European School of High Energy Physics 2010, Raseborg, Finland

QCD
Lecture 2

The Ultraviolet

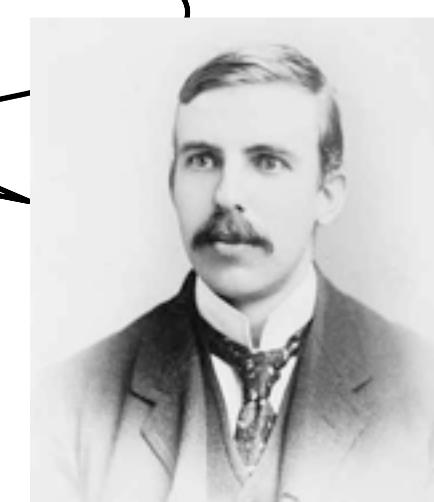
P. Skands

From Partons ...

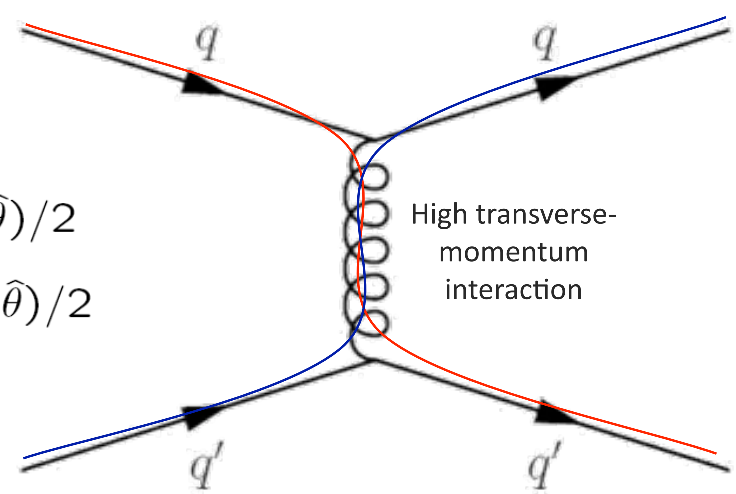
- Main Tool
  - Lowest-Order Matrix Elements calculated in a fixed-order perturbative expansion → parton-parton scattering cross sections

L=...

$$\mathrm{qq'} \to \mathrm{qq'} \quad : \quad \frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi}{\hat{s}^2} \frac{4}{9} \alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$



$$\hat{s} = (p_1 + p_2)^2$$
$$\hat{t} = (p_1 - p_3)^2 = -\hat{s}(1 - \cos\hat{\theta})/2$$
$$\hat{u} = (p_1 - p_4)^2 = -\hat{s}(1 + \cos\hat{\theta})/2$$



Question: what is the colour factor?

L → LanHEP/FeynRules → MadGraph/CompHEP/CalcHEP/… → partons

... to Pions


q
q
High transverse
momentum
interaction
q'
q'
Reality is more complicated

Complications

LO = ***Leading Order*** and ***Totally Inclusive***

Radiative corrections

- **Additional jets** change signal topology
- **K factors** change cross sections (total *and* differential)

Complications

LO = ***Perturbative*** and ***Factorized***

Hadronization, Underlying Event,
Beam Remnants, Hadron Decays, ...

- **No major changes** to event rates or topologies
- Aparatus > 1fm away from interaction point
- Important for **calibration** and **precision**

Overview

1. Fundamentals of QCD
2. QCD in the Ultraviolet
3. QCD in the Infrared
4. Monte Carlo Generators
5. Jets & Matching
6. Getting (kick)started with PYTHIA 8

Asymptotic Freedom

At High Energies

QCD is weak → quarks and gluons almost free

Smaller coupling

→ Perturbation theory better behaved

J.D. "BJ" Bjorken
(b. 1934)

Beware the Bjorken Scaling

Small absolute value of coupling, but …

Singular enhancements in soft/collinear regions

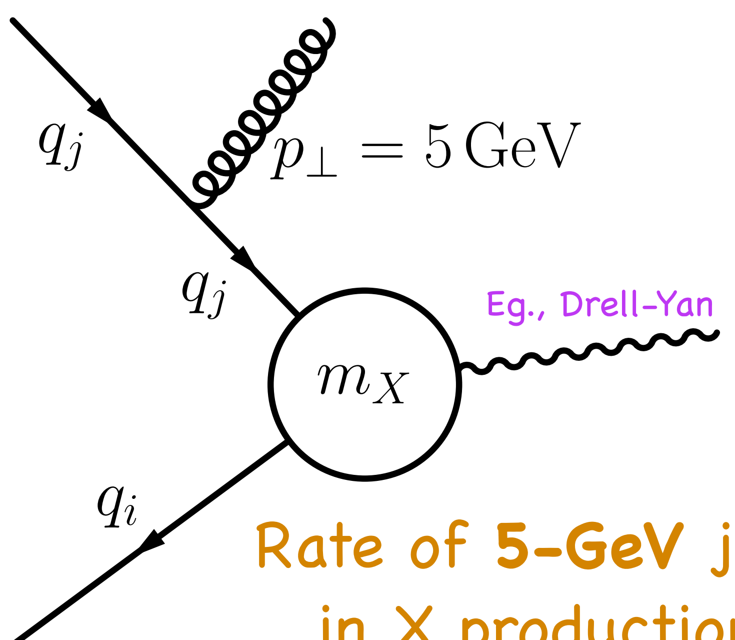
+ Dynamics ≈ conformal (Bjorken scaling)

⇒ Soft/collinear enhancements also scale …

Conformal QCD

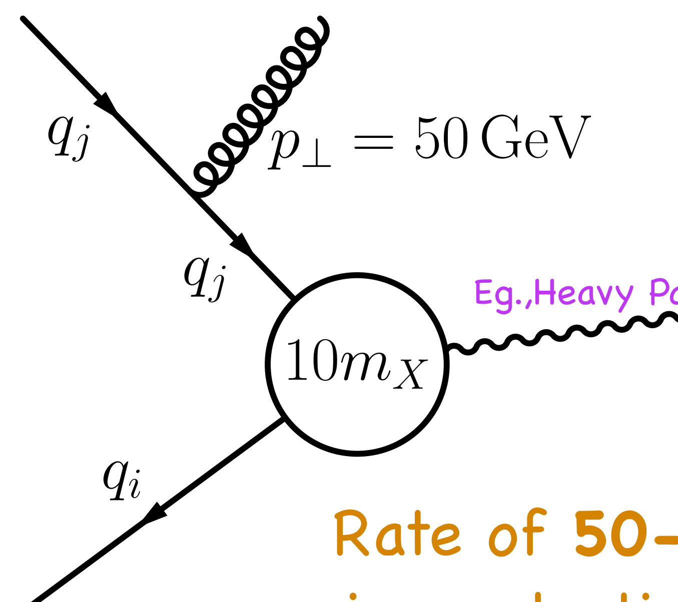
Bremsstrahlung

Rate of bremsstrahlung jets mainly depends on the **RATIO** of the jet p_T to the "hard scale"



Rate of **5-GeV** jets in X production

$\approx$



Rate of **50-GeV** jets in production of 10X

See, e.g., Plehn, Rainwater, PS: PLB645(2007)217
Plehn, Tait: 0810.2919 [hep-ph]
Alwall, de Visscher, Maltoni: JHEP 0902(2009)017

Conformal QCD

Naively, brems suppressed by $\alpha_s \approx 0.1$

Truncate at fixed order = LO, NLO, ...

But beware the jet-within-a-jet-within-a-jet ...

