

PHENOMENOLOGICAL STUDIES ON SUPERSYMMETRY AND THE STRONG FORCE

Peter Zeiler Skands

October 8, 2004

Lund



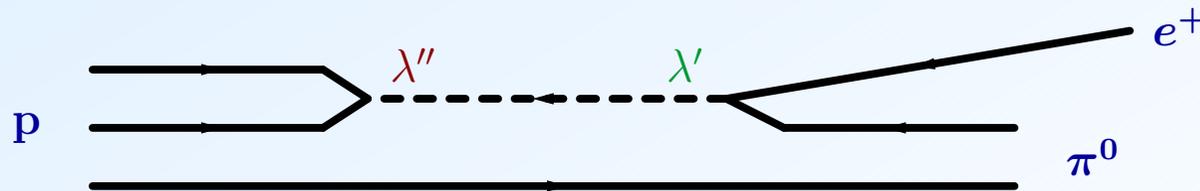
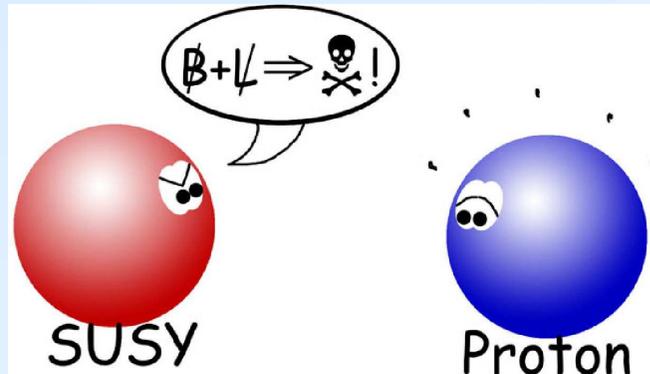
Paper I

Lepton Number Violating Supersymmetry – Decays and First Studies with PYTHIA.

- I “Searching for L-Violating Supersymmetry at the LHC”.
By P. Skands.
LU TP 01-32, Oct 2001.
Published in European Physical Journal C23 (2002) 173.

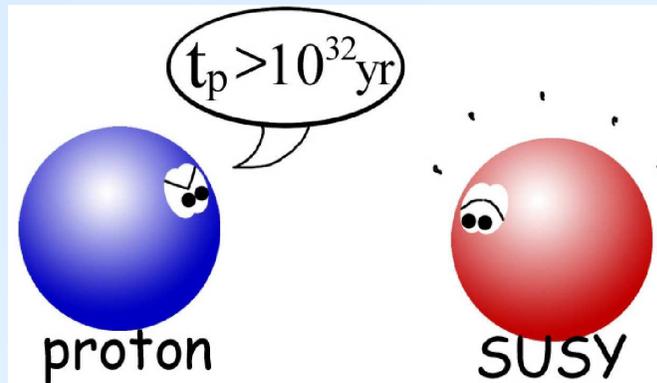
Lepton Number Violating Supersymmetry

- ☛ General MSSM contains renormalizable **Lepton** and **Baryon** Number violating operators.

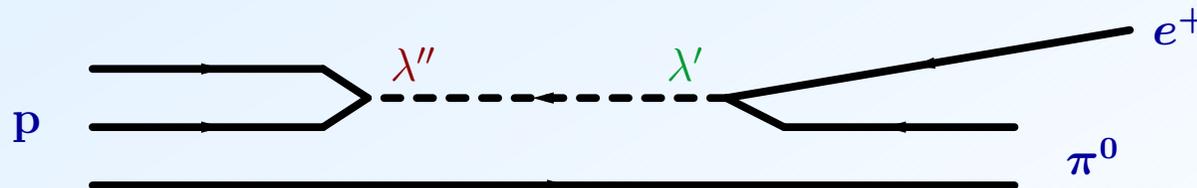


Lepton Number Violating Supersymmetry

- ☛ General MSSM contains renormalizable **Lepton** and **Baryon** Number violating operators.



- ☛ Proton lifetime bound \rightarrow *either* **LVN** or **BNV**, not both!
(could also be *neither* \equiv **R-parity conservation**.)

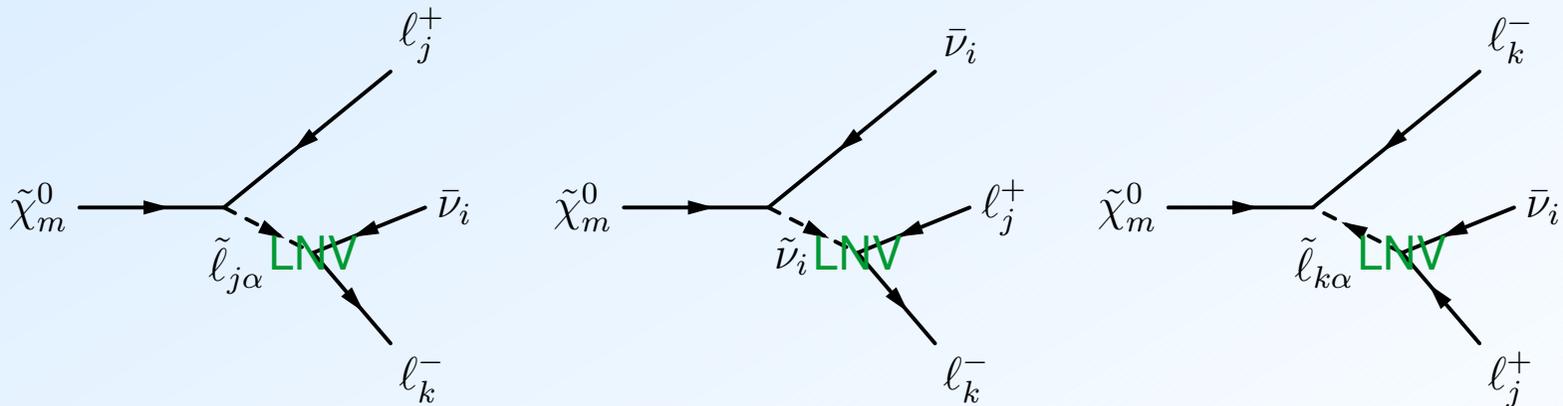


Lepton Number Violating Supersymmetry

- We have considered ~ 1200 decay processes ($1 \rightarrow 2$ and $1 \rightarrow 3$) of sparticles to particles, induced at tree level by the **trilinear LNV** terms in the superpotential:

$$W_{\text{LNV}} = \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \epsilon_{ij} U_i H_2$$

