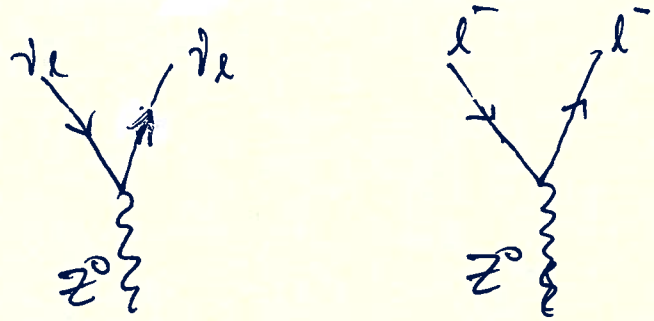
Neutral Currents and Z-lepton Vertices



Z-lepton vertices that conserve lepton numbers and electric charge.

So, the Z leptonic decays would be

$$Z^0 \to e^+e^-,\ \mu^+\mu^-,\ \tau^+\tau^-,$$
$$\nu_e\bar{\nu}_e,\ \nu_\mu\bar{\nu}_\mu,\ \nu_\tau\bar{\nu}_\tau.$$

General nomenclature

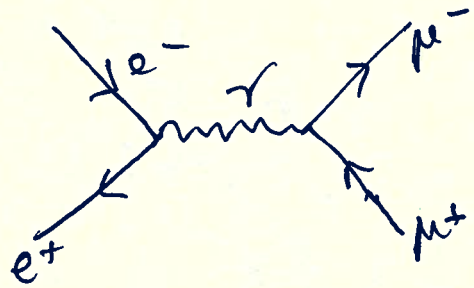
$Z \to l^+l^-$ (decay to charged leptons)

$Z \to \nu\bar{\nu}$ (decay to neutral leptons)

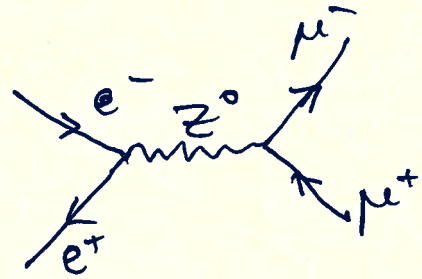
↑ invisible decay

Electroweak Unification

We have talked about $e^+e^- \to \mu^+\mu^-$ as a standard reaction used for many purposes.



Also



These are the two dominant "Electroweak" processes.

Higher order processes:

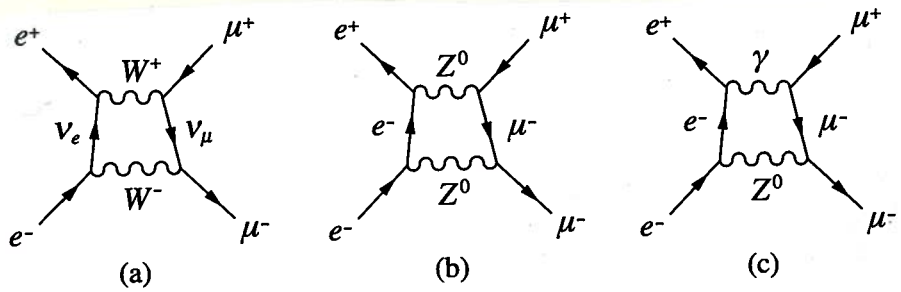


Figure 9.1 Some of the higher-order diagrams that contribute to the reaction $e^+ + e^- \to \mu^+ + \mu^-$ in the unified theory of the electroweak interaction.

Z-quark Vertices

If we assume the quark mixing of the d and s states as in the case of W-quark interactions, then the vertices would be

$$uuZ^0,\ ccZ^0,\ d'd'Z^0 \text{ and } s's'Z^0$$

Substituting $d' = d\cos\theta_c + s\sin\theta_c$

and $s' = -d\sin\theta_c + s\cos\theta_c$

we get

(a) $$d'd'Z^0 = (d\cos\theta_c + s\sin\theta_c)(d\cos\theta_c + s\sin\theta_c)Z^0$$
$$= ddZ^0\cos^2\theta_c + ssZ^0\sin^2\theta_c + (dsZ^0 + sdZ^0)\times \sin\theta_c\cos\theta_c$$

and

(b) $$s's'Z^0 = (-d\sin\theta_c + s\cos\theta_c)(-d\sin\theta_c + s\cos\theta_c)Z^0$$
$$= ddZ^0\sin^2\theta_c + ssZ^0\cos^2\theta_c - (dsZ^0 + sdZ^0)\times \sin\theta_c\cos\theta_c$$

so that

$$d'd'Z^0 + s's'Z^0 = ddZ^0 + ssZ^0$$

One can re-interpret (a) and (b) as $d'd'Z^0$ and $s's'Z^0$ contributing to ddZ^0 and ssZ^0 vertices as well as dsZ^0 and sdZ^0 vertices.

