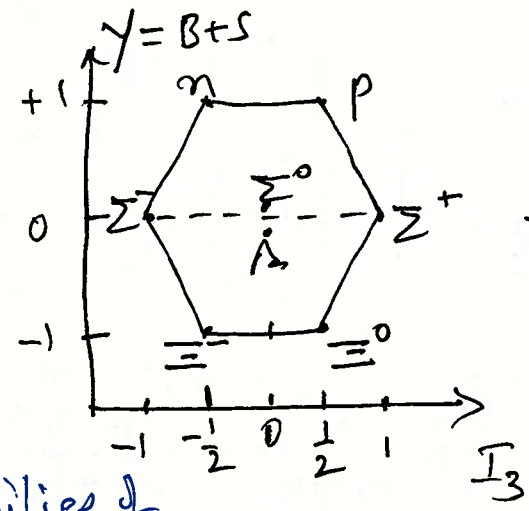
Table 14.3: $q\bar{q}$ quark-model assignments for the observed heavy mesons. Mesons in bold face are included in the Meson Summary Table.

$n\,^{2s+1}\ell_J$	J^{PC}	I = 0 $c\bar{c}$	I = 0 $b\bar{b}$	I = $\frac{1}{2}$ $c\bar{u}$, $c\bar{d}$; $\bar{c}u$, $\bar{c}d$	I = 0 $c\bar{s}$; $\bar{c}s$	I = $\frac{1}{2}$ $b\bar{u}$, $b\bar{d}$; $\bar{b}u$, $\bar{b}d$	I = 0 $b\bar{s}$; $\bar{b}s$	I = 0 $b\bar{c}$; $\bar{b}c$
$1\,^1S_0$	0^{-+}	$\eta_c(1S)$	$\eta_b(1S)$	D	$D_s^{\pm}$	B	B_s^0	$B_c^{\pm}$
$1\,^3S_1$	1^{--}	$J/\psi(1S)$	$\Upsilon(1S)$	D^*	$D_s^{*\pm}$	B^*	B_s^*	
$1\,^1P_1$	1^{+-}	$h_c(1P)$		$D_1(2420)$	$D_{s1}(2536)^{\pm}$			
$1\,^3P_0$	0^{++}	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$D_0^*(2400)$	$D_{s0}^*(2317)^{\pm\dagger}$			
$1\,^3P_1$	1^{++}	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$		$D_{s1}(2460)^{\pm\dagger}$			
$1\,^3P_2$	2^{++}	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$	$D_2^*(2460)$	$D_{s2}(2573)^{\pm}$			
$1\,^3D_1$	1^{--}	$\psi(3770)$						
$2\,^1S_0$	0^{-+}	$\eta_c(2S)$						
$2\,^3S_1$	1^{--}	$\psi(2S)$	$\Upsilon(2S)$					
$2\,^3P_{0,1,2}$	$0^{++}, 1^{++}, 2^{++}$		$\chi_{b\,0,1,2}(2P)$					

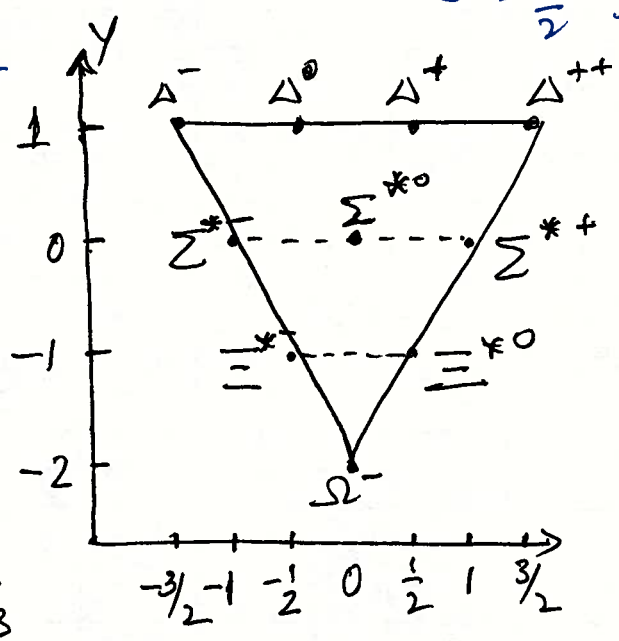
† The masses of these states are considerably smaller than most theoretical predictions. They have also been considered as four-quark states (See the "Note on Non-$q\bar{q}$ Mesons" at the end of the Meson Listings). The $D_{s1}(2460)^{\pm}$ and $D_{s1}(2536)^{\pm}$ are mixtures of the $1^{+\pm}$ states.

