MC Tools: Matching & Tuning

P. Skands (CERN-TH)
Merging Parton Showers and Matrix Elements

Matching

Note: tough subject
Not required to understand everything
Don’t loose yourselves in the details,
Just try to understand the overall reasoning
power: $Q^2_{\text{max}} = s$;  
wimpy: $Q^2_{\text{max}} = m^2_{\perp}$;  
tune A: $Q^2_{\text{max}} = 4m^2_{\perp}$

$m_t = 175$ GeV,  
$m_{\tilde{g}} = 608$ GeV,  
$m_{\tilde{u}_L} = 567$ GeV

(T. Plehn, D. Rainwater, P. Skands)
A Naive Proposal

**Born × Shower**

<table>
<thead>
<tr>
<th>$X^{(2)}$</th>
<th>$X+1^{(2)}$</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X^{(1)}$</td>
<td>$X+1^{(1)}$</td>
<td>$X+2^{(1)}$</td>
</tr>
<tr>
<td>Born</td>
<td>$X+1^{(0)}$</td>
<td>$X+2^{(0)}$</td>
</tr>
</tbody>
</table>

**$X+1 @ LO$**

<table>
<thead>
<tr>
<th>$X+1^{(2)}$</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X+1^{(1)}$</td>
<td>$X+2^{(1)}$</td>
</tr>
<tr>
<td>$X+1^{(0)}$</td>
<td>$X+2^{(0)}$</td>
</tr>
</tbody>
</table>

Fixed-Order Matrix Element

Fixed-Order ME above $p_T$ cut & nothing below

Shower Approximation

Fixed-Order ME above $p_T$ cut & Shower Approximation below
### A Naive Proposal

#### Born × Shower

<table>
<thead>
<tr>
<th>( X^{(2)} )</th>
<th>( X+1^{(2)} )</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X^{(1)} )</td>
<td>( X+1^{(1)} )</td>
<td>( X+2^{(1)} )</td>
</tr>
<tr>
<td>Born</td>
<td>( X+1^{(0)} )</td>
<td>( X+2^{(0)} )</td>
</tr>
</tbody>
</table>

#### \( X+1 \) @ LO × Shower

<table>
<thead>
<tr>
<th>( X+1^{(2)} )</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X+1^{(1)} )</td>
<td>( X+2^{(1)} )</td>
</tr>
<tr>
<td>( X+1^{(0)} )</td>
<td>( X+2^{(0)} )</td>
</tr>
</tbody>
</table>

- Fixed-Order Matrix Element
- Shower Approximation

\[ \text{Fixed-Order ME above } p_T \text{ cut} \]
\[ \& \text{ nothing below} \]

\[ \text{Fixed-Order ME above } p_T \text{ cut} \]
\[ \& \text{ Shower Approximation below} \]
A Naïve Proposal

Born × Shower + (X+1) × shower

Double Counting of terms present in both expansions

Worse than useless

Fixed-Order ME above $p_T$ cut & nothing below

Fixed-Order ME above $p_T$ cut & Shower Approximation below
Cures

Tree-Level Matrix Elements

PHASE-SPACE SLICING (a.k.a. CKKW, MLM, …)

UNITARITY (a.k.a. merging, PYTHIA, VINCIA, …)

NLO Matrix Elements

SUBTRACTION (a.k.a. MC@NLO)

UNITARITY + SUBTRACTION (a.k.a. POWHEG, VINCIA)

+ WORK IN PROGRESS …

NLO + multileg tree-level matrix elements
NLO multileg matching
Matching at NNLO
Phase-Space Slicing
Matching to Tree-Level Matrix Elements

A.K.A. CKKW, CKKW-L, MLM
Phase Space Slicing
(with “matching scale”)

**Born × Shower**

+ shower veto above $p_T$

$X^{(2)}$ $X+1^{(2)}$ ...
$X^{(1)}$ $X+1^{(1)}$ $X+2^{(1)}$ $X+3^{(1)}$ ...
Born $X+1^{(0)}$ $X+2^{(0)}$ $X+3^{(0)}$ ...

