Cross Section

← the probability for an interaction to occur

In the classical sense,

Cross-section ≡ geometrical cross sectional area of a sphere of a characteristic radius "a"

$$\sigma \sim \pi a^2$$

$$\sigma_{atom} \sim \pi a_0^2 \sim 3 \times 10^8 \text{ b} \qquad a_0 \sim 1 \text{ Å}$$

$$\sigma_{nucleus} \sim \pi a_N^2 \sim 30 \text{ mb} \qquad a_N \sim 1 \text{ fm}$$

$$1 \text{ b} = 10^{-24} \text{ cm}^2$$

$$\sigma_{atom} / \sigma_{nucleus} \sim 10^{10}$$

In quantum mechanics,

$$\sigma \sim |\langle f | H | i \rangle|^2$$

Mean Free Path

← Average distance between scatterings

Consider a material with mass density ρ and atomic weight A, then the number of nuclei per unit volume of the medium is

$$\frac{N_A \cdot \rho}{A} \quad ; \quad N_A \equiv \text{Avogadro's number}$$

If you have N particles incident on such a material, then the number of scatterings in a thickness dx,

$$dN = N \cdot \frac{N_A \cdot \rho}{A} \cdot \sigma \cdot dx$$

$\frac{N_A \cdot \rho}{A} \cdot dx$ ≡ No. of nuclei per unit area

$$\Rightarrow N(x) = N(0)\, e^{-\frac{N_A \cdot \rho}{A} \cdot \sigma \cdot x}$$

$$= N(0)\, e^{-x/\langle L \rangle}$$

Mean Free Path $\lambda = \langle L \rangle = 1 \Big/ \left(\frac{N_A \cdot \rho \cdot \sigma}{A}\right)$ cm

$$\langle L\rho \rangle = 1 \Big/ \left(\frac{N_A \cdot \sigma}{A}\right) \text{ g/cm}^2$$

Interactions of Photons

Photons, unlike heavy charged particles, get attenuated in matter easily.

The number of photons lost from a monoenergetic beam of N photons, while passing through a thickness dx of a material.

$$dN = -\mu N dx \; ; \quad N = N_0 e^{-\mu x}$$

where μ = mass attenuation coefficient, and is related to the probability that a photon will be scattered or absorbed in the materia

$\therefore$ The intensity of a photon varies as

$$N = N_0 e^{-\mu x}$$

i.e., decrease exponentially with thickness of the material traversed.

Note: Contrast with energy attenuation of charged particles during radiation loss

Photoelectric Effect

- First explained by Einstein in 1905.
- An interaction between photon and the atom

$$\gamma + \text{atom} \rightarrow \text{atom}^+ + e^-$$

- Photons with energy $E_\gamma >> E_b$, the binding energy of an electron in the atom, may be absorbed and an atomic electron ejected with kinetic energy $T = E_\gamma - E_b$.
- "Resonant" absorption occurs with E_γ near the bound state energies of the electrons,

$$E_n = -13.6\,\text{eV} \cdot \frac{Z^2}{n^2} \qquad \left(n = \text{principal quantum number}\right)$$

- The photoelectric cross section

$$\sigma_{pe} = \frac{32\pi}{3} \cdot \sqrt{2} \cdot Z^5 \alpha^4 \left(\frac{m}{\hbar\omega}\right)^{7/2} \cdot (\alpha\lambda\!\!\!^{-})^2$$

$$\sigma_{pe} \propto \frac{Z^5}{E_\gamma} \quad \text{for high energies}$$

- Main application in PMTs

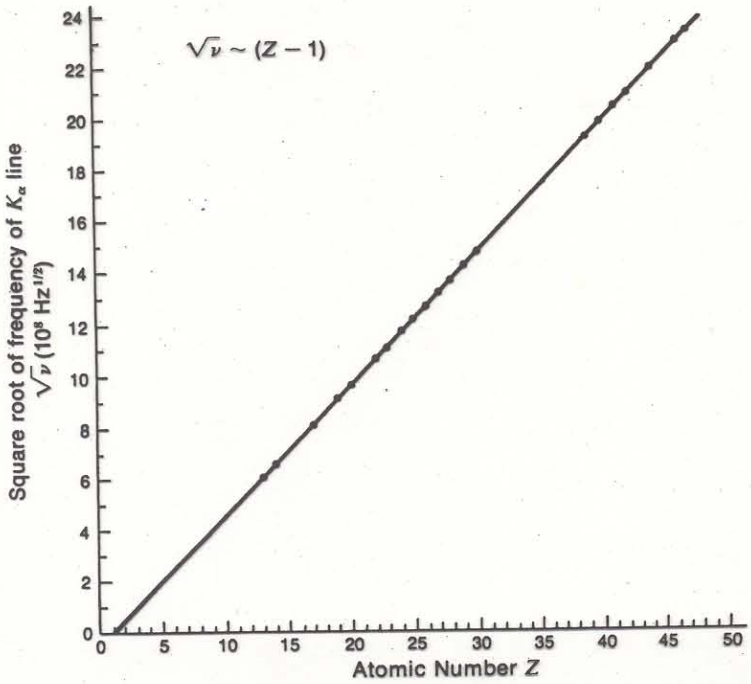
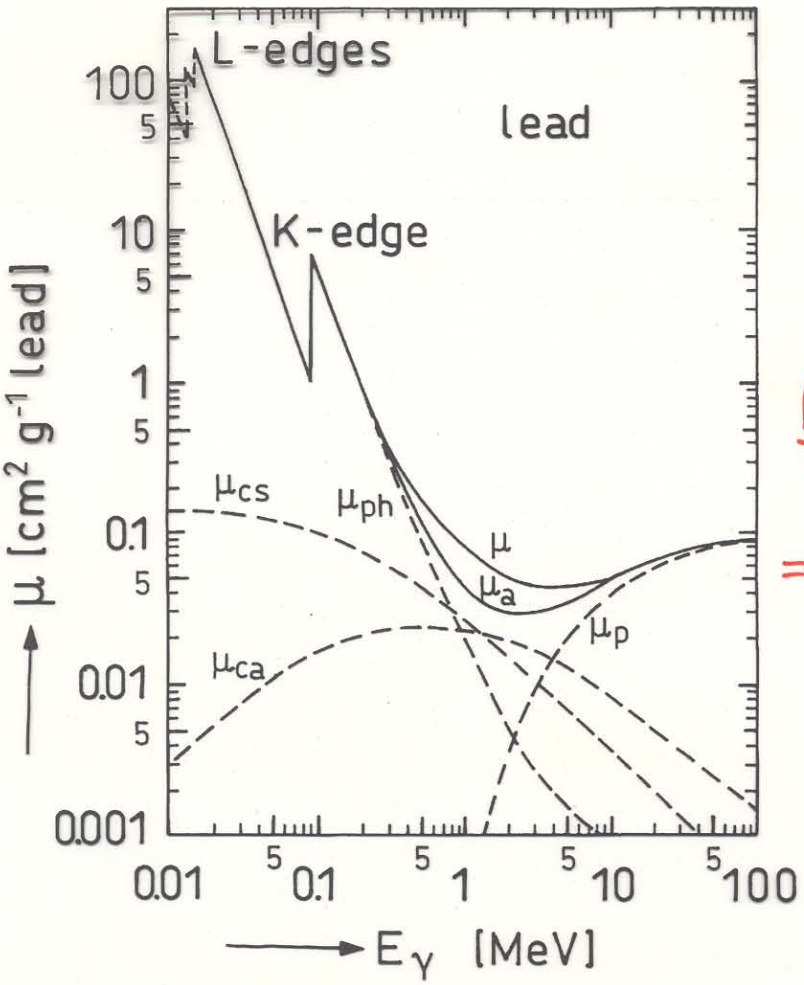


Fig. 2.3. Data on the relationship between the x-ray wavelength and the atomic number. The $\hbar\omega = E \sim Z^2$ behavior expected from Eq. 2.6 is observed. (From Ref. 1.7, with permission.)



