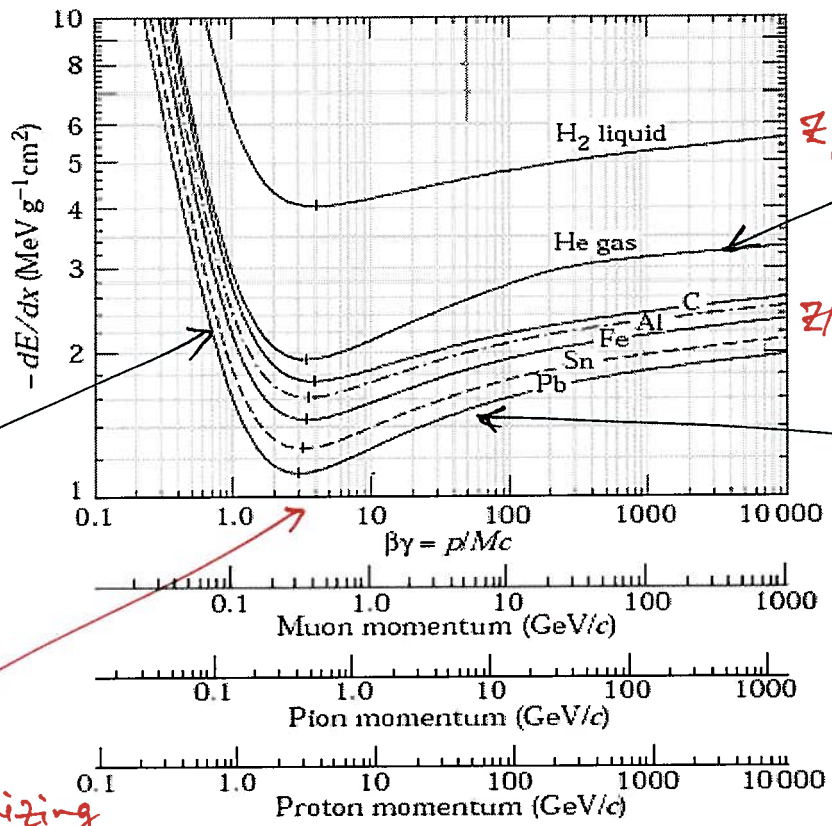
$$-\frac{dE}{dx} = \kappa \cdot z^2 \frac{Z}{A} \cdot \frac{1}{\beta^2} \cdot \left[\frac{1}{2} \ln \frac{2 m_e c^2 \gamma^2 \beta^2}{I^2} E_{kin}^{max} - \beta^2 - \frac{\delta}{2} \right]$$



$Z/A = 1$

Fermi Plateau

$Z/A \sim 0.5$

Kinematical Term

$\frac{dE}{dx} \propto \frac{1}{\beta^2}$

$\frac{dE}{dx} \propto \ln \beta^2 \gamma^2$

"Relativistic Rise"