**X+1 @ LO × Shower**

with 1 jet above $p_T$

$X+1^{(2)}$ ...
$X+1^{(1)}$ $X+2^{(1)}$ $X+3^{(1)}$ ...
$X+1^{(0)}$ $X+2^{(0)}$ $X+3^{(0)}$ ...

Fixed-Order Matrix Element

Shower Approximation

Fixed-Order ME above $p_T$ cut & nothing below

Fixed-Order ME above $p_T$ cut & Shower Approximation below
Phase Space Slicing
(with “matching scale”)

Born × Shower + X+1 @ LO × Shower
+ shower veto above p_T
with 1 jet above p_T

X+1 now correct in both soft and hard limits

Attention!
Must use the SAME p_T cut in both samples

But still ... : \( \alpha_s \) and “splitting functions” usually discontinuous

Fixed-Order Matrix Element

Fixed-Order ME above p_T cut & nothing below

Shower Approximation

Fixed-Order ME above p_T cut & Shower Approximation below
Multi-Leg Slicing
(a.k.a. CKKW or MLM matching)

Keep going

Veto all shower emissions above “matching scale”
(except for the highest-multiplicity matrix element)

LO: when all jets hard
LL: for soft emissions

→ Multileg Tree-level matching

MLM: Michelangelo L Mangano
Vetoed Parton Showers
(used in Phase Space Slicing, a.k.a. CKKW or MLM matching)

**Common** (at ME level):
1. Generate one ME sample for each of $\sigma_n(p_{Tcut})$ (using large, fixed $\alpha_s^0$)
2. Use a jet algorithm (e.g., $k_T$) to determine an approximate shower history for each ME event
3. Construct the would-be shower $\alpha_s$ factor and reweight
   \[ w_n = \frac{\text{Prod}[\alpha_s(k_{Ti})]}{\alpha_s^0^n} \]
   $\rightarrow$ “Renormalization-improved” ME weights

**CKKW and CKKW-L**
1. Apply Sudakov $\Delta(t_{\text{start}},t_{\text{end}})$ for each reconstructed internal line
   (NLL for CCKW, trial-shower for CKKW-L)
2. Accept/Reject: $w_n \times= \text{Prod}[\Delta_i]$  
3. Do parton shower, vetoing any emissions above cutoff

**MLM**
1. Do normal parton showers
2. Cluster showered event (cone)
3. Match ME partons to jets
4. If \{all partons matched $\&\&$ $n_{\text{partons}} = n_{\text{jets}}$\} Accept : Reject;
Subtraction
Matching to Born+NLO
Matrix Elements

A.K.A. MC@NLO, POWHEG, VINCIA [incl X+n @ LO]
**MC@NLO**

**Subtraction**

**Born x Shower**

<table>
<thead>
<tr>
<th>$X^{(2)}$</th>
<th>$X+1^{(2)}$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X^{(1)}$</td>
<td>$X+1^{(1)}$</td>
<td>$X+2^{(1)}$</td>
</tr>
<tr>
<td>Born</td>
<td>$X+1^{(0)}$</td>
<td>$X+2^{(0)}$</td>
</tr>
</tbody>
</table>

**NLO**

<table>
<thead>
<tr>
<th>$X^{(2)}$</th>
<th>$X+1^{(2)}$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X^{(1)}$</td>
<td>$X+1^{(1)}$</td>
<td>$X+2^{(1)}$</td>
</tr>
<tr>
<td>Born</td>
<td>$X+1^{(0)}$</td>
<td>$X+2^{(0)}$</td>
</tr>
</tbody>
</table>

... Fixed-Order Matrix Element

... Shower Approximation
MC@NLO

Subtraction

**Born** \( \times \) **Shower**

\[
\begin{array}{ccc}
X^{(2)} & X+1^{(2)} & \ldots \\
X^{(1)} & X+1^{(1)} & X+2^{(1)} & X+3^{(1)} & \ldots \\
\text{Born} & X+1^{(0)} & X+2^{(0)} & X+3^{(0)} & \ldots \\
\end{array}
\]

