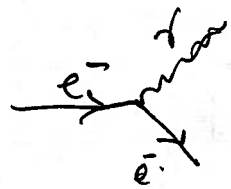
A bit more on Feynman Diagrams

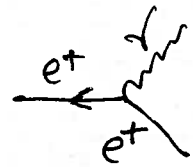
Electromagnetic Interactions:

Basic EM processes (vertices) involve emission or absorption of a photon by an electron or a positron or e^+e^- pair production or annihilation involving a photon

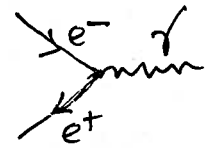
$e^- \to e^- + \gamma$



$e^+ \to e^+ + \gamma$



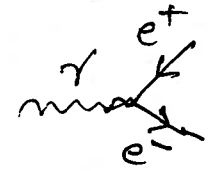
$e^+ + e^- \to \gamma$



$e^- + \gamma \to e^-$

$e^+ + \gamma \to e^+$

$\gamma \to e^+ + e^-$



These basic processes cannot occur for free particles since energy conservation will be violated.

Take the process, $e^- \to e^- + \gamma$, for example.

Let E = Energy, $\vec{k}$ = 3-momentum, $k = |\vec{k}|$

In the rest frame of the electron emitting the photon

$$e^-(E_0, 0) \to e^-(E_k, -\vec{k}) + \gamma(ck, \vec{k})$$

(Note: Momentum conservation is imposed.)

$k = \frac{2\pi}{\lambda} = \frac{p}{\hbar}$

Now $E_0 = mc^2$, $E_k = \sqrt{k^2c^2 + m^2c^4}$

$$\Delta E = E_k + kc - E_0$$

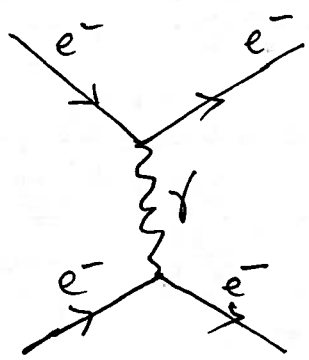
And, it is easy to see that $kc < \Delta E < 2kc \;\forall k$

3a

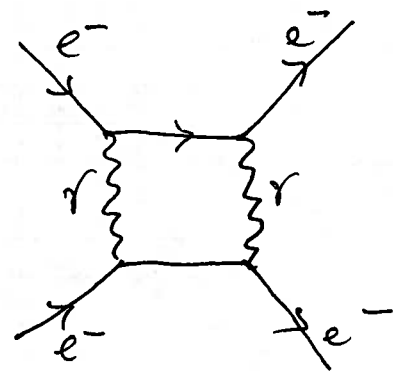
Similarly, one can show that energy conservation is violated for all these basic processes. Two or more of these basic processes have to be combined in such a way that energy conservation is violated in a "virtual process" lasting a short period τ compatible with the uncertainty principle

$$\tau \Delta E \sim \hbar$$

So, we come to processes such as, e-e scattering



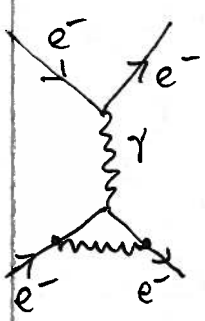
One-photon exchange

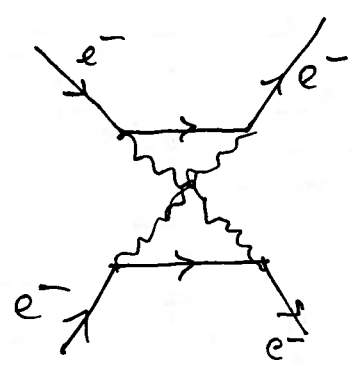


Two-photon exchange

Box Diagrams

Remember there are actually two "time-ordered" diagrams, for this process

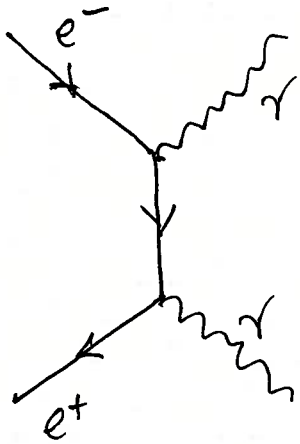




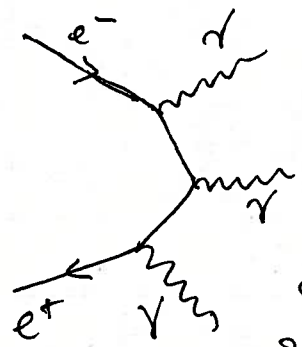
There are other two photon processes that can appear as corrections.