Baryons

$J^P = \frac{1}{2}^+$ Octet



$J^P = \frac{3}{2}^+$ Decuplet



Families of Super-multiplets with same Baryon number, Spin and Parity.

needs to be antisymmetric

Baryon wavefunction

$$\psi = \psi(\text{space})\,\psi(\text{spin})\,\psi(\text{flavor})\,\psi(\text{color})$$

TABLE 6.5 The states of the $L = 0, \frac{1}{2}^+$ octet of light baryons.

Quark composition	Observed state	I_3	I	S
uud	$p(938)$	1/2	1/2	0
udd	$n(940)$	−1/2	1/2	0
uds	$\Lambda(1116)^{\#}$	0	0	−1
uus	$\Sigma^+(1189)$	1	1	−1
uds	$\Sigma^0(1193)^{\#}$	0	1	−1
dds	$\Sigma^-(1197)$	−1	1	−1
uss	$\Xi^0(1315)$	1/2	1/2	−2
dss	$\Xi^-(1321)$	−1/2	1/2	−2

The $\Lambda(1116)$ and the $\Sigma^0(1193)$ differ in that in the former the *ud* pair is in a spin-0 state, while in the latter it is in a spin-1 state, as discussed at the end of Section 6.2.2.

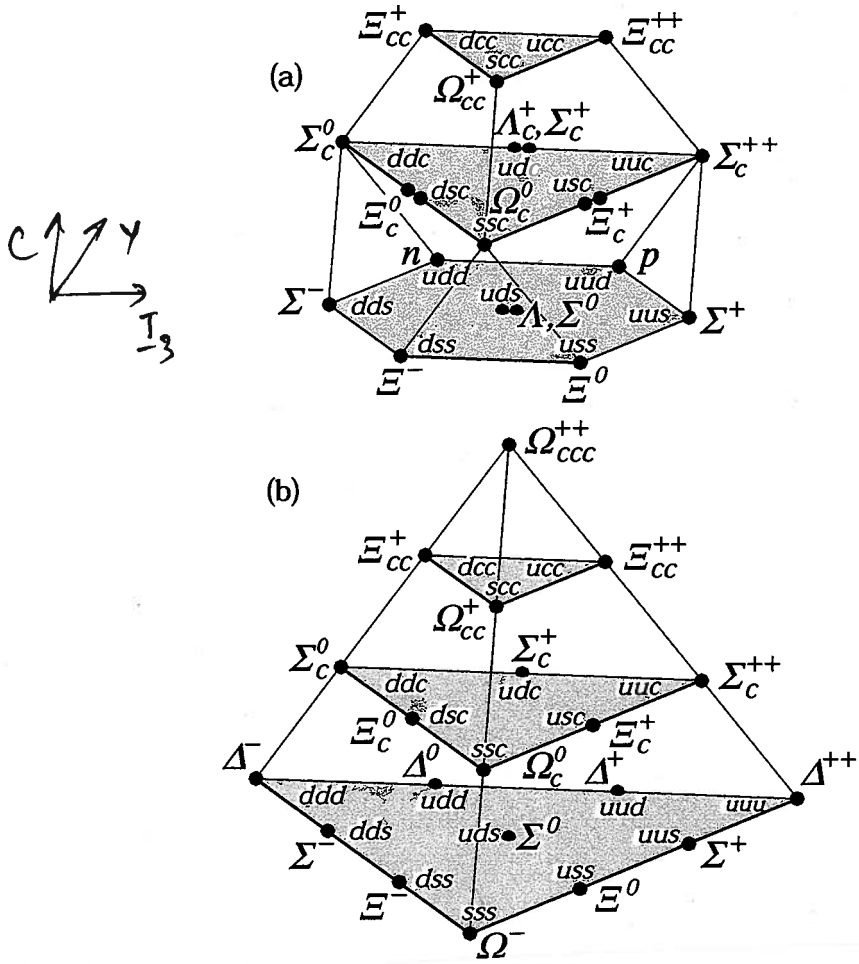
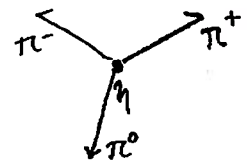


TABLE 6.6 The states of the $L=0, \frac{3}{2}^+$ decuplet of light baryons.