**NLO** - **Shower**

\[
\begin{array}{ccc}
X^{(2)} & X+1^{(2)} & \ldots \\
X^{(1)} & X+1^{(1)} & X+2^{(1)} & X+3^{(1)} & \ldots \\
\text{Born} & X+1^{(0)} & X+2^{(0)} & X+3^{(0)} & \ldots \\
\end{array}
\]

Expand shower approximation to NLO analytically, then subtract:

Fixed-Order ME minus Shower Approximation *(NOTE: can be < 0!)*
Add

Born + shower-subtracted $O(\alpha_s)$ matrix elements

\[
\begin{array}{ccc}
X^{(2)} & X+1^{(2)} & \ldots \\
X^{(1)} & X+1^{(1)} & X+2^{(1)} & X+3^{(1)} & \ldots \\
\text{Born} & X+1^{(0)} & X+2^{(0)} & X+3^{(0)} & \ldots \\
\end{array}
\]

→ NLO + parton shower

( however, the “correction events” can have $w<0$)

MC@NLO
Subtraction

NLO: for $X$ inclusive
LO for $X+1$
LL: for everything else

Note 1: NOT NLO for $X+1$
Note 2: Multijet tree-level matching still superior for $X+2$
**MC@NLO**

**Negative Weights**

**Born $\times$ Shower**

<table>
<thead>
<tr>
<th>$X^{(2)}$</th>
<th>$X^{(1)}$</th>
<th>Born</th>
<th>$X^{(0)}$</th>
<th>$X+1^{(2)}$</th>
<th>$X+1^{(1)}$</th>
<th>$X+2^{(1)}$</th>
<th>$X+3^{(1)}$</th>
<th>$X+1^{(0)}$</th>
<th>$X+2^{(0)}$</th>
<th>$X+3^{(0)}$</th>
<th>$\ldots$</th>
</tr>
</thead>
</table>

**NLO - Shower**

<table>
<thead>
<tr>
<th>$X^{(2)}$</th>
<th>$X^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X+1^{(2)}$</td>
<td>$X+1^{(1)}$</td>
</tr>
</tbody>
</table>

Born $X+1^{(0)}$ $X+2^{(0)}$ $X+3^{(0)}$ $\ldots$

Expand shower approximation to NLO analytically, then subtract:

Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)
PYTHIA / POWHEG / VINCIA
(Unitarity + Subtraction)

Born $\times$ First-Order Corrected Shower

Use exact (process-dependent) splitting function for first splitting(s)

Fixed-Order ME minus Shower Approximation
NLO Matching in 1 Slide

► First Order Shower expansion

$$\int d\Phi_2 \quad \text{Born} \quad \int_{Q_{\text{had}}^2}^s \frac{d\Phi_3}{d\Phi_2} \quad \text{LL} \quad \delta (\mathcal{O} - \mathcal{O}(\{p\}_3))$$

Unitarity of shower $\Rightarrow$ 3-parton real = + 2-parton “virtual”

► 3-parton real correction ($A_3 = |M_3|^2/|M_2|^2 + \text{finite terms; } \alpha, \beta$)

$$x_{+1}^{(0)} = x_{+1}^{(0)} - \left( \frac{x_{+1}^{(0)}}{\text{Born}} + \frac{4\pi\alpha_s \hat{C}_F}{s} \left( \alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) \right) \text{Born}$$

$$= -\frac{4\pi\alpha_s \hat{C}_F}{s} \left( \alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) |M_2^{(0)}|^2 \quad \Rightarrow \quad \text{Finite terms cancel in 3-parton } \mathcal{O}$$

