Quantum Chromodynamics (QCD)

Quarks and Gluons, Jets

* From Quark Model, we saw that the static properties of hadrons can be understood in terms of quark substructure

  i.e., charge, spin, parity, other quantum nos

* Dynamical effects are manifested in Scattering experiments.

* Interestingly, lepton annihilation reactions such as $e^+e^- \rightarrow$ hadrons

  and lepton deep-inelastic scattering such as $e^-p \rightarrow e^-p$

  i.e., high energy scattering

  give us a lot of information on quarks, gluons and structure of hadrons, mainly proton and neutron.

In ep, en or eN scattering, we are looking for eq scattering, essentially.

QCD

is the theory of strong interactions in the Standard Model (SM).

Not tested to the same level of precision as QED (tested to extraordinary precision) but impressive agreement with lots of experimental data.

QCD similar to QED in some ways but very different in others

(1) Spin-1 gluons ($m=0$) mediate forces — Spin-1 photons ($m=0$) mediate forces

(2) Gauge invariance holds and so gauge theories are used.

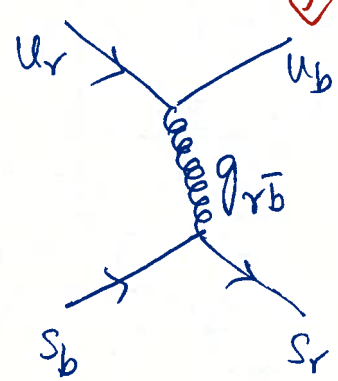
(3) couples to color charges — couples to electric charge

BUT

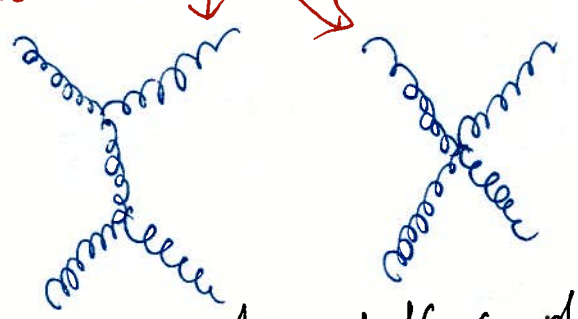
difference ⟹ (4) non-zero color charge — zero electric charge

So gluon can couple to itself while photon does not.

So, in addition to



we have



gluon self-coupling

No analogue in QED

For the gluon, $I_3^c = I_3^c(r) - I_3^c(b) = \frac{1}{2}$

$Y^c = Y^c(r) - Y^c(b) = 1$

Colour charges

Any quark can exist in one of the three colors r, g, b. Similar to spin states.

Color state χ_c correspond to a color hypercharge Y^c, Color Isospin charge I_3^c.

For a composite system:

$$\psi = \psi(r)\,\chi_{spin}\,\chi_c$$

Needs to be antisymmetric w.r.t. interchange of quarks of same flavor

Δ^{++} has 3 u quarks in symmetric state. Color makes it antisymmetric

Quarks	I_3^c	Y^c	Antiquarks	I_3^c	Y^c
r	$1/2$	$1/3$	$\bar{r}$	$-1/2$	$-1/3$
g	$-1/2$	$1/3$	$\bar{g}$	$1/2$	$-1/3$
b	0	$-2/3$	$\bar{b}$	0	$2/3$

Hadrons must have $I_3^c = Y^c = 0$. Must be Color-singlets

$\Rightarrow$ color confinement

$$\chi_B^c = \alpha_1 r_1 g_2 b_3 + \alpha_2 g_1 r_2 b_3 + \alpha_3 b_1 r_2 g_3 + \alpha_4 b_1 g_2 r_3 + \alpha_5 g_1 b_2 r_3 + \alpha_6 r_1 b_2 g_3$$

For color confinements, the antisymmetric combination

$$\chi_B^c = \frac{1}{\sqrt{6}}\left(r_1 g_2 b_3 - g_1 r_2 b_3 + b_1 r_2 g_3 - b_1 g_2 r_3 + g_1 b_2 r_3 - r_1 b_2 g_3\right)$$

is required.

Quarks come in three colors r, g, b and form an $SU(3)$ color group.

Gluons $\longrightarrow$ 8 colors (bicolored?)

$r\bar{g}, r\bar{b}, g\bar{r}, g\bar{b}, b\bar{r}, b\bar{g}, \frac{1}{\sqrt{2}}(r\bar{r}-g\bar{g}),$

$\frac{1}{\sqrt{2}}(r\bar{r}+g\bar{g}-2b\bar{b})$ $SU(3)$ color Octet

and $\frac{1}{\sqrt{3}}(r\bar{r}+g\bar{g}+b\bar{b})$ $SU(3)$ color singlet

$\Longleftrightarrow$ colorless and so cannot mediate between color charges.

$\therefore$ 8 colored Gluons are taken to exist.

Because gluons carry color and self-couple, strong interaction is markedly different from electromagnetic interaction.

$\Rightarrow$ Color Confinement

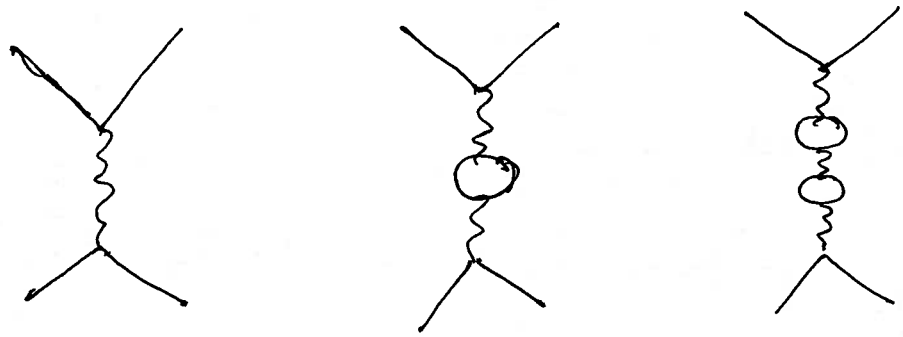
i.e., observed states are colorless so can't observe free quarks or gluons.

$\Rightarrow$ Asymptotic freedom

At short distances the strong interaction becomes weaker and the quarks asymptotically become free.

QED

... etc.



The loops are from vacuum polarization

∴ charge is modified by the vacuum polarization loop and so are the interactions

The effective charge is dependent on the separation and so has to be defined at a particular momentum transfer or virtual photon momentum $q^2 = -Q^2 = -\mu^2$.