e^+e^- Annihilation

$e^+ + e^- \to \gamma + \gamma$



In order to conserve energy and momentum, at least two photons must be produced in the annihilation process.



$e^+ + e^- \to \gamma + \gamma + \gamma$

lowest order diagram for 3 photon production

Since there are 3 vertices, there are $3! = 6$ time-orderings of this process.

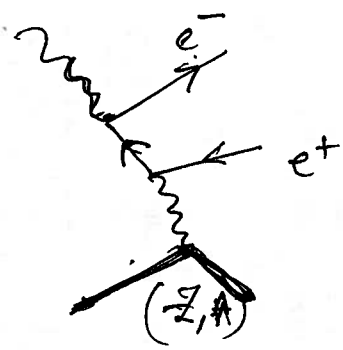
A process $e^+ + e^- \to p\gamma$ is of the order p and has probability $\sim \alpha^p$

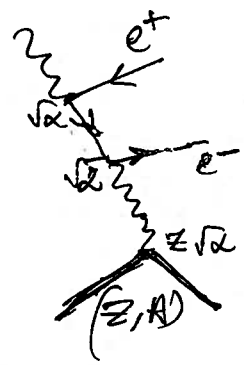
$$\therefore R = \frac{\text{Rate}\,(e^+e^- \to 3\gamma)}{\text{Rate}\,(e^+e^- \to 2\gamma)} = O(\alpha)$$

e^+e^- Pair Production

$\gamma \to e^+ + e^-$ cannot conserve energy and momentum simultaneously, and so cannot occur in free space. However, it is allowed in the presence of a nucleus.

$$\gamma + (Z, A) \to e^+ + e^- + (Z, A)$$





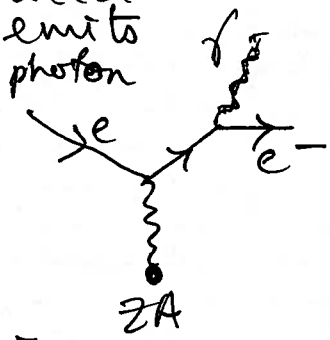
Rate $\sim (Z\sqrt{\alpha}\cdot\sqrt{\alpha}\cdot\sqrt{\alpha})^2$

$\approx Z^2\alpha^3$

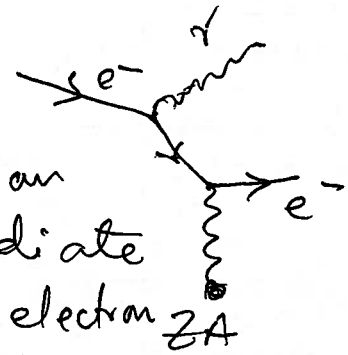
Bremsstrahlung

$$e^- + (Z, A) \to e^- + \gamma + (Z, A)$$

electron accelerated in the field of the nucleus emits a real photon

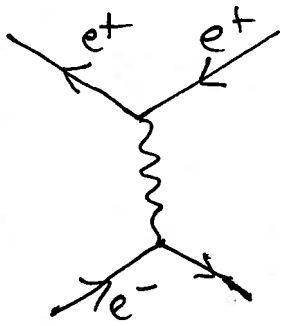


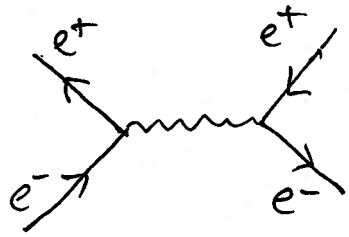
Another way of representing the nucleus.



Here, an intermediate virtual electron is involved which is "off-mass shell".

