PHYSICS OF PARTICLE DETECTION

- We detect particles by exploiting some sort of interaction of the particles with the material in the detector

- Different physical processes are involved in the detection of charged and neutral particles.

- Charged particles passing through matter
  - lose energy
    - → primarily due to interactions with atomic electrons (dominant loss)
  - Scatter
    - → primarily due to interactions/scattering off atomic nuclei

  Other Processes:
  - Production of em radiation
    - → Čerenkov, Transition Radiation, Scintillation, bremsstrahlung (for electrons and high energy muons)

- Photons are detected/measured via photoelectric effect, Compton scattering or pair production
- Neutrons & neutrinos via strong & weak interactions respectively.

Single Scattering

Coulomb collision of a charged particle with charge "e", mass m, velocity βc by a nucleus with atomic number "Ze" at an impact parameter b.

→ scatters off at an angle θ

Coulomb force $\sim 1/r^2$ and active on time scales where the incident particle is near the scattering center.

$$F(b) = Ze \cdot e / b^2$$

$$\Delta t = 2b/v$$

Net momentum impulse,

$$\Delta p_T \sim F(b) \cdot \underbrace{\Delta t}_{\text{characteristic time}} = 2Ze^2/bv$$

Scattering angle, $\theta \sim \dfrac{\Delta p_T}{p} = \dfrac{2Ze^2}{pbv} \sim \dfrac{2Z\alpha}{pbv}$

$$\theta_R \sim \frac{Z\alpha}{b} \cdot \frac{1}{T} = \frac{U(b)}{T} \leftarrow \text{P.E.}/\text{K.E}$$

Scattering Cross Section

In a real experiment, we cannot control the impact parameter b. So, "b" corresponding to all areas transverse to the incident beam are equally probable.

Scattering Probability,

$$dP \sim db = d\sigma$$

transverse area element

$$d\sigma \sim b \cdot db \cdot d\phi = \frac{d\sigma}{d\Omega} \cdot d\Omega$$

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left(\frac{db}{d\theta}\right)$$

Using $\theta \sim \frac{2Z\alpha}{pvb}$,

Power law $\Rightarrow \sim 1/\theta^4$ characteristic of scattering of point particles

$$\frac{d\sigma}{d\Omega} \sim \left(\frac{Z\alpha}{Mv^2}\right)^2 / \theta^4$$

Minimum ~~Multiple~~ Scattering Angle

Extent of the electron cloud field $\sim \mathring{A}$
$\sim 10^{-8}$ cm

(Nucleus $\sim$ fm $= 10^{-13}$ cm)

If the incident particle is outside the ~ Bohr radius, the charge/field of the nucleus is screened by the electrons.

$\Rightarrow$ No effect of the nuclear charge can be seen.

$\Rightarrow$ A minimum scattering angle

$$\theta_{min} \sim \frac{2Z\alpha}{pv\,a_0}$$

Since there is a θ_{min}, even though we have $1/\theta^4$ behaviour, the integration won't diverge.

Multiple Coulomb Scattering

Total cross section

$$\sigma = \int_{\theta_{min}}^{\infty} \frac{d\sigma}{d\Omega} 2\pi\theta \cdot d\theta \sim \frac{1}{\theta^4} = \int_0^{a_0} 2\pi b\, db$$

$$= \pi a_0^2 \sim \frac{1}{\theta_{min}^2}$$

Mean scattering angle is the angle weighted by the angular distribution

$$\langle \theta^2 \rangle = \frac{\int \theta^2 \frac{d\sigma}{d\Omega} \cdot d\Omega}{\int \frac{d\sigma}{d\Omega} \cdot d\Omega} = \frac{\int d\theta/\theta}{\int d\theta/\theta^3}$$

$$\sim 2\,\theta_{min}^2 \left[\ln \frac{\theta_{max}}{\theta_{min}} \right]$$

When the particles traverse the medium, let us say there are N scattering along the way, on average

$$\langle \theta_{MS}^2 \rangle = N \langle \theta^2 \rangle$$

$$N = \frac{N_0 \rho \sigma}{A} \cdot dx = \frac{dx}{\langle L \rangle}$$

Mean free path between scattering

$$\langle \theta_{MS}^2 \rangle = \frac{N_0 \rho \, dx}{A} \cdot 2\pi \left[\frac{2 Z \alpha}{p \beta c}\right]^2 \ln(\;)$$