$\beta\gamma \approx 3-4$

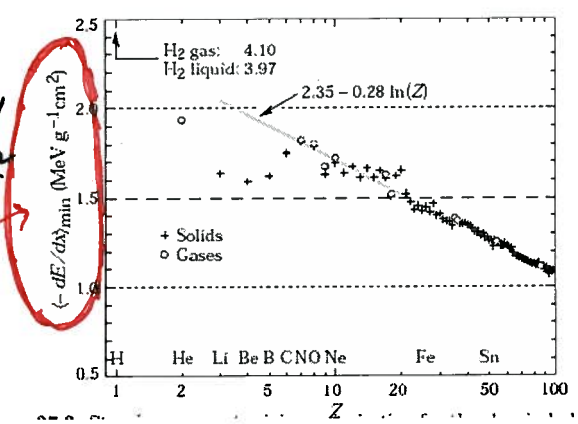
Minimum Ionizing Particles, MIPs

To first approximation, medium characterized by Z/A ← electron density

E.g.,

$$-\frac{dE}{dx}\Big|_{min}^{Fe} = 1.48 \text{ MeV/gm/cm}^2$$

$$= 11.65 \text{ MeV/cm}$$



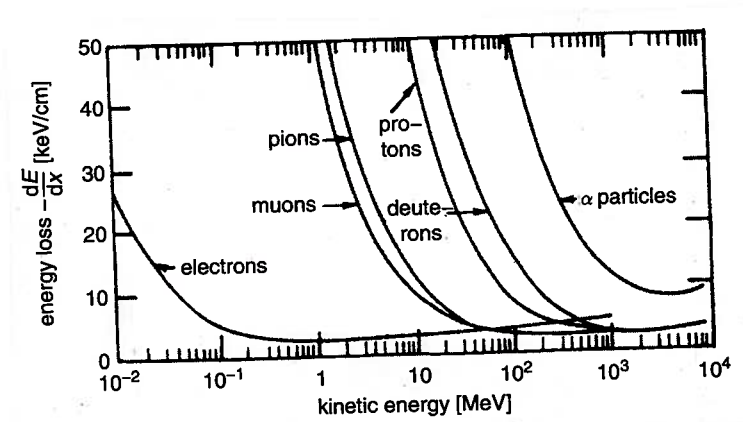
Z-dependence

Stopping Power at Minimum Ionization

Table 1.1. *Average energy loss of minimum-ionising particles in various materials [10–12]; gases for standard pressure and temperature*

Absorber	$\frac{dE}{dx}\Big\|_{min} \left[\frac{MeV}{g/cm^2}\right]$	$\frac{dE}{dx}\Big\|_{min} \left[\frac{MeV}{cm}\right]$
Hydrogen (H_2)	4.10	$0.37 \cdot 10^{-3}$
Helium	1.94	$0.35 \cdot 10^{-3}$
Lithium	1.64	0.87
Beryllium	1.59	2.94
Carbon (Graphite)	1.75	3.96
Nitrogen	1.82	$2.28 \cdot 10^{-3}$
Oxygen	1.80	$2.57 \cdot 10^{-3}$
Air	1.82	$2.35 \cdot 10^{-3}$
Carbon dioxide	1.82	$3.60 \cdot 10^{-3}$
Neon	1.73	$1.56 \cdot 10^{-3}$
Aluminium	1.62	4.37
Silicon	1.66	3.87
Argon	1.52	$2.71 \cdot 10^{-3}$
Titanium	1.48	6.72
Iron	1.45	11.41
Copper	1.40	12.54
Germanium	1.37	7.29
Tin	1.26	9.21
Xenon	1.25	$7.32 \cdot 10^{-3}$
Tungsten	1.15	22.20
Platinum	1.13	24.24
Lead	1.13	12.83
Uranium	1.09	20.66
Water	1.99	1.99
Lucite	1.95	2.30
Shielding concrete	1.70	4.25
Quartz (SiO_2)	1.70	3.74

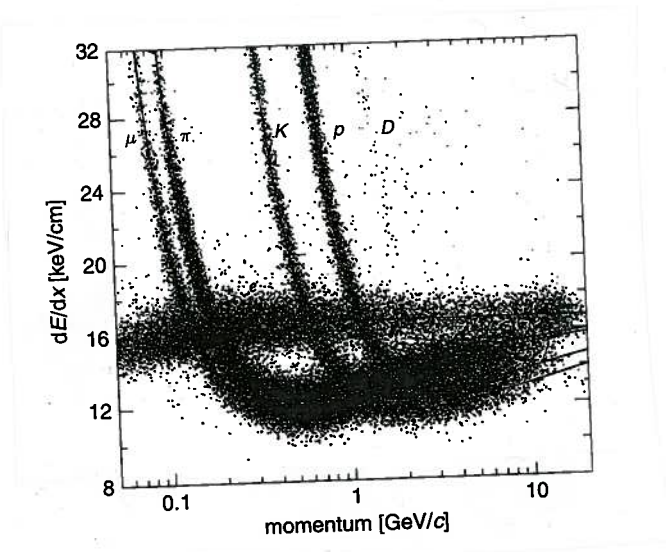
$\frac{dE}{dx}$ for different types of particles →

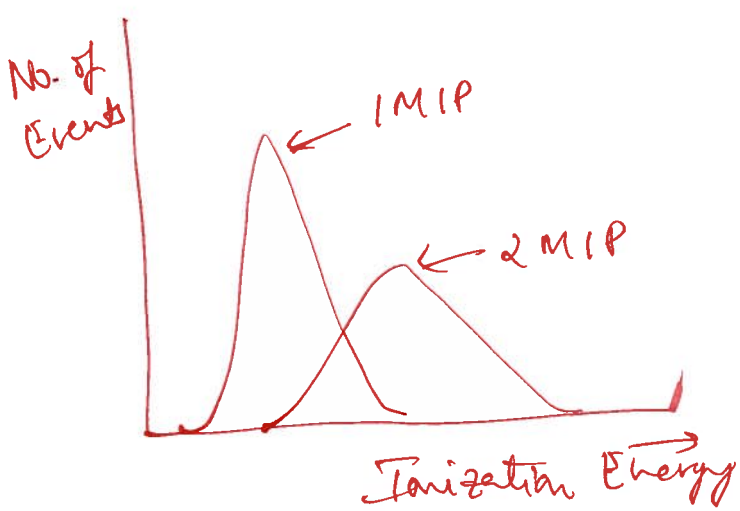


This aids in Particle ID

i.e., determining what particle went through the detector

Measured $\frac{dE}{dx}$ in the PEP4 TPC (Time Projection Chamber)