Example: fully leptonic $\tilde{\chi}^0$ decay:



Lepton Number Violating Supersymmetry

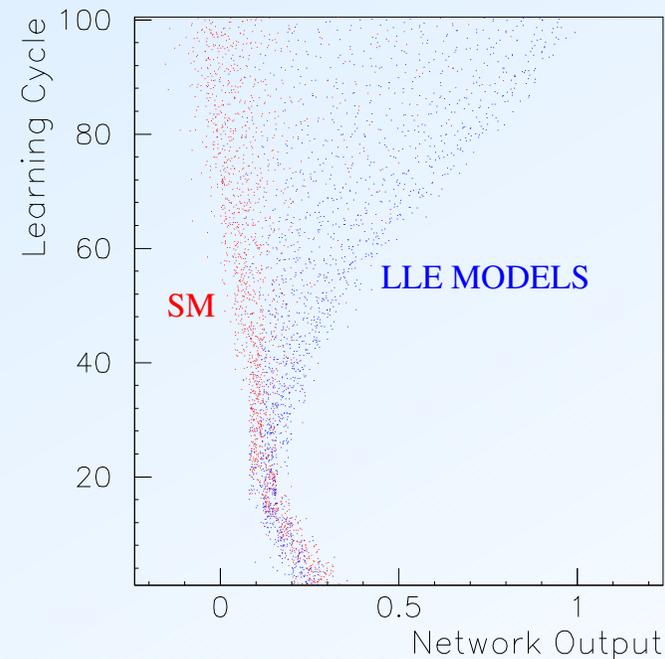
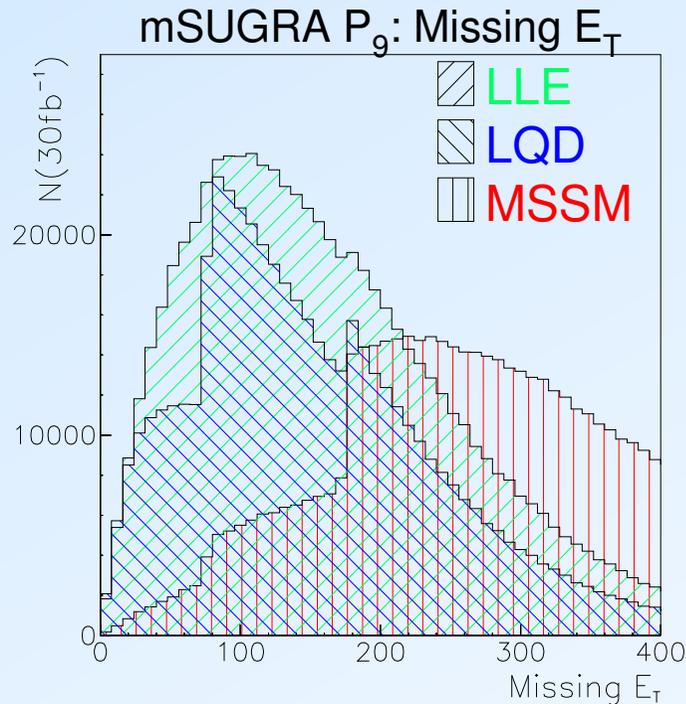
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$$W_{\text{LNV}} = \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \epsilon_{ij} H_1 H_2$$

- ☁ Matrix Elements implemented in PYTHIA for partial width calculations (only SUSY particles and t and b quarks treated as massive).
- ☁ Initial decay products distributed isotropically in phase space, \oplus subjected to QCD and QED bremsstrahlung and (string) hadronization.

Lepton Number Violating Supersymmetry

- ☛ Second part: primitive LHC study, based on cuts and neural networks, focussing on experimental triggers and overall discovery potential for LNV-SUSY at LHC.



- ☛ → Discovery with 30 fb^{-1} data down to $\sigma = 10^{-10} \text{ mb}$.

Paper II

Baryon Number Violating Supersymmetry – Hadronization and String Topologies.

II “Baryon Number Violation and String Topologies”.

By T. Sjöstrand and P. Skands.

LU TP 02-46, Dec 2002.

Published in Nuclear Physics B 659 (2003) 243.

Baryon Number Violating Supersymmetry

- ☁ This time, ~ 200 decay channels \rightarrow PYTHIA.
- ☁ Again, sometimes they had bothersome expressions...