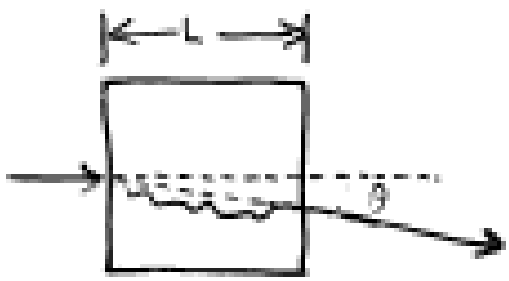
$$\theta = \sqrt{\langle \theta_{MS}^2 \rangle} = \frac{E_S}{p\beta} \sqrt{\frac{dx}{X_0}}$$

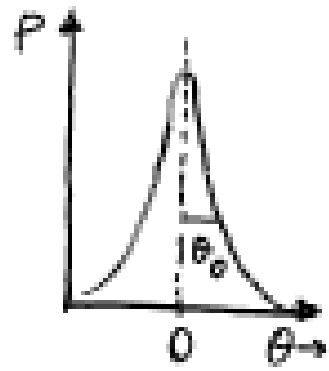
$$E_S = \sqrt{\frac{4\pi}{\alpha}} \, (mc^2) = 21 \text{ MeV}$$

A 20 GeV pion, for example, in one radiation length, suffers a mean scattering by angle $\theta \sim 0.001$ rad or 1 mrad

Multiple Coulomb Scattering

- A charged particle traversing a medium can suffer many small angle scatters in the Coulomb potentials of nuclei





$$P(\theta) = \frac{1}{\sqrt{2\pi}\,\theta_0} \exp\left(-\frac{\theta^2}{2\theta_0^2}\right)$$

$$\theta_0 = \sqrt{\langle\theta^2\rangle} = \frac{13.6\ \text{MeV}}{\beta c p}\, z \sqrt{\frac{L}{X_0}}\left[1 + 0.038 \ln\frac{L}{X_0}\right]$$

p: momentum in MeV/c, βc: velocity, z: charge of the scattered particle, X_0 = Radiation length

$$\boxed{\theta_0 \simeq \frac{13.6}{\beta c p}\sqrt{\frac{L}{X_0}}}$$

Energy Loss by Ionization

consider a heavy charged particle with charge ze and velocity v passing through some material.

The momentum transfered to an electron in the material, at a distance r and impact parameter b is,

$$\Delta p = \int_0^{\infty} F_\perp \, dt = \int_{-\infty}^{+\infty} \frac{ze \cdot e}{r^2} \cdot \frac{b}{r} \cdot \frac{dx}{v}$$

$$= \frac{ze^2}{v} \int_{-\infty}^{+\infty} \frac{b \cdot dx}{\left(\sqrt{b^2+x^2}\right)^3} = \frac{ze^2}{v \cdot b} \underbrace{\int_{-\infty}^{+\infty} \frac{d(x/b)}{\left(\sqrt{1+(x/b)^2}\right)^3}}_{=2}$$

$$= \frac{2ze^2}{b \cdot v} \quad \text{--- (1)}$$

$\therefore$ The energy transfer is,

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{\left(2ze^2/bv\right)^2}{2m} = \frac{2z^2e^4}{mb^2v^2} \quad \text{--- (2)}$$

If we consider a cylindrical shell between b and b+db and in thickness dx, the energy transfered to electrons becomes,

$$-\Delta E(b) = \frac{2Z^2e^4}{mb^2v^2} \cdot n_e \cdot 2\pi b \cdot db \cdot dx$$

$$-\frac{dE}{dx} = \frac{4\pi n_e Z^2 e^4}{mv^2} \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{4\pi Z^2 e^4}{mv^2} \ln \frac{b_{max}}{b_{min}} \quad -(3)$$