Bhabha Scattering (e^+e^- Scattering)

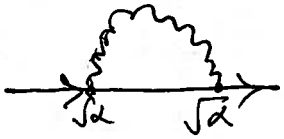




"Annihilation" diagram through a virtual photon

Higher Order Terms

For any process, higher order terms can come from multiple photon exchange or from "Self-energy" terms.

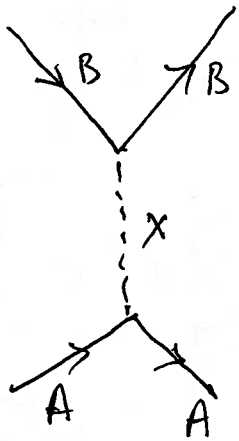


Vacuum Polarization

The self-energy terms are taken care of in the calculations by the process of "renormalization" (which we will not talk about here).

Particle Exchange and Range of Forces

Forces are associated with exchange of particles. So the range of forces depend on the mass of the particle exchanged.



Let us consider,

$$A + B \rightarrow A + B$$

A and B have masses m_A, m_B and equal coupling to X $(= g)$

At the lower vertex, $A(M_A c^2, 0) \rightarrow A(E_A, \vec{p}) + X(E_X, -\vec{p})$ in the rest frame of A

where $E_A = \sqrt{p^2c^2 + M_A^2c^4}$; $E_X = \sqrt{p^2c^2 + M_X^2c^4}$

Difference in energy between final and initial states is,

$$\Delta E = E_X + E_A - M_A c^2$$

For $p \rightarrow \infty$, $\Delta E \rightarrow 2pc$

For $p \rightarrow 0$, $\Delta E \rightarrow M_X c^2$

$\therefore$ for all p, $\Delta E \geq M_X c^2$

This energy violation is allowed for $\tau \sim \hbar / \Delta E$

So the distance or range, x is allowed to travel before being absorbed,

$$c\tau \sim R = \hbar / M_x c \qquad \boxed{R = \frac{\hbar}{M_x c}}$$

Range of the interaction

The electromagnetic interaction, mediated by the massless photon, has an <u>infinite range</u>.

The strong force between quarks also has infinite range, because gluons are massless. (But, hadron interactions are complicated since it is not single gluon exchange. They have effectively short range interactions. We will discuss later.)

What about the weak interaction?

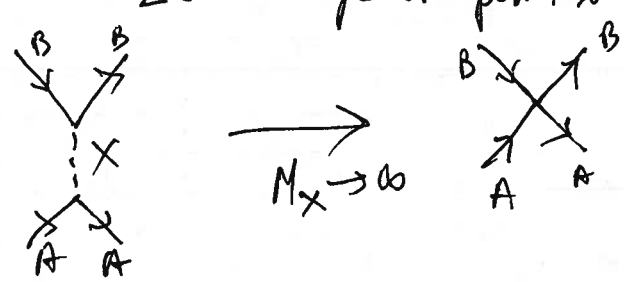
$$M_W \sim 80 \text{ GeV}, \quad M_Z \sim 91 \text{ GeV}$$

$$R_{W,Z} \sim \frac{\hbar c}{M_{W,Z} c^2} \sim 2 \times 10^{-3} \text{ fm}$$

$(1 \text{ fm} = 10^{-15} \text{ m})$

Very small compared to de Broglie wavelengths of particles involved

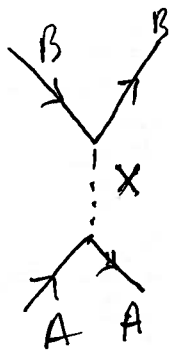
Can approximate as zero-range or point interaction



The Yukawa Potential and Short range forces

Hideki Yukawa introduced the idea of short range forces being due to massive particle exchange, in 1935, to explain the observed short-range nuclear forces. In the process, he predicted the existence of such a field quantum particle with mass ~300 me (or 150 MeV/c^2)

Let us go back to the picture of particles A and B scattering off of each other.