Know your signal
Especially if looking for decay jets of similar $p_\perp$

Example: 100 GeV can be "soft" at the LHC

SUSY pair production at 14 TeV, with $M_{SUSY} \approx$ 600 GeV

LHC - sps1a - m~600 GeV

Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	σ_{tot}[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive **X + 1 "jet"** →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive **X + 2 "jets"** →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

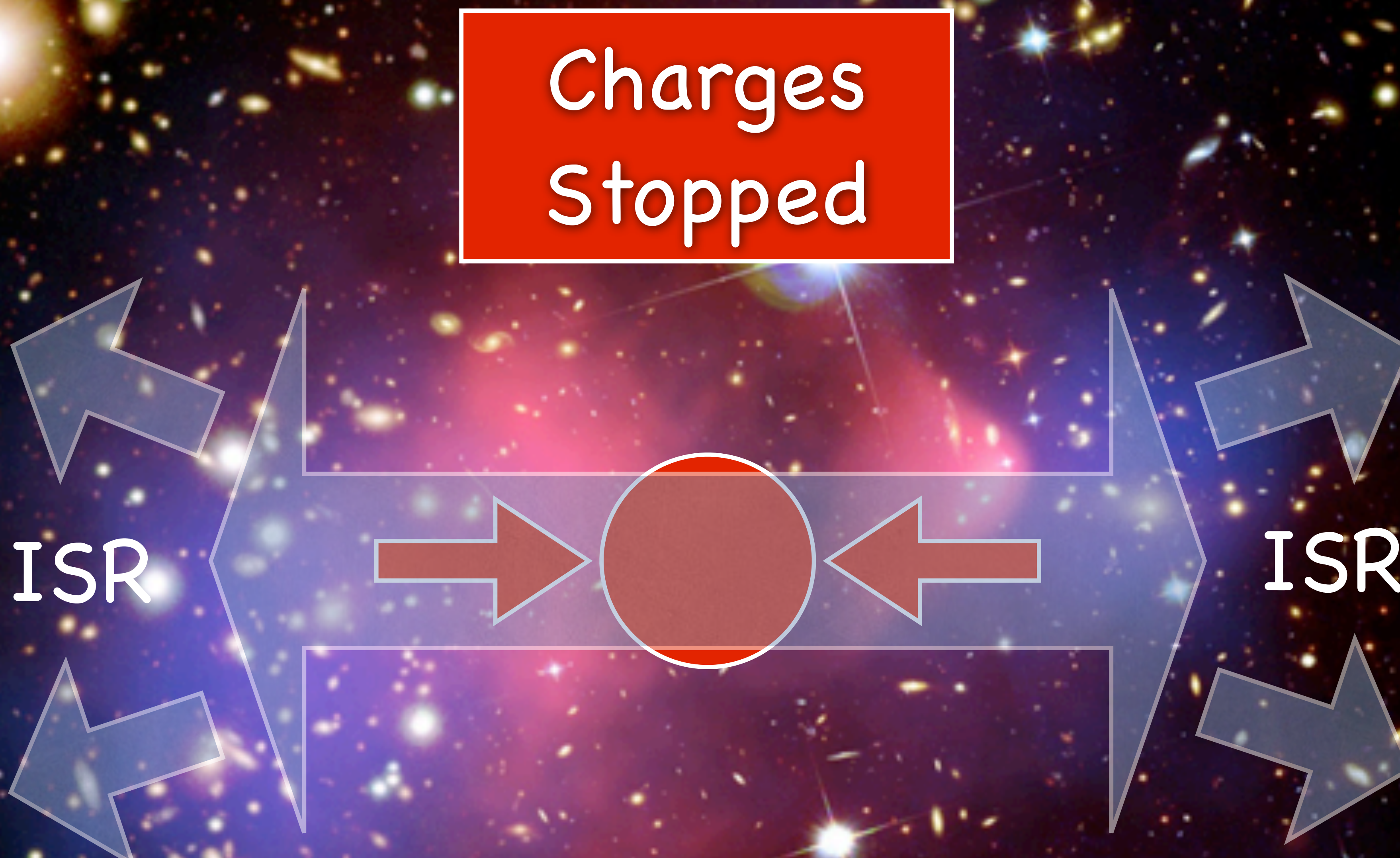
(Computed with SUSY-MadGraph)

σ for X + jets much larger than naive estimate

σ for 50 GeV jets ≈ larger than total cross section → not under control

Caused by the conformal nature of quantum fluctuations inside fluctuations inside fluctuations ...

Brems



The harder they stop, the harder the fluctations that continue to become strahlung

The Ultraviolet

Factorization

Factorization and Infrared Safety

Matrix Elements (fixed order pQCD)

LO, NLO, and all that

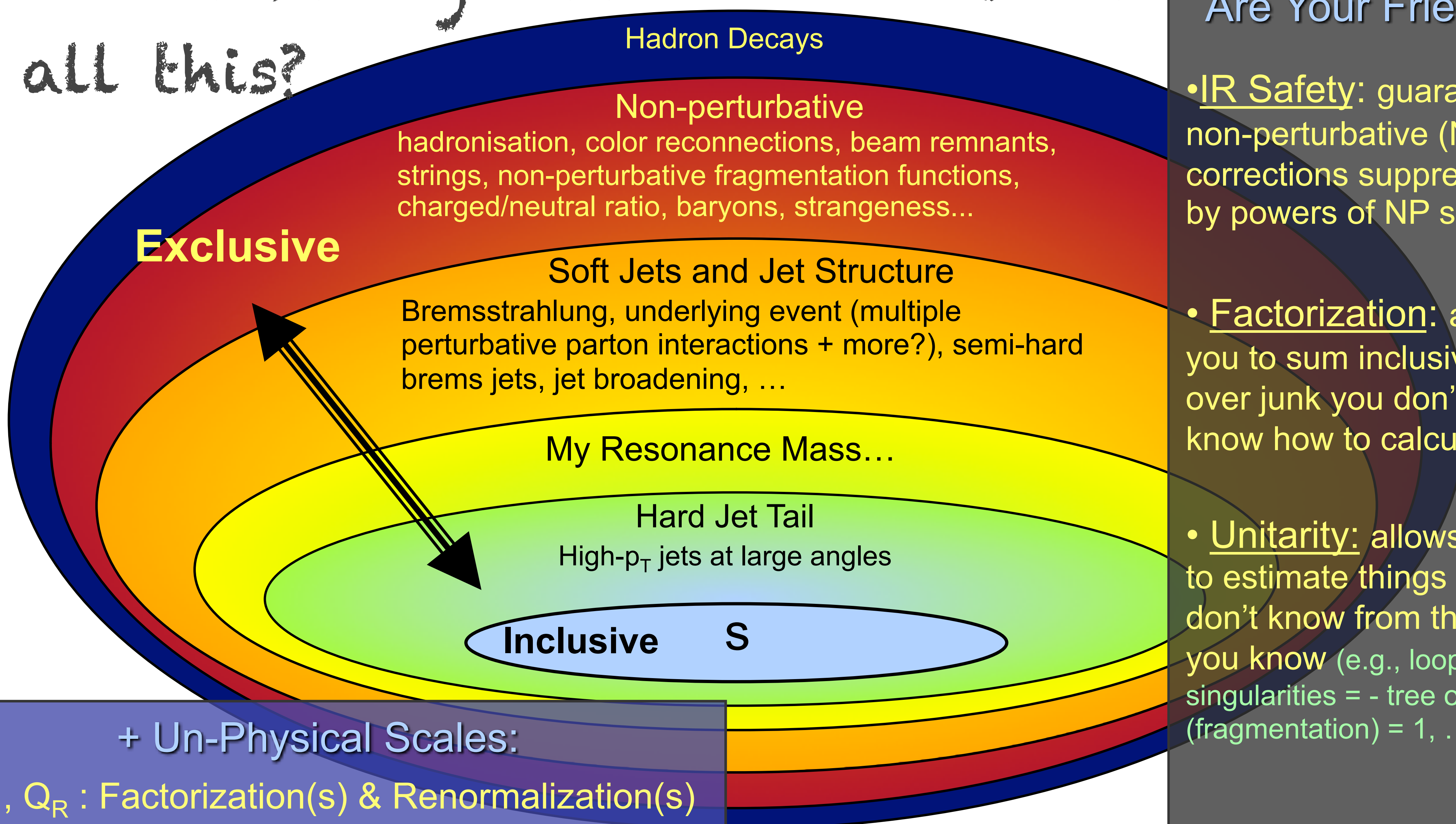
Region of applicability

Beyond Fixed Order

PDFs, Fragmentation functions, resummation

Collider Energy Scales

Do we really need to calculate all this?