► 2-parton virtual correction (same example)

$$x^{(0)} = \frac{x^{(0)}}{\text{Born}} + \int_0^s \frac{d\Phi_3}{d\Phi_2} \text{ LL} + \int_{Q_{\text{had}}^2}^s \frac{d\Phi_3}{d\Phi_2} \quad x_{+1}^{(0)}$$

$$= \frac{\alpha_s \hat{C}_F}{2\pi} \left( 2I_{q\bar{q}}^{(1)} (e, s) - 4 - 2I_{q\bar{q}}^{(1)} (e, s) + \frac{19 + \alpha + \frac{2}{3} \beta}{4} \right) \text{ Born}$$

$$= \frac{\alpha_s}{\pi} \left( 1 + \frac{1}{3} \left( \alpha + \frac{2}{3} \beta \right) \right) \quad \text{Born} \quad \Rightarrow \quad \text{Finite terms cancel in 2-parton } \mathcal{O} \text{ (normalization)}$$
NLO Matching in 1 Slide

- **First Order Shower expansion**

  \[ \int d\Phi_2 \left| M_2^{(0)} \right|^2 \int_{Q_{\text{had}}^2}^s \frac{d\Phi_3}{d\Phi_2} A_{q\bar{q}}(...)(\mathcal{O} - \mathcal{O}(\{p\}_3)) \]

  Unitarity of shower \( \Rightarrow \) 3-parton real = + 2-parton “virtual”

- **3-parton real correction** (\( A_3 = \left| M_3 \right|^2/\left| M_2 \right|^2 + \text{finite terms; } \alpha, \beta \))

  \[
  w_3^{(R)} = \left| M_3^{(0)} \right|^2 - \left( A_3^{0}(...) + \frac{4\pi\alpha_s \hat{C}_F}{s} \left( \alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) \right) \left| M_2^{(0)} \right|^2
  \]

  \[
  = -\frac{4\pi\alpha_s \hat{C}_F}{s} \left( \alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) \left| M_2^{(0)} \right|^2
  \]

  Finite terms cancel in 3-parton \( \mathcal{O} \)

- **2-parton virtual correction** (same example)

  \[
  w_2^{(V)} = 2\text{Re} \left[ M_2^{(1)} M_2^{(0)*} \right] + \left| M_2^{(0)} \right|^2 \int_0^s \frac{d\Phi_3}{d\Phi_2} A_{q\bar{q}}(...) + \int_{Q_{\text{had}}^2}^s \frac{d\Phi_3}{d\Phi_2} w_3^{(R)}
  \]

  \[
  = \frac{\alpha_s \hat{C}_F}{2\pi} \left( 2I_{q\bar{q}}^{(1)}(\epsilon, s) - 4 - 2I_{q\bar{q}}^{(1)}(\epsilon, s) + \frac{19 + \alpha + \frac{2}{3}\beta}{4} \right) \left| M_2^{(0)} \right|^2
  \]

  \[
  = \frac{\alpha_s}{\pi} \left( 1 + \frac{1}{3} \left( \alpha + \frac{2}{3}\beta \right) \right) \left| M_2^{(0)} \right|^2
  \]

  Finite terms cancel in 2-parton \( \mathcal{O} \) (normalization)
Approaches on the Market

**Hw/Py standalone**
1st order matching for many processes, especially resonance decays

**Alpgen + Hw/Py**
MLM + HW or PY showers
NOTE: If you just write “AlpGen” on a plot, we assume AlpGen standalone! (no showering or matching!) - very different from Alp+Py/Hw

**MadGraph + Hw/Py**
MLM-slicing + HW or PY showers

**Sherpa**
CKKW-slicing + CS-dipole showers

**Ariadne**
CKKW-L-slicing + Lund-dipole showers

**MC@NLO**
NLO with subtraction, 10% w<0
+ Herwig showers

**POWHEG**
NLO with unitarity; 0% w<0
+ “truncated” showers + HW or PY

**(Vincia+Py8)**
NLO + multileg with unitarity
+ dipole-antenna showers

Still only for LEP
Constraints
and Tuning
Constraining Models

- A wealth of data available at lower energies
- Used for constraining (‘tuning’) theoretical models (E.g., Monte Carlo Event Generators)
Constraining Models

- A wealth of data available at lower energies
- Used for constraining (‘tuning’) theoretical models (E.g., Monte Carlo Event Generators)

- The low-energy LHC runs are giving us a unique chance to fill in gaps in our knowledge at lower energies
- Which model would you trust more? One that also describes SPS, RHIC, Tevatron, Low-Energy LHC? Or one that doesn’t?