One has $e = e_0 \left(1 + e_0^2 A_1(Q^2) + e_0^4 A_2(Q^2) + \dots\right)_{Q^2=\mu^2}$

e_0 = bare-charge with no loops

and for the scattering amplitude,

$$-iM(e_0^2) = e_0^2 \left[F_1(Q^2) + e_0^2 F_2(Q^2) + O(e_0^4) + \dots\right]$$

$A_1, F_1, \dots$ etc are infinities

But, through the magic of renormalization (or reparametrization,

$$-iM(e^2) = e^2 \left[F_1'(Q^2) + e^2 F_1'(Q^2) + O(e^4) + \dots\right]$$

So one can show that

$$e^2(Q^2) = e_0^2 \left(\frac{1}{1 + I(q^2)} \right)$$

where $I(q^2) = \frac{\alpha}{3\pi} \log\left(\frac{M^2}{-q^2}\right)$

M^2 is some large arbitrary cut-off parameter

~~And for $Q^2 = -q^2$~~ for large $Q^2 = -q^2$

$$\alpha(Q^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \log \frac{Q^2}{M^2}}$$

Choosing a reference momentum (or renormalization) μ,

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{Q^2}{\mu^2}\right)} \quad \text{for large } Q^2.$$

This is the "Running Coupling constant" for QED which describes how effective charge depends on the separation of charged particles.

$\therefore$ As Q^2 increases, $\alpha(Q^2)$ increases

i.e., as separation between charges decreases, $\alpha(Q^2)$ or force increases.

QCD

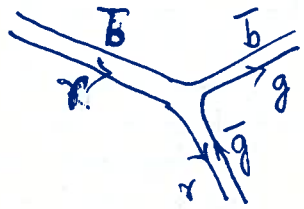
We would have similar processes here

etc.

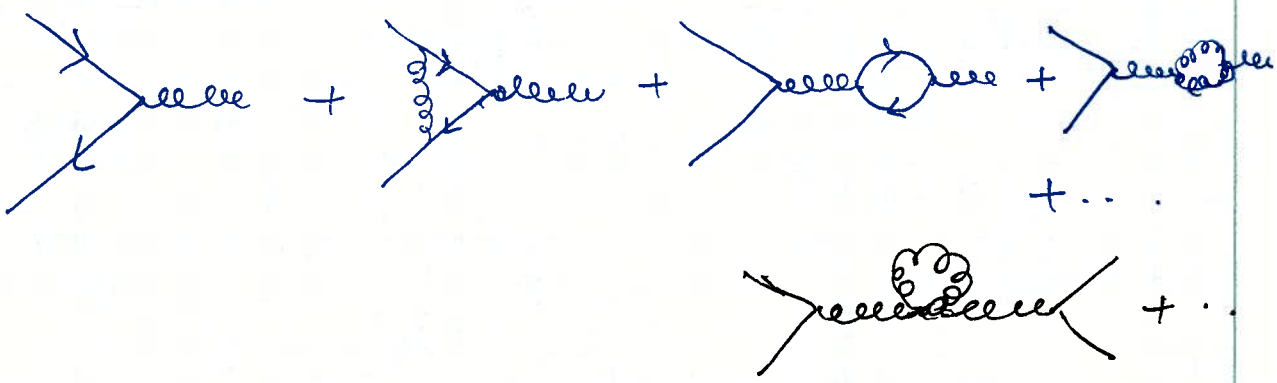
Also

Vacuum Polarization

Gluon Splitting into two gluons



$r\bar{b} \to r\bar{g} + g\bar{b}$



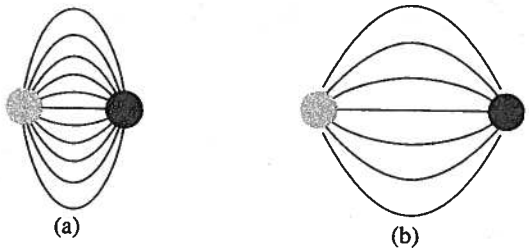


Fig. 6.26. The electrostatic field-lines between two equal and opposite charges. The lines going to infinity are not drawn for simplicity.

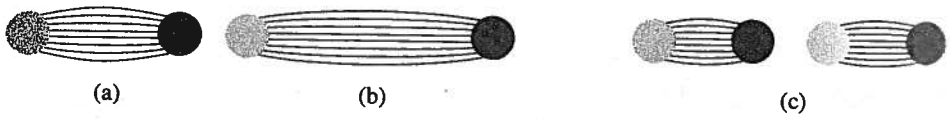
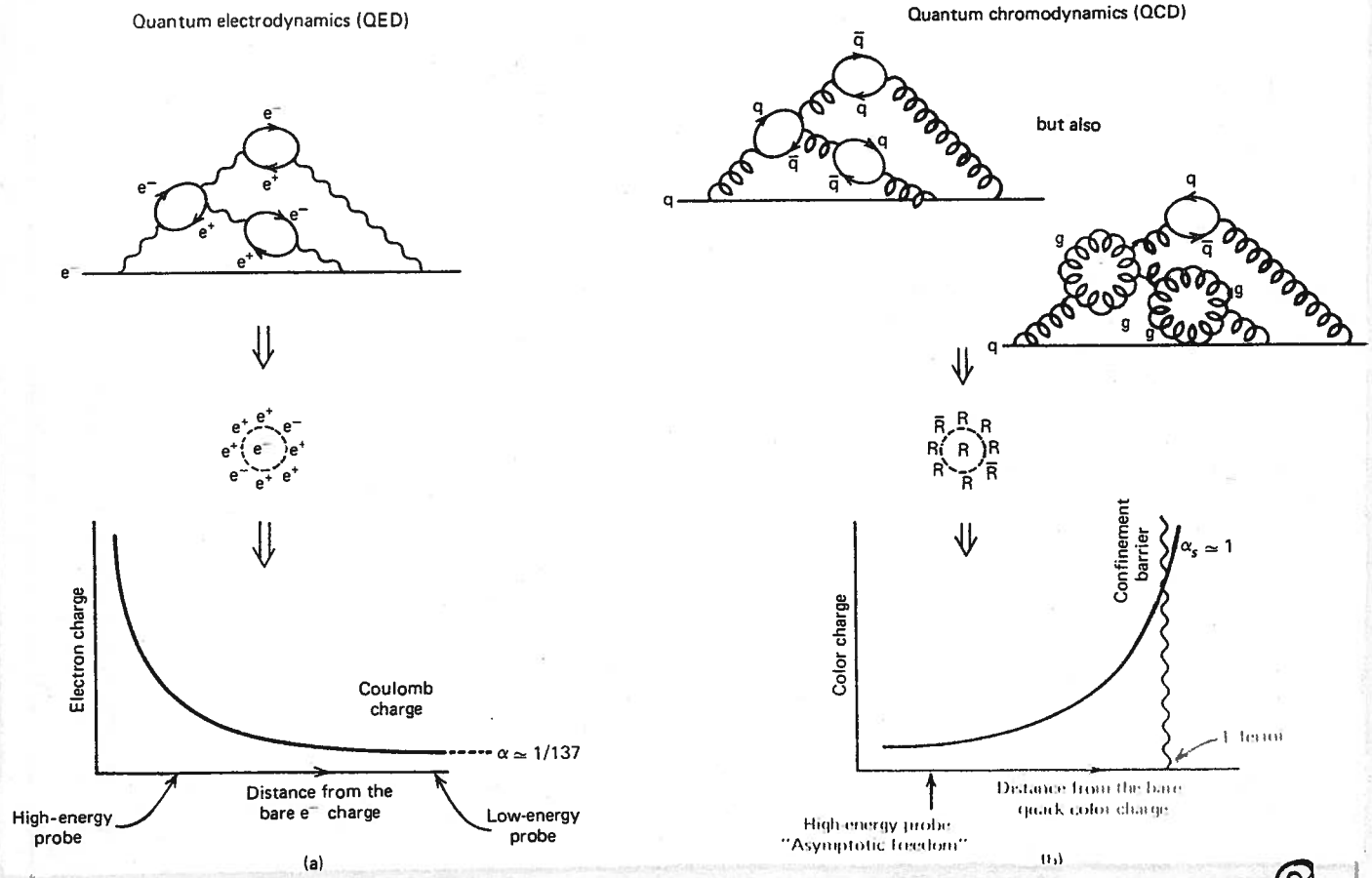


Fig. 6.27. Sketch of the colour field-lines between a quark and an antiquark.