The Electroweak Unification

The various coupling constants in EW interaction are not independent but should be related by the requirement that the infinities in the higher-order diagrams, in, for e.g., $e^+e^- \to \mu^+\mu^-$ should cancel. This is accomplished by satisfying the unification condition

$$\frac{e}{2\sqrt{2\epsilon_0}} = g_W \sin\theta_W = g_Z \cos\theta_W \qquad (1)$$

where θ_W is the weak mixing angle or the Weinberg angle and is the given by

$$\cos\theta_W = M_W / M_Z \qquad (0 < \theta_W < \pi/2) \qquad (2)$$

The Anomaly condition relates the electric charges of the quarks and leptons

$$\underbrace{\sum_{l}}_{\text{all leptons}} Q_l + 3 \overbrace{\underbrace{\sum_{a}}_{\text{all quarks}}}^{\text{color factor}} Q_a = 0 \qquad (3)$$

The relations between the EM and weak coupling strengths is obtained using the principle of gauge invariance.

(See Appendix D)

Historically, the EW unification relation was used to predict the W, Z masses before their discovery.

In the low energy limit, where the charged current can be approximated as a contact interaction,

$$M_W^2 = \frac{\sqrt{2}\, g_W^2}{G_F} = \frac{\pi \alpha}{\sqrt{2}\, G_F \sin^2\theta_W} \qquad (4)$$

and so

$$M_Z^2 = \frac{\pi \alpha}{\sqrt{2}\, G_F \sin^2\theta_W \cos^2\theta_W} \qquad (5)$$

θ_W can be measured / determined by comparing NC and CC processes at $E \ll M_W, M_Z$.

for contact interaction NC, one has

$$G_Z = \sqrt{2}\, \frac{g_Z^2}{M_Z^2} \qquad (6)$$

$$\Rightarrow \frac{G_Z}{G_F} = \frac{g_Z^2}{g_W^2} \cdot \frac{M_W^2}{M_Z^2} = \sin^2\theta_W \qquad (7)$$

rates of measured NC and CC reactions at low energies

$$\Rightarrow \sin^2\theta_W = 0.227 \pm 0.014$$

Substituting the $\sin^2\theta_W$ in (4) and (5)

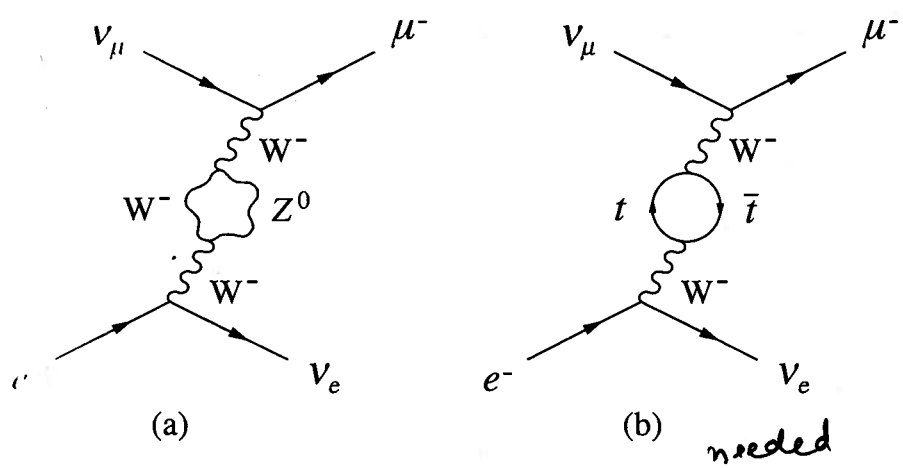
$$\Rightarrow \quad M_W = 78.3 \pm 2.4 \text{ GeV}/c^2$$

$$M_Z = 89.0 \pm 2.0 \text{ GeV}/c^2$$

Current experimental values

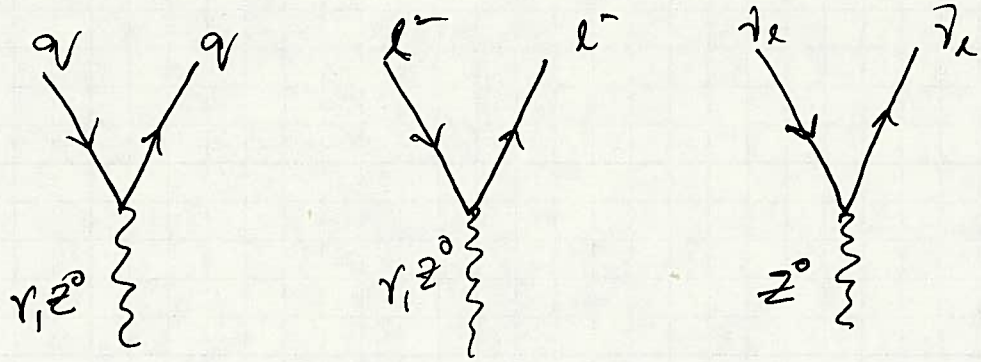
$$M_W = 80.392 \pm 0.029 \text{ GeV}$$

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

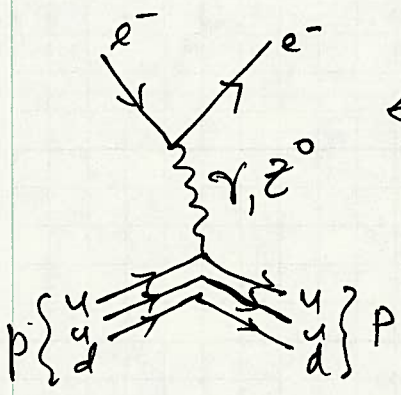


needed

Figure 9.6 Two of the higher-order contributions to inverse muon decay [illegible] in obtaining the Fermi coupling constant G_F as a low-energy limit.