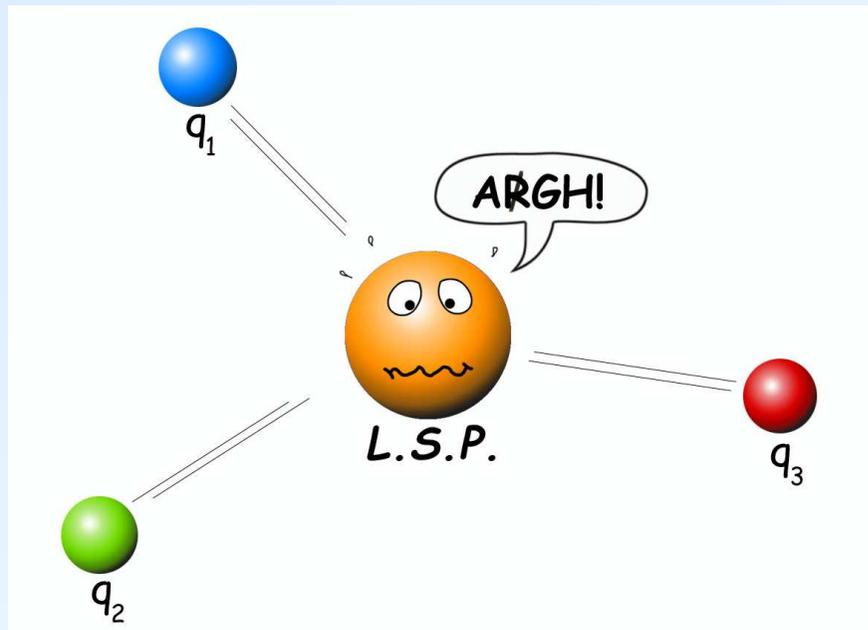
$$\begin{aligned}
\frac{\Gamma(\tilde{\chi}^0 \rightarrow \bar{u}_i \bar{d}_j \bar{d}_k)}{|\overline{M}(\tilde{\chi}^0 \rightarrow \bar{u}_i \bar{d}_j \bar{d}_k)|^2} &= \frac{1}{(2\pi)^3} \frac{1}{32M_{\tilde{\chi}^0}^3} \int dm_{12}^2 \int dm_{23}^2 |\overline{M}(\tilde{\chi}^0 \rightarrow \bar{u}_i \bar{d}_j \bar{d}_k)|^2 \\
&= \frac{1}{N_c! \lambda_{ijk}''^2} \\
&\sum_{\alpha=1}^2 |Q_{\alpha R}^{2i-1}|^2 R(\tilde{u}_{i\alpha}^*, m_{jk}^2)(m_{jk}^2 - m_j^2 - m_k^2) \left((a^2(\tilde{u}_{i\alpha}^*) + b^2(\tilde{u}_{i\alpha}^*))(m_{\tilde{\chi}^0}^2 + m_i^2 - m_{jk}^2) + 4a(\tilde{u}_{i\alpha}^*)b(\tilde{u}_{i\alpha}^*)m_i m_{\tilde{\chi}^0} \right) \\
&+ \sum_{\alpha=1}^2 |Q_{\alpha R}^j|^2 R(\tilde{u}_{j\alpha}^*, m_{ik}^2)(m_{ik}^2 - m_i^2 - m_k^2) \left((a^2(\tilde{u}_{j\alpha}^*) + b^2(\tilde{u}_{j\alpha}^*))(m_{\tilde{\chi}^0}^2 + m_j^2 - m_{ik}^2) + 4a(\tilde{u}_{j\alpha}^*)b(\tilde{u}_{j\alpha}^*)m_j m_{\tilde{\chi}^0} \right) \\
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&+ 2Q_{1R}^i Q_{2R}^i S(\tilde{u}_{i1}^*, \tilde{u}_{i2}^*, m_{jk}^2, m_{jk}^2)(m_{jk}^2 - m_j^2 - m_k^2) \left((a(\tilde{u}_{i1}^*)a(\tilde{u}_{i2}^*) + b(\tilde{u}_{i1}^*)b(\tilde{u}_{i2}^*))(m_{\tilde{\chi}^0}^2 + m_i^2 - m_{jk}^2) \right. \\
&\quad \left. + 2(a(\tilde{u}_{i1}^*)b(\tilde{u}_{i2}^*) + a(\tilde{u}_{i2}^*)b(\tilde{u}_{i1}^*))m_i m_{\tilde{\chi}^0} \right) \\
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\end{aligned}$$

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- ☁ But all that worked like before.

Baryon Number Violating Supersymmetry

- ☞ This time, ~ 200 decay channels \rightarrow PYTHIA.
- ☞ Again, sometimes they had bothersome expressions...
- ☞ But all that worked like before.
- ☞ The real challenge was the colour flows!



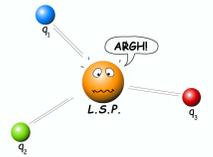
- ☞ How do such systems hadronise?

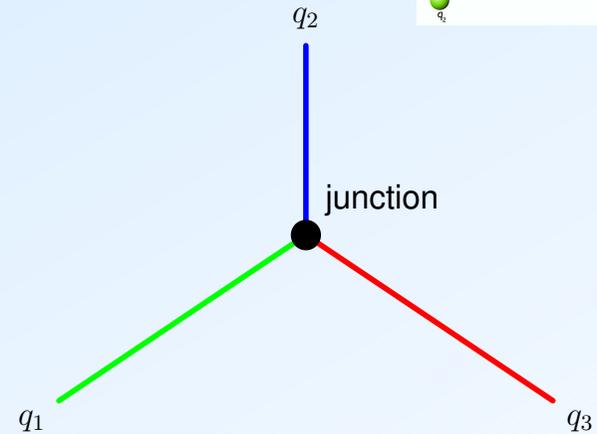
Baryon Number Violating SuSy

New: 3 (antisym) colour carriers at large momentum separation – no corresponding (perturbative) coupling in SM!

“Ordinary” string (e.g. $Z^0 \rightarrow q\bar{q}$):



“Baryonic” string (e.g. ):