Use physical arguments to estimate b_{min}, b_{max}

The maximum transferable energy in a head-on collision is $(2\gamma m_e v)^2/2m_e = 2\gamma^2 m v^2$

$$2\gamma^2 m v^2 = \frac{2Z^2e^4}{m b_{min}^2 v^2} \quad \text{using (2)} \quad \therefore b_{min} = \frac{Ze^2}{\gamma m v^2}$$

The interaction must take place in a time short compared to the orbital period of the atomic electrons

$$\tau_{int} = \frac{b}{\gamma v} = \frac{1}{\nu} \quad \therefore b_{max} = \frac{\gamma v}{\nu}$$

$$\boxed{\frac{dE}{dx} = \frac{4\pi n_e z^2 e^4}{m v^2} \ln \frac{\gamma^2 m v^3}{z e^2 \nu}}$$

Bohr's Classical Formula

A semiclassical approach for b_{min}, b_{max}:

Assume min. impact parameter to be half the de Broglie wavelength of the electron

$$b_{min} = \frac{h}{2p} = \frac{h}{2\gamma m v}$$

As before,

$$b_{max} = \frac{\gamma v}{\nu} = \frac{h \cdot \gamma v}{h\nu} = \frac{h \cdot \gamma v}{I}$$

I = Ionization potential

$$\boxed{\frac{dE}{dx} = \frac{4\pi n_e z^2 e^4}{m v^2} \ln \frac{2\gamma^2 m v^2}{I}}$$

Quantum Treatment:

A proper treatment of energy loss must take into account (1) that energy transfers to atomic electrons occur in discrete amount and (2) the wave nature of particles. Such a treatment was done by Bethe & Bloch in early 1930's.

$$-\frac{dE}{dx} = n_e \int_0^\infty W \frac{d\sigma}{dW}(E, W)\, dW$$

$\frac{d\sigma}{dW}(E, W)$ = cross section for an incident particle with energy E to lose an amount of energy W in the collision with an electron. The transition probabilities for excitation or ionization of an electron are calculated using first order perturbation theory. The incident particle is treated as a plane wave.

$$-\frac{dE}{dx} = \frac{4\pi n_e z^2 e^4}{m v^2}\left[\frac{1}{2}\ln \frac{2 m v^2 \gamma^2 E_{kin}^{max}}{I^2} - \beta^2\right]$$

Bethe - Bloch Formula

$$-\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \cdot \frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\gamma^2\beta^2 E_{kin}^{max}}{I^2} - \beta^2 - \frac{\delta}{2} - \frac{C}{Z}\right]$$

Correction needed at very low energies

N_A = Avogadro Number

r_e = Bohr Radius $= e^2/m_e c^2$

z = charge of the incoming particle

Z = Atomic number of the material

A = Mass number "

δ = Density Effect Parameter

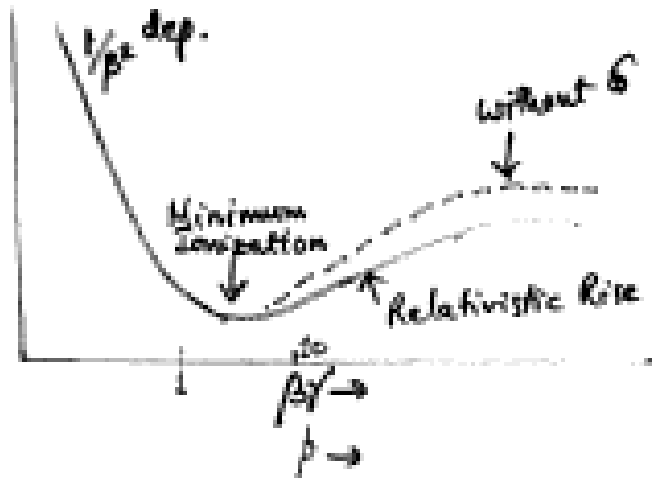
Also, used $n_e = Z \cdot \frac{N_A}{A} \cdot \rho$