So, in any process between charged particles, where a photon is exchanged, a Z^0 boson can be exchanged



← ep scattering

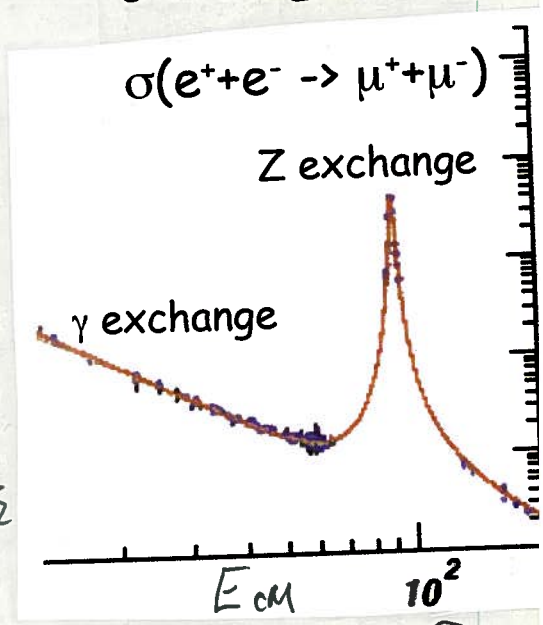
At lower energies photon exchange dominates i.e., $E^2 \ll M_Z^2$

and $\sigma_\gamma \simeq \alpha^2/E^2$; $\sigma_Z \simeq G_Z^2 E^2$

For higher energy Z^0 exchanges becomes increasingly important, with a resonance at $E_{CM} = M_Z$.

The cross section near the resonance is given by the Breit-Wigner form

$$\sigma(e^+e^- \to X) = \frac{12\pi M_Z^2}{E_{CM}^2} * \frac{\Gamma(Z^0 \to e^+e^-)\,\Gamma(Z^0 \to X)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$



Gauge Invariance

Gauge invariance is a fundamental property of field theories used in particle physics, where the physical results of the interactions they mediate are independent of the local scale or gauge of the potentials involved.

In QED, the theory is invariant under gauge transformations of the form

$$\psi(\vec{r},t) \rightarrow \psi'(\vec{r},t) = e^{-i\cdot q f(\vec{r},t)}\,\psi(\vec{r},t)$$

q = charge

$f(\vec{r},t) \equiv$ an arbitrary function

This is change of an electron state into a new state of the electron. $e^- \rightarrow e^-$ or $e^- \rightarrow e^- \gamma$ where the photon is emitted and absorbed, effected from the EM gauge interactions.

One can define gauge transformations where not only the phase of the wavefunction changes, but also the nature of the particle itself leading to, for example, $e^- \rightarrow \nu_e W^-$, $\nu_e \rightarrow e^- W^+$ for CC

and let us $e^- \rightarrow e^- W^0$, $\nu_e \rightarrow \nu_e W^0$ for NC

However, the NC processes have different strengths (and form) from CC processes. So, we can't trivially add a W^0.

The resolution comes from defining the photon and the Z^0 boson as mixtures of W^0 and another neutral boson B^0,

i.e., $\gamma = B^0 \cos\theta_W + W^0 \sin\theta_W$

and $Z^0 = -B^0 \sin\theta_W + W^0 \cos\theta_W$

$\theta_W \equiv$ weak mixing angle

B^0 is such that γ is the observed photon.

This can be achieved by the gauge transformation

$$\psi_\ell(\vec{r},t) \rightarrow \psi_\ell'(\vec{r},t) = e^{-i g_Z y_\ell f(\vec{r},t)} \psi(\vec{r},t)$$

where g_Z is the weak coupling constant associated with Z.

y_ℓ's are constants to be determined.

These can be chosen so that

$$\gamma = B^0 \cos\theta_W + W^0 \sin\theta_W$$

has exactly the couplings of the photon, provided

$$\frac{e}{2\sqrt{2G_0}} = g_W \sin\theta_W = g_Z \cos\theta_W$$

(Details in Appendix D of Martin & Shaw)

The Higgs Field and the Particle Masses

The gauge invariance implies that the spin-1 gauge bosons have zero masses. This is ok for QED and QCD. where photons and gluons have zero masses.
Not so with $W^{\pm}$, Z bosons.

$\Rightarrow$ Gauge invariance is not exact.

$\Rightarrow$ EW symmetry is broken. But, How?

The Higgs Mechanism

- A new type of scalar field called the Higgs field is assumed.
- Particles (Gauge Bosons $W^{\pm}$, Z and the fermions) interact with the Higgs field, and these interactions are gauge-invariant
- The Higgs field has a non-zero value in its vacuum state. This "Vacuum Expectation Value" is not invariant under a gauge transformation. So, the interactions are gauge invariant but the vacuum of the Higgs field is not and the symmetry is "spontaneously broken".

The Spontaneous symmetry breaking and the Higgs mechanism can be better understood by considering Ferromagnetism.

At $T > T_c$, Magnetization $= \emptyset$ (zero)
(Curie Temp.) Forces between electron spins are rotationally invariant.