$I = I_0 e^{-\mu x}$

$\frac{\mu}{\rho} = \frac{N_A}{A} \cdot \sum_i \sigma_i$

$= \frac{N_A}{A} \left(\sigma_{pE} + Z \sigma_c + \sigma_{pair} \right)$

$\frac{\mu}{\rho} \equiv$ Mass attenuation coefficient

Fig. 1.14 d. Energy dependence of the mass attenuation coefficient μ and mass absorption coefficient μ_a for photons in lead [63, 73, 74, 75].

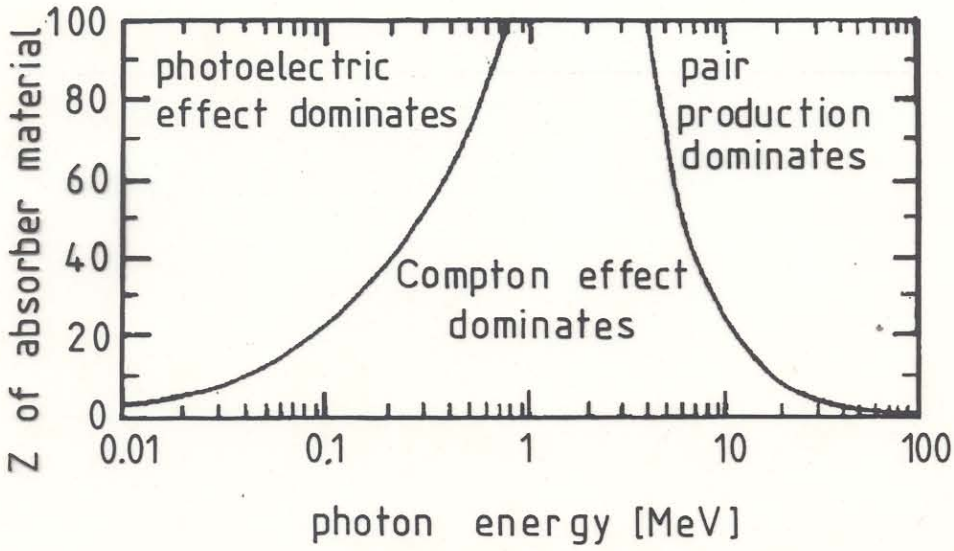


Fig. 1.15. Ranges, in which the photoelectric effect, Compton effect, and pair production dominate as a function of the photon energy and the target charge Z [37, 65, 68].

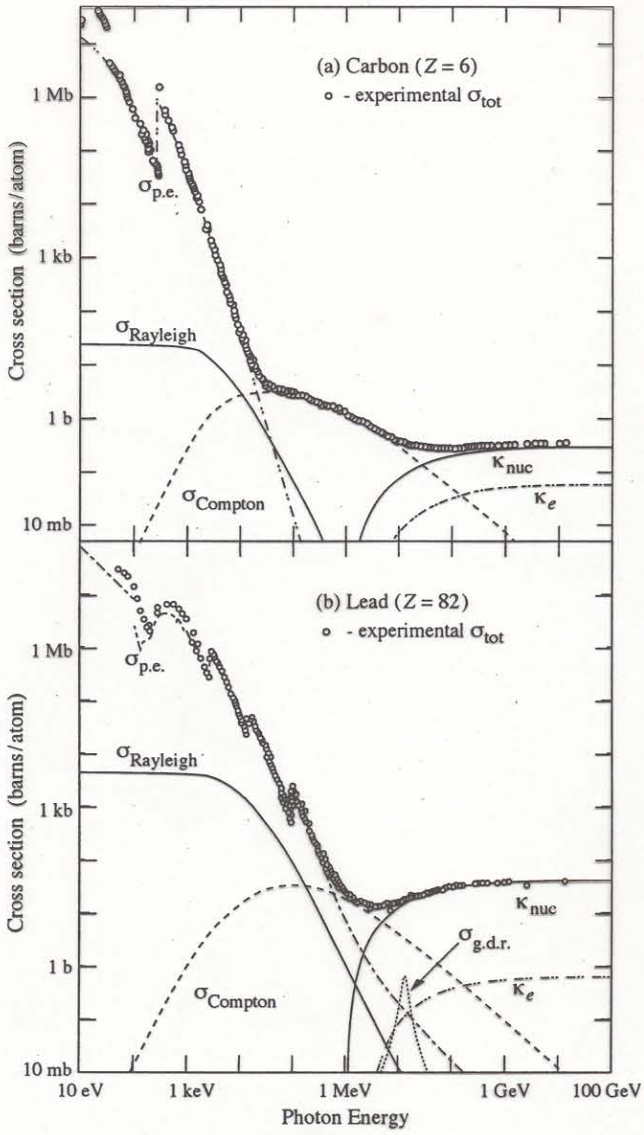


Figure 27.14: Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes:

$\sigma_{\text{p.e.}}$ = Atomic photoelectric effect (electron ejection, photon absorption)

σ_{Rayleigh} = Rayleigh (coherent) scattering–atom neither ionized nor excited

σ_{Compton} = Incoherent scattering (Compton scattering off an electron)

κ_{nuc} = Pair production, nuclear field

κ_e = Pair production, electron field

$\sigma_{\text{g.d.r.}}$ = Photonuclear interactions, most notably the Giant Dipole Resonance [46]. In these interactions, the target nucleus is broken up.

