A new model for parton showers and the underlying event.

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Based on:
Overview

Introduction:
- Parton Showers and the Underlying Event.
- Why develop a new model?

The new framework.
- $p_{\perp}$-ordered showers: FSR and ISR.
- Interleaved multiple interactions.
- Model tests.

Outlook.
Basic Philosophy — Parton Showers

1 hadron collision =

$\left(2 \rightarrow 2 \oplus \text{ISR} \oplus \text{FSR} \oplus \text{UE}\right) \otimes \text{hadronisation etc.}$

Eff. resum. of multiple (semi-)soft gluon emission effects

- ISR: Initial–State Radiation (spacelike).
- FSR: Final–State Radiation (timelike).
- UE: Underlying Event – any additional (perturbative) activity.

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Basic Philosophy — Parton Showers

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Eff. resum. of multiple (semi-)soft gluon emission effects

- \( 2 \rightarrow 2 \): ‘hard subprocess’ (on–shell).
- ISR: Initial–State Radiation (spacelike).
- FSR: Final–State Radiation (timelike).
- UE: Underlying Event – any additional (perturbative) activity.

\[ Q^2_{\text{max}} \]

NB: no doublecounting!

⇒ For QCD: the hard \( 2 \rightarrow 2 \equiv \) most virtual \( \sim \) shortest distance, everything else is softer.
Existing Showers: Pros and Cons

Essential difference: ordering variables.
consider e.g. gluon emission off a $q_1\bar{q}_2$ system.

$\ln k^2_\perp$

$P\gamma\text{IA}/\text{JETSET}$

$m^2 (-m^2$ for ISR)
High–virtuality ems. first. (may be at ‘small’ angles.)

$\ln k^2_\perp$

$E^2 \theta^2$
Large–angle ems. first. (may be soft.)

$\ln k^2_\perp$

$p^2_\perp$
Large–$p_\perp$ ems. first.
Existing Showers: Pros and Cons

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consider e.g. gluon emission off a $q_1\bar{q}_2$ system.

\[ m^2 \quad (m^2 \text{ for ISR}) \]

**PYTHIA/JETSET**

High–virtuality ems. first. (may be at ‘small’ angles.)

**HERWIG**

Large–angle ems. first. (may be soft.)

\[ \sim E^2 \theta^2 \]

**ARIADNE**

Large–$p_\perp$ ems. first.

Another important difference is the way recoils are assigned, i.e. how the on–shell kinematics prior to the branching is reinterpreted to include the virtual (branching) leg.
HERWIG: \[ Q^2 \approx E^2 (1 - \cos \theta) \approx E^2 \theta^2 / 2 \]
+ angular ordering \(\Rightarrow\) coherence inherent
- emissions not ordered in hardness
- emissions do not cover full phase space (messy kinematics)
- kinematics constructed at the very end

PYTHIA: \[ Q^2 = m^2 \text{ (timelike)} \text{ or } = -m^2 \text{ (spacelike)} \]
+ convenient merging with ME
\(\pm\) emissions ordered in (some measure of) hardness
- coherence by brute force \(\Rightarrow\) approximate
- kinematics constructed when daughter masses known
ARIADNE: $Q^2 = p_{\perp}^2$, (final-state) dipole emission

+ $p_{\perp}$ ordering $\Rightarrow$ coherence inherent
+ Lorentz invariant
+ emissions ordered in hardness
+ kinematics constructed after each branching
  (partons explicitly on-shell until they branch)
+ showers can be stopped and restarted at any $p_{\perp}$ scale.
  $\Rightarrow$ good for ME/PS matching (L-CKKW, real+fictitious showers)
**Existing Showers: Pros and Cons**

**ARIADNE:** $Q^2 = p_{\perp}^2$, (final-state) dipole emission

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- $g \rightarrow q\bar{q}$ artificial
- Not so suited for $pp$ on its own: ISR is primitive in ARIADNE.
Why Develop a New Shower?