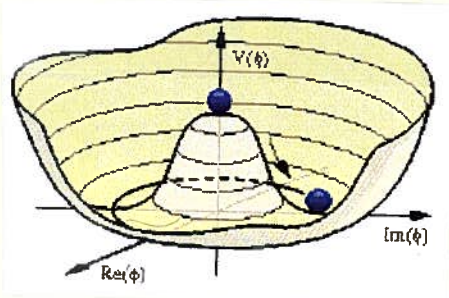
But @ $T < T_c$, The spins align in a specific direction so there is a nonzero magnetization

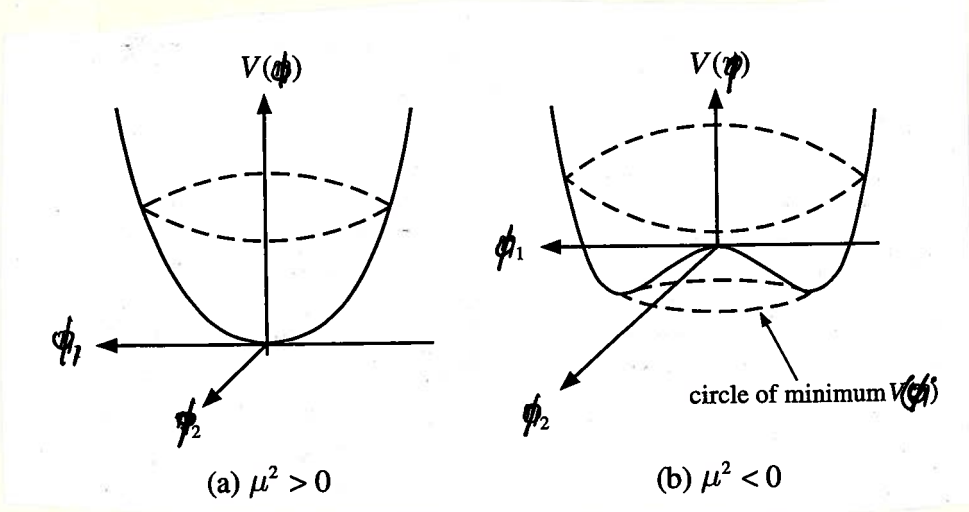
$$M \neq 0$$

and breaks rotational invariance

However, M could have been any other direction and the rotational invariance of the interaction itself is intact.

The Higgs Mechanism (Cont'd)





(a) $\mu^2 > 0$ (b) $\mu^2 < 0$

$$V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4$$

$\phi(\vec{r},t)$ is a complex scalar field i.e., $\phi = \phi_1 + i\phi_2$

$V(\phi)$ is the potential energy density

λ and μ^2 are real parameters

the interactions are invariant under the transformations

$$\phi(\vec{r},t) \rightarrow \phi(\vec{r},t)e^{i\beta} \quad ; \beta = \text{arbitrary phase parameter.}$$

$$\mu^2 < 0, \quad \phi(\vec{r},t) = \phi_0 = \left(-\frac{\mu^2}{2\lambda}\right)^{1/2} e^{i\theta} \qquad 0 \leq \theta \leq 2\pi$$

θ is an arbitrary direction in the complex ϕ-plane. Assume $\theta = 0$.

Not invariant under transformation.

Consequences of the Higgs Mechanism

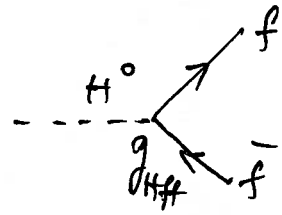
- $W^{\pm}, Z^{0}$ bosons acquire masses by interacting with the non-zero VEV of the Higgs field (interaction of gauge fields)

  and $\dfrac{M_W}{M_Z} = \cos\theta_W$.

- A neutral H^{0} boson should exist.

- Quarks and leptons acquire masses through the interaction with the Higgs field, through a dimensionless coupling constant

$$g_{Hff} = \sqrt{2}\, g_W \left(\frac{m_f}{M_W}\right)$$



The Standard Model (in detail)

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Dirac} + \mathcal{L}_{\rm mass} + \mathcal{L}_{\rm gauge} + \mathcal{L}_{{\rm gauge}/\psi} \ . \tag{1}$$

Here,

$$\mathcal{L}_{\rm Dirac} = i\bar{e}^i_{\rm L}\partial\!\!\!/ e^i_{\rm L} + i\bar{\nu}^i_{\rm L}\partial\!\!\!/ \nu^i_{\rm L} + i\bar{e}^i_{\rm R}\partial\!\!\!/ e^i_{\rm R} + i\bar{u}^i_{\rm L}\partial\!\!\!/ u^i_{\rm L} + i\bar{d}^i_{\rm L}\partial\!\!\!/ d^i_{\rm L} + i\bar{u}^i_{\rm R}\partial\!\!\!/ u^i_{\rm R} + i\bar{d}^i_{\rm R}\partial\!\!\!/ d^i_{\rm R} \ ; \tag{2}$$

$$\mathcal{L}_{\rm mass} = -v\left(\lambda^i_e\bar{e}^i_{\rm L}e^i_{\rm R} + \lambda^i_u\bar{u}^i_{\rm L}u^i_{\rm R} + \lambda^i_d\bar{d}^i_{\rm L}d^i_{\rm R} + {\rm h.c.}\right) - M^2_{\rm W}W^+_\mu W^{-\mu} - \frac{M^2_{\rm W}}{2\cos^2\theta_W}Z_\mu Z^\mu \ ; \tag{3}$$