I

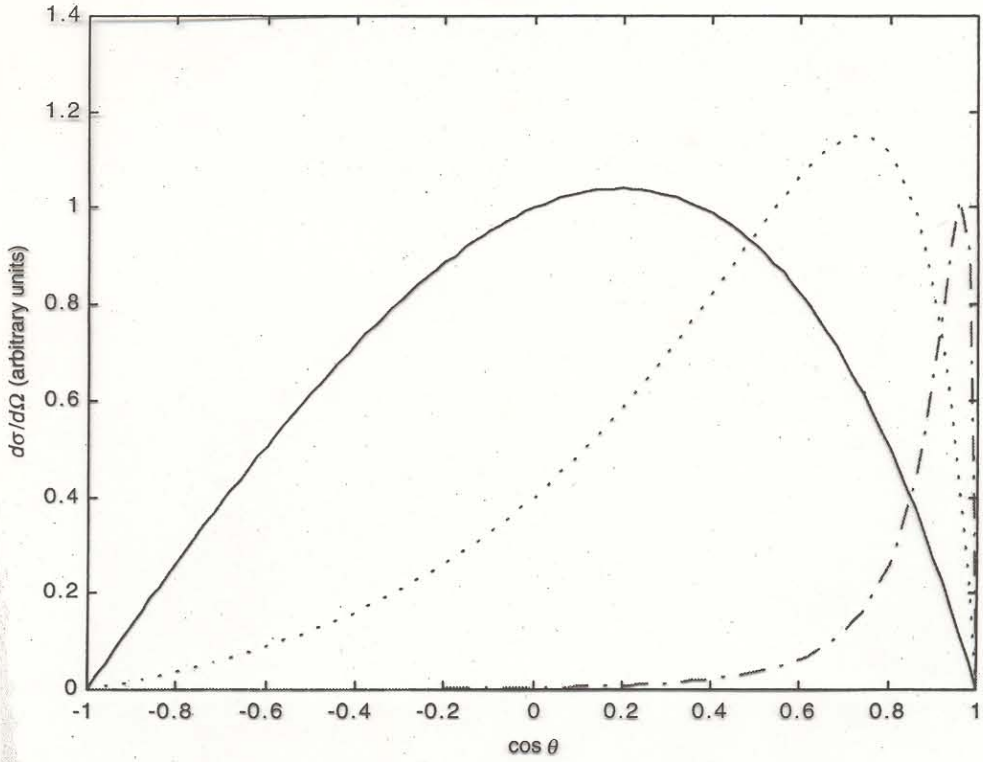
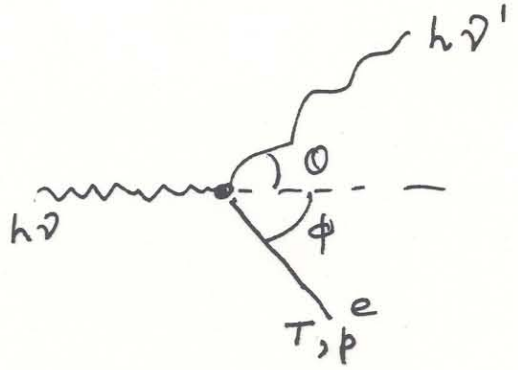


Fig. 2.6. Angular distribution of the photoelectron with respect to the incident photon direction for several recoil electron velocities. —, $\beta=0.1$; · · ·, $\beta=0.5$; –·–·, $\beta=0.9$.

Compton Effect

- Scattering of an incident photon off a quasi-free atomic electron.

$$\gamma + e \rightarrow \gamma' + e'$$



From energy momentum conservation,

$$h\nu = h\nu' + T$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p\cos\phi$$

$$0 = \frac{h\nu'}{c}\sin\theta - p\sin\phi$$

Using these and

$$T = \sqrt{p^2c^2 + m_e^2c^4} - m_ec^2,$$

One can derive a number of useful relations

The frequency of the scattered photon:

$$\nu' = \frac{\nu}{1 + \epsilon(1-\cos\theta)} \quad \text{—— (1)} \qquad \epsilon = \frac{h\nu}{m_e c^2}$$

The kinetic energy of the recoil electron:

$$T = m_e c^2 \cdot \frac{\epsilon^2 (1-\cos\theta)}{1+\epsilon(1-\cos\theta)} \quad \text{—— (2)}$$

The electron recoil angle:

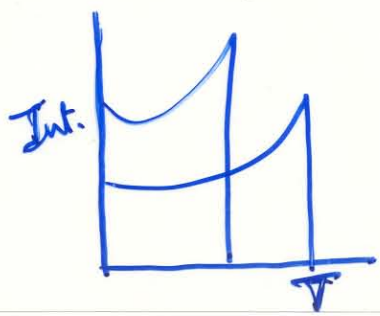
$$\cos\phi = (1+\epsilon)\left[\frac{1-\cos\theta}{2+\epsilon(\epsilon+2)(1-\cos\theta)}\right]^{1/2} \quad \text{—— (3)}$$

For backscattering $(\theta = \pi)$, (2) becomes,

$$T = T^{max} = \frac{2\epsilon^2}{1+2\epsilon} \cdot m_e c^2 = \frac{2\epsilon}{1+2\epsilon} \cdot E_\gamma$$

For $\epsilon \gg 1$, i.e., $E_\gamma = h\nu \gg m_e c^2$,

$$T^{max} \simeq E_\gamma$$



$$E_\gamma - \frac{m_e c^2}{2}$$

Compton Edge

The cross section for compton scattering can be calculated using QED.

Klein - Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \cdot \frac{1}{[1+\epsilon(1-\cos\theta)]^2} \left(1+\cos^2\theta + \frac{\epsilon^2(1-\cos\theta)^2}{1+\epsilon(1-\cos\theta)}\right)$$

$$\sigma_c = \int \frac{d\sigma}{d\Omega} \cdot d\Omega$$

(Probability/electron for a compton scattering to occur.)

$$\sigma_{cs} = \frac{E_\gamma'}{E_\gamma} \cdot \sigma_c$$

scattered cross section

$$\sigma_{ca} = \sigma_c - \sigma_{cs}$$

absorption cross section

Pair Production

Production of electron-positron pairs is possible in the coulomb field of a nucleus (or an atomic electron) if $E_\gamma > E_{th}$

From energy & momentum conservation, the threshold energy can be calculated.

$$E_\gamma \geq 2 m_e c^2 + 2 \cdot \frac{m_e^2}{m_{nucleus}} c^2$$

But $m_{nucleus} >> m_e$

$\therefore E_\gamma \geq 2 m_e c^2$ is the threshold

$$\sigma_{pair} = 4 \alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} \right) \quad cm^2/atom$$

$$\boxed{\sigma_{pair} \approx \frac{7}{9} \cdot \frac{A}{N_A} \cdot \frac{1}{X_0}}$$

Scintillators

Inorganic

→ high light output
~25 eV/γ in NaI(Tl)

→ slow decay time/response
~μs (CsI(Tl)) ; 0.62μs in BaF_2

→ Inorganic crystals

Organic

→ low light output
~100 eV/γ required

→ fast decay time/response
~22ns in Anthracene

→ Organic crystals
→ Organic liquids
→ Plastics
a few ns decay time

+ Gaseous scintillators

Noble gases: Xe, Kr, Ar, He (also N_2)

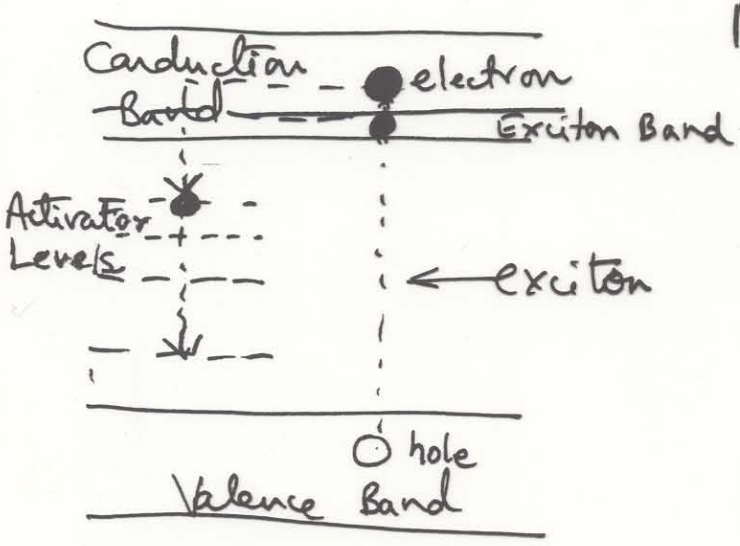
- Response extremely rapid (< 1ns)
- Light emitted in UV (where PMTs are inefficient)
- Used in experiments with heavy charged particles or fission fragments.