α_s, the Strong Interaction Coupling Constant

Gluon self-coupling and gluon loops are also involved.

Gluons smear the color charge producing "antiscreening effect. (opposite to that of quarks and in QED).

So color charge decreases with decreasing distance.

Strength of the interaction decreases with " ".

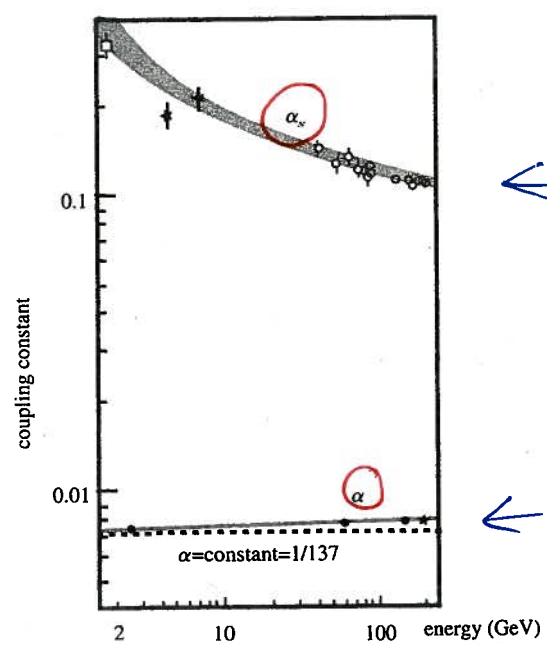
So quarks become "Asymptotically free"

The "Running" Coupling Constant

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi} \cdot (33 - 2n_f) \ln\left(\frac{|Q^2|}{\mu^2}\right)}$$

Evolution of α and α_s

$\alpha_s(M_Z) = 0.118 \pm 0.00$



large variation. decrease with μ

Small variation (slow increase)

If $\Lambda^2 = \mu^2 \exp\left[\frac{12\pi}{(33-2n_f)\alpha_s(\mu^2)}\right]$

$\Lambda \equiv \Lambda_{QCD}$

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\ln\left(\frac{|Q^2|}{\Lambda^2_{QCD}}\right)}$$

n_f = number of quark flavors

For $Q^2 >> \Lambda^2$, $\alpha_s(Q^2)$ is small, so perturbative methods can be used (free quarks and gluons)

For $Q^2 << \Lambda^2$, then α_s is large (strongly bound states of hadrons)

$\Lambda_{QCD} \approx 400$ MeV.

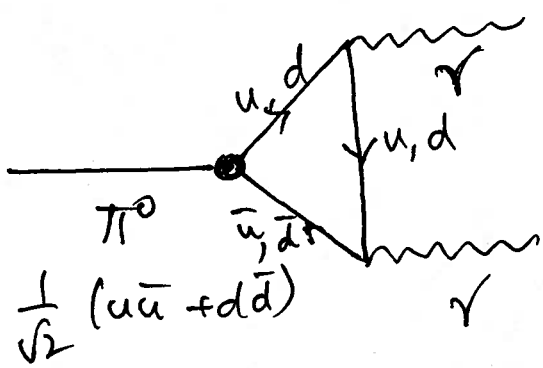
QCD, Color and The Decay of the π^0

1949-50 $\pi^0 \to \gamma\gamma$ rate measured by Steinberger

1967 Veltman finds π^0 does not decay using the modern field theory

1968-70 Adler, Bell and Jackiw "fix" the theory but the decay rate is off by a factor of 9

1973-4 Gellmann and Fritzsch use QCD with 3 colors to get the correct decay rate



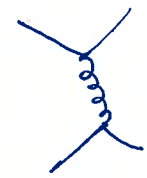
The triangle diagram for π^0 decay.

Each color (r, g, b) contributes an amplitude.

So the decay rate goes up by a factor of 9.

Quark - Antiquark Potential

At short distances,



$r \lesssim 0.1$ fm,

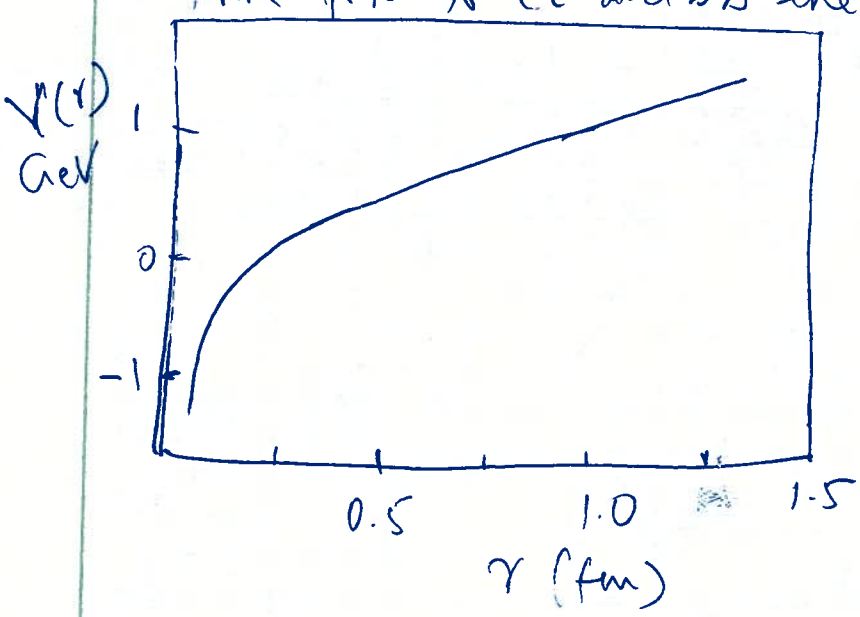
interaction is dominated by one-gluon exchange

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} \quad \text{or} \quad V(r) \sim -\frac{\alpha_s}{r}$$

For $r \gtrsim 0.1$ fm the strength of the interaction increases rapidly; higher order interactions become important.