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}(G^a_{\mu\nu})^2 - \frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{WZA} \ , \tag{4}$$

where

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_3 f^{abc}A^b_\mu A^c_\nu \\ W^\pm_{\mu\nu} &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \ . \end{aligned} \tag{5}$$

and

$$\begin{aligned} \mathcal{L}_{WZA} = \ & ig_2\cos\theta_{\rm W}\left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu\right)\partial^\mu Z^\nu + W^+_{\mu\nu}W^{-\mu}Z^\nu - W^-_{\mu\nu}W^{+\mu}Z^\nu\right] \\ & + ie\left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu\right)\partial^\mu A^\nu + W^+_{\mu\nu}W^{-\mu}A^\nu - W^-_{\mu\nu}W^{+\mu}A^\nu\right] \\ & + g_2^2\cos^2\theta_{\rm W}\left(W^+_\mu W^-_\nu Z^\mu Z^\nu - W^+_\mu W^{-\mu}Z_\nu Z^\nu\right) \\ & + g_2^2\left(W^+_\mu W^-_\nu A^\mu A^\nu - W^+_\mu W^{-\mu}A_\nu A^\nu\right) \\ & + g_2 e\cos\theta_{\rm W}\left[W^+_\mu W^-_\nu\left(Z^\mu A^\nu + Z^\nu A^\mu\right) - 2W^+_\mu W^{-\mu}Z_\nu A^\nu\right] \\ & + \frac{1}{2}g_2^2\left(W^+_\mu W^-_\nu\right)\left(W^{+\mu}W^{-\nu} - W^{+\nu}W^{-\mu}\right) \ ; \end{aligned} \tag{6}$$

and

$$\mathcal{L}_{{\rm gauge}/\psi} = -g_3 A^a_\mu J^{\mu a}_{(3)} - g_2\left(W^+_\mu J^\mu_{W^+} + W^-_\mu J^\mu_{W^-} + Z_\mu J^\mu_Z\right) - eA_\mu J^\mu_A \ , \tag{7}$$

where

$$\begin{aligned} J^{\mu a}_{(3)} &= \bar{u}^i\gamma^\mu T^a_{(3)}u^i + \bar{d}^i\gamma^\mu T^a_{(3)}d^i \\ J^\mu_{W^+} &= \frac{1}{\sqrt{2}}\left(\bar{\nu}^i_{\rm L}\gamma^\mu e^i_{\rm L} + V^{ij}\bar{u}^i_{\rm L}\gamma^\mu d^j_{\rm L}\right) \\ J^\mu_{W^-} &= \left(J^\mu_{W^+}\right)^* \\ J^\mu_Z &= \frac{1}{\cos\theta_{\rm W}}\left[\frac{1}{2}\bar{\nu}^i_{\rm L}\gamma^\mu\nu^i_{\rm L} + \left(-\frac{1}{2} + \sin^2\theta_{\rm W}\right)\bar{e}^i_{\rm L}\gamma^\mu e^i_{\rm L} + (\sin^2\theta_{\rm W})\bar{e}^i_{\rm R}\gamma^\mu e^i_{\rm R}\right. \\ & \quad + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_{\rm W}\right)\bar{u}^i_{\rm L}\gamma^\mu u^i_{\rm L} + \left(-\frac{2}{3}\sin^2\theta_{\rm W}\right)\bar{u}^i_{\rm R}\gamma^\mu u^i_{\rm R} \\ & \quad \left. + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_{\rm W}\right)\bar{d}^i_{\rm L}\gamma^\mu d^i_{\rm L} + \left(\frac{1}{3}\sin^2\theta_{\rm W}\right)\bar{d}^i_{\rm R}\gamma^\mu d^i_{\rm R}\right] \\ J^\mu_A &= (-1)\bar{e}^i\gamma^\mu e^i + \left(\frac{2}{3}\right)\bar{u}^i\gamma^\mu u^i + \left(-\frac{1}{3}\right)\bar{d}^i\gamma^\mu d^i \ . \end{aligned} \tag{8}$$

The Higgs Boson

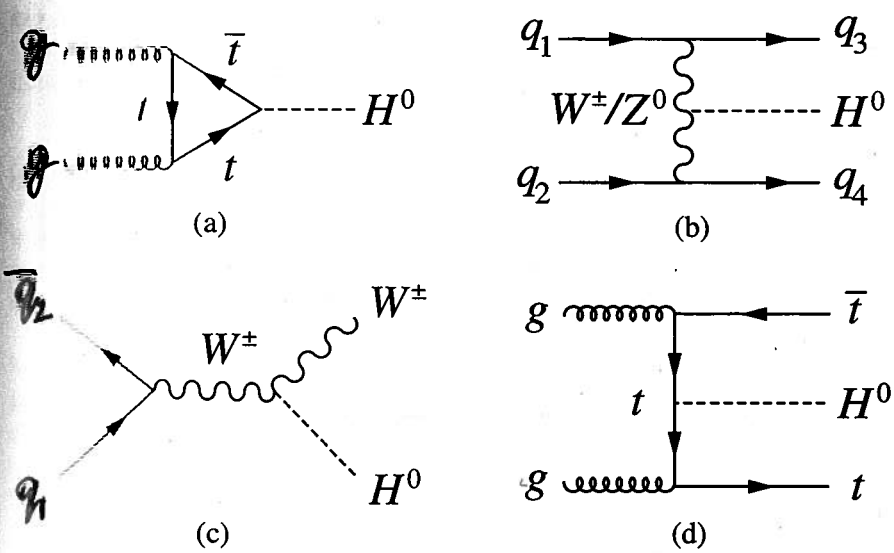


Figure 9.23 Production mechanisms for a standard model Higgs boson at the LHC.