+ Glasses

- Lithium or Boron silicates (Cerium activated)
- low light output (< 25-30% of anthracene)
- Used in ^1_0n detection, generally.

Inorganic Scintillators

- The scintillation mechanism is an effect of the lattice. (crystal structure of the material)
- Inorganic crystals, mainly alkali halides (CsI, NaI, KI, ..) with an activator impurity such as Tl



Most commonly used:

$NaI(Tl)$

non-alkali materials:

BGO (Bismuth Germanate $Bi_4Ge_3O_{12}$)

BaF_2, $ZnS(Ag)$

- Incident particles can transfer energy by:
  - exciting an electron from the valence band to the conduction band
    → Creates a free electron & a free hole
  - exciting an electron from the valence band to the exciton band
    → Creates a bound e-h pair (exciton)

- free electrons, holes and the excitons can move freely through the lattice.
- Electrons combining with holes (or returning to the valence band) results in photon emission
  - Frequency of the emitted radiation, response time depend on the band gap and the details of electron migration in the lattice.
- The dopants (or activator impurities) create additional activation centers (energy levels) in the forbidden gap.

  These can be excited [by the excitons moving through the lattice], which then decay with photon emission.

  The addition of the dopants thus increase the light yield, provide faster response and can also help match the radiation wavelength to the photocathode spectral sensitivity.

  The decay time $\propto e^{-E/kT}$

  $E \equiv$ Energy of the excited impurity level.
- Sometimes the de-excitation can be radiationless (the impurity center becomes a trap.).

Table 14. *Properties of scintillating inorganic crystals*

Scintillator	NaI(Tl)	LiI(Eu)	CsI(Tl)	$Bi_4Ge_3O_{12}$	BaF_2[a]		CeF_3	$PbWO_4$
Density (g/cm^3)	3.67	4.06	4.51	7.13	4.9		6.16	8.3
Melting point (°C)	650	450	620				1443	
Decay time (μs)	0.23	1.3	1.0	0.35	0.62	6×10^{-4}	0.03	0.01
λ_{max}(emission) (nm)	410	470	550	480	310	225	310	470
Light yield (photons/MeV)	4×10^4	1.4×10^4	5.2×10^4	8200	6500	2000	1000	100
Radiation length X_0 (cm)	2.59	2.2	1.86	1.12	2.05		1.7	0.89
Critical energy (MeV)	12.5		10.2	8.8	12		13.6	8.5
Molière radius (cm)	4.3		3.8	2.7	3.4		2.63	2.2
Refractive index n	1.85		1.8	2.15	1.56		1.68	2.16
$(dE/dx)_{min}$ (MeV/cm)	4.13	5.1	5.1	8.07	5.72		7.7	13.0
Temperature coefficient of light output (%K^{-1})	−(0.22–0.9)		<0.2	−1.7	−0.6			
Radiation damage	fair		fair	medium	very small		small	small
Hygroscopic	yes		weak	no	no		no	no

[a] BaF_2 has two scintillating components.

Organic Scintillators

← Scintillation is a molecular process

- Organic crystals (Anthracence $C_{14}H_{10}$
  Naphthalene $C_{10}H_8$)
- Organic Liquids (Mixtures of one or more in a solvent
  e.g., p-Terphenyl ($C_{18}H_{14}$) in Toluene)
- Plastics (most popular & most widely used)
  polyvinyltoluene, polystyrene ← plastic solvents
  Typical solutes: $C_{18}H_{14}$

∴ Organic scintillators are typically 2 or 3 active component mixtures in an organic base.

Primary fluorescence agent is excited by the energy loss of particles. The primary scintillation light from the deexcitation is emitted in the UV range. However, the fluorescence agent is opaque to its own light (short absorption length). So, a second fluorescent agent which absorbs the UV light and re-emits at longer wavelengths is added. (Wavelength Shifter). The emission spectrum of the 2nd agent is matched to the spectral sensitivity of the light receiver.

Light Output

Average energy loss / photon for incident electrons

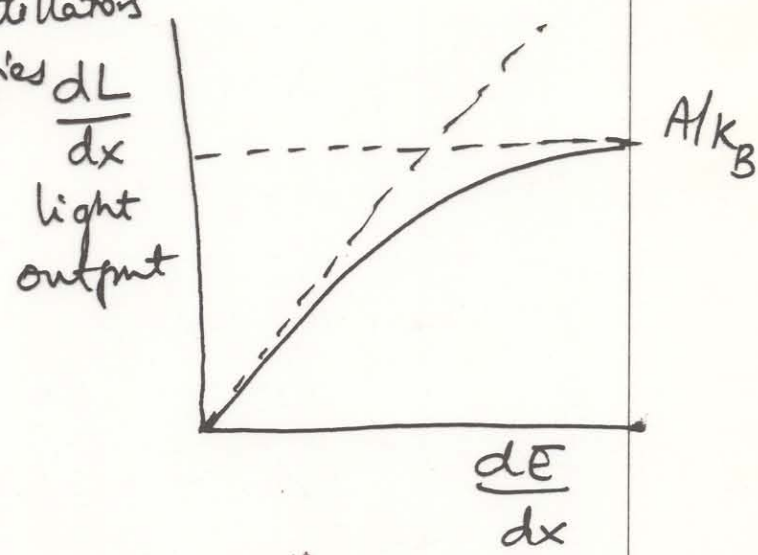
Scintillator	ε (eV/photon)
Anthracene	60
NaI	25
Plastic	100
BGO	300

* Light output lower for heavier particles at same E
* efficiency of photodetection is also important.
  For photomultipliers, efficiency ~30%.

Linearity of Response

Particularly, for plastic scintillators
$L \not\propto E$, at high energies

Semi-empirical model
by Birks (1951)



$$\frac{dL}{dx} = \frac{A \cdot dE/dx}{1 + k_B \cdot dE/dx}$$

$A \equiv$ Absolute scintillation efficiency
$k_B =$ Birks density parameter

→ Higher the ionization density, higher the quenching interactions, lower the light.