These Things Are Your Friends

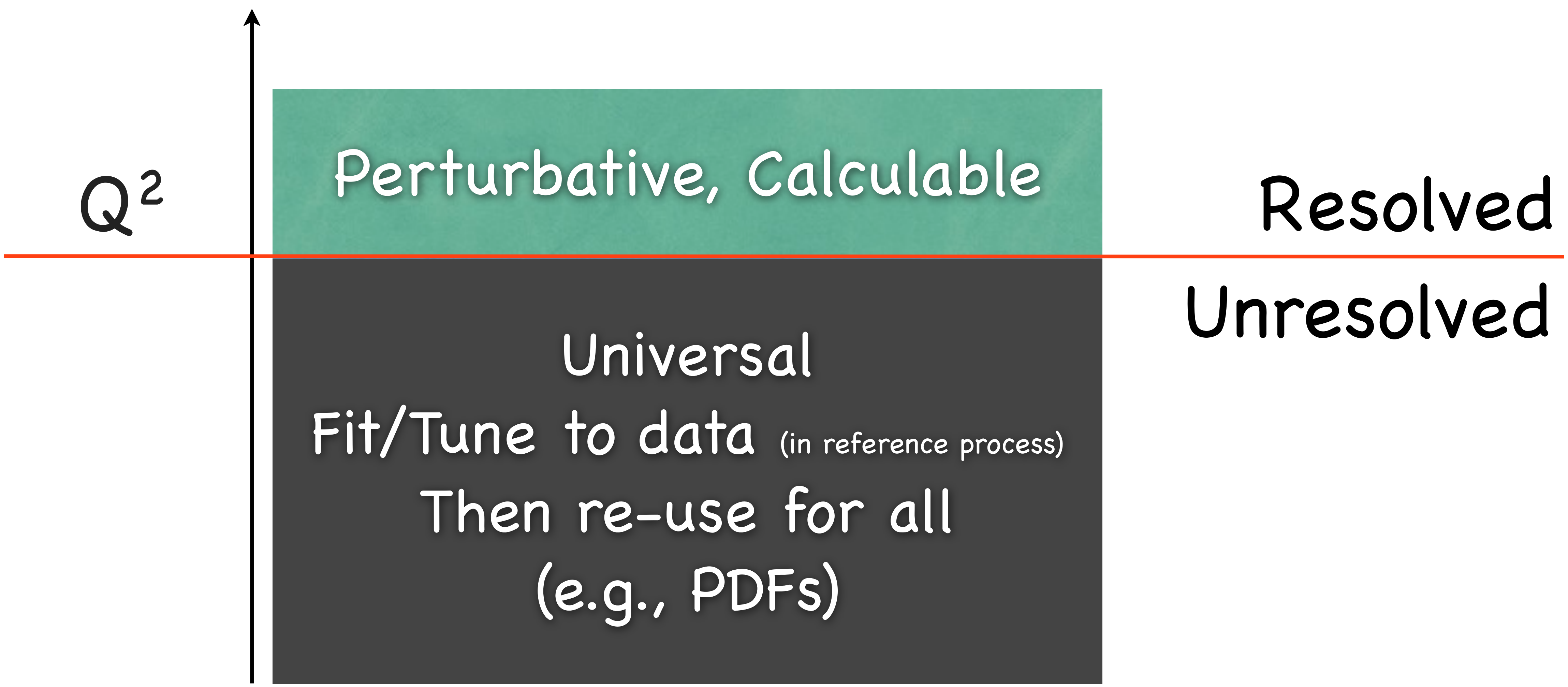
- IR Safety: guarantees non-perturbative (NP) corrections suppressed by powers of NP scale
- Factorization: allows you to sum inclusively over junk you don't know how to calculate
- Unitarity: allows you to estimate things you don't know from things you know (e.g., loop singularities = - tree ones; P (fragmentation) = 1, …)

+ Un-Physical Scales:

- Q_F , Q_R : Factorization(s) & Renormalization(s)
- Q_E : Evolution(s)