But wait ... which gaps?
Gaps

• QCD pheno evolving rapidly
  • The models that were tested 20 years ago are not the models of today
  • Capabilities of experiments are different today than 20 years ago (resolution, coverage, systematics,…)
  • We define new observables, new quantities of interest, as knowledge evolves (e.g., IR safety)
  • Also learned some hard lessons about data preservation and about ‘truth’ corrections
3 Kinds of Tuning

1. Fragmentation Tuning
   - **Non-perturbative:** hadronization modeling & parameters
   - **Perturbative:** jet radiation, jet broadening, jet structure

2. Initial-State Tuning
   - **Non-perturbative:** PDFs, primordial $k_T$
   - **Perturbative:** initial-state radiation, initial-final interference

3. Underlying-Event & Min-Bias Tuning
   - **Non-perturbative:** Multi-parton PDFs, Beam Remnant fragmentation, Color (re)connections, collective effects, impact parameter dependence, …
   - **Perturbative:** Multi-parton interactions, rescattering
Pure pQCD - the “parton” level

Default PYTHIA 8 - No Hadronization

Theory vs LEP

Theory/LEP
Hadron Level

Default PYTHIA 8 + Hadronization

Theory vs LEP

UV
IR
1-T
Obl
C
D

Theory/LEP

UV
IR
UV
IR
UV
IR
These results

Obtained with $\alpha_s(M_Z) \approx 0.14 \neq$ World Average $= 0.1176 \pm 0.0020$

Value of $\alpha_s$

Depends on the order and scheme
- MC $\approx$ Leading Order + LL resummation
- Other leading-Order extractions of $\alpha_s \approx 0.13 - 0.14$
- Plus uncertainty from different effective scheme

Not so crazy

Tune/measure even pQCD parameters with the actual generator.
Sanity check = consistency with other determinations at a similar formal order, within the uncertainty at that order (including an (unknown) scheme redefinition to go to 'MC scheme')
1. Fragmentation Tuning

Constrain incalculable model parameters

Good model $\rightarrow$ good fit. Bad model $\rightarrow$ bad fit $\rightarrow$ improve model
Before

**PYTHIA 8.100**

- \( N_{\text{ch}} \)
- Mesons
- Baryons
- \( \ln(1/x) \)
After

**PYTHIA 8.135**

$n_{Ch}$  
Mesons  
Baryons  
$\ln(1/x)$
Fragmentation

- Normal MC Tuning Procedure:
  - Fragmentation and Flavour parameters constrained at LEP, then used in pp/\bar{p}p (Jet Universality)
  - But pp/\bar{p}p is a very different environment, at the infrared level!
Fragmentation

• Normal MC Tuning Procedure:
  • Fragmentation and Flavour parameters constrained at LEP, then used in pp/ppbar (Jet Universality)

• Check fragmentation \textit{in situ} at hadron colliders
  • $N$ and $p_T$ spectra (and $x$ spectra normalized to ‘jet’/minijet energy?)
    \textbf{Identified particles} highly important to dissect fragmentation
  • Fully Exclusive $\rightarrow$ Particle-Particle CORRELATIONS
  • (How) do the spectra change with (pseudo-)rapidity? (forward = synergy with cosmic ray fragmentation, different dominating production/fragmentation mechanisms as fct of rapidity? E.g., compare LHCb with central?)
  • How do they change with event activity? (cf. heavy-ion ~ central vs peripheral collisions, hard trigger event (UE))
Tuning the Initial State

2. Initial state

Constrain $\Lambda$ (or $\alpha_s$) and "primordial $k_T$"

Similar to fitting PDF functions

Main reference:

Drell-Yan $p_T$, + Jets
(also DIS)

Complication:

Initial-Final interference!

Figure 1: Comparisons to the CDF and DØ measurements of the $p_T$ of Drell-Yan pairs [51, 52]. Insets show the high $p_T$ tails. Left: virtuality-ordered showers. Right: $p_T$-ordered showers. See [43] for.