Incorporate several of the good points of the dipole formalism within the shower approach

± explore alternative $p_\perp$ definitions
+ $p_\perp$ ordering $\Rightarrow$ coherence inherent
+ ME merging works as before
  (unique $p_\perp^2 \leftrightarrow Q^2$ mapping; same $z$)
+ $g \rightarrow q\bar{q}$ natural
+ kinematics constructed after each branching
  (partons explicitly on-shell until they branch)
+ showers can be stopped and restarted at any $p_\perp$ scale
  $\Rightarrow$ well suited for ME/PS matching
  (not yet worked-out for ISR+FSR)
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  ⇒ well suited for ME/PS matching
  (not yet worked-out for ISR+FSR)
+ allows to combine $p_\perp$ evolutions of showers and multiple interactions → common (competing) evolution of ISR, FSR, and MI!

≡ ‘Interleaved Multiple Interactions’
Basic Philosophy — Multiple Interactions

Consider perturbative QCD $2 \rightarrow 2$ scattering:

(dominated by $t$-channel gluon exchange – IR divergent: \( \frac{d\sigma}{dp_\perp^2} \propto \frac{1}{p_\perp^4} \))

\[
\sigma_{2 \rightarrow 2}(p_{\perp \min}) = \int_{p_{\perp \min}}^{\sqrt{s}/2} \frac{d\sigma}{dp_\perp} \, dp_\perp \propto \frac{1}{p_{\perp \min}^2}
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1. Multiple interactions (MI)!
   - Must exist (hadrons are composite!)
   - $\sigma_{\text{tot}}$: hadron-hadron collisions.
   - $\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n$
   - $\sigma_{2\rightarrow2}$: parton-parton collisions.
   - $\sigma_{2\rightarrow2} = \sum_{n=0}^{\infty} n \sigma_n$
   - $\sigma_{2\rightarrow2} > \sigma_{\text{tot}} \iff \langle n \rangle > 1$

2. Breakdown of pQCD, colour screening.

$\lambda \sim 1/p_{\perp}$

$p_{\perp 0} \sim 2$ GeV

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Multiple Interactions — Evidence

Basic idea: expect four pair-wise balancing jets in double parton scattering (DPS) but not in double bremsstrahlung emission.

**AFS**: 4-jet events at $E_\perp > 4$ GeV in 1.8 units of $\eta$. Project out 2 pairs of jets and study imbalancing variable, $I = p_{\perp 1}^2 + p_{\perp 2}^2$. Excess of events with small $I$.

**CDF**: Extraction by comparing double parton scattering (DPS) to a mix of two separate scatterings. Sample: 14000 $\gamma/\pi^0 + 3j$ events. Strong signal observed, 53% DPS.

**+ Indirect**: KNO violation, pedestal effect, Fwd–Bwd asymmetry, ...

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Why Develop a New UE Model?

- Need to understand correlations and fluctuations. From QCD point of view:
  
  **many interesting questions remain unanswered.**

- Any reliable extrapolation to LHC energies will require a good understanding of the physics mechanisms.

  **Simple parametrizations not sufficient.**

- Random and systematic fluctuations in the underlying activity can impact precision measurements as well as New Physics searches:

  **more reliable understanding is needed.**

- Lots of fresh data from Tevatron:

  → **great topic for phenomenology right now!**
The New Framework

This led us to develop a new sophisticated model for UE (and min–bias) → JHEP 0403 (2004) 053.

But still each interaction was considered separately, with its set of ISR and FSR.

That’s probably not the way it happens in real life...

The new picture: start at the most inclusive level, \(2 \rightarrow 2\). Add exclusivity progressively by evolving everything downwards in one common sequence:

→ Interleaved evolution

(→ also possible to have interactions intertwined by the ISR activity?)
The New Framework

The building blocks:

- $p_\perp$—ordered initial–state parton showers. ✓
- $p_\perp$—ordered final–state parton showers. ✓
- $p_\perp$—ordered multiple interactions. ✓
- $p_\perp$ used as scale in $\alpha_s$ and in PDF’s. ✓
- (Model for) correlated multi–parton densities. ✓
- Beam remnant hadronization model. ✓
- Model for initial state colour correlations. (✓ — but far from perfect!)
- Other phenomena? (e.g. colour reconnections (✓), ...)
- Realistic tunes to data (not yet!)
Multiple Interactions: Some Details

Correlated PDF’s:
- Momentum and Energy in parent hadron conserved.
- Sum rules for valence quarks respected.
  (Can’t kick the same quark out twice!)
- Sea quarks knocked out → ‘companion quarks’.