Factorization

Subdivide a calculation

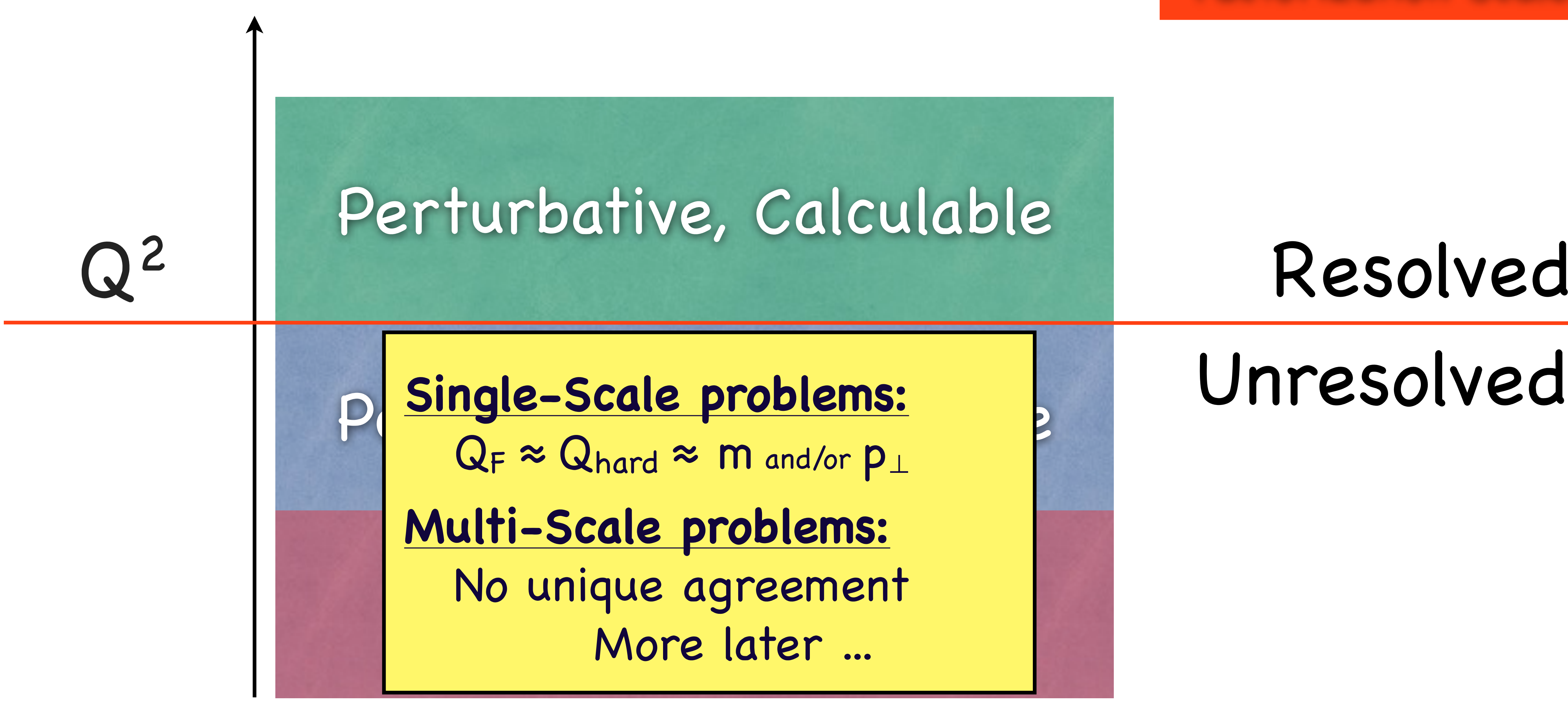


Factorization

Subdivide a calculation

Dependence on

Factorization Scale



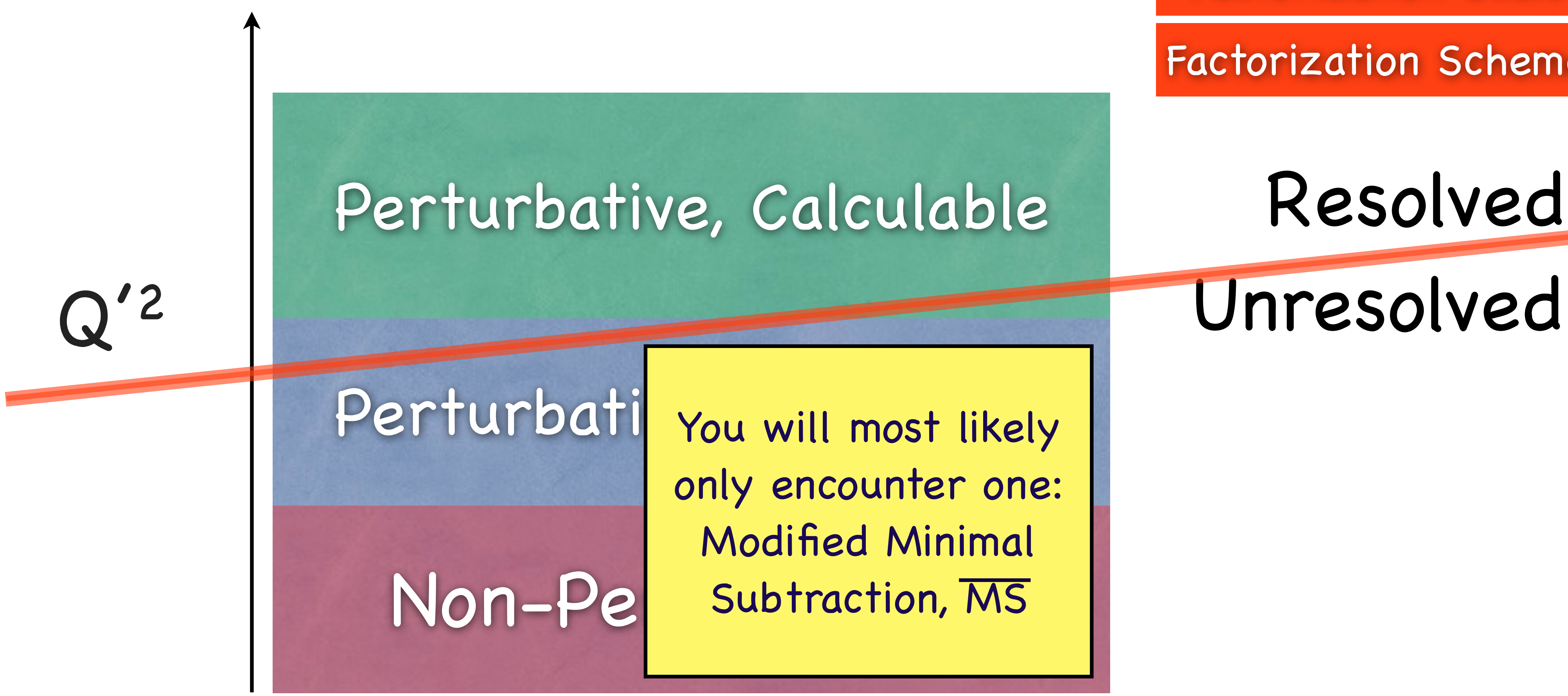
Factorization

Subdivide a calculation

Dependence on

Factorization Scale

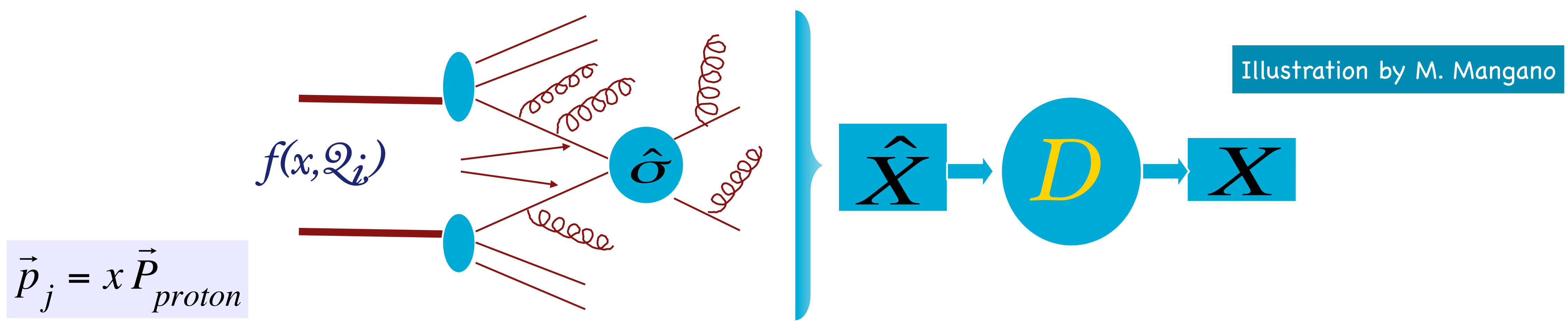
Factorization Scheme



Factorization Theorem

Factorization: expresses the independence of long-wavelength (soft) emission on the nature of the hard (short-distance) process.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b} \sum_f \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{\mathrm{d}\hat{\sigma}_{ab \to f}(x_a, x_b, f, Q_i^2, Q_f^2)}{\mathrm{d}\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$



$f_a(x_a, Q_i^2)$ Parton distribution functions (PDF)

$D(\hat{X}_f \to X, Q_i^2, Q_f^2)$ Fragmentation Function (FF)

- sum over long-wavelength histories leading to a with x_a at the scale Q_i^2 (ISR)

- Sum over long-wavelength histories from $\hat{X}_f$ at Q_f^2 to X (FSR and Hadronization)

+ (At H.O. each of these defined in a specific scheme, usually $\overline{\mathrm{MS}}$)

Matrix Elements

Fixed-Order perturbative QCD

QCD at Fixed Order

Distribution of observable: O

In production of X + anything

Sum over identical amplitudes, then square

Momentum configuration

Fixed Order
(all orders)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}}\bigg|_{\mathrm{ME}} = \sum_{k=0} \int \mathrm{d}\Phi_{X+k} \left| \sum_{\ell=0} M^{(\ell)}_{X+k} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

Phase Space

Cross Section differentially in O

Sum over "anything" ≈ legs

Matrix Elements for X+k at (l) loops

Evaluate observable → differential in O

Truncate at k=0, l=0

→ **Born Level = First Term**

Lowest order at which X happens

QCD at Fixed Order

Distribution of observable: O

In production of X + anything

Fixed Order
(all orders)

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}}\right|_{\mathrm{ME}} = \sum_{k=0} \int \mathrm{d}\Phi_{X+k} \left| \sum_{\ell=0} M_{X+k}^{(\ell)} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

Sum over identical amplitudes, then square

Momentum configuration

Phase Space

Cross Section differentially in O

Sum over "anything" ≈ legs

Matrix Elements for X+k at (l) loops

Evaluate observable → differential in O

Truncate at k=n, l=0

→ **Leading Order** for X + n

Lowest order at which X + n happens

QCD at Fixed Order

Distribution of observable: O

In production of X + anything

Sum over identical amplitudes, then square

Momentum configuration

Fixed Order (all orders)

$$\left.\frac{d\sigma}{d\mathcal{O}}\right|_{ME} = \sum_{k=0} \int d\Phi_{X+k} \left| \sum_{\ell=0} M^{(\ell)}_{X+k} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

Phase Space

Cross Section differentially in O

Sum over "anything" ≈ legs

Matrix Elements for X+k at (l) loops

Evaluate observable → differential in O

Truncate at k+l = n

→ N^nLO for X

Includes $N^{n-1}LO$ for X+1, $N^{n-2}LO$ for X+2, ...