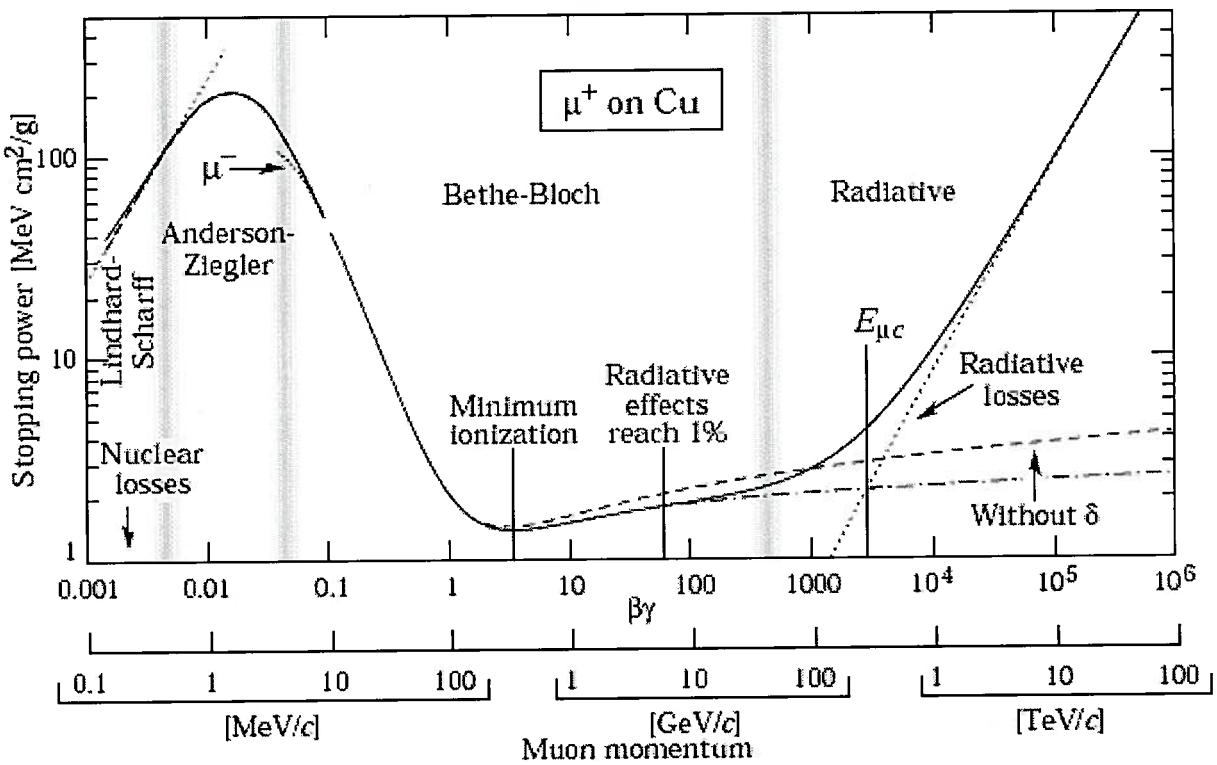




← Allows separation of 2 particles going through the detector from one particle. e.g., $\gamma \to e^+e^-$ The opening angle can be small.