and converted dE/dx to energy / gm · cm^{-2}

$$K = 4\pi N_A r_e^2 m_e c^2 = .307 \text{ MeV}/\text{g}\cdot\text{cm}^{-2}$$

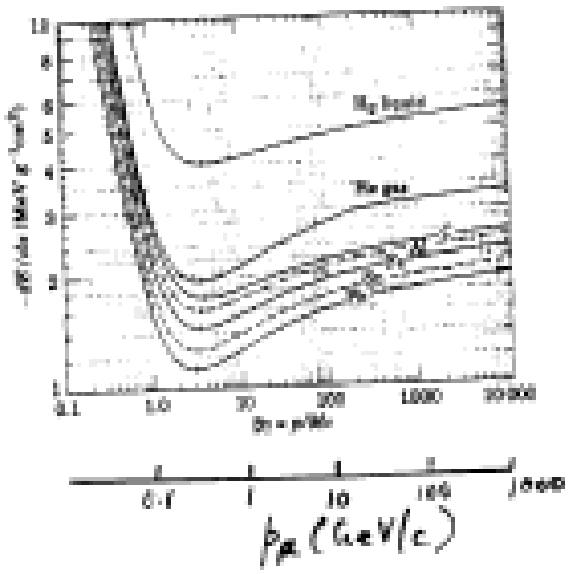
$$-\frac{dE}{dx} = K \; z^2 \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2} \cdot \left[\frac{1}{2} \ln \frac{2 m_e c^2 \gamma^2 \beta^2 E_{kin}^{max}}{I^2} - \beta^2 - \frac{\delta}{2} \right]$$

- $\frac{dE}{dx}$ first falls as $\frac{1}{\beta^2}$
- Reaches minimum at $\beta\gamma \simeq 4$ (MIP)
- Then rises due to $\ln \gamma^2$ term (contribution by more distant collisions)



- Relativistic rise cancelled by "density effect", screening of more distant atoms

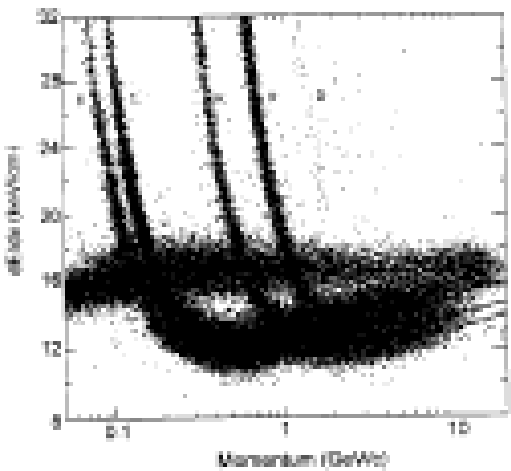
  → Fermi Plateau



$\frac{1}{\beta^2}$ dependence

$$\left.\frac{dE}{dx}\right|_{min}^{Fe} = 1.48\ \mathrm{MeV/g\,cm^{-2}}$$

$$= 11.55\ \mathrm{MeV/cm}$$



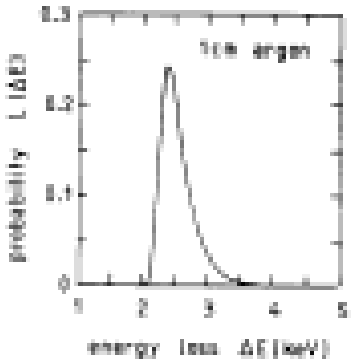
Measured dE/dx in the PEP4 TPC

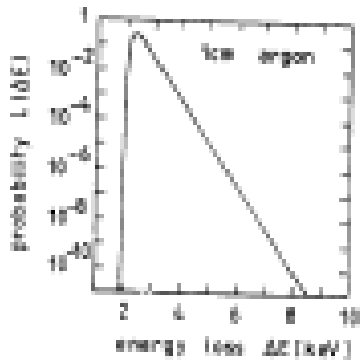
Mass dependence of dE/dx

$\Rightarrow$ Particle ID

$$\frac{dE}{dx} \propto \frac{1}{\beta^2} \propto \frac{M^2}{p^2}$$

The Bethe-Bloch formula gives us the mean/average energy loss. In thin absorbers, strong fluctuations around the average energy loss exist.