Fixed-Order Monte Carlo

(e.g., AlpGen, CalcHEP, CompHEP, MadGraph, ...)

High-dimensional problem (phase space)

d≥5 ➔ Monte Carlo integration

“Monte Carlo”: N. Metropolis, first Monte Carlo calculation on ENIAC (1948), basic idea goes back to Enrico Fermi

“Experimental” distribution of observable O in production of X:

Fixed Order (all orders)

$$\left.\frac{d\sigma}{d\mathcal{O}}\right|_{ME} = \sum_{k=0} \int d\Phi_{X+k} \left| \sum_{\ell=0} M^{(\ell)}_{X+k} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

k : legs

ℓ : loops

{p} : momenta

Principal virtues

1. Stochastic error $O(N^{-1/2})$ independent of dimension
2. Full (perturbative) quantum treatment at each order
3. (KLN theorem: finite answer at each (complete) order)

Note 1: For k larger than a few, need to be quite clever in phase space sampling

Note 2: For $k+\ell > 0$, need to be careful in arranging for real-virtual cancellations

Loops and Legs

Another representation

$X^{(2)}$	$X{+}1^{(2)}$	...		
$X^{(1)}$	$X{+}1^{(1)}$	$X{+}2^{(1)}$	$X{+}3^{(1)}$	...
Born	$X{+}1^{(0)}$	$X{+}2^{(0)}$	$X{+}3^{(0)}$	...

Loops

Legs

Loops and Legs

Another representation

Born

M. Born
(1882-1970)
Nobel 1954

Loops

$X^{(2)}$	$X+1^{(2)}$	...		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	...
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	...

Legs

Loops and Legs

Another representation

X+2 @ LO

Loops					
	$X^{(2)}$	$X+1^{(2)}$	...		
	$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	...
	Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	...

Legs

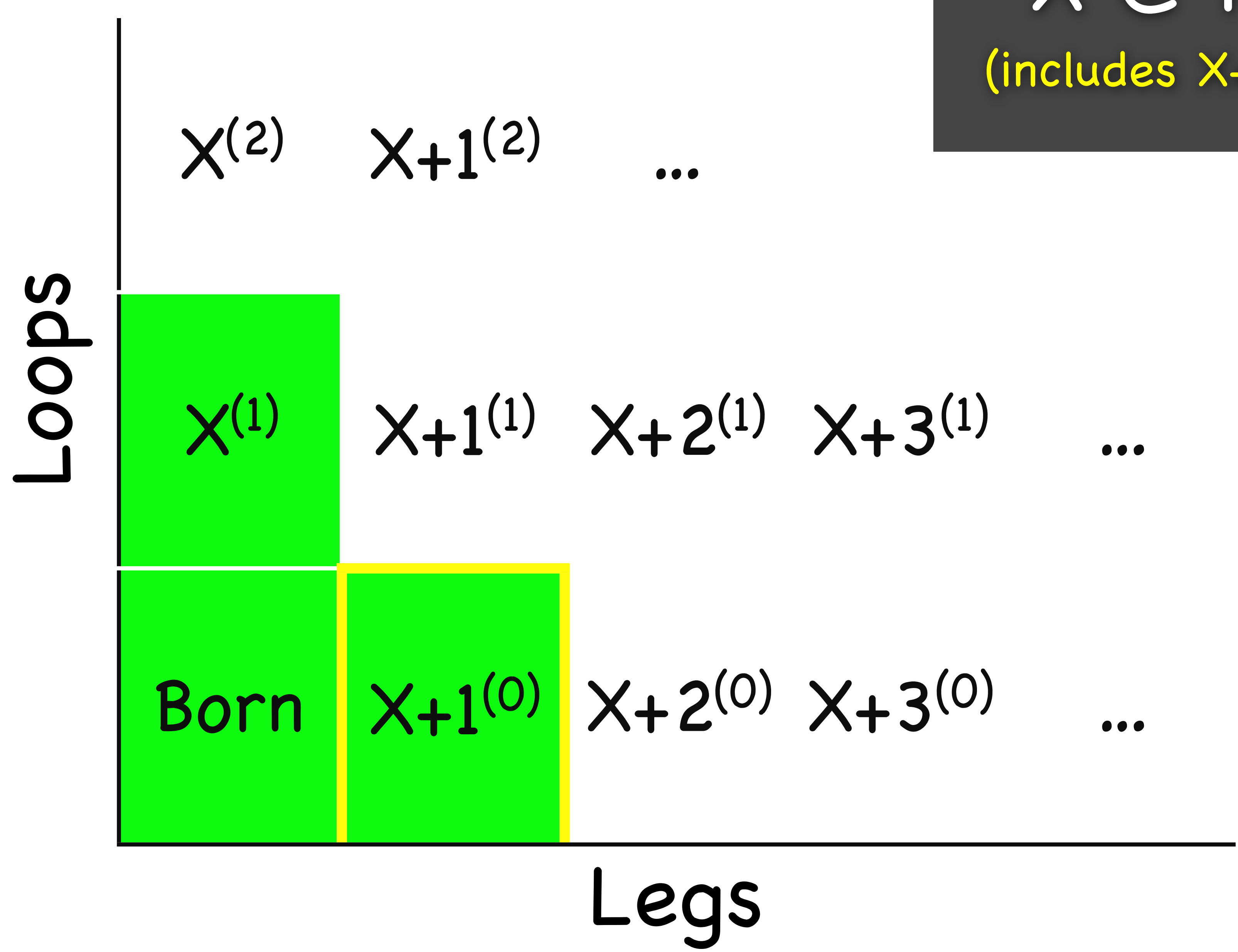
Note: $\sigma \to \infty$ if both jets not resolved