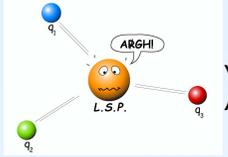


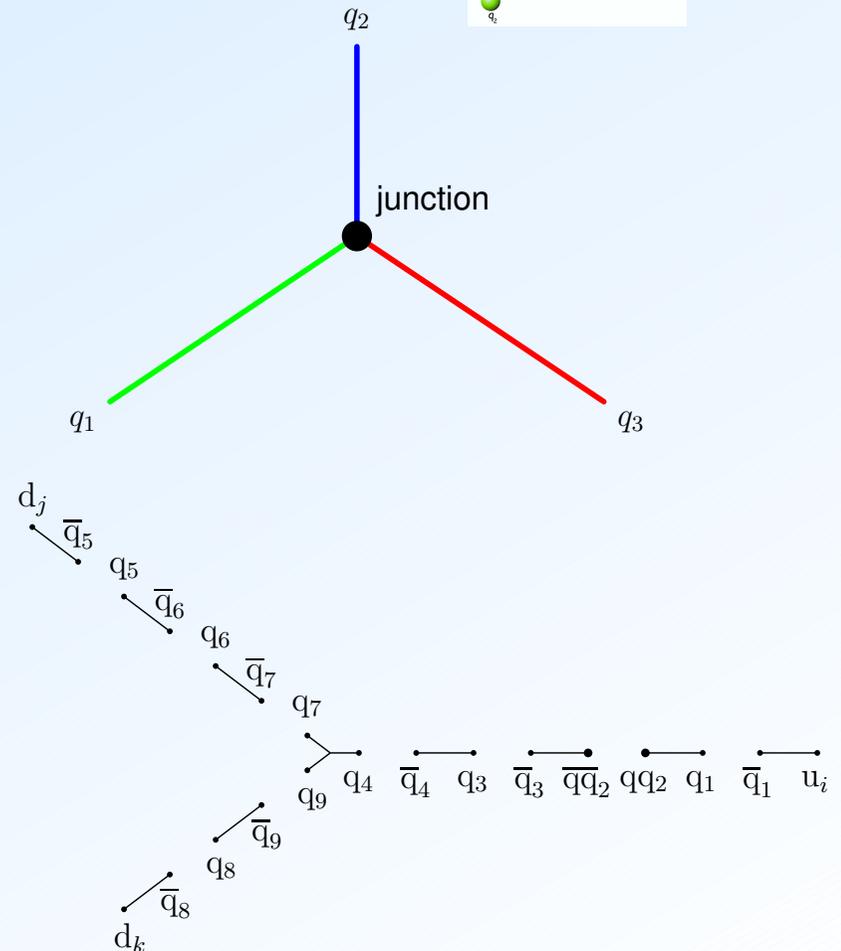
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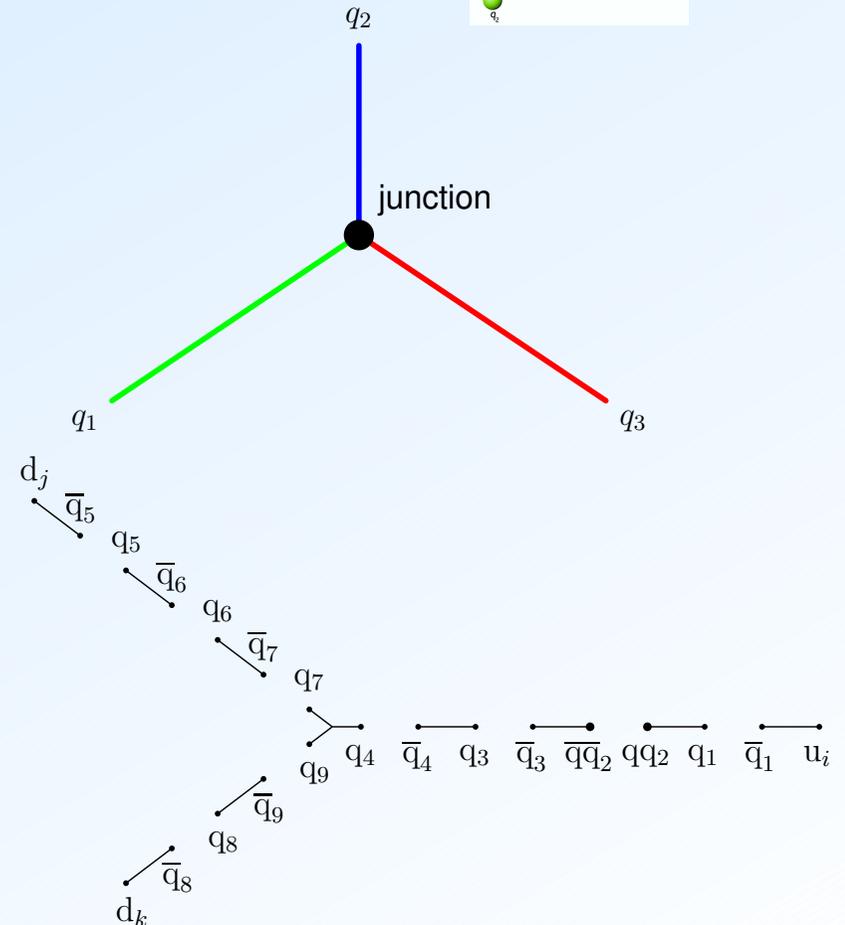
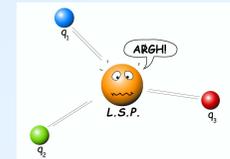
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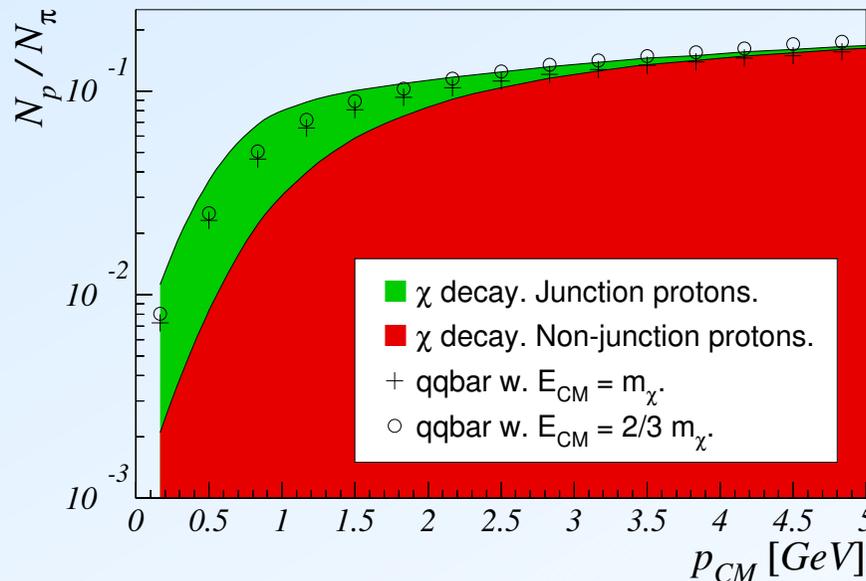
“Ordinary” string (e.g. $Z^0 \rightarrow q\bar{q}$):



“Baryonic” string (e.g. $L.S.P. \rightarrow q_1 q_2 q_3$):



 $B \equiv$ string topologies w/ junction(s).