Quark composition	Observed state	I_3	I	S
uuu	$\Delta^{++}(1232)^{\#}$	3/2	3/2	0
uud	$\Delta^{+}(1232)^{\#}$	1/2	3/2	0
udd	$\Delta^{0}(1232)^{\#}$	−1/2	3/2	0
ddd	$\Delta^{-}(1232)^{\#}$	−3/2	3/2	0
uus	$\Sigma^{+}(1383)$	1	1	−1
uds	$\Sigma^{0}(1384)$	0	1	−1
dds	$\Sigma^{-}(1387)$	−1	1	−1
uss	$\Xi^{0}(1532)$	1/2	1/2	−2
dss	$\Xi^{-}(1535)$	−1/2	1/2	−2
sss	$\Omega^{-}(1672)$	0	0	−3

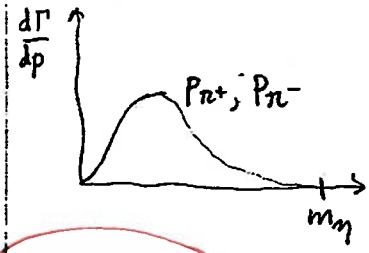
We do not show the small mass differences between the various charged and neutral states of the Δ as these are not all precisely determined.



$\eta \to \pi^+(p_1) + \pi^-(p_2) + \pi^0(p_3)$ Act on this with $\hat{C}$

$\eta \to \pi^+(p_2) + \pi^-(p_1) + \pi^0(p_3)$ must be same state.

So the π^+ and π^- must have <u>same</u> momentum distributions.



Individual events will have different p_{π^+}, p_{π^-}, but their distributions are same.

<u>Positronium</u> $= e^+e^-$ bound state (like H atom)

$V(r) = -\frac{\alpha}{r}$ ($\alpha = e^2$ in natural units)

$E_n = -\frac{(13.6\,\text{eV})}{2n^2}$ only difference from H atom.

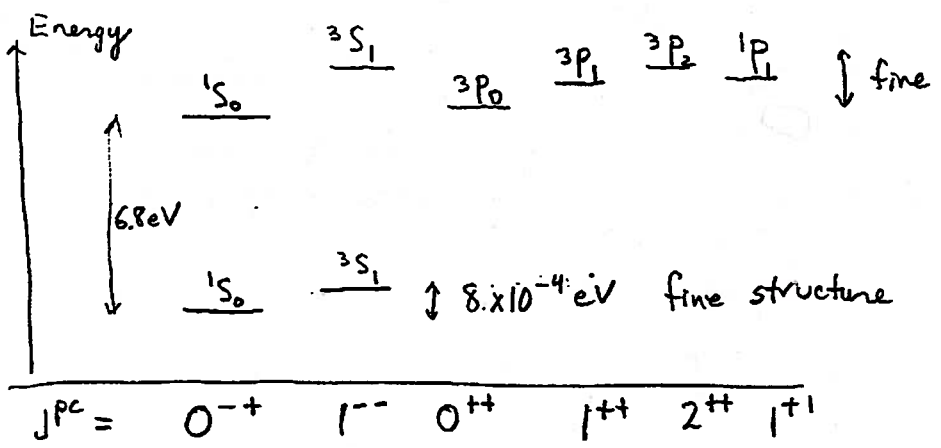
For each n, $L = 0, 1, \ldots, n-1$. $\Big\}$ $J = 0, \ldots, n$

Also, $S = S_{e^+} + S_{e^-} = 0$ or 1.

$P = \underbrace{P_{e^+} P_{e^-}}_{-1} (-1)^L = (-1)^{L+1}$

$C = (-1)^{L+S}$

The $n=1$ and $n=2$ energy levels:



Charmonium ($c\bar{c}$) and Bottomonium ($b\bar{b}$) have similar structure, but larger binding energy.