Loops and Legs

Another representation

X @ NLO

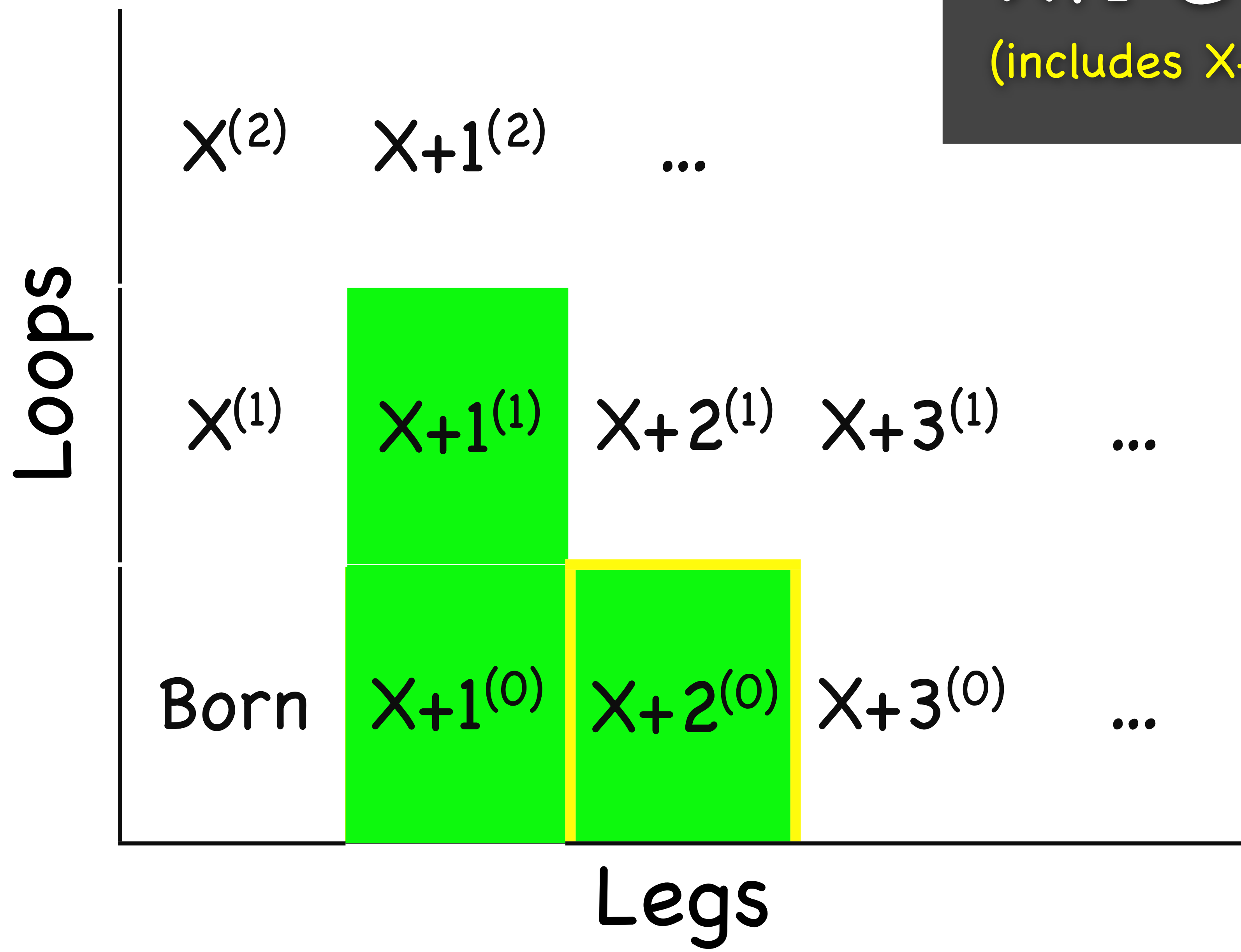
(includes X+1 @ LO)



Note: X+1 jet observables only correct at LO

Loops and Legs

Another representation



X+1 @ NLO

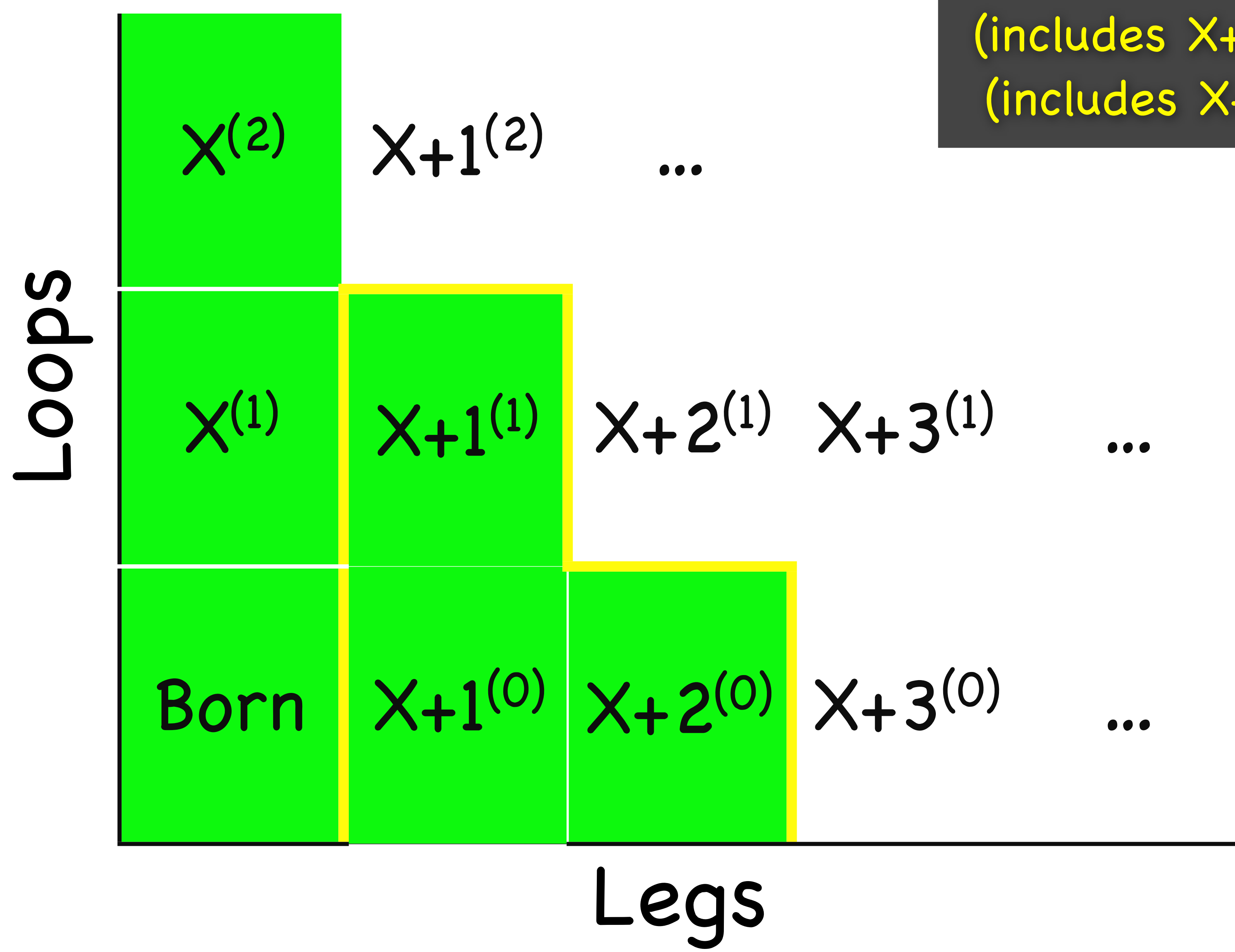
(includes X+2 @ LO)

Note: $\sigma \to \infty$ if no jet resolved

Note: X+2 jet observables only correct at LO

Loops and Legs

Another representation



X @ NNLO
(includes X+1 @ NLO)
(includes X+2 @ LO)

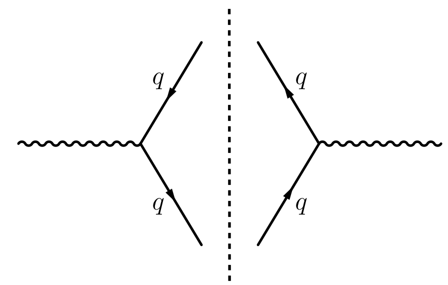
$\sigma \to \sigma_{NNLO}$
if no jet resolved

Note: X+2 jet observables only correct at LO

Cross sections at LO

Born:

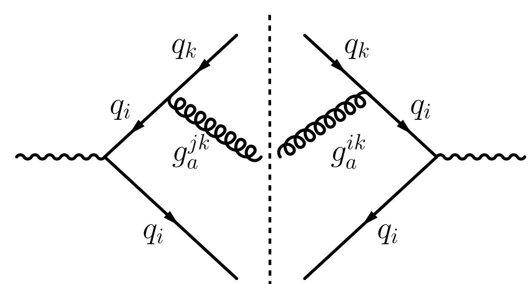
$$\sigma_{\text{Born}} = \int |M_X^{(0)}|^2$$



$X^{(2)}$	$X{+}1^{(2)}$	...
$X^{(1)}$	$X{+}1^{(1)}$	...
Born	$X{+}1^{(0)}$	$X{+}2^{(0)}$

Born + n

$$\sigma^{\text{LO}}_{\text{X+1}}(R) = \int_R |M^{(0)}_{X+1}|^2$$



$X^{(2)}$	$X{+}1^{(2)}$	...
$X^{(1)}$	$X{+}1^{(1)}$	...
Born	$X{+}1^{(0)}$	$X{+}2^{(0)}$

Infrared divergent → Must be regulated

R = some Infrared Safe phase space region

(Often a cut on p⊥ > n GeV)

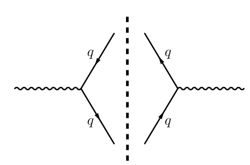
Careful not to take it too low!