For small $\frac{dE}{dx}$, $\frac{dL}{dx} \simeq A \cdot \frac{dE}{dx}$; Large $\frac{dE}{dx}$, $\frac{dL}{dx} \simeq A/k_B$

Light Transfer to Photodetectors:

- Light Guides:
  transfer by total internal reflection
- Wavelength Shifter (WLS) bars
  Absorb and re-emit at longer wavelengths to match photodetector sensitivity.
- Fibers:
  - Clear fibers (Total internal reflection)
  - WLS fibers
  - Scintillating fibers (used in tracking)

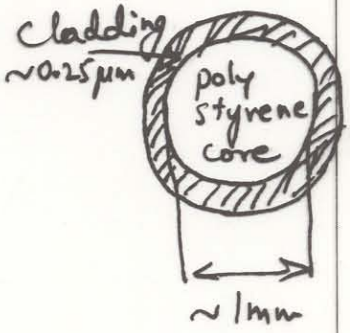
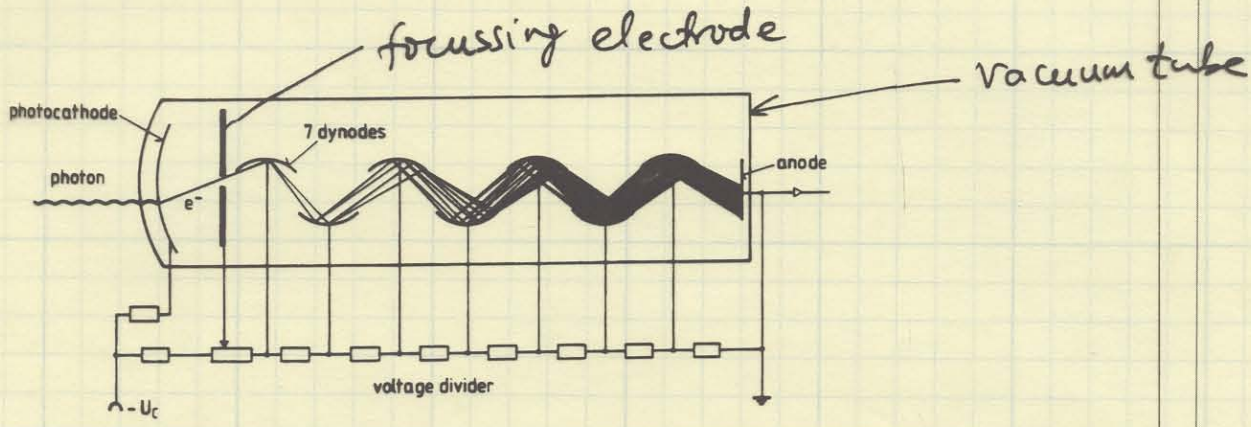


Photo Detectors

- Photo Multiplier Tube (PMT)



- Photons from the scintillator impinge on the photocathode, electrons are emitted via photoelectric effect. [photocathode coated with a low-work function material]

$$Q.E. = N_{p.e.} / N_{photons}$$ ~25% @ 400nm in Cs-K with Sb

- The emitted photoelectrons are accelerated and focused onto the first dynode of the tube. For every electron incident more secondary electrons are produced at the dynode (BeO, Mg-O-Cs)

dynode gain $g = 3-5$

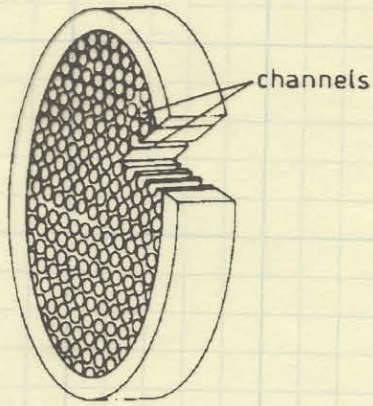
For $g = 4$, $N = 10$

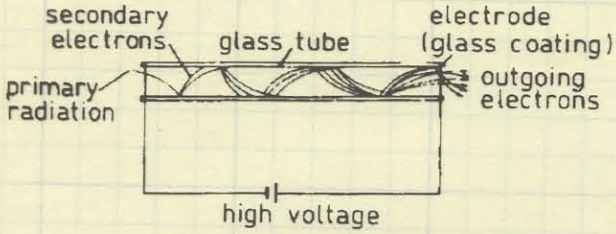
Total gain for N dynodes $\} M = \prod_{i=1}^{N} g_i$ $\quad M = 4^{10} \simeq 10^6$

- The amplified current is collected at the anode.

- PMs are sensitive to B fields, even earth's field (30–60 μT). Mu-metal shielding used.

Micro Channel Photomultipliers (Plates)





MicroChannel Plates contain a large number of small tubes / channels (dia 10-50 μm, length 5-10 mm)

- Glass tubes coated inside with a resistive layer (holes in lead-glass plate)

~ 1000 V applied across

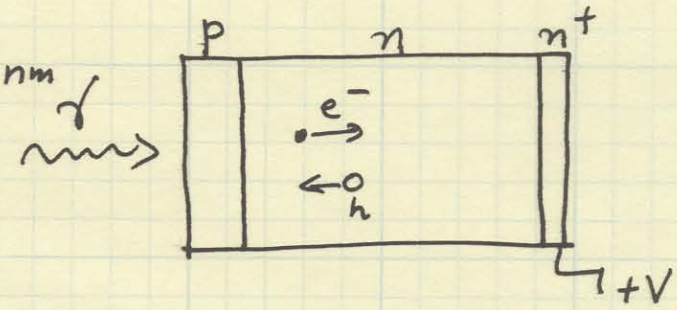
- Each channel acts as a continuous dynode (See figure). Amplifications ~ $10^5 - 10^6$ possible
- Multi-step micro-channel PMT's are possible
- Bent single-channel electron multipliers for space applications
- Fast signal; low path-length fluctuations, hence, smaller transit-time spread ≈ 50 ps as opposed to 200 ps for conventional PMT
- less sensitive to B-field

Photo Diodes

- High Q.E.

  ~80% @ $\lambda \sim 700$ nm

gain $G = 1$



Avalanche Photo Diodes (APD)

Add a high voltage (high electric field) region to effect avalanche multiplication.

High reverse bias voltage ~ 100–200V

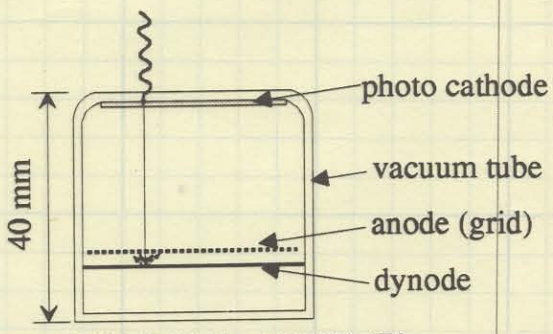
$G \sim 100$

thin depletion region
large field gradient

Photo Triodes (vacuum tube Triode)

Single stage PMT
not Silicon!

$G \sim 10$



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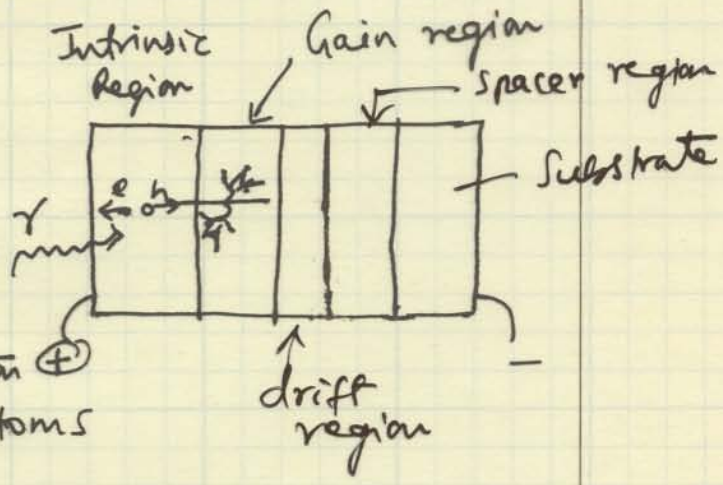
works in axial B-fields of 1 T