S. Martin

C-parity and positronium annihilation

Para-positronium $= {}^1S_0$, $J^{PC} = 0^{-+}$ state (typo in MS, 2 lines before 5.64)

Ortho positronium $= {}^3S_1$, $J^{PC} = 1^{--}$ state.

So, since a $\gamma\gamma$ state has $C=+1$, and $\gamma\gamma\gamma$ has $C=-1$,

$$\left.\begin{array}{ll} {}^1S_0 \to \gamma\gamma & \text{allowed} \\ {}^1S_0 \to \gamma\gamma\gamma & \text{forbidden} \end{array}\right\} \text{ \underline{if} } C \text{ is conserved}$$

Experimental result:

$$\frac{\Gamma({}^1S_0 \to \gamma\gamma\gamma)}{\Gamma({}^1S_0 \to \gamma\gamma)} < 1 \times 10^{-5} \quad \text{for para-positronium.} \checkmark$$

Also,

$$\left.\begin{array}{ll} {}^3S_1 \to \gamma\gamma & \text{forbidden} \\ {}^3S_1 \to \gamma\gamma\gamma & \text{allowed} \end{array}\right\} \text{ \underline{if} } C \text{ is conserved}$$

So, para and ortho are distinguished by how many γ's in final state:

${}^1S_0 \to \gamma\gamma$ and ${}^3S_1 \to \gamma\gamma\gamma$.

Since each extra γ implies an extra factor of $\alpha \simeq \frac{1}{137}$, we expect

$$R = \frac{\tau({}^1S_0)}{\tau({}^3S_1)} \sim \alpha \sim \frac{1}{137}.$$

In reality,

$$R = \frac{1.25 \times 10^{-10}\ \text{sec}}{1.42 \times 10^{-7}\ \text{sec}} \approx 0.88 \times 10^{-3}\ \text{sec} \approx 0.12\,\alpha.$$

O.K. for an order-of-magnitude

S. Martin

Heavy Flavor Quarkonia

Charmonium, Bottomonium
$(c\bar{c})$ $(b\bar{b})$

c, b, t are heavy flavor quarks

[t cannot form hadrons lives too short to form hadrons

Strong analogy with positronium e^+e^- bound state except for energy scale.

Same spectroscopic notations can be used to label the energy levels n, J, L and S.

$\Rightarrow {}^{2S+1}L_J$ and J^{PC}

$$P = P_q P_{\bar{q}} (-1)^L = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

TABLE 6.9 Predicted $c\bar{c}$ and $b\bar{b}$ states with principal quantum numbers $n = 1$ and 2 and radial quantum number $n_r = n - L$, compared with experimentally observed states. Masses are given in MeV/c^2.

${}^{2S+1}L_J$	n	n_r	J^{PC}	$c\bar{c}$ state	$b\bar{b}$ state
${}^1S_0$	1	1	0^{-+}	$\eta_c(2980)$	$\eta_b(9300)$#
${}^3S_1$	1	1	1^{--}	$J/\psi(3097)$	$\Upsilon(9460)$
${}^3P_0$	2	1	0^{++}	$\chi_{c0}(3415)$	$\chi_{b0}(9859)$
${}^3P_1$	2	1	1^{++}	$\chi_{c1}(3511)$	$\chi_{b1}(9893)$
${}^3P_2$	2	1	2^{++}	$\chi_{c2}(3556)$	$\chi_{b2}(9913)$
${}^1P_1$	2	1	1^{+-}	$h_c(3526)$#	
${}^1S_0$	2	2	0^{-+}	$\eta_c(3638)$	
${}^3S_1$	2	2	1^{--}	$\psi(3686)$	$\Upsilon(10023)$

State is not well established and its quantum number assignments are unknown.

First observed in e^+e^- and pBe

observed in pNucleus

Energy Spectra for Positronium and charmonium very similar.

