RADIATION IN HIGH-$\tilde{s}$ FINAL STATES

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Overview

- QCD @ high energy: scales, logs & hands
- Tevatron: ttbar production
- LHC: ttbar production
- LHC: SUSY pair production
Collider Energy Scales

**HARD SCALES:**
- $s$: collider energy
- $p_{T,jet}$: extra activity
- $Q^2_X$: signal scale ($t\bar{t}$bar)
- $m_X$: large rest masses

\[ (\hat{s}, \hat{m}^2, \ldots) \]
Collider Energy Scales

**HARD SCALES:**
- $s$: collider energy
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- $Q_X$: signal scale (ttbar)
- $m_X$: large rest masses

**SOFT SCALES:**
- $\Gamma$: decay widths
- $m_p$: beam mass
- $\Lambda_{QCD}$: hadronisation
- $m_i$: small rest masses

**"ARBITRARY" SCALES:**
- $Q_F$, $Q_R$: Factorisation & Renormalisation
Approximations to QCD

1. Fixed order matrix elements: Truncated expansion in $\alpha_s$:
   - Full interference and helicity structure included to given order.
   - Divergences appear as low-$p_T$ log divergences.
   - Difficulty (computation time) increases rapidly with final state multiplicity $\Rightarrow$ limited to $2 \rightarrow 5/6$.

2. Parton Showers: infinite series in $\alpha_s$ (but only singular terms = collinear approximation):
   - Resums logs to all orders $\Rightarrow$ excellent at low $p_T$.
   - Factorisation $\Rightarrow$ Exponentiation $\Rightarrow$ Arbitrary multiplicity
   - Easy match to hadronisation models
   - Interference terms neglected + simplified helicity structure $\Rightarrow$ large uncertainties away from singular regions.
A handwaving argument

• Quantify: what is a soft jet?
A handwaving argument

• **Quantify**: what is a soft jet?

• **Handwavingly**, leading logs are:

  \[ \alpha_s \log^2 \left( \frac{Q_F^2}{p^2,\text{jet}} \right) \to \mathcal{O}(1) \text{ for } \frac{Q_F}{p^\perp,\text{jet}} \sim 6 \]

• So, **very roughly**, logs become large for jet \( p_T \) around 1/6 of the hard scale.
ttbar + jets @ Tevatron

Process characterized by:

- Threshold production (mass large compared to $s$)
- A 50-GeV jet is reasonably hard, in comparison with hard scale ~ top mass

**SCALES [GeV]**

$s = (2000)^2$

$Q_{\text{Hard}}^2 \sim (175)^2$

$50 < p_{T,\text{jet}} < 250$

$\rightarrow$ **RATIOS**

$Q_{H}^2/s = (0.1)^2$

$1/4 < p_T / Q_H < 2$
ttbar + jets @ Tevatron

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Hard tails:
- Power Showers (solid green & blue) surprisingly good (naively expect collinear approximation to be worse!)
- Wimpy Showers (dashed) drop rapidly around top mass.

Soft peak: logs large @ ~ m_{top}/6 ~ 30 GeV → fixed order still good for 50 GeV jets (did not look explicitly below 50 GeV yet)

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\[Q^2_s = (2000)^2\]
\[Q^2_H = (175)^2\]

\[50 < p_{T,\text{jet}} < 250\]
ttbar + jets @ LHC

Process characterized by:

• Mass scale is small compared to \( s \)

• A 50-GeV jet is still hard, in comparison with hard scale \( \sim \) top mass, but is now soft compared with \( s \).

**SCALES [GeV]**

\[
\begin{align*}
\text{s} &= (14000)^2 \\
Q^2_{\text{Hard}} &\sim (175+\ldots)^2 \\
50 &\leq p_{T,jet} < 450
\end{align*}
\]

**RATIOS:**

\[
\begin{align*}
Q^2_{\text{H}}/s &= (0.02)^2 \\
1/5 &< p_T / Q_{H} < 2.5
\end{align*}
\]
ttbar + jets @ LHC

Hard tails: More phase space → more radiation.
- Power Showers still reasonable (but caution advised!)
- Wimpy Showers (dashed) drop catastrophically around top mass.

- Soft peak: logs slightly larger (scale larger than mtop, since not threshold dominated here) → but fixed order still reasonable for 50 GeV jets.

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$50 < p_{T,\text{jet}} < 450$

RATIOS

$Q^2_H/s = (0.02)^2$

$1/5 < p_T / Q_H < 2.5$
SUSY + jets @ LHC

Process characterized by:  (SPS1a)

• Mass scale is again large compared to $s$

• But a 50-GeV jet is now soft, in comparison with hard scale $\sim$ SUSY mass.

**SCALES [GeV]**

$s = (14000)^2$

$Q^2_{\text{Hard}} \sim (600)^2$

$50 < p_{T,\text{jet}} < 450$

**RATIOS**

$Q^2_{H} / s = (0.05)^2$

$1/10 < p_T / Q_H < 1$
**Hard tails**: Still a lot of radiation ($p_T$ spectra have moderate slope)

- Parton showers less uncertain, due to higher signal mass scale. Drop of wimpy showers happens later $\sim 600$ GeV.

- **Soft peak**: logs BIG: fixed order breaks down for $\sim 100$ GeV jets. Reconfirmed by parton showers $\rightarrow$ universal limit below 100 GeV.

- (2 jet sample: matrix element blowing up $\rightarrow$ artificially large norm. difference?)
$p_T$ of hard system

(Equivalent to $p_T,Z$ for Drell-Yan)

- $t\bar{t} + 1$ jet @ LHC
  - $p_T$ of ($t\bar{t}$) system

- $\sim g + 1$ jet @ LHC
  - $p_T$ of ($\sim g$) system

- $\sim uL + 1$ jet @ LHC
  - $p_T$ of ($\sim uL$) system

$\Rightarrow$ Resummation necessary

Bulk of cross section sits in peak sensitive to multiple emissions.
Conclusions

- **SUSY-MadGraph** soon to be public.
- Comparisons to **PYTHIA** $Q^2$- and $p_T^2$- ordered showers $\Rightarrow$ **New illustrations of old wisdom:**
  
  - **Hard jets** (= hard in comparison with signal scale) $\Rightarrow$ collinear approximation misses relevant terms $\Rightarrow$ use matrix elements with explicit jets $\Rightarrow$ interference & helicity structure included.
  
  - **Soft jets** (= soft in comparison with signal process, but still e.g. 100 GeV for SPS1a) $\Rightarrow$ large logarithms $\Rightarrow$ use resummation / parton showers to resum logs to all orders.
Conclusions

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• Comparisons to PYTHIA Q2 and pT-ordered showers → New illustrations of old wisdom:
  - Hard jets (= hard in comparison with signal scale) → collinear approximation misses relevant terms → use matrix elements with explicit jets → interference & helicity structure included.
  - Soft jets (= soft in comparison with signal process, but still e.g. 100 GeV) → large logarithms → use resummation / parton showers to resum logs to all orders.