Hadronization:
- Possible to have composite objects in the beam remnants, e.g. diquarks.
- Addressing ‘baryonic’ colour topologies → ‘string junctions’ in the colour confinement field.

Colour Correlations:
- The big question! Seems Nature likes a very high degree of correlation (cf. ‘Tune A’ of old model!).
- Several possibilities investigated, so far without success.
Whole framework.

- Produced a few rough tunes to ‘Tune A’ at the Tevatron, using charged multiplicity distribution and $\langle p_\perp \rangle(n_{ch})$, the latter being highly sensitive to the colour correlations.
- Similar overall results are achieved (not shown here), but $\langle p_\perp \rangle(n_{ch})$ still difficult.
- Anyway, these were only rough tunes...

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Outlook

New sophisticated framework for $p_\perp$-ordered *interleaved* parton showers and multiple interactions has been developed.

Good overall performance, still only primitive studies carried out, except for FSR.

Colour correlations still a headache. We thought perhaps *intertwined* showers would yield a more correlated colour flow, but preliminary studies do not indicate intertwining at perturbative energies to be a frequent phenomenon.
New sophisticated framework for $p_{\perp}$-ordered \textit{interleaved} parton showers and multiple interactions has been developed. Good overall performance, still only primitive studies carried out, except for FSR. Colour correlations still a headache. We thought perhaps intertwined showers would yield a more correlated colour flow, but preliminary studies do not indicate intertwining at perturbative energies to be a frequent phenomenon.

But nobody said hadron collisions were easy...
**Model Tests: FSR**

- **FSR algorithm.**
  - Tested on ALEPH data (G. Rudolph).

<table>
<thead>
<tr>
<th>Distribution of interv.</th>
<th>nb.of interv.</th>
<th>$\sum \chi^2$ of model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphericity</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>Aplanarity</td>
<td>16</td>
<td>23</td>
</tr>
<tr>
<td>1–Thrust</td>
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<td>60</td>
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<tr>
<td>Thrust$_{\text{minor}}$</td>
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<td>26</td>
</tr>
<tr>
<td>jet res. $y_3(D)$</td>
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<td>10</td>
</tr>
<tr>
<td>$x = 2p/E_{cm}$</td>
<td>46</td>
<td>207</td>
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<tr>
<td>$p_{\perp\text{in}}$</td>
<td>25</td>
<td>99</td>
</tr>
<tr>
<td>$p_{\perp\text{out}} &lt; 0.7$ GeV</td>
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<td>29</td>
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<tr>
<td>$p_{\perp\text{out}}$</td>
<td>(19)</td>
<td>(590)</td>
</tr>
<tr>
<td>$x(B)$</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>sum $N_{\text{dof}}$</td>
<td>190</td>
<td>497</td>
</tr>
</tbody>
</table>

- (Also, generator is not perfect. Adding 1% to errors $\Rightarrow \sum \chi^2 = 234$. i.e. generator is ‘correct’ to $\sim 1\%$)
ISR algorithm.

- Less easy to test. We looked at $p_\perp$ of $Z^0$ at Tevatron.
- Compared “Tune A” with an ‘intermediate scenario’ (“Rap”), and three rough tunes of the new framework.
- Description is improved (but there is still a need for a large primordial $k_\perp$).
**MI — Indirect Verifications**

Basic idea:

- Hadronization alone produces roughly Poissonian fluctuations in multiplicity.

- Additional soft interactions can ‘mess up’ colour flow → larger fluctuations.

\[ \langle n_{\text{ch}} \rangle = 35.6, \]
\[ \sigma_{n_{\text{ch}}} = 19.6. \]

- UA5: (900 GeV)

\[ \text{UA5 data} \]

- Tune A double Gaussian

- at most one hard int.

+ forward–backward correlations (UA5, E735)

+ pedestal effect (UA1, CDF, H1), ...