Read-out of lead-glass end-cap calorimeter in OPAL, DELPHI

Visible Light Photon Counter (VLPC)

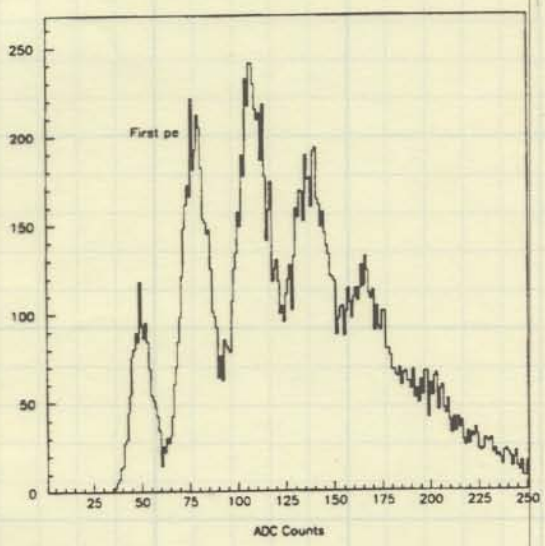
A variant of the solid-state photomultiplier
Si:As Impurity Band Conduction Avalanche Diode

Hole drifts towards highly doped region and ionizes a donor atom
→ free electron
multiplication by ionization of more neutral donor atoms



Q.E. ~ 70% ; G ~ 20,000 (DØ VLPC for the fiber tracker)
Rate capability ~ 10 MHz.

Bias voltage ~ 7V
Operating Temperature ~ 7-14 K



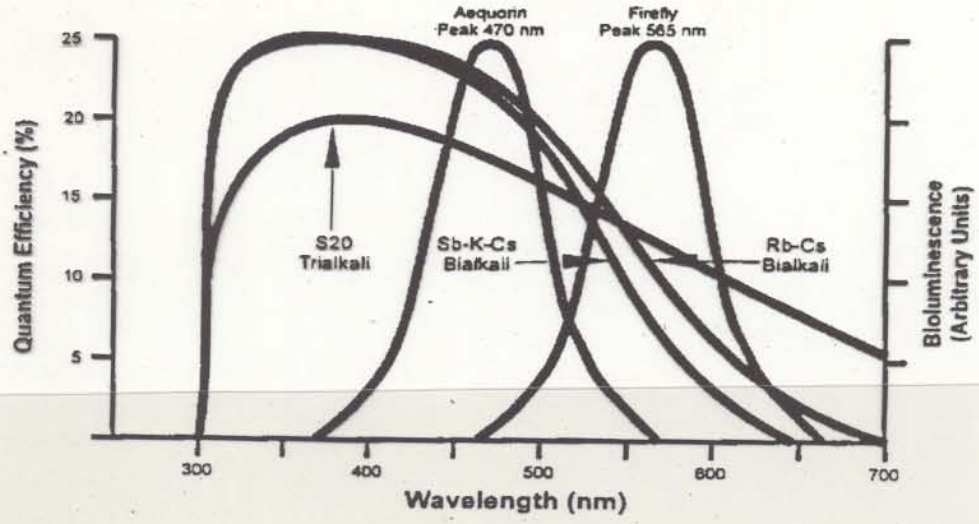
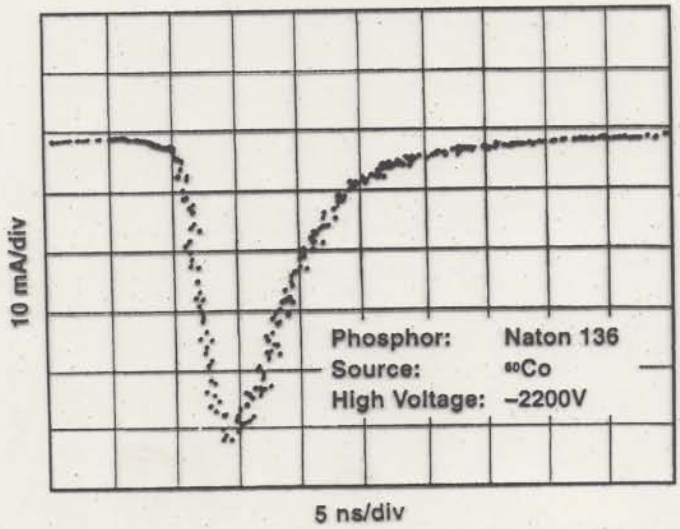


Fig. 2.9. Photocathode 'quantum efficiency' as a function of λ of the light impinging on a PMT. (From Ref. 2.10.)



Typical Anode Output with a Burle 8575 Photomultiplier Tube.

Fig. 2.13. Oscilloscope trace of a PMT current pulse output. Note the rise time of ~ 2 ns and the pulse width of ~ 10 ns full width at half maximum, FWHM. The peak current is ~ 50 mA and the signal pulse height spread is ~ 10%. (From Ref. 2.10.)

Scintillating Fiber Tracker

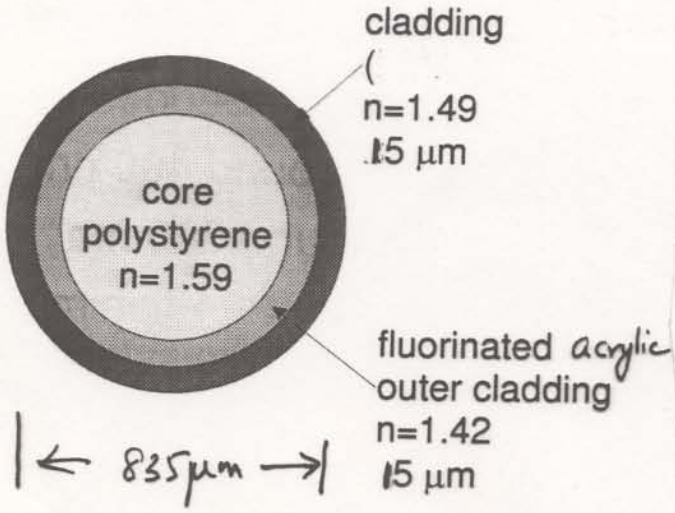
- Plastic fibers
- Capillary fibers filled with liquid scintillator

Advantages

- High Geometrical flexibility
- fine granularity
- low mass
- fast response (~ns)
  → first level trigger

DØ Fiber Tracker

Multi-clad fibers



The addition of second cladding increases light trapping by 70%.