For $r \geq 1$ fm $V(r) \sim \lambda r$

In fact, this is shown experimentally from fits to $c\bar{c}$ and $b\bar{b}$ energy levels



$$V(r) = \frac{a}{r} + br$$

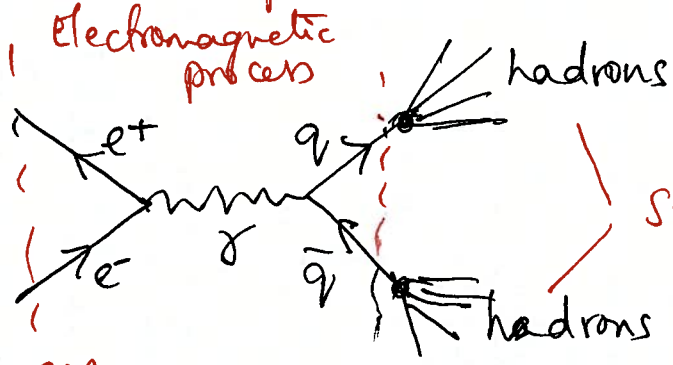
$a = 0.30$

$b = 0.23\ \text{GeV}^2$

e^+e^- Annihilation

$e^+e^- \longrightarrow$ hadrons

provided evidence for color and gluons

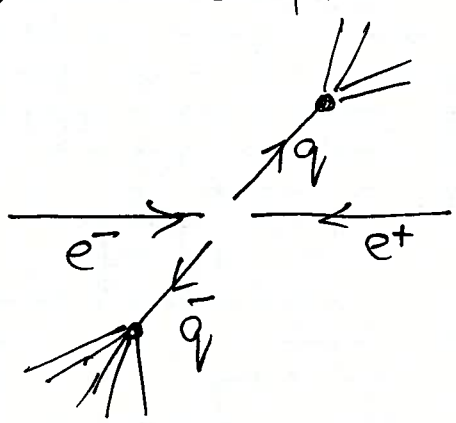


Strong Process called "Fragmentation"

$e^+e^- \xrightarrow{em} q\bar{q}$
$\hookrightarrow$ hadron jet
$\hookrightarrow$ hadron jet

Strong

In the CM frame



The two quarks (normally called "partons" in practice which includes gluons)

parton $\equiv$ quark / gluon)

are back-to-back in CM frame.

So the jets will be back-to-back in the CM frame, due to conservation of momentum.

$$\boxed{P_{jet} = \sum_i P_i}$$

summed over all particles in jet

In two jet events observed, the directions of the hadronic "jet" pretty much give you the final parton direction.

Since the fragmentation process is pretty much removed or decoupled from the ~~bad~~ production of $q\bar{q}$ from hard interaction, and the jets from fragmentation point almost exactly to the initial quark and antiquark directions, the jet angular distributions of $e^+e^- \to q\bar{q} \to$ jet jet will be same as $e^+e^- \to \mu^+\mu^-$ (point particles in initial and final states)

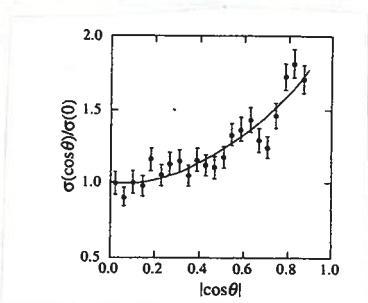
$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to \mu^+\mu^-) = \frac{\pi\alpha^2}{8E^2}(1+\cos^2\theta)$$

(θ is the production angle of either muon w.r.t initial electron direction in CM frame)

For $e^+e^- \to q\bar{q}$ of a given flavor

for color quark charge

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to q\bar{q}) = \frac{3 \cdot e_q^2 \pi\alpha^2}{8E^2}(1+\cos^2\theta)$$



$\leftarrow \frac{d\sigma}{d\cos\theta} \propto (1+\cos^2\theta)$

Data: 2-jet events from e^+e^- at DESY

Curve: theoretical prediction.

Integrating the differential cross section over all angles θ,

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{\pi\alpha^2}{3E^2}$$

$$\sigma(e^+e^- \to q\bar{q}) = N_c\, e_q^2\, \sigma(e^+e^- \to \mu^+\mu^-)$$

$$R_0 = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_{\text{flavor}} e_q^2$$

Including 3-jet and other $\sim\alpha_s$ correction

$$R = R_0\left(1 + \alpha_s/\pi\right)$$

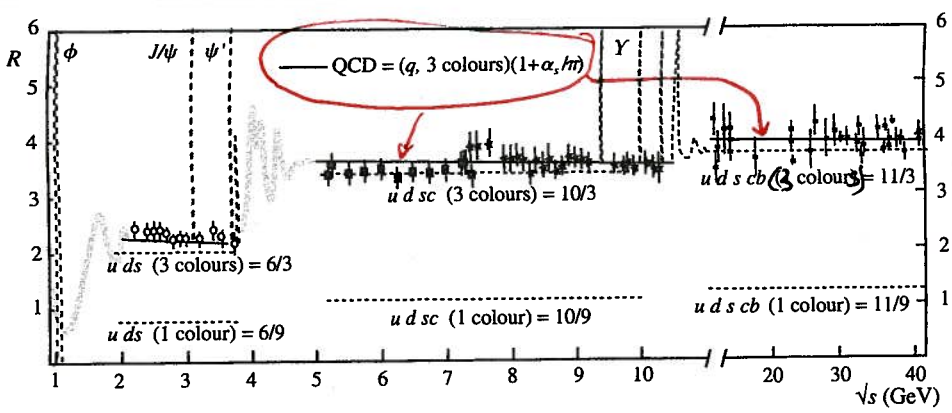
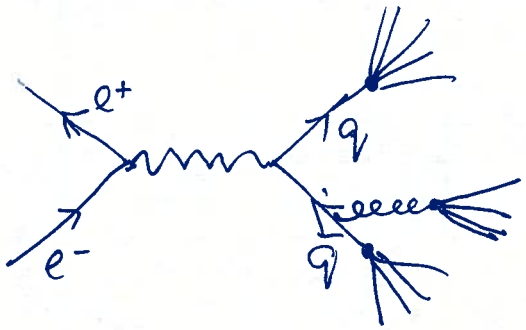
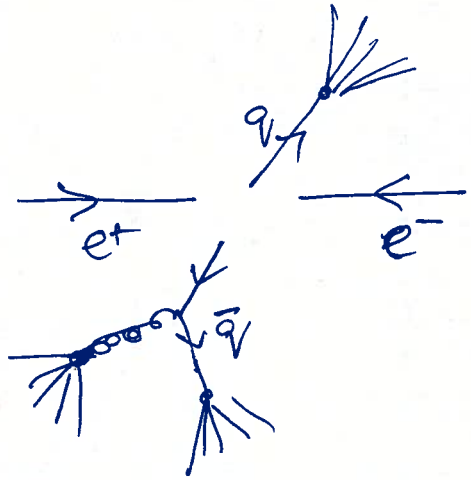


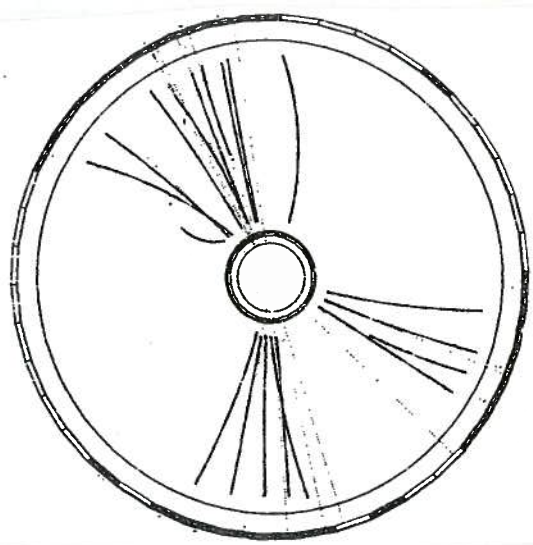
Fig. 6.3. Ratio R of hadronic to point-like cross section in e^+e^- annihilation as a function of $\sqrt{s}$. (Yao *et al.* 2006)

Three jet events & the Discovery of the Gluon

In $e^+e^- \to$ hadrons, 2 jet events are dominant. But, sometimes, a gluon can be emitted by an outgoing quark or an antiquark. So, we can get 3 jet events.