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Consider branching $a \rightarrow bc$ in lightcone coordinates $p^\pm = E \pm p_z$

\[
\begin{align*}
   p_b^+ &= z p_a^+ \\
   p_c^+ &= (1 - z) p_a^+ \\
   p^- \text{ conservation}
\end{align*}
\]

\[\implies m_a^2 = \frac{m_b^2 + p_-^2}{z} + \frac{m_c^2 + p_-^2}{1 - z}\]

**Timelike branching:**

\[Q^2 = m_a^2 > 0\]

\[p_- = z (1 - z) Q^2\]

**Spacelike branching:**

\[m_a = 0\]

\[Q^2 = -m_b^2 > 0\]

\[p_- = (1 - z) Q^2\]

**Guideline, not final $p_-$!**
**$p_\perp$-ordered showers: General Strategy (1)**

1) Define

\[
p_{\perp \text{evol}}^2 = z(1 - z)Q^2 \text{ for FSR}
\]

\[
p_{\perp \text{evol}}^2 = (1 - z)Q^2 \text{ for ISR}
\]

2) Find list of *radiators* = partons that can radiate. *Evolve* them all *downwards* in $p_{\perp \text{evol}}$ from common $p_{\perp \text{max}}$

\[
dP_a = \frac{dp_{\perp \text{evol}}^2}{p_{\perp \text{evol}}^2} \frac{\alpha_s(p_{\perp \text{evol}}^2)}{2\pi} P_{a\rightarrow bc}(z) \, dz \, \exp \left( - \int_{p_{\perp \text{evol}}^2}^{p_{\perp \text{max}}^2} \right)
\]

\[
dP_b = \frac{dp_{\perp \text{evol}}^2}{p_{\perp \text{evol}}^2} \frac{\alpha_s(p_{\perp \text{evol}}^2)}{2\pi} \frac{x' f_a(x', p_{\perp \text{evol}}^2)}{x f_b(x, p_{\perp \text{evol}}^2)} P_{a\rightarrow bc}(z) \, dz \, \exp (- \cdots)
\]

*Pick* the one with *largest* $p_{\perp \text{evol}}$ to undergo branching; also gives $z$.

3) Derive

\[
Q^2 = p_{\perp \text{evol}}^2 / z(1 - z) \text{ for FSR}
\]

\[
Q^2 = p_{\perp \text{evol}}^2 / (1 - z) \text{ for ISR}
\]
4) Find *recoiler* = parton to take recoil when radiator is pushed off-shell
usually nearest colour neighbour for FSR
incoming parton on other side of event for ISR

5) Interpret $z$ as energy fraction (not lightcone)
in radiator+recoiler rest frame for FSR,
in mother-of-radiator+recoiler rest frame for ISR,
so that Lorentz invariant

$$(2E_i/E_{\text{cm}} = 1 - m_{jk}^2/E_{\text{cm}}^2)$$

and straightforward match to matrix elements

6) Do kinematics based on $Q^2$ and $z$,
   a) assuming yet unbranched partons on-shell
   b) shuffling energy–momentum from recoiler as required

7) Continue evolution of all radiators from recently picked $p_{{\perp\text{evol}}}$. Iterate until no branching above $p_{{\perp\text{min}}}$.\
⇒ One combined sequence $p_{{\perp\text{max}}} > p_{{\perp1}} > p_{{\perp2}} > \ldots > p_{{\perp\text{min}}}$.
**p⊥–ordered showers: Some Details**

**FSR Evolution:**
- Massive quarks: $p_{\perp \text{evol}}^2 = z(1 - z)(m^2 - m_Q^2)$
  $\Rightarrow m^2 \rightarrow m_Q^2$ when $p_{\perp \text{evol}}^2 \rightarrow 0$.
- Special treatment of narrow resonances (e.g. top).

**ISR Evolution:**
- Massive quarks: $p_{\perp \text{evol}}^2 = (1 - z)(Q^2 + m_Q^2) = m_Q^2 + p_{\perp \text{LC}}^2$
  $\Rightarrow$ Light–Cone $p_{\perp \text{LC}}^2 \rightarrow 0$ when $p_{\perp \text{evol}}^2 \rightarrow m_Q^2$.
- Backwards evolution uses correlated pdf’s at scales where more than 1 interaction is resolved.

**Both ISR and FSR:**
- ME merging by veto for many SM+MSSM processes.
- Gluon polarization $\rightarrow$ asymmetric $\varphi$ distribution.
Q: What are the pdf’s for a proton with 1 valence quark, 2 sea quarks, and 5 gluons knocked out of it?