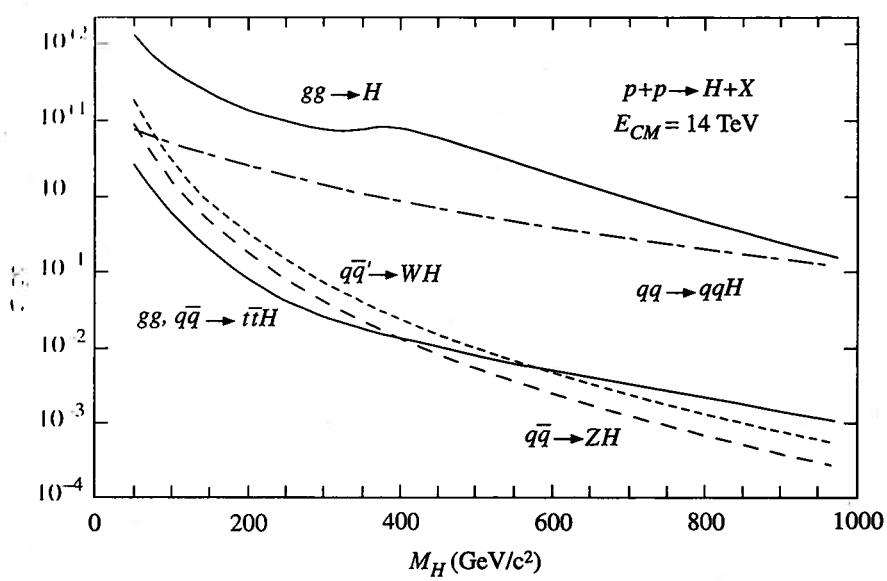


Figure 9.24 Production cross-sections of the standard model Higgs boson at the LHC at $E_{CM} = 14$ TeV. (With kind permission from Springer Science and Business Media, Kunszt et al., *Zeitschrift für Physik C*, **74**, 1997, 479)

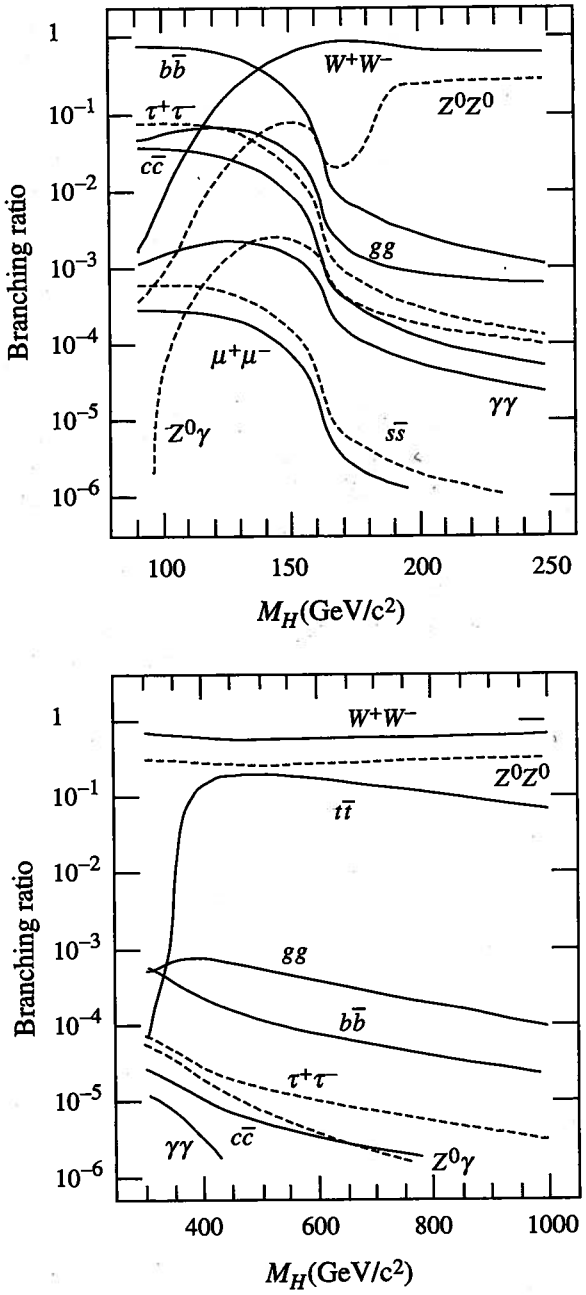


Figure 9.17 The branching ratios of a standard-model Higgs boson H^0 for all decays with branching ratios greater than 10^{-6}. (With kind permission from Springer Science and Business Media, Kunszt *et al.*, *Zeitschrift für Physik C*, **74**, 1997, 479)

$e^+e^- \rightarrow Z + H$

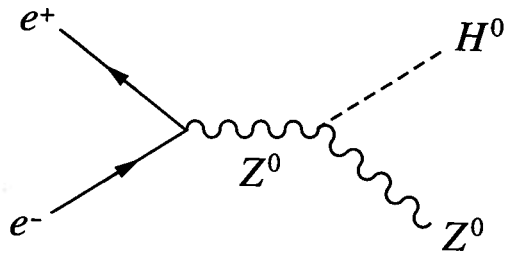


Figure 9.21 Dominant mechanism for Higgs boson production in e^+e^- annihilation

For $M_H < 2M_W$, the main decay would be $H \rightarrow b\bar{b}$

limits on $M_H > 113.5$ GeV/c^2 at 95% C.L. from LEP experiments

LEP C.M. energy ~ 208 GeV.