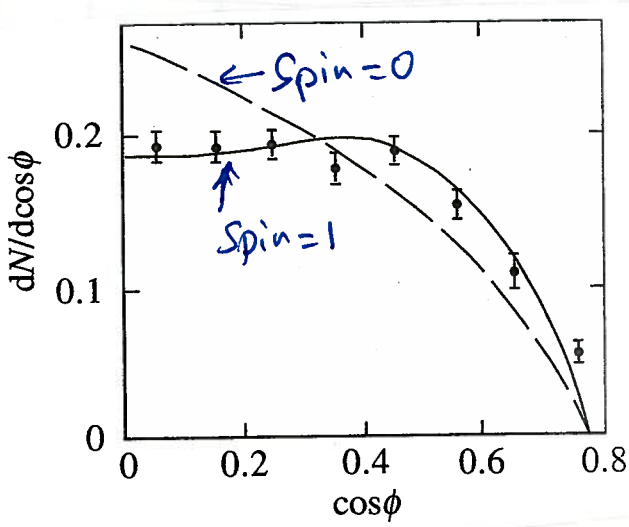






← "Evidence for the Gluon"

3-Jet events seen in JADE detector at DESY



ϕ is the angle of the gluon candidate jet from TASSO collaboration (1980)

Data in agreement with Spin-1 gluon

QCD in $p\bar{p}$ Collisions

$A + B \to C + X$

$$\sigma_{A+B\to C+X} = \sum_{ij} \int dx_1 dx_2 f_i^A(x_1, \mu_F) f_j^B(x_2, \mu_F) * \sigma_{ij\to C}\left[\hat{s}, \mu_F, \mu_R, \alpha_s(\mu_R)\right]$$

$\hat{s} = x_1 x_2 s$ $\qquad$ i, j are the partons involved in hard scattering.

$f(x, \mu_F)$ are the Parton Structure functions

σ_{ij} is the cross-section for the parton-parton hard process (could be $q\bar{q}$, qg, gg, qq etc...)

μ_F and μ_R are scale parameters.

- Dijet and multi-jet production and jet production with photons, electroweak bosons extensively studied to test QCD prediction

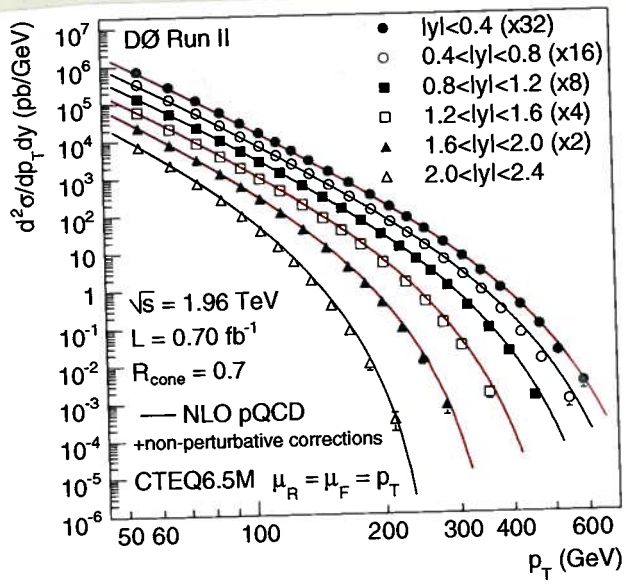
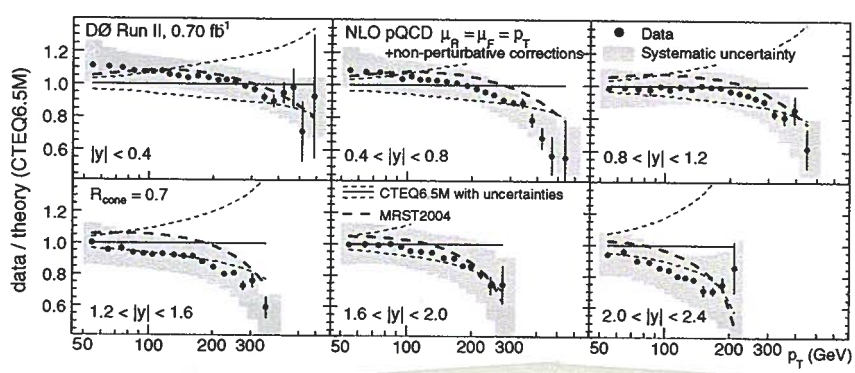


FIG. 1 (color online). The inclusive jet cross section as a function of jet p_T in six $|y|$ bins. The data points are multiplied by 2, 4, 8, 16, and 32 for the bins $1.6 < |y| < 2.0$, $1.2 < |y| < 1.6$, $0.8 < |y| < 1.2$, $0.4 < |y| < 0.8$, and $|y| < 0.4$, respectively.



Lepton – Nucleon Scattering

Electron and muon scattering have been used to explore the structure of nucleons.

Elastic scattering provided measurements of the size of the proton.

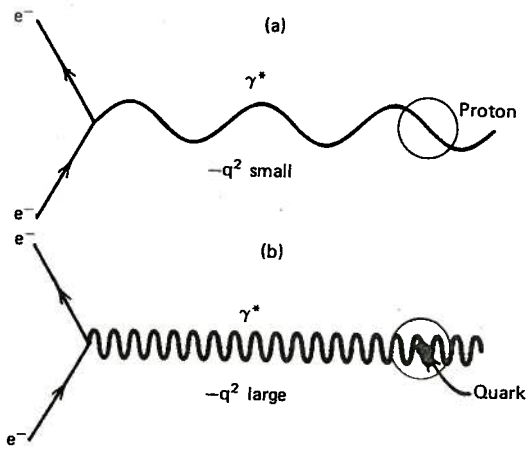


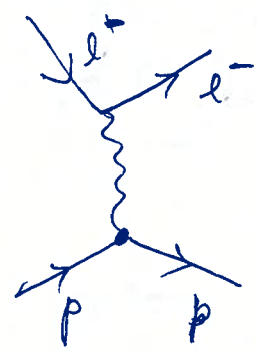
Fig. 9.1 (a) Elastic ep → ep scattering in which a large-wavelength "photon beam" measures the size of the proton through the elastic form factor analysis. (b) In deep inelastic scattering a short-wavelength "photon beam" resolves the quarks within the proton provided $\lambda\left(\approx 1/\sqrt{-q^2}\right) \ll 1\text{F}$.