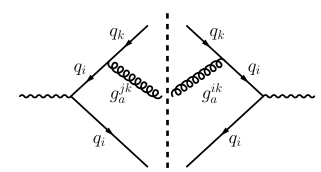
if σ(X+n) ≈ σ(X) you got a problem

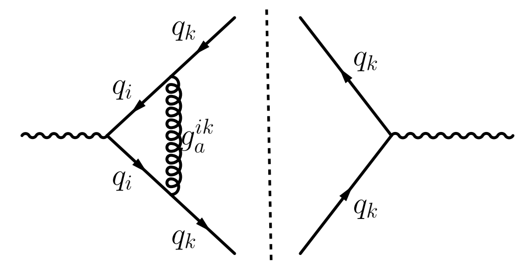
perturbative expansion not reliable

Cross sections at NLO

NLO:







$X^{(2)}$	$X+1^{(2)}$	...
$X^{(1)}$	$X+1^{(1)}$	...
Born	$X+1^{(0)}$	$X+2^{(0)}$

$$\sigma_{\mathrm{X}}^{\mathrm{NLO}} = \int |M_X^{(0)}|^2 + \int |M_{X+1}^{(0)}|^2 + \int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]$$

(note: Not the 1-loop diagram squared)

KNL Theorem (Kinoshita-Lee-Nauenberg)

Singularities cancel at complete order (only finite terms left over)

Lemma: only after some hard work

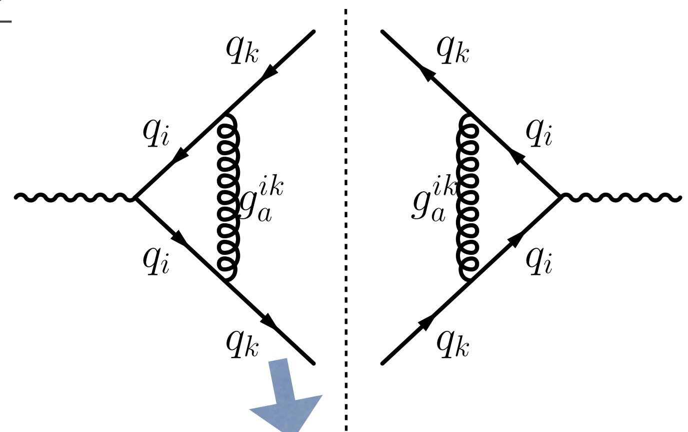
$$= \sigma_{\mathrm{Born}} + \mathrm{Finite}\left\{\int |M_{X+1}^{(0)}|^2\right\} + \mathrm{Finite}\left\{\int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]\right\}$$

$$\sigma_1(e^+e^- \to q\bar{q}(g)) = \sigma_0(e^+e^- \to q\bar{q})\left(1 + \frac{\alpha_s(E_{CM})}{\pi} + O(\alpha_s^2)\right)$$

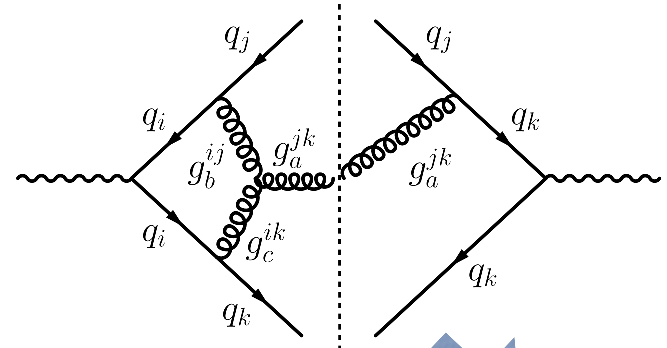
Cross Sections at NNLO

NNLO

1-Loop × 1-Loop

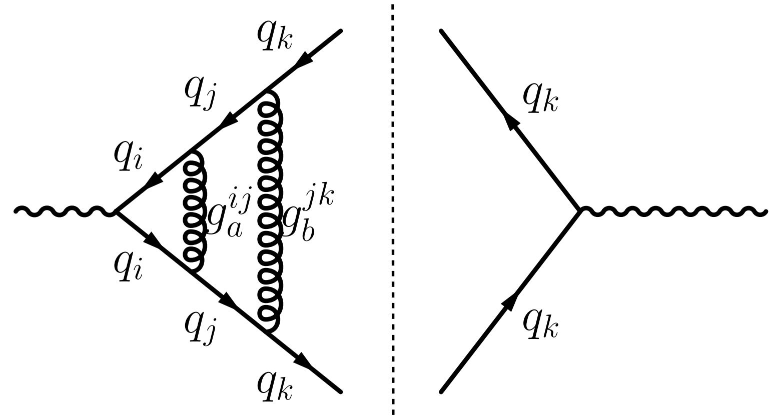


1-Loop × Real (X+1)

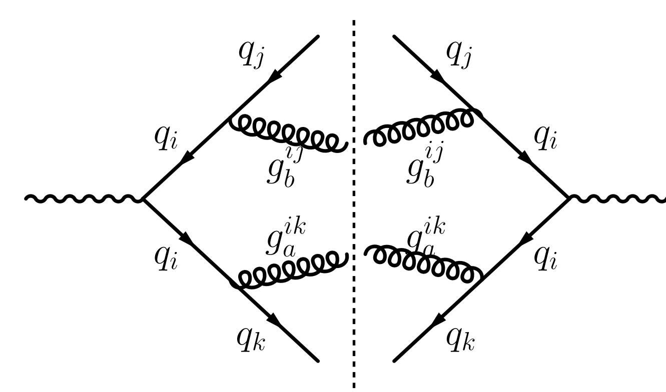


$X^{(2)}$	$X+1^{(2)}$	...
$X^{(1)}$	$X+1^{(1)}$	...
Born	$X+1^{(0)}$	$X+2^{(0)}$

$$\sigma_{\mathrm{X}}^{\mathrm{NNLO}} = \sigma_X^{\mathrm{NLO}} + \int \left(|M_X^{(1)}|^2 + 2\mathrm{Re}[M_X^{(2)} M_X^{(0)*}] \right) + \int 2\mathrm{Re}[M_{X+1}^{(1)} M_{X+1}^{(0)*}] + \int |M_{X+2}^{(0)}|^2$$



Two-Loop × Born Interference



Real × Real (X+2)

Fixed-Order QCD

What kind of observables can we evaluate this way?