Finding the Higgs at the Tevatron

PHYSICAL REVIEW D, VOLUME 62, 074022

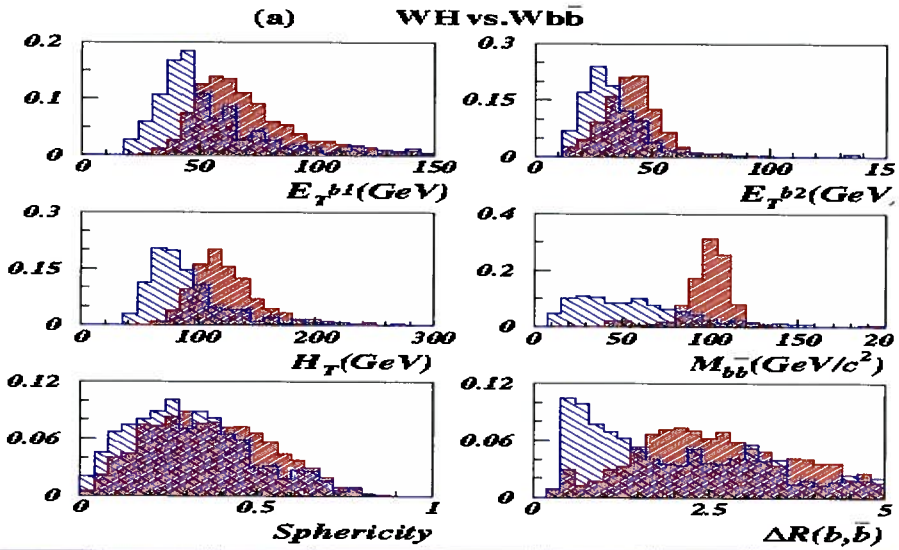
Strategy for discovering a low-mass Higgs boson at the Fermilab Tevatron

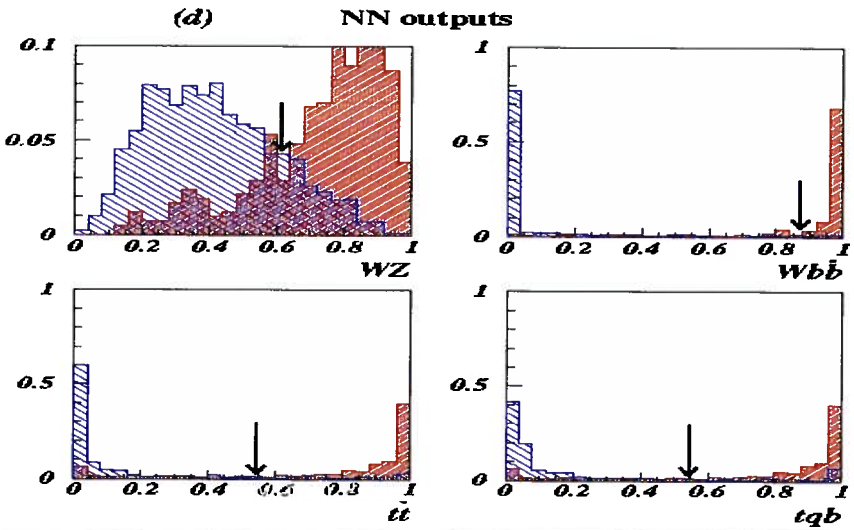
Pushpalatha C. Bhat
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

Russell Gilmartin and Harrison B. Prosper
Department of Physics, Florida State University, Tallahassee, Florida 32306

(Received 14 January 2000; published 12 September 2000)