"Deep inelastic scattering" with high energy lepton probes were used to shed light on the internal structure of the nucleons

$$e^- + p \rightarrow e^- + p$$

$$\text{or } \mu^- + p \rightarrow \mu^- + p$$

Note: Here we are assuming a fairly low momentum probe scattering off of the proton so it sees the proton as a whole



Studying the angular distribution and energy of the outgoing lepton tells us about the target (proton)

For scattering of an electron with momentum p off of a static point charge is given by Rutherford scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_R \propto \frac{1}{p^4 \sin^4 \theta/2}$$

If you have a charge distribution with an extent then,

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R \cdot G_E(q^2)$$

$\vec{q} = \vec{p} - \vec{p}'$

$$G_E(q^2) = \int d^3\underline{r}\, \rho(r)\, e^{i\underline{q}\cdot\underline{r}}$$

Form factor ~~(denoted $G_E(q^2)$ in Martin & Shaw)~~

$\rho(r)$ is spherically symmetric density distribution.

$G_E(q^2) \to 0$ as $q^2 \to \infty$
(increasingly oscillating exponential factor)

For $q=0$; $G_E(q^2) = 1$ $\quad \because \int \rho(r)\, d^3r = 1$ (normalized distribution)

From the measurement of crosssection $\frac{d\sigma}{d\Omega}$, one can calculate the $G_E(q^2)$, the form factor. from which you can learn about the charge distribution that caused the scattering.

$$r_E^2 = \overline{r^2} = \int d^3\underline{r}\, r^2 \rho(r) = -6 \left.\frac{d\, G_E(q^2)}{dq^2}\right|_{q^2=0}$$

G_E is the form factor associated with electric charge distribution.

We have to take into account the magnetic moment distribution within the proton as well.

Then, one has

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \left(\frac{E'}{E}\right) \left[G_1(Q^2) \cos^2 \theta/2 + 2\tau G_2(Q^2) \sin^2 \theta/2 \right]$$

where $G_1 = \dfrac{G_E^2 + \tau G_M^2}{1+\tau}$; $G_2 = G_M^2$; $\tau = Q^2/4M^2$

M = proton mass

$(p, E) \rightarrow$ initial electron momenta, energy $\quad Q^2 = (\vec{p} - \vec{p}')^2 - (E - E')^2$

$(p', E') \rightarrow$ final " "

$$G_E(0) = 1 \text{ and } G_M(0) = \mu_p = 2.79 \text{ nuclear magnetons}$$

G_E and G_M have been measured in many scattering experiments (starting with famous ones by Hofstadter et. al in 1956 which was the 1st measurement of proton size).

At low Q^2, G_E dominates in $\frac{d\sigma}{d\Omega}$ since G_2 term is suppressed by $\tau \ll$

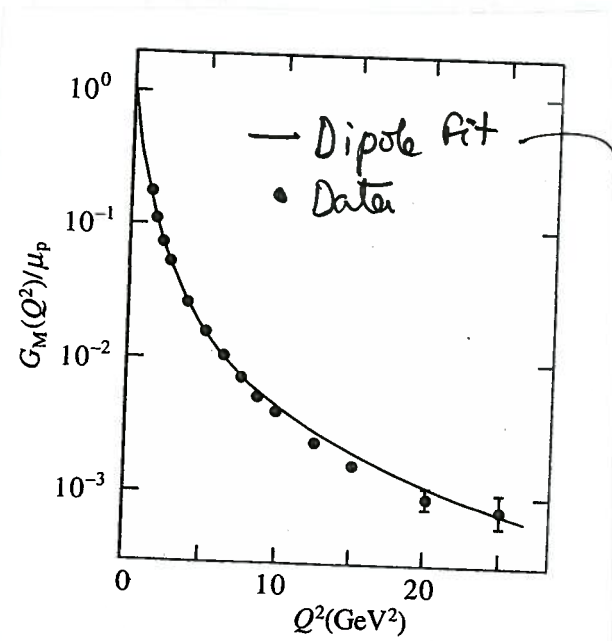
$\Rightarrow$ precise measurement of G_E

$\Rightarrow$ RMS charge radius of the proton

World average $r_E = 0.85 \pm 0.02$ fm

For $0.02 \leq Q^2 \leq 3\ \text{GeV}^2$, both G_E and G_M are measurable (but distinguished by varying θ) ← see eqn for $\frac{d\sigma}{d\Omega}$

This gives a so called "Dipole fit"



$$G_E(Q^2) \approx \frac{G_M(Q^2)}{\mu_p}$$

$$\approx G_D = \frac{1}{(1+Q^2/\beta^2)^2}$$

$$\beta = 0.84\ \text{GeV}$$

For $Q^2 > 3\ \text{GeV}^2$, G_E becomes negligible.

$\therefore G_M$ can be measured with good accuracy.

→ in good agreement with G_D from the Dipole fit.

Note that the form factors fall rapidly with Q^2.

If proton were a point particle, then,

$$G_E(Q^2) = G_M(Q^2) = 1, \text{ independent of } Q^2$$

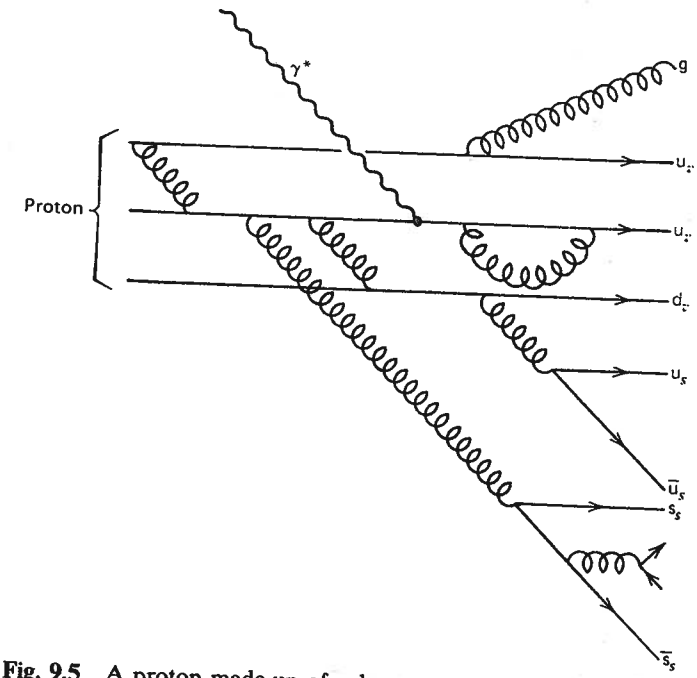


Fig. 9.5 A proton made up of valence quarks, gluons, and slow debris consisting of quark–antiquark pairs.

What does a photon see inside a proton in deep inelastic scattering?