For depth L

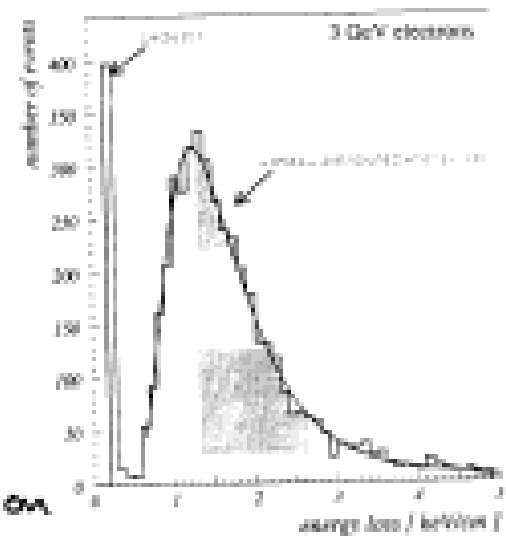
$$\langle \Delta E \rangle = \left\langle \frac{dE}{dx} \right\rangle L$$

↑ Most probable energy loss

$$P(\lambda) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\lambda + e^{-\lambda})}$$ (Landau distribution)

$$\lambda = \frac{\Delta E - \langle \Delta E \rangle}{\langle \Delta E \rangle}$$

deviation from the m. p. energy loss

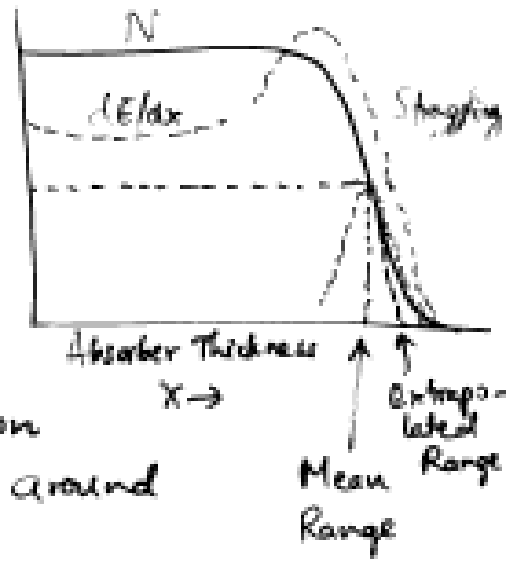


For thick absorbers, the energy loss distribution → Gaussian

Range & Straggling

Energy loss is statistical in nature.

Two identical particles with the same initial energy will have different number of collisions and hence different energy loss.



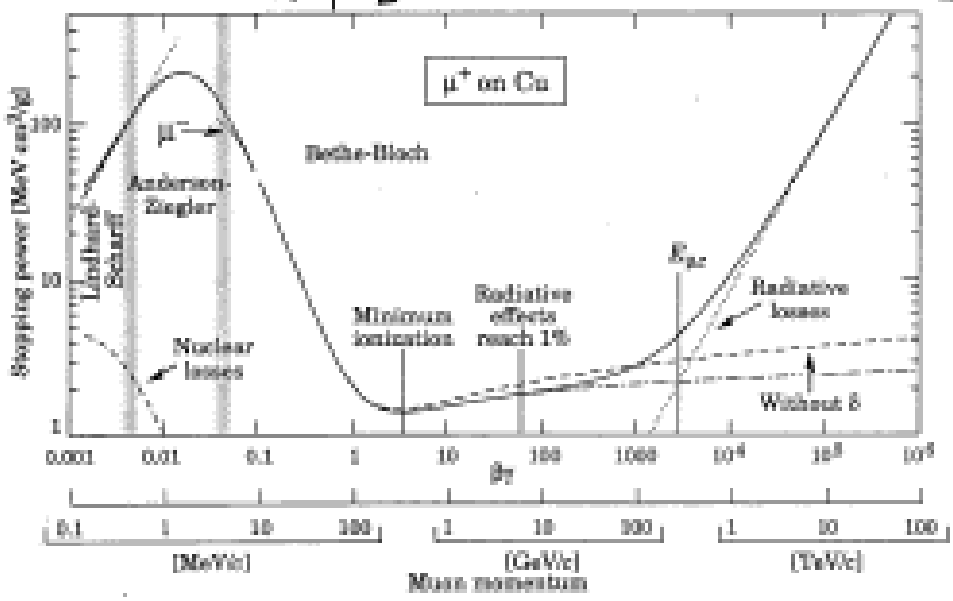
⇒ statistical distribution of Ranges centred around a "mean range"