Paper III

The SUSY Les Houches Accord. – Standardising SUSY calculations.

- III “SUSY Les Houches Accord: Interfacing SUSY Spectrum Calculators, Decay Packages, and Event Generators”.

By P. Skands, B.C. Allanach, H. Baer, C. Balázs, G. Bélanger, F. Boudjema, A. Djouadi, R. Godbole, J. Guasch, S. Heinemeyer, W. Kilian, J-L. Kneur, S. Kraml, F. Moortgat, S. Moretti, M. Mühlleitner, W. Porod, A. Pukhov, P. Richardson, S. Schumann, P. Slavich, M. Spira, G. Weiglein.

LU TP 03-39, Nov 2003.

Published in Journal of High Energy Physics 07 (2004) 036.

The SUSY Les Houches Accord

-  Problem: lots of people doing SuSy calculations today!
- Spectrum Calculators: ~ 7 programs.
 - Relic Density Codes: ~ 3 programs.
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- ☞ This gave rise to some problems...
- ☞ So why not make an *Accord*? I.e. agree on a standard set of conventions for SUSY theories, with standard file structures \rightarrow unambiguous communication.

The SUSY Les Houches Accord

- ☞ At Les Houches 2003, the organisers let me gather a lot of experts in a room, to discuss this. I made sure nobody could get out for some hours.
- ☞ Next day, we had another long meeting, and another one every day after that, for almost two weeks.

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- ☞ Now please use it!

Paper IV

Bilinear Lepton Number Violating Supersymmetry – Measuring Neutrino Mixing at the LHC?

IV “Measuring Neutrino Mixing angles at LHC”.

By W. Porod and P. Skands.

LU TP 03-50, ZU-TH 20/30, Jan 2004. [hep-ph/0401077]

In Beyond the Standard Model Working Group: Summary report, 3rd Les Houches Workshop: Physics at TeV Colliders, Les Houches, France, 26 May - 6 Jun 2003, B. C. Allanach *et al.* [hep-ph/0402295].

Bilinear L -violation

$$W_{\text{SUSY}} = W_{\text{MSSM}} + \epsilon_i L_i H_2$$

(Occurs e.g. when R -parity is broken spontaneously)

In context of neutrino masses, the important consequences are:



EW symmetry is broken by **Higgs** and **sneutrino vev's**,

$$\langle \nu_i \rangle = v_i \text{ (i.e. } m_W^2 = \frac{1}{4} g^2 (v_d^2 + v_u^2 + v_1^2 + v_2^2 + v_3^2)\text{)}.$$



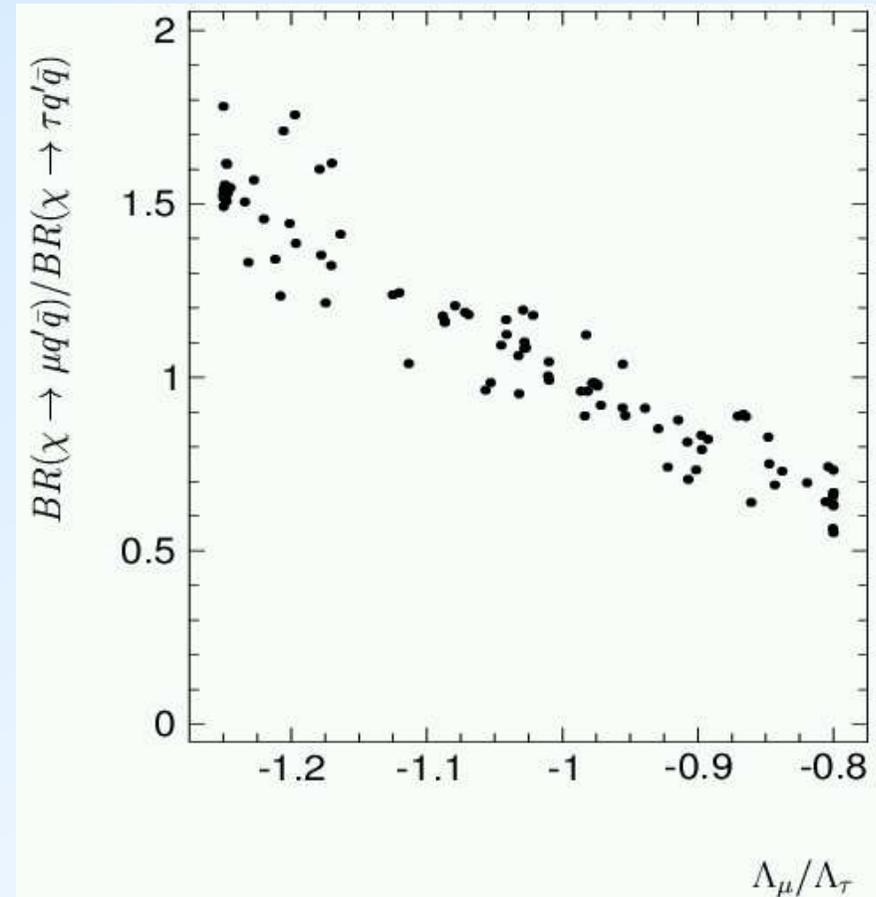
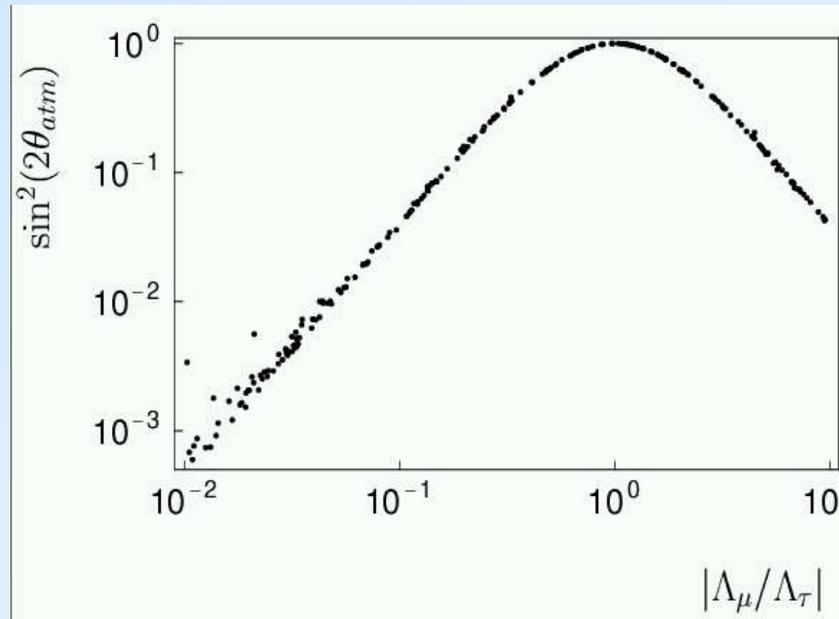
Neutrinos mix with **neutralinos** $\rightarrow 7 \times 7$ mixing:

$$\text{In block form: } M_N = \begin{pmatrix} 0 & m_{(3 \times 4)} \\ m_{(4 \times 3)}^T & M_{(4 \times 4)} \end{pmatrix}$$

Measuring a ν angle...

Mixing depends on

$$\Lambda_i = \mu v_i + v_d \epsilon_i$$



- ☞ BRPV couplings also responsible for **LSP decay**.
- ☞ \rightarrow Ratio of $\tilde{\chi}_1^0$ semileptonic branching ratios is **strongly correlated** with Λ_i/Λ_j !

Paper V

High Energy Proton Collisions 1

– Improving the Description of Underlying
and Minimum Bias Events.

V “Multiple Interactions and the Structure of Beam Remnants”.

By T. Sjöstrand and P. Skands.

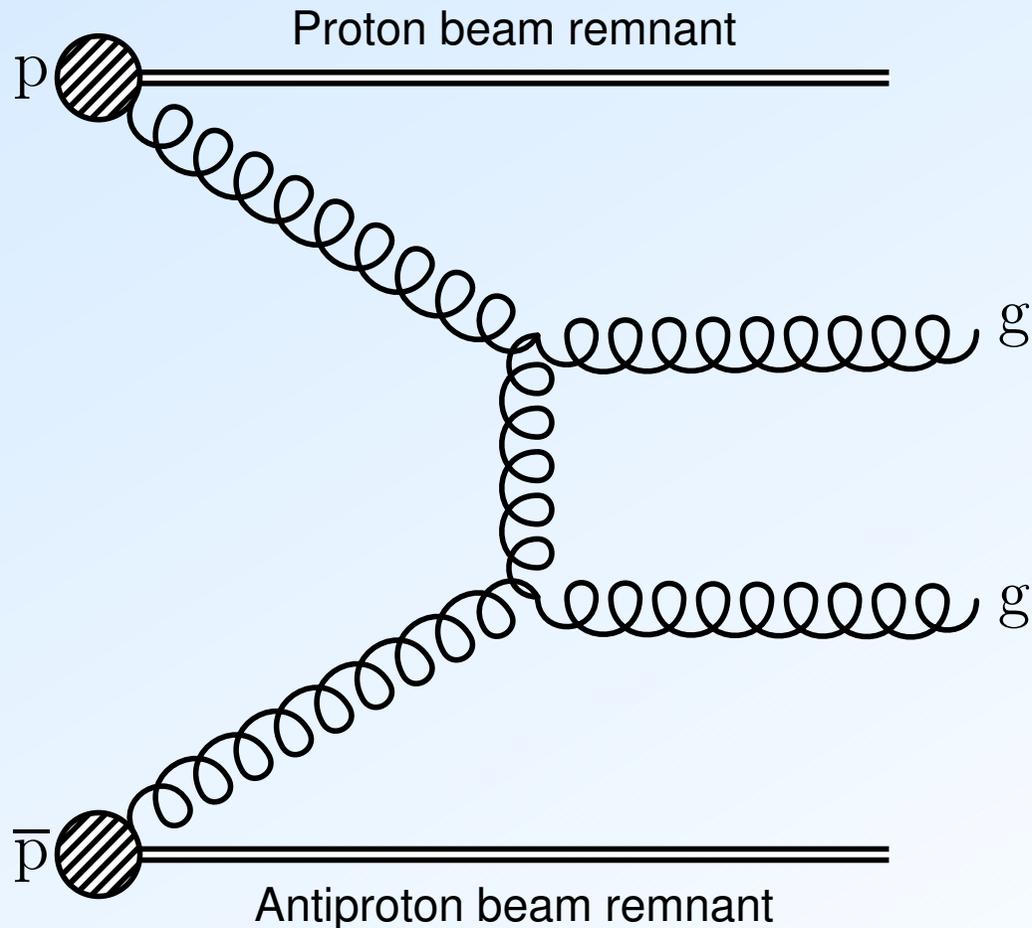
LU TP 04-01, Feb 2004.

Published in Journal of High Energy Physics 03 (2004) 053.

High Energy Proton Collisions 1

The Motivation:

Example: minimum-bias at the Tevatron



High Energy Proton Collisions 1

The Motivation:

Real life is more complicated...