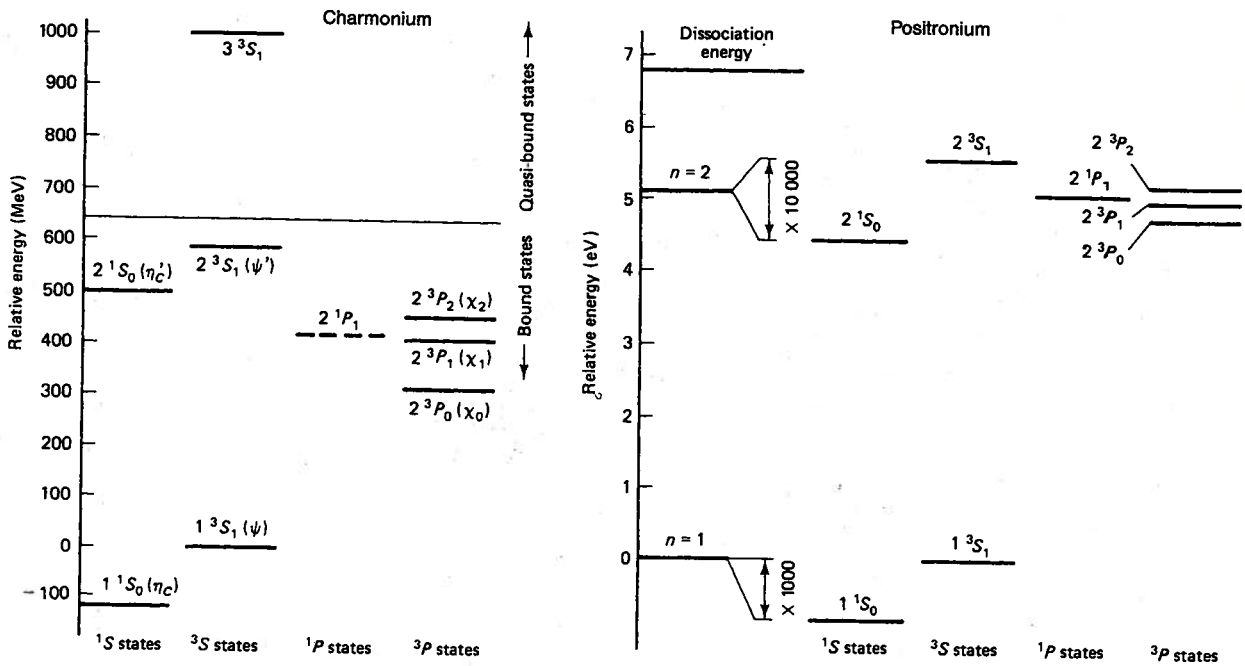


Fig. 5.6 Spectrum of energy levels in positronium and charmonium. Note that the scale is greater by a factor of 100 million for charmonium. In positronium, the various combinations of angular momentum cause only minuscule shifts in energy (shown by expanding the vertical scale), but in charmonium the shifts are much larger. All energies are given with reference to the 1^3S_1 state. At 6.8 eV positronium dissociates. At 633 MeV above the energy of the ψ charmonium becomes quasi-bound, because it can decay into D^0 and $\overline{D}^0$ mesons. (Source: Bloom, E. and Feldman, G. (May 1982) 'Quarkonium', *Scientific American*, p. 66, reprinted by permission.)

But note that the energy scales are ~ 100's of MeV in $c\bar{c}$ and eV in e^+e^-

Charm quark was discovered through the J/ψ (~3.1 GeV) state

Most conveniently produced by forming $J^{PC} = 1^{--}$ states in e^+e^-

Since $J^{PC}(\gamma) = 1^{--}$

$\therefore$ e^+e^- annihilate in a virtual photon, converts to the meson V^0 $(c\bar{c})$.