→ Range – Straggling

$$R(E) = \int_{\bar{E}}^{0} \frac{1}{-dE/dx} \cdot dE$$

Review of Energy Loss / Stopping Power

$$-\frac{dE}{dx} = \kappa \cdot z^2 \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2 \gamma^2 \beta^2 E_{kin}^{max}}{I^2} - \beta^2 - \frac{\delta}{2} - \frac{C}{Z}\right]$$



- Bethe-Bloch equation is valid for $\beta \gg \alpha Z$
- At low energies, $\frac{dE}{dx} \propto \frac{1}{\beta^2}$, reaches a broad minimum at $\beta\gamma \approx 4$. (Minimum Ionizing Particles)
- In light absorbers $-\left.\frac{dE}{dx}\right|_{min} = 1.5 - 2\ \mathrm{MeV/g\,cm^{-2}}$
- Relativistic rise for $\gamma > 4$ ($\sim \ln\gamma$).
- Density effect reduces the relativistic rise and eventually saturates
- ϵ dependent corrections at low β; Radiative losses at high E.

Energy Loss of Electrons

Recall the Bethe-Bloch formula for heavy charged particles:

$$-\frac{dE}{dx} = K \cdot z^2 \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \gamma^2 \beta^2 \cdot E_{kin}^{max}}{I^2} - \beta^2 - \cancel{\frac{\delta}{2} - \frac{C}{Z}} \right]$$

ignore for now

For relativistic particles, $\beta \to 1$
with $E_{kin}^{max} = 2 m_e c^2 \gamma^2 \beta^2$,

$$-\frac{dE}{dx} \approx K \cdot \frac{Z}{A} \cdot \left[\ln \frac{2 m_e c^2}{I} + 2 \ln \gamma - 1 \right]$$

When the incident particle is an electron, one cannot distinguish between the primary and the secondary electron after the collision.

$$E_{kin}^{max} = \frac{1}{2} (E - m_e c^2) \simeq \frac{1}{2} E \quad \text{for } E \gg m_e$$

$$-\frac{dE}{dx} = K \cdot \frac{Z}{A} \cdot \left[\ln \frac{2 m_e c^2}{I} + \frac{3}{2} \ln \gamma - \frac{1}{2} \ln 4 \right]$$

Bremsstrahlung

A charged particle accelerated or decelerated in the Coulomb field of a nucleus can emit a fraction of its energy as real photons.

The energy loss due to radiation can be calculated as:

$$-\left(\frac{dE}{dx}\right)_{rad} = N \int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E,\nu)\, d\nu$$

$$\boxed{-\left(\frac{dE}{dx}\right)_{rad} = 4\alpha N_A r_e^2 \cdot z^2 \cdot \frac{Z^2}{A} \cdot E \cdot \ln \frac{183}{Z^{1/3}}}$$

$\alpha = e^2/\hbar c$; $r_e^2 = e^2/mc^2$; m = mass of the incident particle

To take into account interactions with electrons,

$Z^2 \rightarrow Z^2 + Z = Z(Z+1)$

For electrons, $z=1$, $m = m_e$, we write

$$-\frac{dE}{dx} = \frac{E}{X_0}$$ $\Leftarrow$ This defines "Radiation Leng

∴ The Radiation Length of a material

$$X_0 = \frac{A}{4\alpha N_A Z(Z+1) r_e^2 \cdot \ln\left(183 Z^{-1/3}\right)} \quad g/cm^2$$