Inelastic Scattering of Leptons off Nucleons

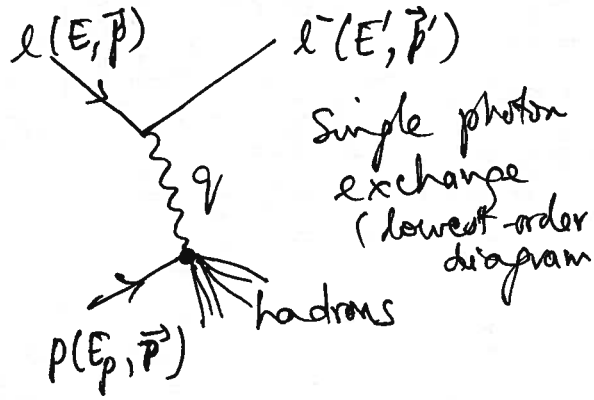
Inelastic electron scattering at high energies provided the first clear evidence for quark structure of protons.

$$e^- + p \to e^- + X$$

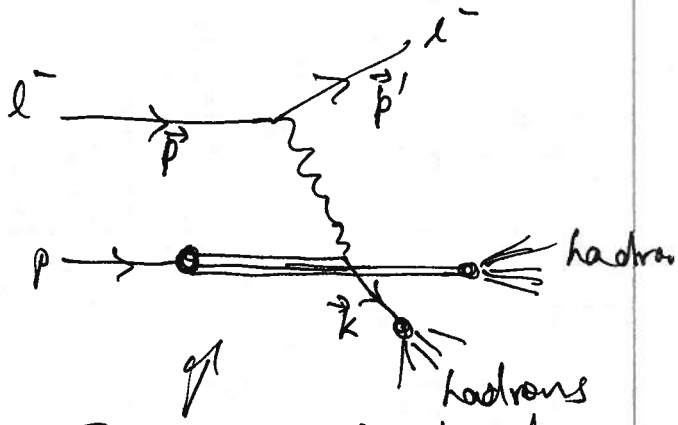
$$\mu^- + p \to \mu^- + X$$

normally written as

$$l^- + p \to l^- + X \; ; \quad l = e, \mu$$



Single photon exchange (lowest-order diagram



Two steps involved

$P_\mu \equiv (E_p, \vec{P})$

(1) $l^- + q \to l^- + q$ (em process)

Squared Momentum transfer $= q_\mu$

$\therefore Q^2 = q_\mu q^\mu$

(2) $q \to$ hadrons (fragmentation) (strong process)

From kinematics, you can show that

$$W^2 = M_p^2 + 2P_\mu q^\mu - Q^2 = M_p^2 + 2M_p\nu - Q^2$$

↳ invariant mass of the hadronic state X

$$2M_p\nu = W^2 + Q^2 - M_p^2$$

$$x = Q^2/2M_p\nu$$

x: Scaling variable

and $\nu = E - E'$ (energy transfer)

In inelastic scattering, both E' and θ are independent variables.

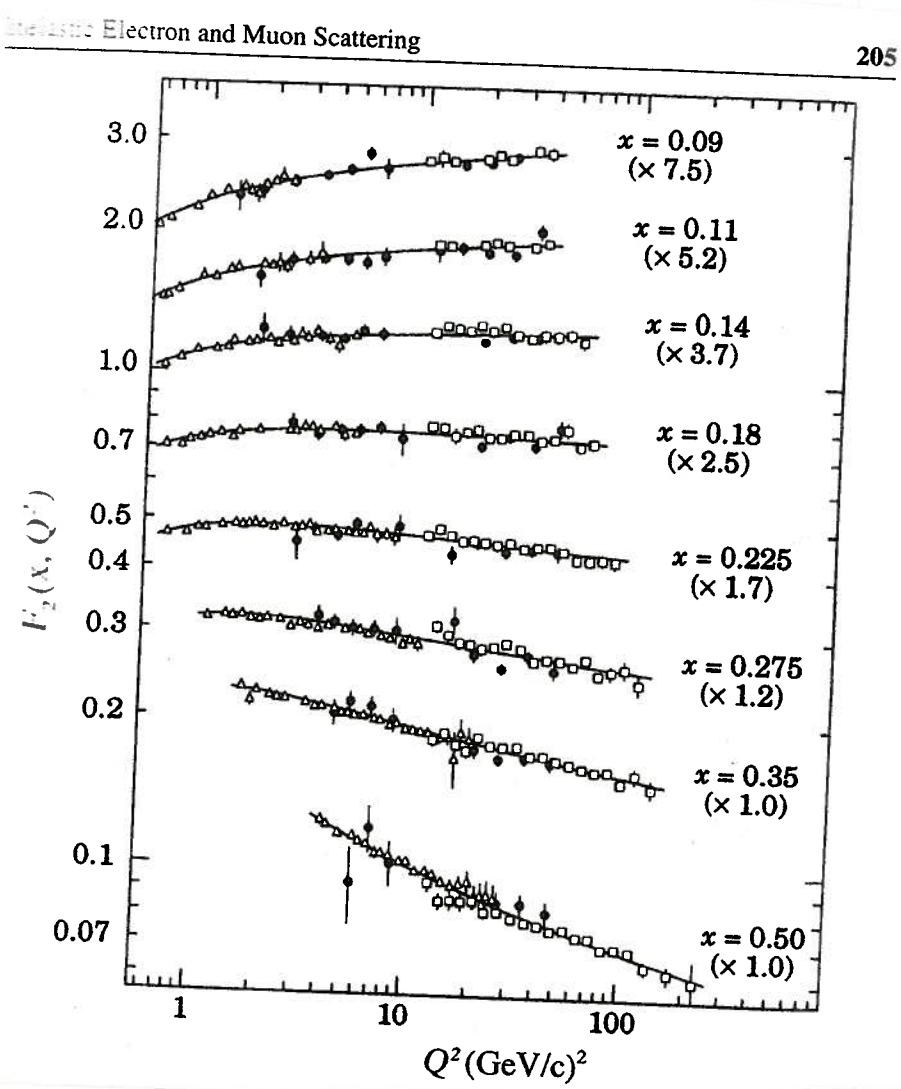
The differential scattering cross section,

$$\frac{d\sigma}{dE'd\Omega'} = \frac{\alpha^2}{4E^2\sin^4\theta/2}\cdot\frac{1}{\nu}\left[\cos^2\theta/2\, F_2(x,Q^2) + \sin^2\theta/2\,\frac{Q^2}{xM^2}F_1(x,Q^2)\right]$$

F_1 and F_2 are the "Structure Functions" that parametrize the interaction of the photon with the proton.

For $Q^2 \gg M^2$, $\boxed{F_{1,2}(x,Q^2) \simeq F_{1,2}(x)}$

Bjorken Scaling or Scale invariance.



⟸ Point-like scatterers or targets

The scaling behavior of inelastic scattering can be explained as due to scattering by point like constituents within the proton.

⇐ First dynamical evidence for quarks whose existence had previously been accounted or inferred from static properties of hadrons.

This scaling is not exact. Violations and their quantitative predictions from QCD have been verified.

Parton Model

Partons are point-like constituents of proton (or hadrons)

name given by Feynman

Proton is a stream of partons.