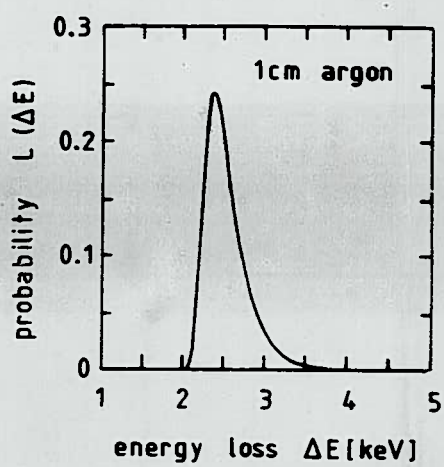
Review of Energy Loss / Stopping Power

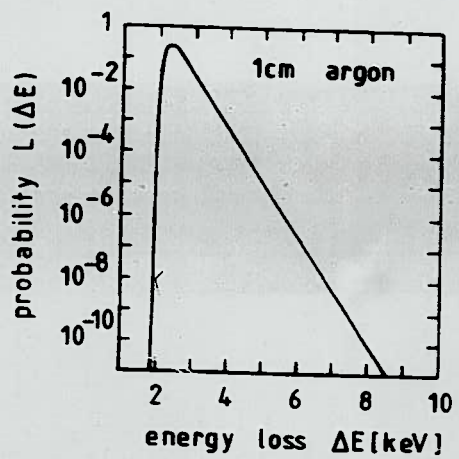
$$-\frac{dE}{dx} = \mathcal{K} \cdot z^2 \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\gamma^2\beta^2 E_{kin}^{max}}{I^2} - \beta^2 - \frac{\delta}{2} - \frac{C}{Z}\right]$$



- Bethe - Bloch equation is valid for $\beta \gg \alpha z$.
- $\frac{dE}{dx} \propto \frac{1}{\beta^2}$ at low energies, reaches a broad minimum at $\beta\gamma \approx 3-4$ (MIPs), has a relativistic rise for $\beta\gamma > 4$ ($\sim \ln \beta^2\gamma^2$)
- In light absorbers, $\left.-\frac{dE}{dx}\right|_{min} = 1.5-2$ MeV/gm/cm^2
- Density effect reduces the relativistic rise and eventually saturates
- At very low energies, various Z dependent corrections must be made; At very high energies, radiative losses become important and eventually dominate.

The Bethe-Bloch formula gives us the mean/average energy loss. In thin absorbers, strong fluctuations around the average energy loss exist. ← Few collisions, some with high energy transfer





For depth L

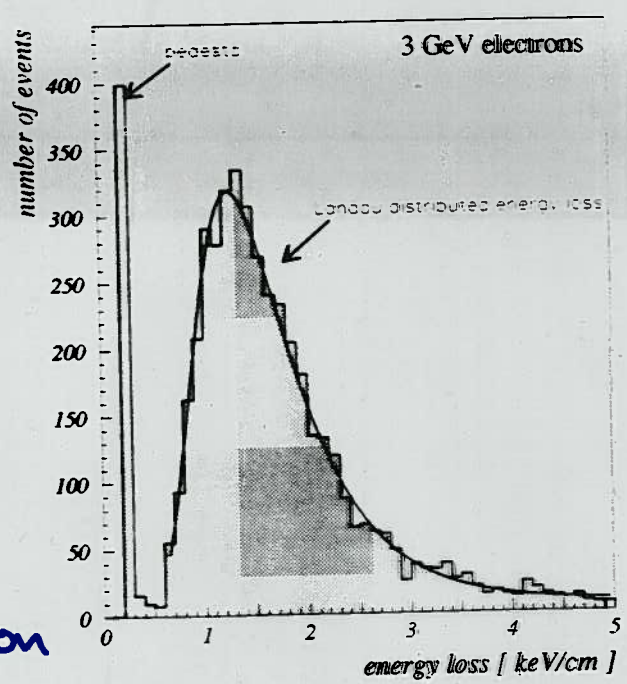
$$\langle \Delta E \rangle = \left\langle \frac{dE}{dx} \right\rangle L$$

↑ Most probable energy loss

$$P(\lambda) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\lambda + e^{-\lambda})}$$ (Landau distribution)

$$\lambda = \frac{\Delta E - \langle \Delta E \rangle}{\langle \Delta E \rangle}$$