Note: $X_0 \propto Z^{-2}$; $X_0 \propto m^2$

For a mixture or a compound

$$X_0 = \frac{1}{\sum_{i=1}^{N} f_i / X_0^i}$$; f_i = fraction of the ith component by weight

X_0^i = Radiation length of the ith component.

$$-\frac{dE}{dx} = \frac{E}{X_0} \Rightarrow E = E_0 e^{-x/X_0} \quad (X_0(Fe) = 1.76cm)$$

Exponential attenuation of energy by radiation

Compare with ionization loss:

$$\left.\frac{dE}{dx}\right|_{ion} \propto Z \quad ; \quad \left.\frac{dE}{dx}\right|_{rad} \propto Z^2$$

$$\left.\frac{dE}{dx}\right|_{ion} \propto \ln E \quad ; \quad \left.\frac{dE}{dx}\right|_{rad} \propto E$$

$(\gamma > 4)$

Direct Pair Production

At high energies, e^+e^- pairs can be produced by virtual photons in the Coulomb field of the nuclei.

e.g., $\mu + \text{nucleus} \rightarrow \mu + e^+ + e^- + \text{nucleus}$

$$-\frac{dE}{dx}\Big|_{pair} = b(Z, A, E) \cdot E$$

b varies only slightly with E

$$\therefore \; -\frac{dE}{dx}\Big|_{pair} \propto E$$

For 100 GeV μ, in Fe

$$-\frac{dE}{dx}\Big|_{pair} = 0.3 \; \frac{\text{MeV}}{\text{g/cm}^2} \qquad \left(= 2.36 \text{ MeV/cm} \right.$$

Energy Loss by Photonuclear Interactions

Charged particles can interact inelastically via virtual photons with nuclei and lose energy

$$-\frac{dE}{dx}\Big|_{photonucl.} = b_{nucl}(Z, A, E) \cdot E$$

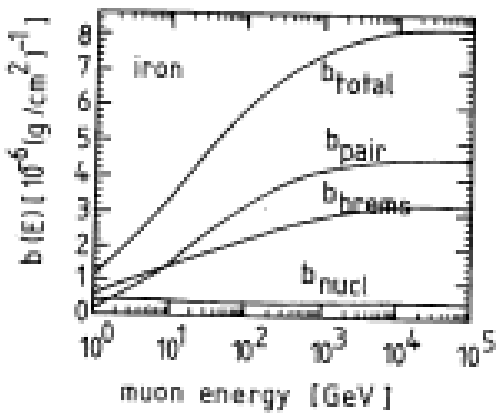
For 100 GeV muons in Fe,

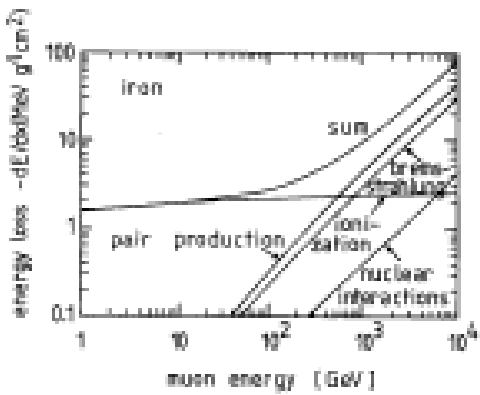
$$-\frac{dE}{dx}\Big|_{photonucl} = 0.04 \frac{MeV}{g/cm^2}$$

Total Energy loss

$$-\frac{dE}{dx}\Big|_{total} = -\frac{dE}{dx}\Big|_{ion} - \frac{dE}{dx}\Big|_{brem} - \frac{dE}{dx}\Big|_{pair} - \frac{dE}{dx}\Big|_{photonucl.}$$

$$= a(Z, A, E) + b(Z, A, E) \cdot E$$


iron
b_{total}
b_{pair}
b_{brems}
b_{nucl}
$b(E)$ [10^{-6} (g/cm^2)$^{-1}$]
muon energy [GeV]


iron
sum
bremsstrahlung
pair production
ionisation
nuclear interactions
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muon energy [GeV]