The Parton momentum then is xP ← proton momentum

Parton momentum fraction $= x = Q^2/2M_p\nu$

$\Rightarrow$ the measured cross section at a given x is proportional to the probability of finding a parton with a fraction x of the proton momentum.

$$\boxed{F_2(x, Q^2) = \sum_a e_a^2 x f_a(x)}$$

same for spin-0 or spin-$\frac{1}{2}$

$f_a(x)dx$ is the probability of finding a parton a of charge e_a with fractional momentum between x and $x+dx$.

The sum is over all parton types a.

On the other hand,

$$F_1(x, Q^2) = 0 \quad \text{for spin-0}$$

Collan-Gross relation → $\boxed{2xF_1(x, Q^2) = F_2(x, Q^2)}$ for spin $\frac{1}{2}$

Favored by data $\Rightarrow$ partons are spin $= \frac{1}{2}$ particles.

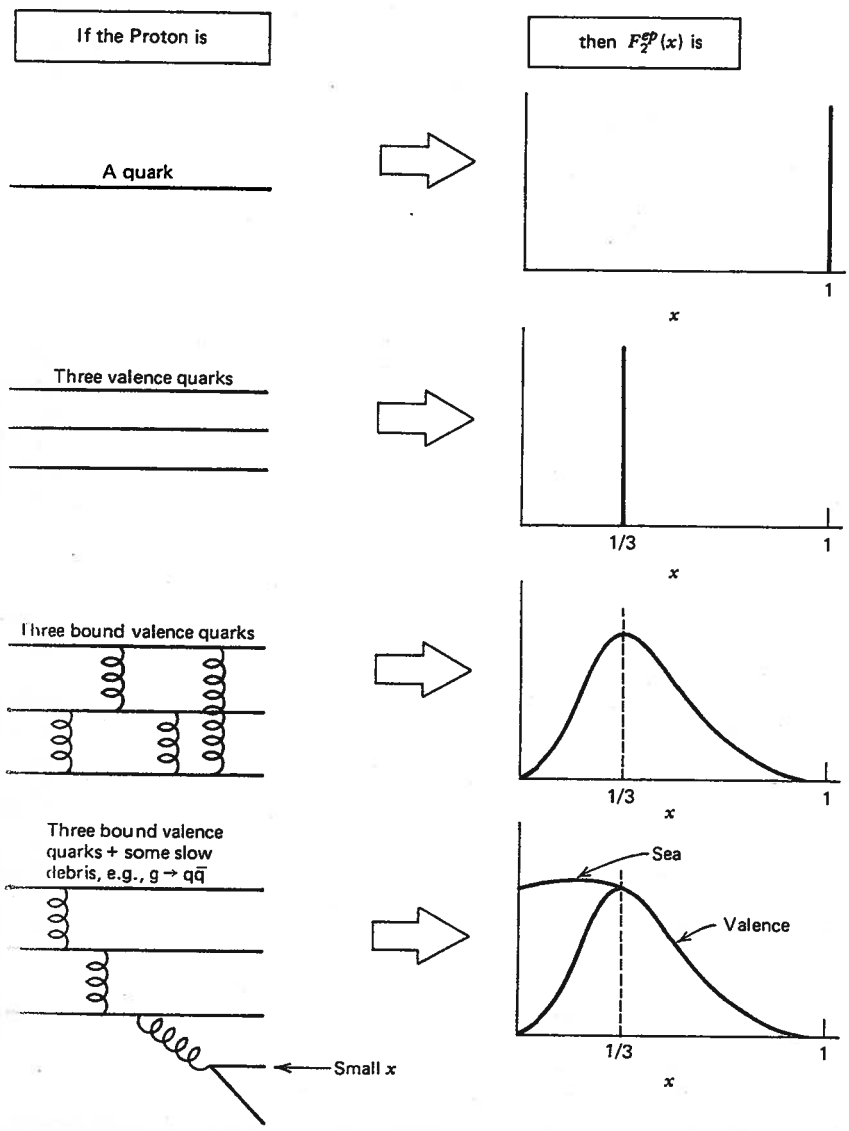


Fig. 9.7 The structure function pictured corresponding to different compositions assumed for the proton.

$$F_2^{ep}(x) = e_u^2 \cdot x\left[u^p(x) + \bar{u}^p(x)\right] + e_d^2 \cdot x\left[d^p(x) + \bar{d}^p(x)\right] + e_s^2 \cdot x\left[s^p(x) + \bar{s}^p(x)\right]$$

$u^p(x)$ are the prob. distributions of u quarks in the proton, etc.

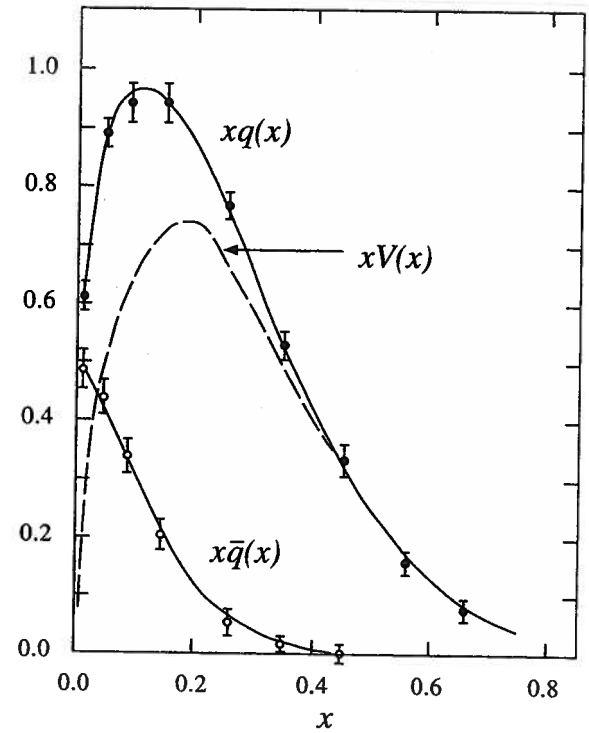
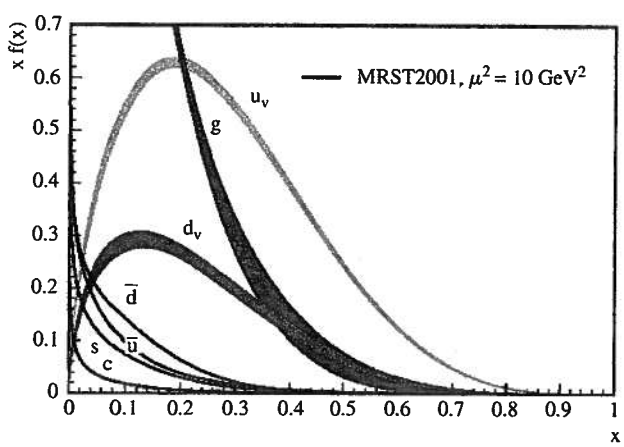


Figure 7.23 Quark and antiquark distributions (7.59a), together with the valence quark distribution (7.59b), measured at a Q^2 value of about $10\,\mathrm{GeV}^2$, from neutrino experiments at CERN and Fermilab.



The momentum fraction of the quarks seems to be ~ 0.5.
The remaining comes from gluons.