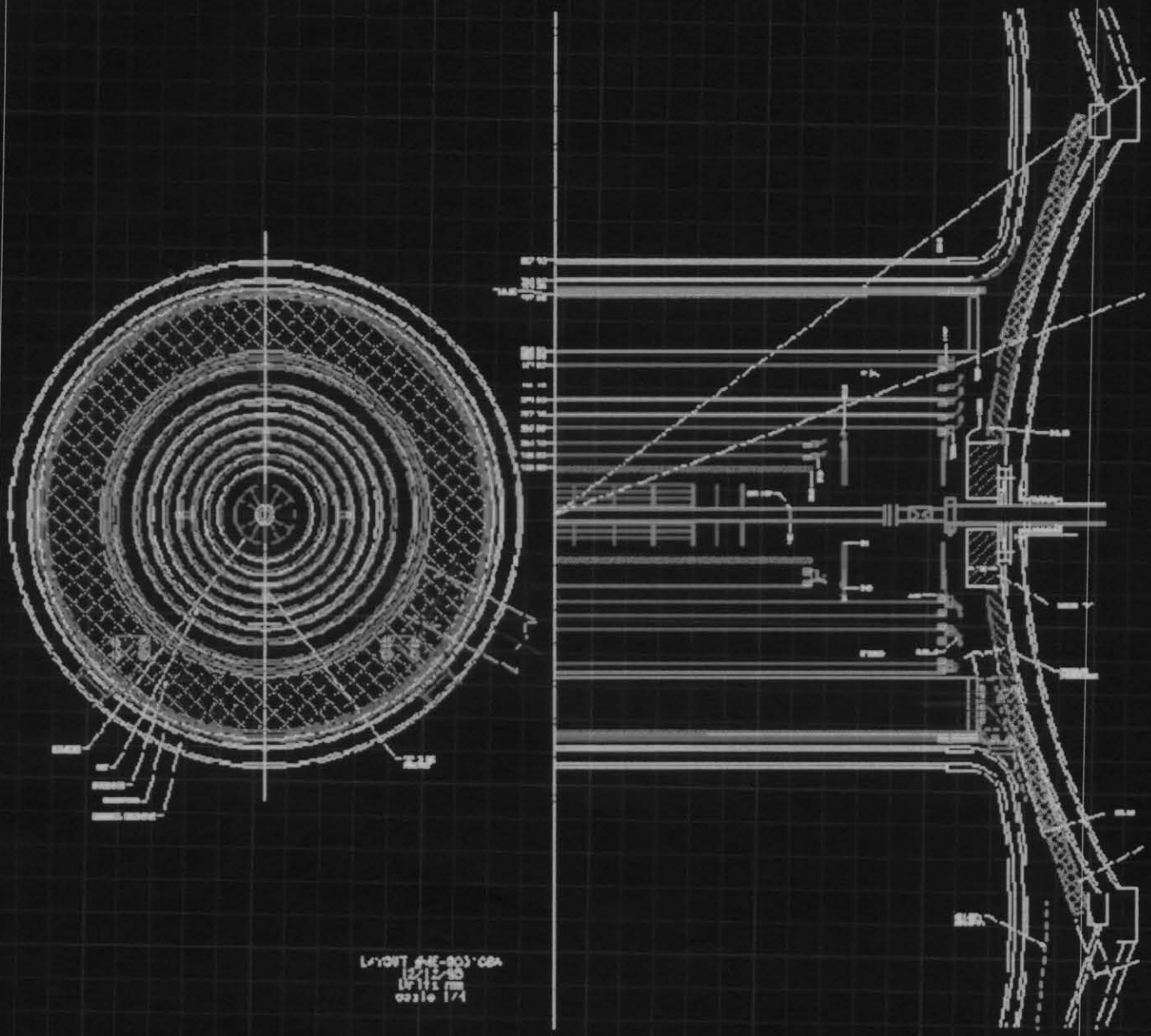
Also improves robustness of the fibers.

Core polystyrene doped with 1% p-terphenyl (PTP) and 1500 ppm 3-hydroxyflavone (3HF)

peak emission $\lambda \sim 530$ nm

The scintillating fibers are mated to 11 m long multiclad fiber wave guides which conduct light to VLPCs on the platform

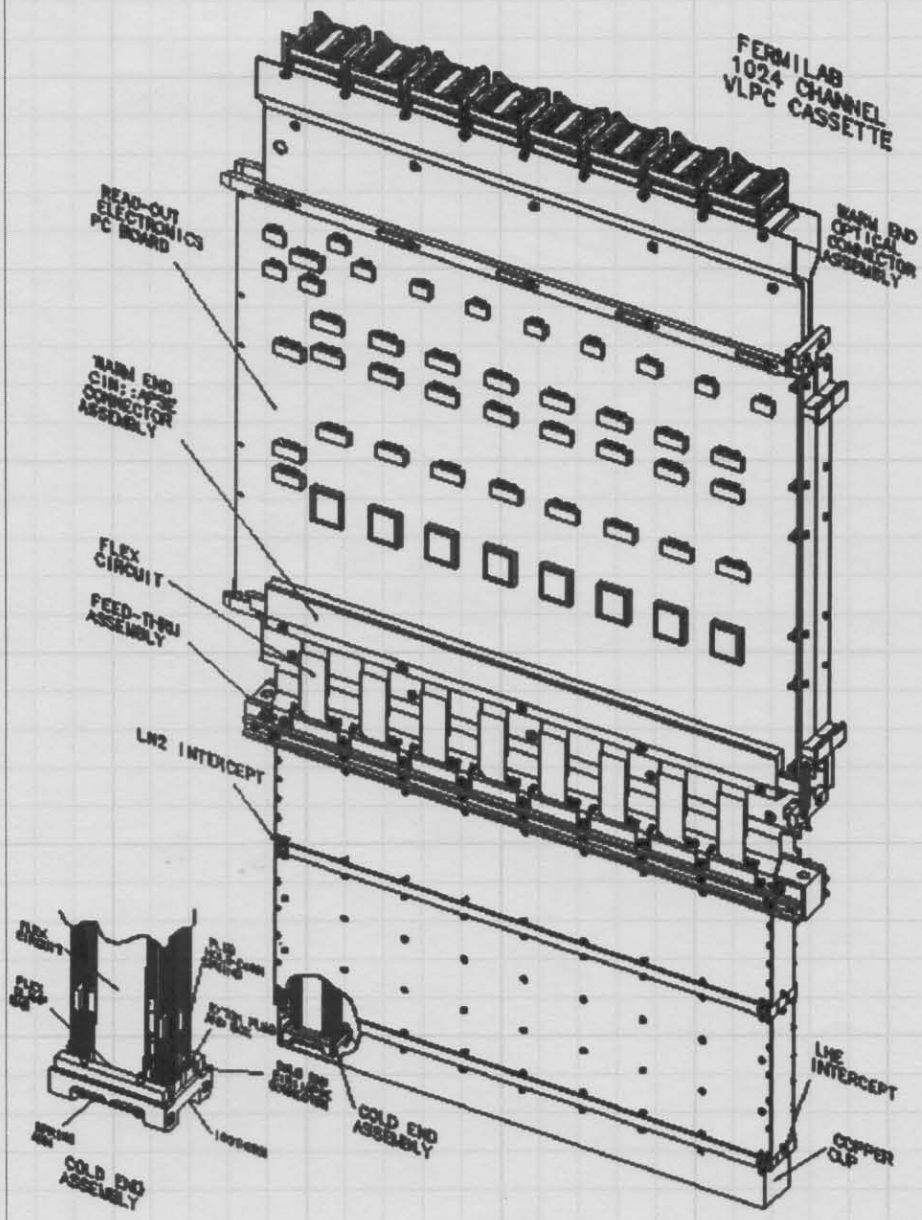
DØ Upgrade Tracker



CFT: Scintillating fibers mounted on 8 concentric cylinders.
Constructed in ribbons each 128 fibres wide
80 sectors in ϕ 71,680 channels

This drawing can be found at - http://d0server1.fnal.gov/projects/scifi/tracker.GIF

The following drawing shows the cassette assembly courtesy of Eric Haggard:

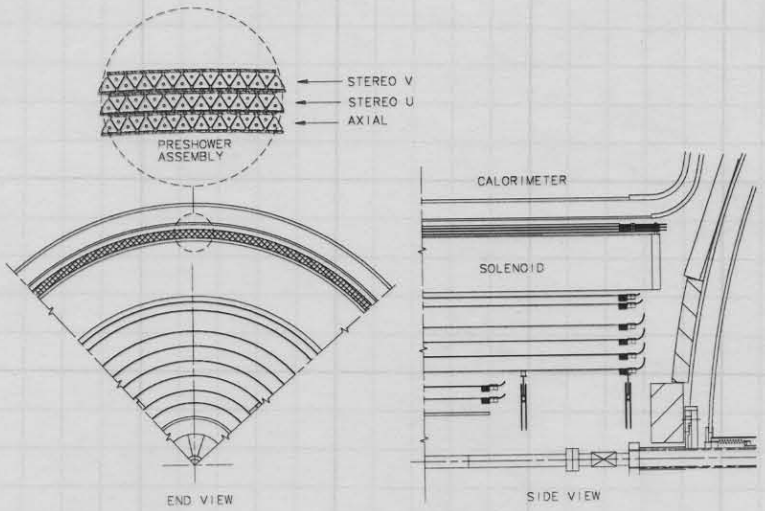


The drawing (gif) can be found at - http://d0server1.fnal.gov/projects/scifi/assembly.GIF

The following drawing shows the cold end of the cassette assembly courtesy of Eric Haggard:

Preshower

DØ



Measures
- energy and position

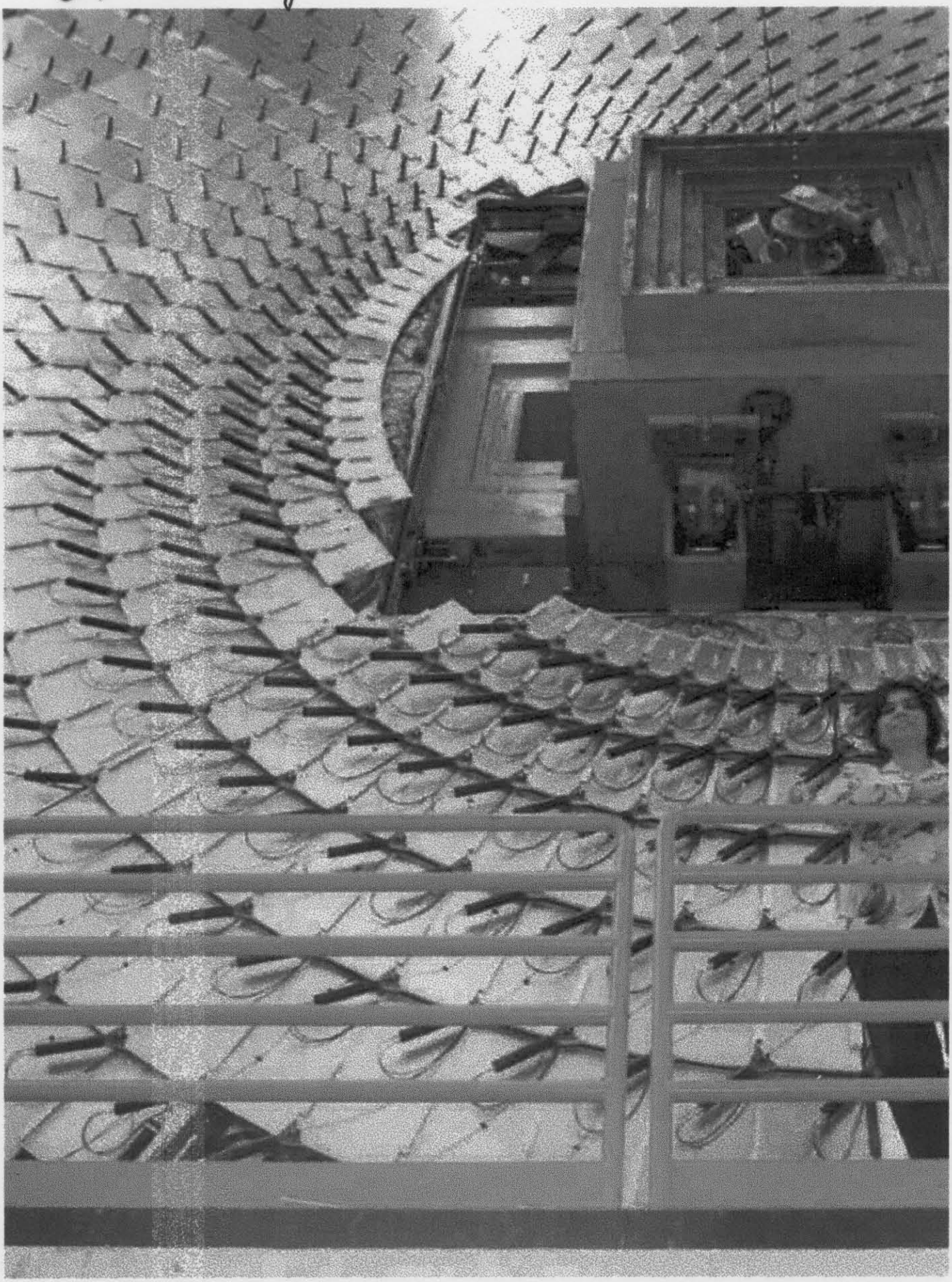
Electron triggering and ID
- γ/π separation

Covers $|\eta| < 2.5$

Level 1 trigger

Consists of
- Triangular scintillators
- WLS readout
- VLPC detection
- SVX-II readout

DØ Muon System: Forward Scintillation Pixels



Time of Flight Measurement

An important application of Scintillators is in measuring the time of flight of particles to use in particle identification.

For a traversed length L,

$$L = vt = \beta c t.$$

$$t = \frac{L}{c} \cdot \frac{1}{\beta}$$

$$\sim \frac{L}{c}\left(1 + \frac{1}{2\gamma^2}\right)$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\sim \left(1 - \frac{1}{2\gamma^2}\right)$$

as $\beta \to 1$.

$$\Delta t \sim \frac{L}{c}\left(\frac{m_1^2 - m_2^2}{2p^2}\right)$$

$$\beta \to 1$$

$$\gamma \sim p/m$$

$$\gamma = E/m \sim p/m$$

$\therefore$ For a given L, and p,

$$\Delta t \propto (m_1^2 - m_2^2)$$

A very useful technique for slow, non-relativistic particles

We will talk about scintillators used as calorimeters

Some examples:

Pure CsI — KTeV
← has a fast component, 6 ns
So, desirable in high-rate experiments

CsI(Tl) — Babar
When Tl is added, response is much slower, but, light yield increases significantly (needed to detect low energy signals).

$PbWO_4$ — dense, fast, radiation hard
Used in CMS

Mainly used as EM calorimeters

High Z is good for shower production
Bremsstrahlung for electrons ($\sigma \propto Z^2$)
photoelectric effect ($\sigma \propto Z^5$)
and pair production ($\sigma \propto Z^2$) for photons

Scintillation Sampling Calorimeters:
Use organic plastic scintillators in the form of fibres or plates

ZEUS, CDF