Possible when $E_{cm} = M_{V^0} = m_{c\bar{c}}$

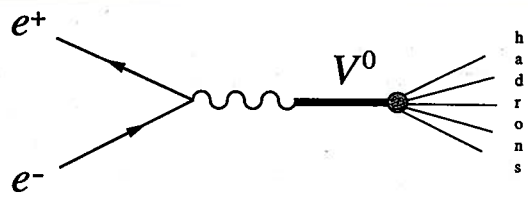
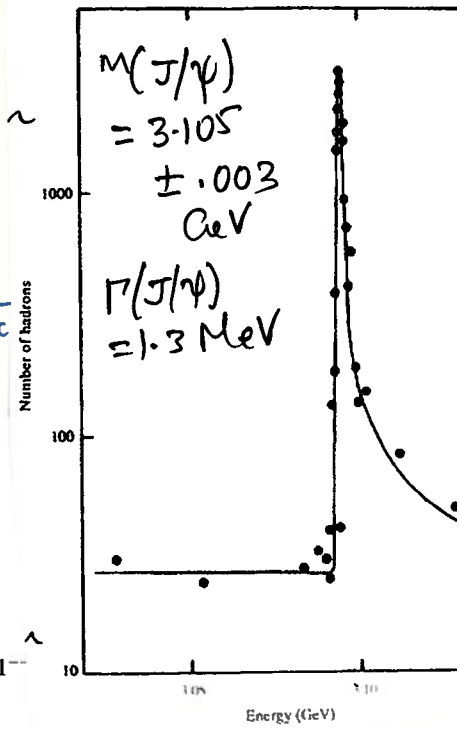
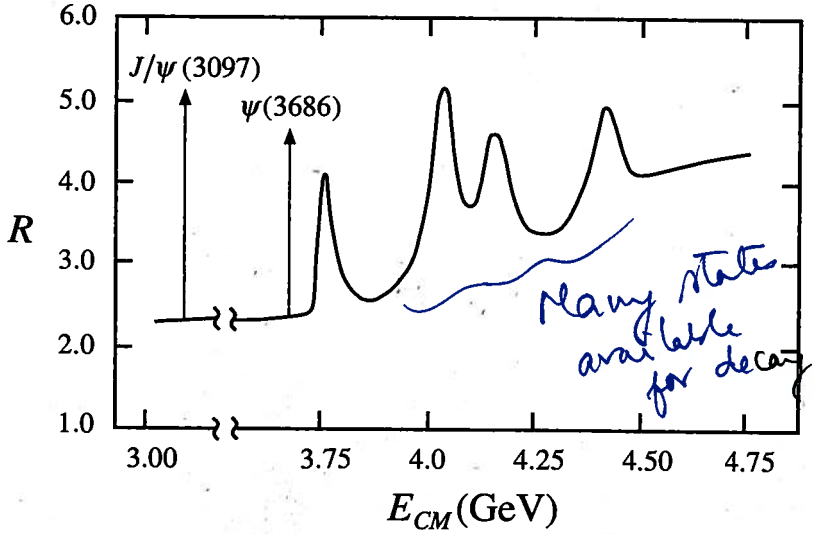


Figure 6.3 Mechanism for the formation of mesons V^0 with quantum numbers $J^{PC} = 1^{--}$ electron–positron annihilation, and their subsequent decay to hadrons.