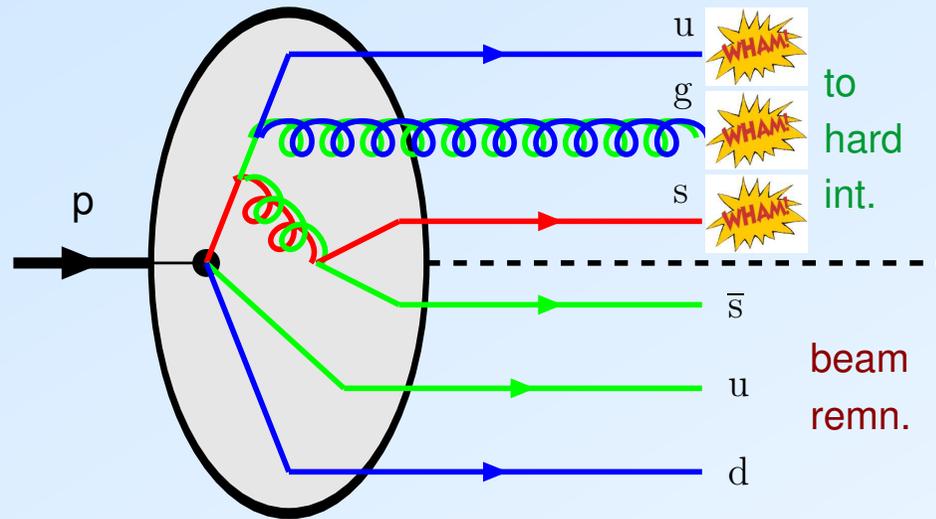


Butch Cassidy and the Sundance Kid. Copyright: Twentieth Century Fox Films Inc.

Why Develop a New MI/UE Model?

- ☁ Need to understand correlations and fluctuations in hadronic collisions. From QCD point of view:
many interesting questions remain unanswered.
- ☁ Any reliable extrapolation to LHC energies will require a good understanding of the physics mechanisms.
Simple parametrizations not sufficient.
- ☁ Random and systematic fluctuations in the underlying activity can impact precision measurements as well as New Physics searches:
more reliable understanding is needed.
- ☁ Lots of fresh data from Tevatron:
→ great topic for phenomenology right now!

Towards a realistic model



☁ How are the hard scattering initiators and beam remnant partons correlated?



- ☞ In impact parameter?
- ☞ In flavour?
- ☞ In longitudinal momentum?
- ☞ In colour?
- ☞ In (primordial) transverse momentum?

The “intermediate” model:

- ☁ Dependence on non-trivial **transverse density profile** of incoming hadrons.
- ☁ Fully **correlated multi-parton densities** calculated event by event, respecting momentum conservation and flavour sum rules, and reducing to standard PDF's for the hardest interaction.
- ☁ Final state multiplicity increased by **ISR and FSR**, for all interactions. (including correlated PDF's for ISR.)
- ☁ Non-vanishing **“primordial” k_{\perp}** for shower initiators.
- ☁ Junction hadronisation (from BNV studies) adapted for description of **baryon beam remnants**.
- ☁ **Colour flow** ambiguous at the non-perturbative level. Interesting (and thorny!) issues here, **to be continued...**

Paper VI

High Energy Proton Collisions 2 – Unifying the Description of Radiation and Interactions.

- VI “Transverse-Momentum-Ordered Showers and Interleaved Multiple Interactions”.
By T. Sjöstrand and P. Skands.
LU TP 04-29, Aug 2004.
Submitted to the European Physical Journal C.

Why Develop a New Shower?



Incorporate several of the good points of the dipole formalism within the shower approach

- ± explore alternative p_{\perp} definitions
- + p_{\perp} ordering \Rightarrow coherence inherent
- + Merging with Matrix Elements unproblematic.
(unique $p_{\perp}^2 \leftrightarrow Q^2$ mapping; same z)
- + $g \rightarrow q\bar{q}$ natural
- + kinematics constructed after each branching
(partons explicitly on-shell until they branch)
- + showers can be stopped and restarted at any p_{\perp} scale
 \Rightarrow well suited for ME/PS matching

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(partons explicitly on-shell until they branch)
- + showers can be stopped and restarted at any p_{\perp} scale
 \Rightarrow well suited for ME/PS matching
- + allows to combine p_{\perp} evolutions of showers and multiple interactions \rightarrow *common (competing) evolution of ISR, FSR, and MI!*

≡ **‘Interleaved Multiple Interactions’**

Proton Collisions... The New Picture

The building blocks:

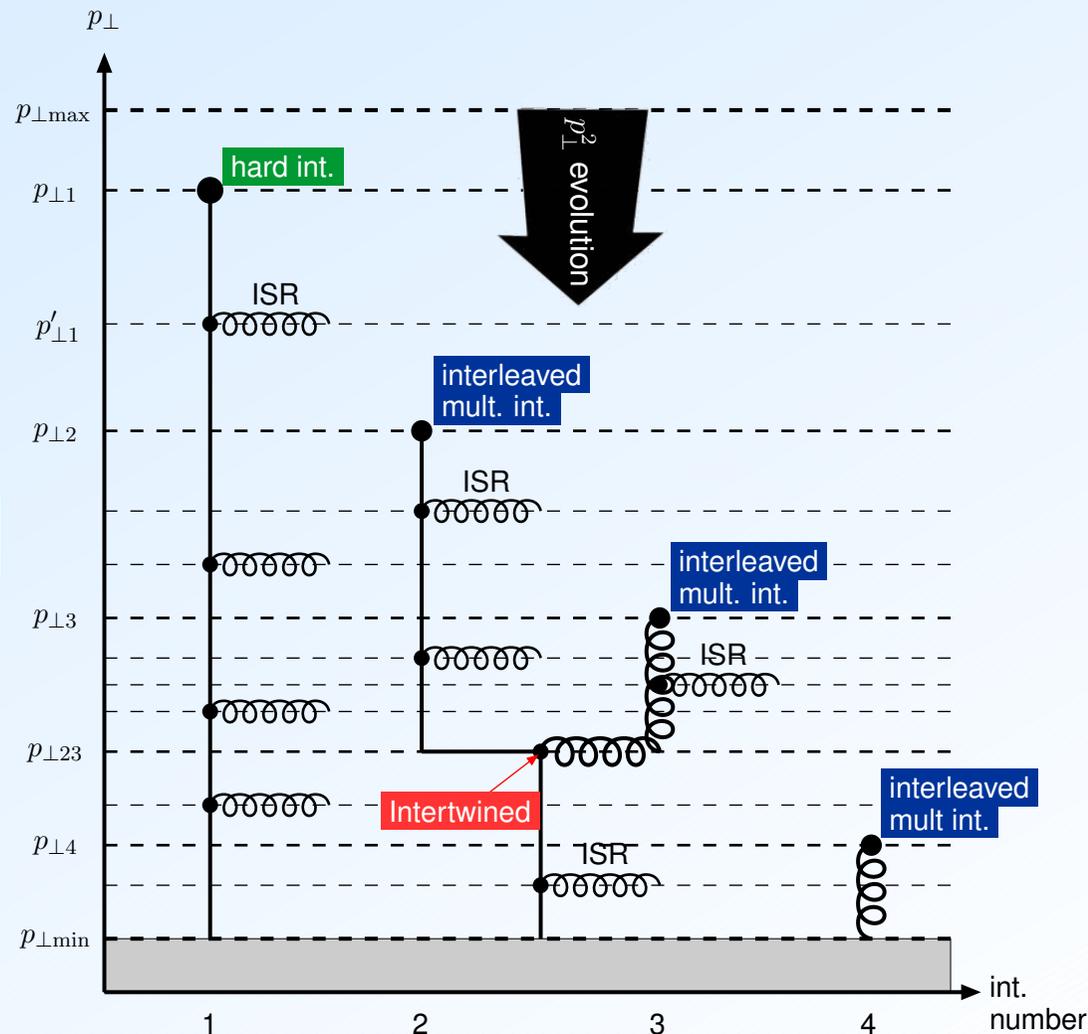
- ☁ p_{\perp} -ordered initial-state parton showers. ✓
- ☁ p_{\perp} -ordered final-state parton showers. ✓
- ☁ p_{\perp} -ordered multiple interactions. ✓
- ☁ p_{\perp} used as scale in α_s and in PDF's. ✓
- ☁ (Model for) correlated multi-parton densities. ✓
- ☁ Beam remnant hadronization model. ✓
- ☁ Model for initial state colour correlations. (✓ — but far from perfect!?)
- ☁ Other phenomena? (e.g. colour reconnections (✓), ...)
- ☁ Realistic tunes to data (not yet!)

Proton Collisions... The New Picture

☁ **The new picture:** start at the most inclusive level, $2 \rightarrow 2$.
 Add exclusivity progressively
 by evolving *everything*
 downwards in *one*
 common sequence:

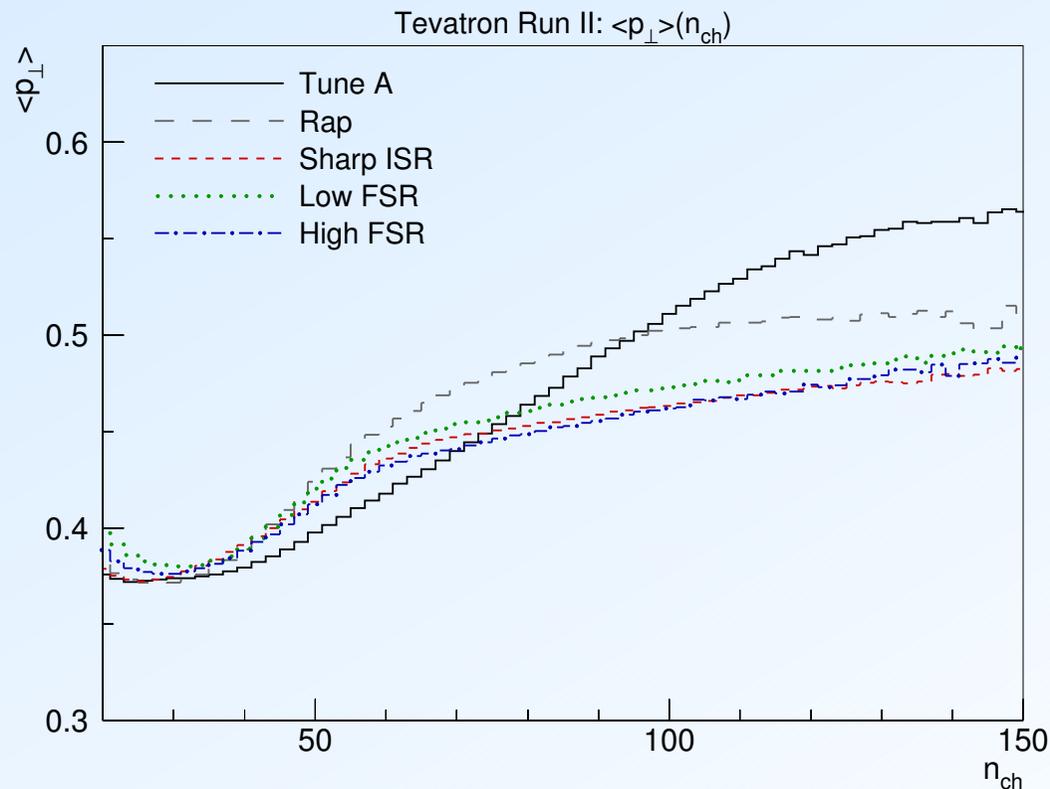
→ **Interleaved evolution**

☁ (→ also possible to have
 interactions **intertwined**
 by the ISR activity?)



Proton Collisions... The New Picture

- ☁ The new description represents a new generation in terms of detail and sophistication of the physics description of hadron collisions.
- ☁ But there is still some way to go...



Plans for the future

Plans for the future

To work hard!