If the exchanged boson X has an integral spin, it will obey Klein-Gordon equation

$$\frac{\partial^2 \psi(\vec{r},t)}{\partial t^2} = \nabla^2 \psi(\vec{r},t) - M_X^2 \psi(\vec{r},t)$$

Assuming a static potential (i.e., time-independent

$$\nabla^2 \phi(\vec{r}) - M_X^2 \phi(\vec{r}) = 0$$

If we set $M_X = 0$, then, for the exchange of a massless boson, we get

$$\nabla^2 \phi(r) = 0 \quad \leftarrow$$ equation for an electrostatic potential

which has solution of the form,

$$\phi(r) \sim -e/r \quad \left[\text{or } \frac{-e}{4\pi\epsilon_0}\frac{1}{r}\right]$$

For $\nabla^2\phi - M_x^2\phi = 0$,

the solution would be,

$$\phi(r) = \frac{g}{4\pi r} \cdot e^{-r/R}$$

where R is the range as we derived before

$R = \frac{\hbar}{M_x c} = \frac{1}{M_x}$ with Natural Units and M_x in GeV

As $M_x \to \emptyset$, $R \to \infty$ $\leftarrow$ Coulomb Force

Nuclear forces have $R \simeq 10^{-15}$ m which gives,

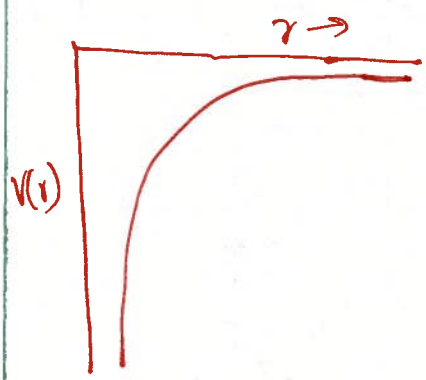
$M_x \sim 200$ MeV. Pions with a mass ~ 140 MeV

were discovered later.

$$V(r) = -\frac{g^2}{4\pi} \cdot \frac{e^{-r/R}}{r}$$

with $\alpha_x = \frac{g^2}{4\pi\hbar c}$ similar to the fine structure constant

$\alpha = e^2/4\pi\hbar c$



Scattering Amplitude

The probability amplitude for scattering of a particle with initial momentum $\vec{q}_i$ to a final state with momentum $\vec{q}_f$ by a potential $V(r)$, is, from Born approximation

$$M(\vec{q}) = \int d^3\vec{r}\; V(\vec{r})\, e^{i\vec{q}\cdot\vec{r}/\hbar}$$

where $\vec{q} = \vec{q}_i - \vec{q}_f$

Using spherical polar coordinates,

$$\vec{q}\cdot\vec{r} = qr\cos\theta \;;\; d^3r = r^2 \sin\theta\, d\theta\, dr\, d\phi$$

By substituting the Yukawa potential for $V(r)$ and integrating over θ and ϕ,

$$M(\vec{q}) = -g^2\left[\int e^{-r/R}\left(\frac{e^{iqr} - e^{-iqr}}{2iq}\right).dr\right]$$

$$= \frac{-g^2\hbar^2}{|\vec{q}|^2 + M_x^2c^2}$$

If the couplings at the two vertices are different (for example, scattering of a particle with coupling strength g, from a massive source with strength, g_0), then,

$$M(\vec{q}) = -gg_0\hbar^2 / |\vec{q}|^2 + M_x^2c^2$$

At higher energies, the recoil energy of the target cannot be neglected.

So $q^2 = (E_f - E_i)^2 - (\vec{q}_f - \vec{q}_i)^2 c^2$

or $q^2 = \Delta E^2 - \Delta \vec{p}^2$

However, one obtains for this case, as well,

$$M(q^2) = \frac{g^2 \hbar^2}{q^2 - M_x^2 c^2} \qquad \left(\text{or } \frac{g_0 g}{q^2 - M_x^2} \right)$$

Also, $\frac{g_0 g}{q^2 + M_x^2}$ if $q^2 = \Delta \vec{p}^2 - \Delta E^2$

If $q^2 << M_x^2 c^2$, then,

$$M(q^2) = \frac{-g^2 \hbar^2}{M_x^2 c^2} = -\left(\frac{g}{M_x c^2}\right)^2 \cdot (\hbar c)^2$$

$= -G$ the Fermi coupling constant

$\Leftarrow$ the point interaction, in the zero range approximation, has one characteristic constant.