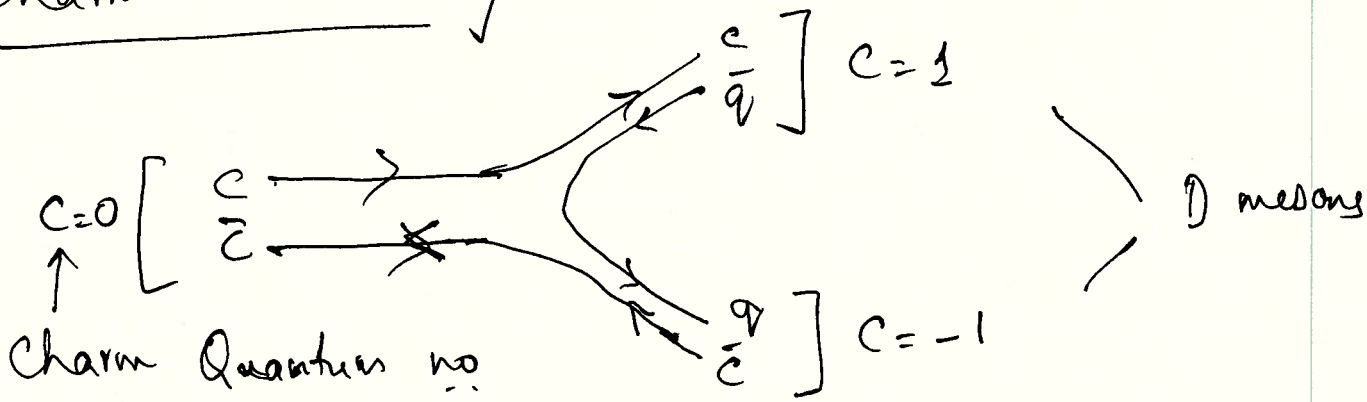


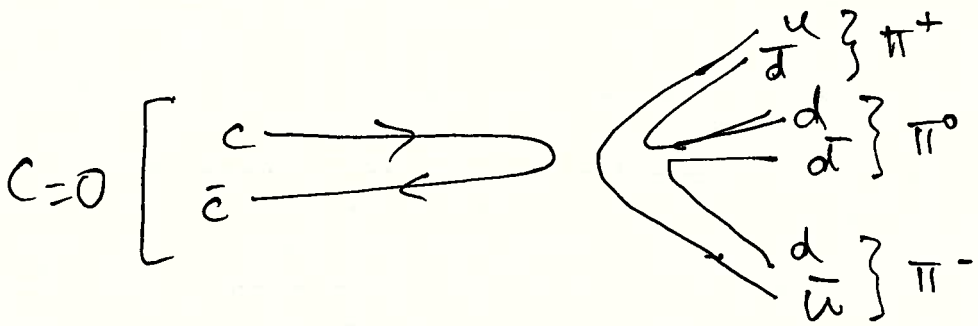
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3E_{CM}^2}$$

Figure 6.4 Measured cross-section ratio (6.50a) in the vicinity of the threshold for producing pairs of charmed particles. The extremely narrow peaks associated with the J/ψ(3097) and ψ(3686) are indicated by arrows because they extend far above the scale shown. In the partially suppressed region between these peaks, the ratio is essentially constant. (Data from Brandelik *et al.*, 1978, and Schindler *et al.*, 1980.)

Charmonium Decay





Decay to non-charmed mesons.

$c\bar{c}$ annihilates into $u\bar{u}$ which hadronize into light hadrons.

Other $c\bar{c}$ states (other than 1^{--} state) that are not produced by the photon-V^0 resonance process are studied by radiative decays of ψ. (BR ~ 2%)