1. Overall momentum conservation (old):

Starting point: simple scaling ansatz in $x$.

For the $n$’th scattering:

$$x \in [0, X] \ ; \ X = 1 - \sum_{i}^{n-1} x_i \implies f_n(x) \sim \frac{1}{X} f_0 \left( \frac{x}{X} \right)$$
Q: What are the pdf’s for a proton with 1 valence quark, 2 sea quarks, and 5 gluons knocked out of it?

Normalization and shape:

✧ If valence quark knocked out.
   → Impose valence counting rule: \( \int_0^X q_{fn}^{\text{val}}(x, Q^2) \, dx = N_{fn}^{\text{val}}. \)

✧ If sea quark knocked out.
   → Postulate “companion antiquark”: \( \int_0^{1-x_s} q_f^{\text{cmp}}(x; x_s) \, dx = 1. \)

✧ But then momentum sum rule is violated:

\[
\int_0^X x \left( \sum_f q_{fn}(x, Q^2) + g_n(x, Q^2) \right) \, dx \neq X
\]

   → Assume sea+gluon fluctuates up when a valence quark is removed and down when a companion quark is added.
**Correlated PDF’s in flavour and $x_i$**

### Remnant PDFs

**Quarks:**

$$q_{fn}(x) = \frac{1}{X} \left[ \frac{N_{f_0}^{val}}{N_{f_0}^{val}} q_{f_0}^{val} \left( \frac{x}{X}, Q^2 \right) + a q_{f_0}^{sea} \left( \frac{x}{X}, Q^2 \right) + \sum_j q_{f_0}^{cmp} \left( \frac{x}{X}; x_{s_j} \right) \right]$$

$$q_{f_0}^{cmp} (x; x_s) = C \tilde{g}(x_s) \frac{x}{x + x_s} P_{g \rightarrow qf\bar{q}} \left( \frac{x_s}{x + x_s} \right) ; \left( \int_0^{1-x_s} q_{f_0}^{cmp} (x; x_s) \, dx = 1 \right)$$

**Gluons:**

$$g_n(x) = \frac{a}{X} g_0 \left( \frac{x}{X}, Q^2 \right)$$

$$a = \frac{1 - \sum_f N_{f_0}^{val} \langle x_{f_0}^{val} \rangle - \sum_{f,j} \langle x_{f_0}^{cmp} \rangle}{1 - \sum_f N_{f_0}^{val} \langle x_{f_0}^{val} \rangle}$$

Companion Distributions

---

Used to select $p_\perp$-ordered $2 \rightarrow 2$ scatterings, and to perform backwards DGLAP shower evolution.
Imagine placing a stick o’ dynamite inside a proton, imparting the 3 valence quarks with large momenta relative to each other.

**‘Ordinary’ colour topology**

(e.g. $Z^0 \rightarrow q\bar{q}$):

**‘Baryonic’ colour topology**

(e.g.):

Need to extend string model to handle baryonic topology.
Fundamental properties of QCD vacuum suggest string picture still applicable.

Baryon wavefunction building and string energy minimization \( \implies \) picture of 3 string pieces meeting at a ‘string junction’.

(Warning: This picture was drawn in a “pedagogical projection” where distances close to the center are greatly exaggerated!)
How does the junction move?

A junction is a topological feature of the string confinement field: \( V(r) = \kappa r \). Each string piece acts on the other two with a constant force, \( \kappa \vec{e}_r \).

\[ \implies \text{in junction rest frame (JRF) the angle is } 120^\circ \text{ between the string pieces.} \]

Or better, ‘pull vectors’ lie at \( 120^\circ \):

\[ p^\mu_{\text{pull}} = \sum_{i=1,N} p_i^\mu - \sum_{j=1}^{i-1} \frac{E_j}{\kappa} \]

(since soft gluons ‘eaten’ by string)

Note: the junction motion also determines the baryon number flow!)
First 2 pieces fragmented outwards–in, junction baryon formed around junction, last string piece fragmented as ordinary $q\bar{q}$ string.

NB: Other topologies also possible (junction–junction strings, junction–junction annihilation).