2. Initial state

Constrain $\Lambda$ (or $\alpha_s$) and "primordial $k_T$"

(Comment): What I learned from the Tevatron

Observe: tune-A predicts $\langle p_{TZ} \rangle \approx 9.7$ GeV (# taken from Y. Gehrstein’s slides)

(Note: the ISR parameters had not been tuned; the ISR renormalization scale had not been touched in Tune A.)

Tune A undershoots $\langle p_{TZ} \rangle$ by $\approx 20\%$. Not too bad for LO+LL at $Q \approx 10$ GeV

$\rightarrow$ this model not optimal at subleading level. Conclusion depends on nature of missing terms: renormalization scale for ISR? Kinematics dependence? a few GeV of "intrinsic $k_T$"?

Answer not clear yet $\rightarrow$ Theoretical uncertainty

$\rightarrow$ what you learn depends on expectation.

Neither "fudging the MC" nor "whining about it" can replace "thinking about it".
Min-Bias & Underlying Event

**Main Parameters**

**Number of MPI**

**Infrared Regularization scale** for the QCD $2 \rightarrow 2$ (Rutherford) scattering used for multiple parton interactions (often called $p_{T0}$) $\rightarrow$ size of overall activity

**Pedestal Rise**

**Proton transverse mass distribution** $\rightarrow$ difference between central (active) vs peripheral (less active) collisions

**Strings per Interaction**

**Color correlations** between multiple-parton-interaction systems $\rightarrow$ shorter or longer strings $\rightarrow$ less or more hadrons per interaction

**Diffraction**

+ (for Min-Bias): **diffractive mixture** and modeling
Dissecting Minimum-Bias

A lab for testing theory models and detector performance with high statistics

This is a schematization to be able to cut down the problem in pieces and model them in a different way. The “pieces” are correlated!

(illustration by F. Krauss)
Minimum-Bias

630 GeV Multiplicity Distribution 1960 GeV

Diffractive ambiguities?

Constrain energy scaling

Low Multiplicity →

High Multiplicity
Minimum-Bias

Average Track $p_T$ vs Multiplicity

What does this tell you?
- Nothing, it’s just min-bias ...
- Hydro in pp? (Core/Corona)
- String reconnections? (Area Law)
- More Minijets?

Low Multiplicity

High Multiplicity
Underlying Event

LHC from 900 to 7000 GeV - ATLAS

Track Density (TRANS)

Not Infrared Safe
Large Non-factorizable Corrections
Prediction off by \( \approx 10\% \)

Sum(pT) Density (TRANS)

(more) Infrared Safe
Large Non-factorizable Corrections
Prediction off by < 10\%

R. Field: “See, I told you!”

Y. Gehrstein: “they have to fudge it again”
But Rivet+Professor (H. Hoeth) shows it fails miserably for UE (Rick Field’s transverse flow as function of jet $p_\perp$):

Where did we go wrong?

Note: not “we have to fudge it again”

MC Distributions courtesy of the Professor tuning Collaboration
Data from the CDF Underlying-Event studies
2 Days Ago … Pythia 8.140

A missing initial-final interference effect (coherence)
“Another change that I find disturbing is the rising tyranny of Carlo. No, I don’t mean that fellow who runs CERN, but the other one, with first name Monte.

The simultaneous increase in detector complexity and in computation power has made simulation techniques an essential feature of contemporary experimentation. The Monte Carlo simulation has become the major means of visualization of not only detector performance but also of physics phenomena. So far so good.

But it often happens that the physics simulations provided by the the MC generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data. All Monte Carlo codes come with a GIGO (garbage in, garbage out) warning label. But the GIGO warning label is just as easy for a physicist to ignore as that little message on a packet of cigarettes is for a chain smoker to ignore. I see nowadays experimental papers that claim agreement with QCD (translation: someone’s simulation labeled QCD) and/or disagreement with an alternative piece of physics (translation: an unrealistic simulation), without much evidence of the inputs into those simulations.”

Authors: Can we do better than the GIGO label? Uncertainty Bands
Users: Account for parameters + pertinent cross-checks and validations