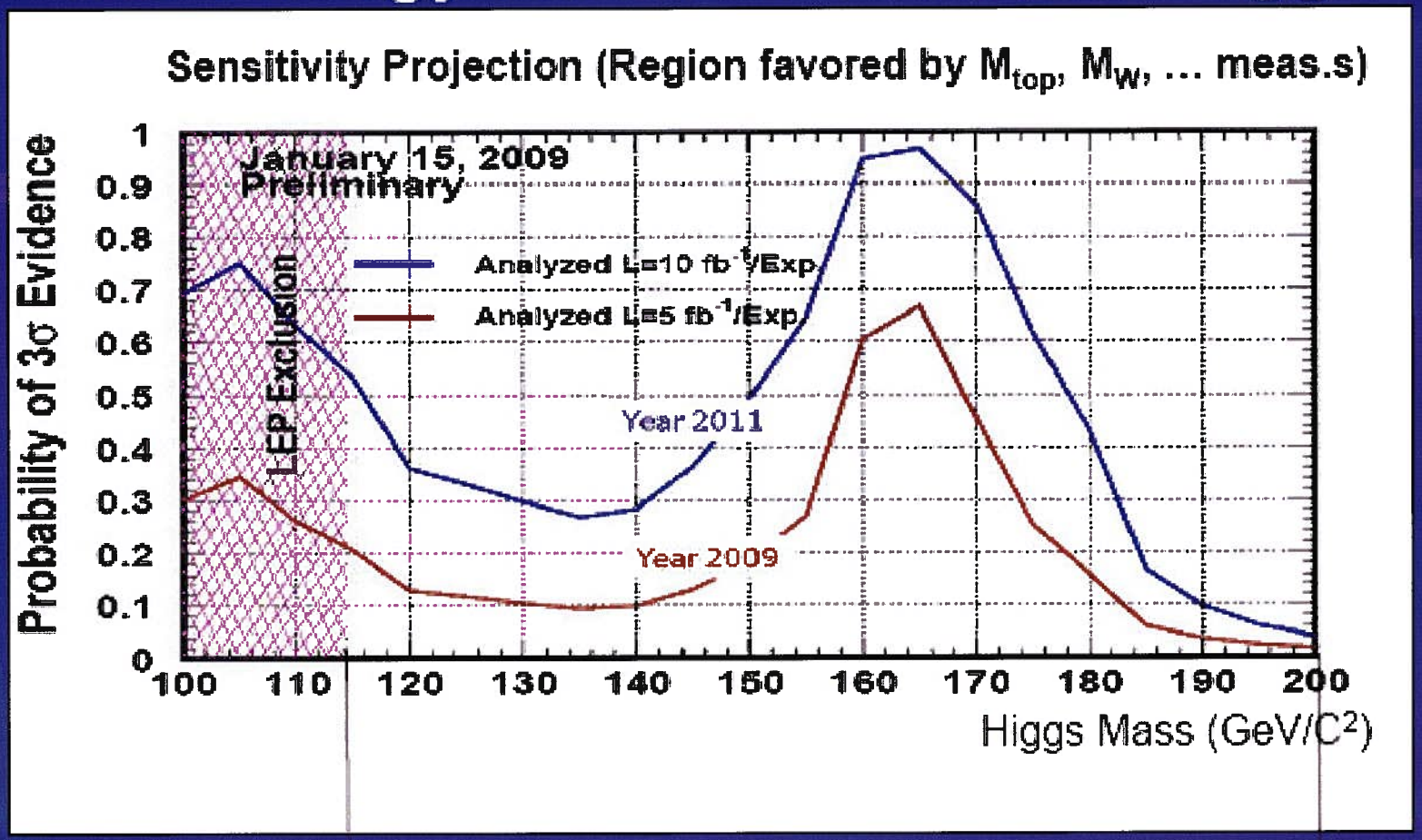
We have studied the potential of the CDF and DØ experiments to discover a low-mass standard model Higgs boson, during run II, via the processes $p\bar{p}\to WH\to l\nu b\bar{b}$, $p\bar{p}\to ZH\to l^+l^-b\bar{b}$ and $p\bar{p}\to ZH\to \nu\bar{\nu}b\bar{b}$. We show that a multivariate analysis using neural networks that exploits all the information contained within a set of event variables leads to a significant reduction, with respect to an equivalent conventional analysis, in the integrated luminosity required to find a standard model Higgs boson in the mass range 90 GeV/c^2<M_H <130 GeV/c^2. The luminosity reduction is sufficient to bring the discovery of the Higgs boson within reach

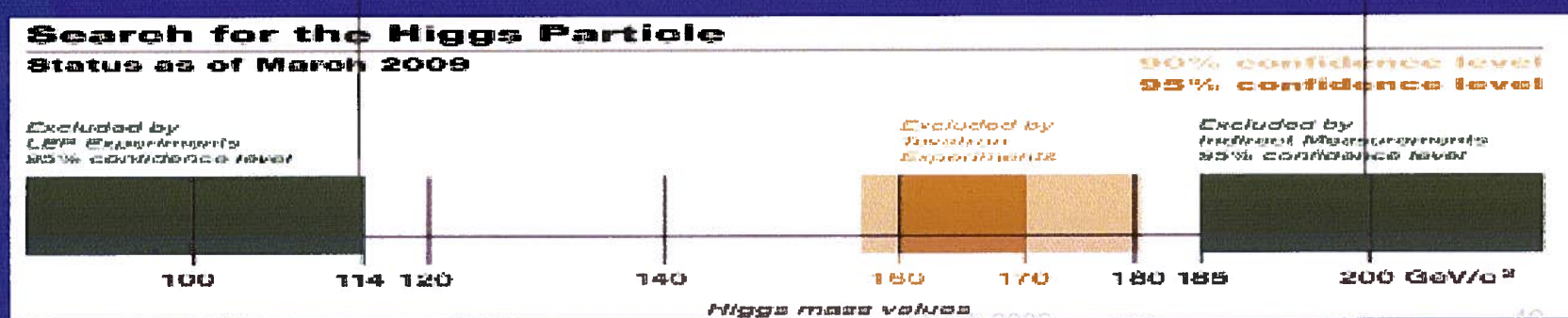




(18)

The Energy Frontier: Tevatron Higgs





(10)