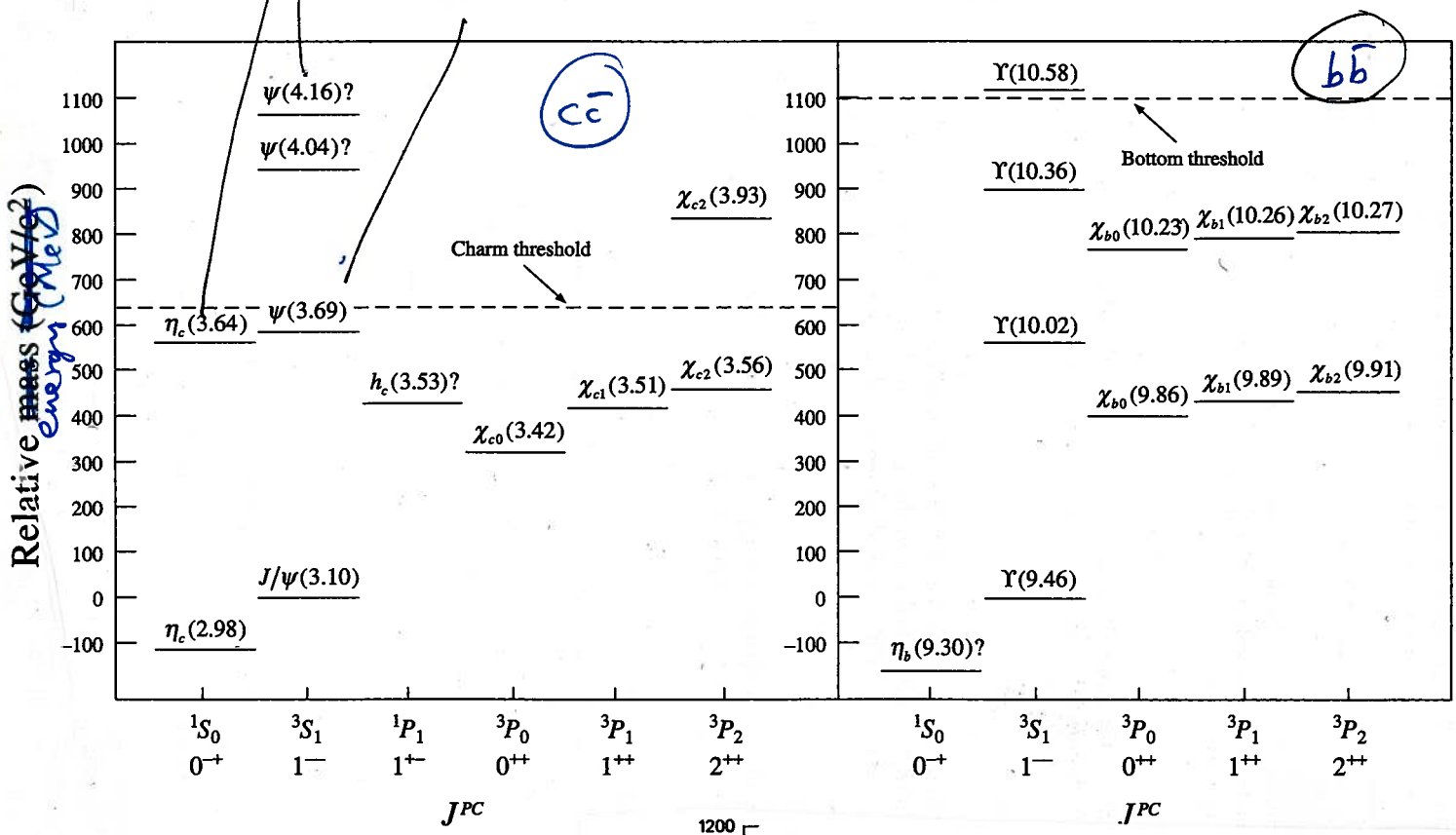
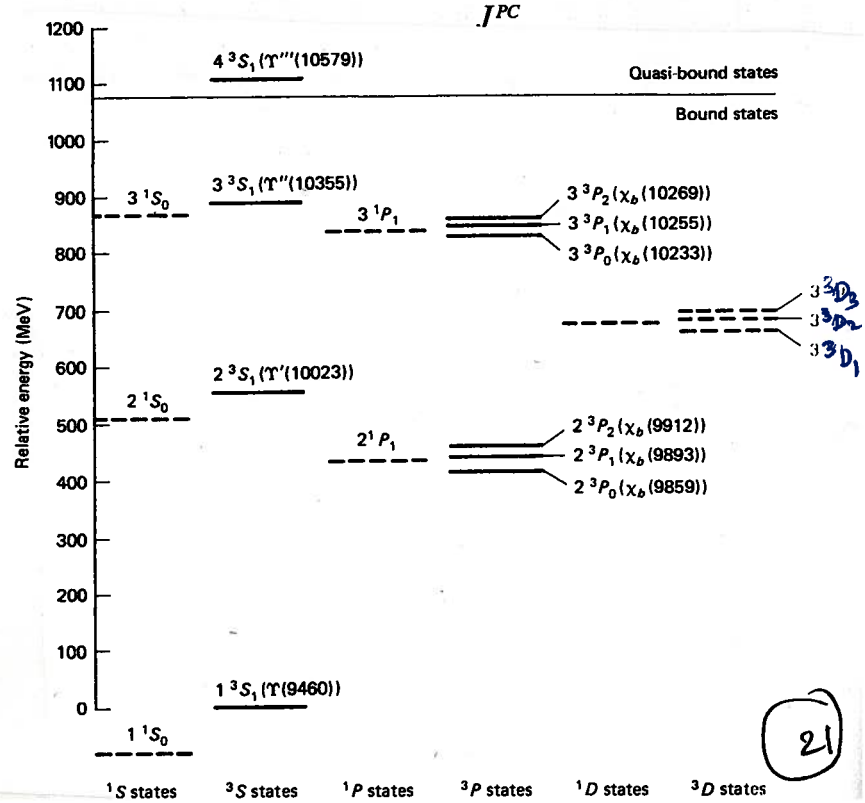
$\psi(3686) \longrightarrow \chi_c + \gamma$ ← 3415, 3511, 3556 MeV.

$\psi(3686) \longrightarrow \eta_c(2980) + \gamma$ → $^1S_0$ ground state.

$\psi(3097) \longrightarrow \eta_c(2980) + \gamma$.

$c\bar{c}$ and $b\bar{b}$

Masses in GeV/c^2



~~Figure 6.6 The observed states of the charmonium ($c\bar{c}$) and that of the $^3S_1$ ground state.~~