deviation from the m. p. energy loss

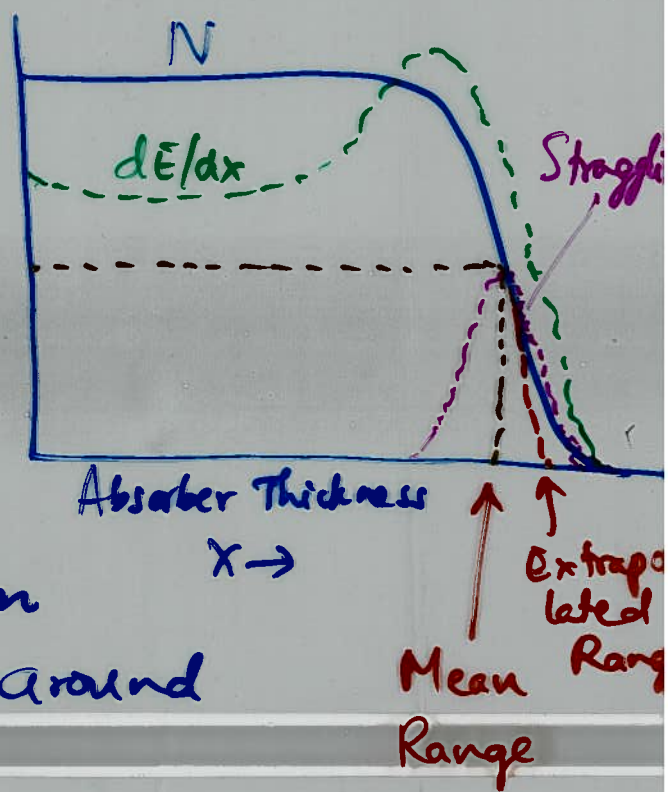


For thick absorbers, the energy loss distribution → Gaussian ← Many collisions because of thick absorber

Range & Straggling

Energy loss is statistical in nature.

Two identical particles with the same initial energy will have different number of collisions and hence different energy loss.



⇒ statistical distribution of Ranges centred around a "mean range".

→ Range – Straggling

$$R(E) = \int_{E}^{\emptyset} \frac{1}{-dE/dx} \cdot dE$$

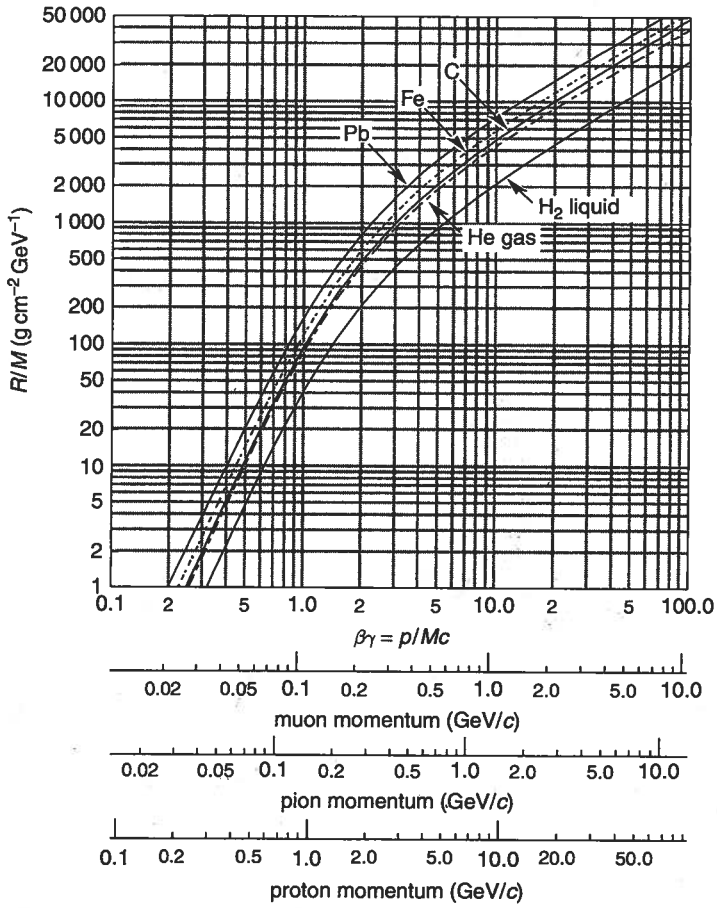


Fig. 1.11. Range of muons, pions and protons in liquid hydrogen, helium gas, carbon and lead [12].

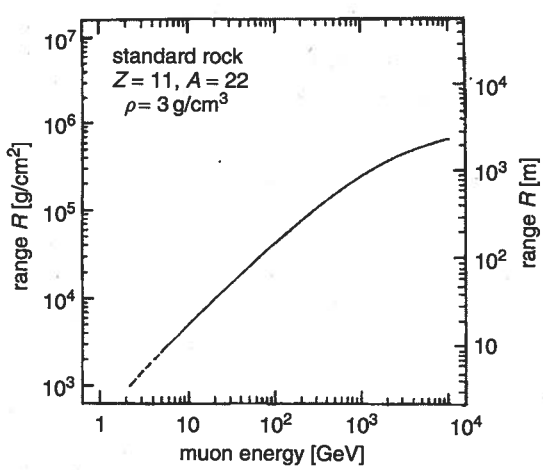


Fig. 1.12. Range of muons in rock [51].