Perturbation theory valid → α_s must be small

→ All $Q_i \gg \Lambda_{QCD}$

Multi-scale: abensence of enhancements from soft/collinear singular (conformal) dynamics

→ All $Q_i/Q_j \approx 1$

All resolved scales $\gg \Lambda_{QCD}$ **AND** no large hierarchies*

*) At "leading twist" (not counting underlying event)

Fixed-Order QCD

All resolved scales >> Λ_{QCD} AND no large hierarchies*

*)At "leading twist" (not counting underlying event)

Trivially untrue for QCD

We're colliding, and observing, hadrons → small scales

We want to consider high-scale processes → large scale differences

→ A Priori, no perturbatively calculable observables in hadron-hadron collisions

Resummed QCD

All resolved scales >> Λ_{QCD} AND no large hierarchies*

*)At "leading twist" (not counting underlying event)

Trivially untrue for QCD

We're colliding, and observing, hadrons → small scales

We want to consider high-scale processes → large scale differences

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b} \sum_{f} \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a, x_b, f, Q_i^2, Q_f^2)}{\mathrm{d}\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-) exclusive cross sections

All resolved scales >> Λ_{QCD} AND X Infrared Safe

*)At "leading twist" (not counting underlying event)

Beyond Fixed Order

Resummation

Parton Densities & Fragmentation Functions

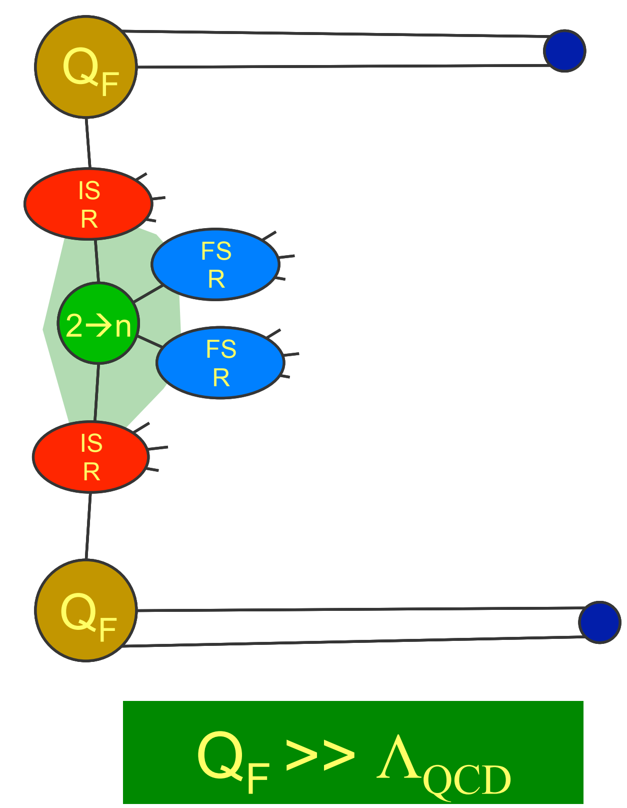
Resummed QCD

▶ Starting point: Matrix Elements

n = a handful
+ resonance
decays

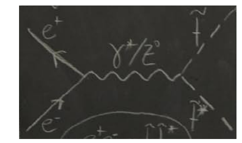
2→n hard parton scattering at (N)LO

\+ Bremsstrahlung ➔ 2→∞ at (N)LL



Bremsstrahlung

“DLA” $\frac{\alpha\, s_{ab}}{s_{ai}s_{ib}}$

$$d\sigma_X = \ldots$$ 

$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}} \frac{ds_{1b}}{s_{1b}}$$

$$d\sigma_{X+2} \sim 2g^2 d\sigma_{X+1} \frac{ds_{a2}}{s_{a2}} \frac{ds_{2b}}{s_{2b}}$$

$$d\sigma_{X+3} \sim 2g^2 d\sigma_{X+2} \frac{ds_{a3}}{s_{a3}} \frac{ds_{3b}}{s_{3b}}$$

Interpretation: the structure evolves

But something’s not right…

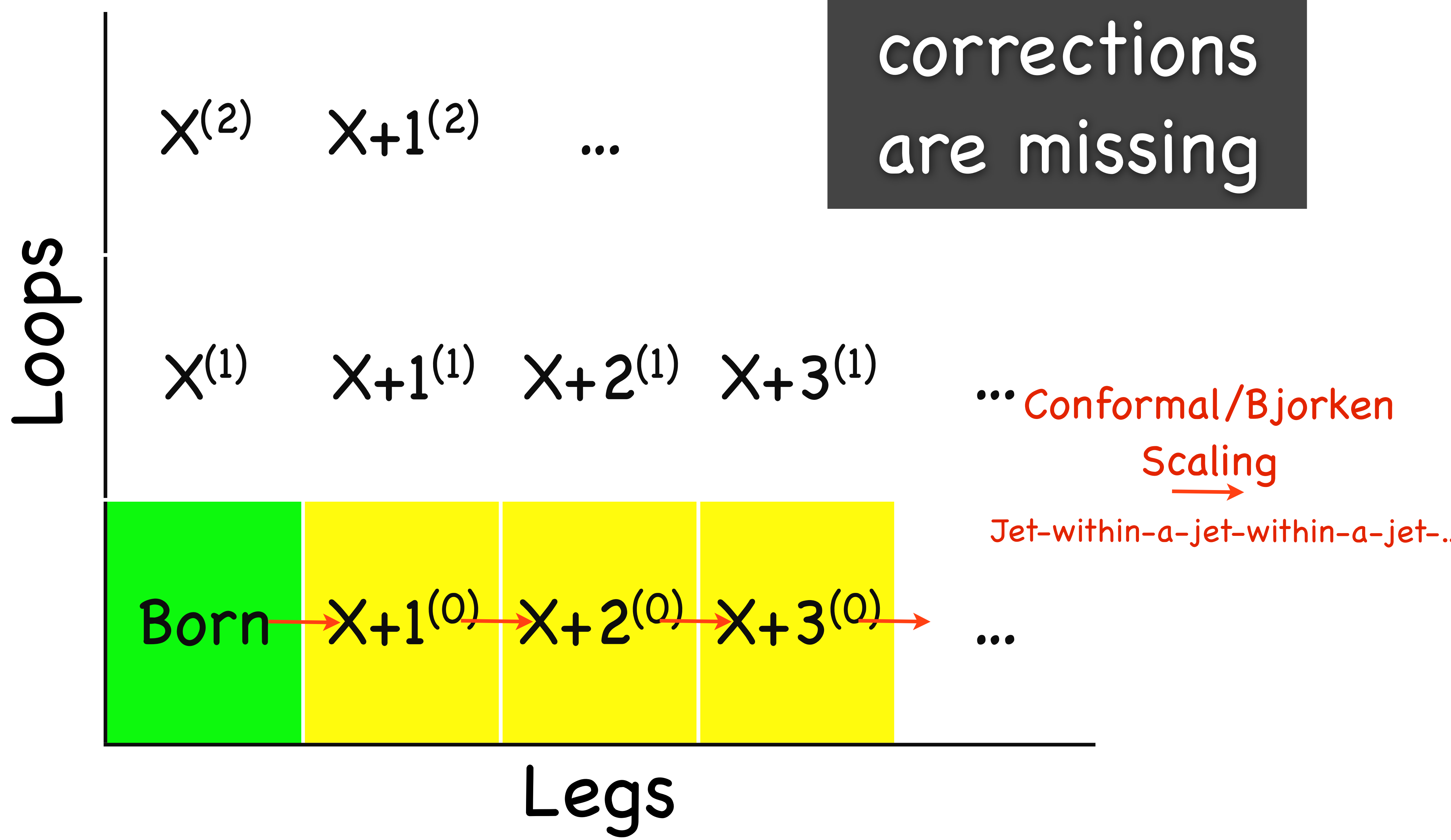
This is an approximation to inifinite-order tree-level cross sections

Total cross section would be infinite …

Loops and Legs

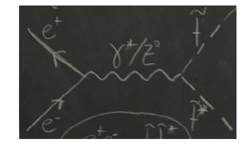
Summation

The Virtual corrections are missing



Resummation

"DLA" $\frac{\alpha\, s_{ab}}{s_{ai}s_{ib}}$

$$d\sigma_X = \ldots$$ 

$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}}\frac{ds_{1b}}{s_{1b}}$$

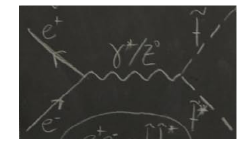
$$d\sigma_{X+2} \sim 2g^2 d\sigma_{X+1} \frac{ds_{a2}}{s_{a2}}\frac{ds_{2b}}{s_{2b}}$$

$$d\sigma_{X+3} \sim 2g^2 d\sigma_{X+2} \frac{ds_{a3}}{s_{a3}}\frac{ds_{3b}}{s_{3b}}$$

- ► Interpretation: the structure evolves! (example: X = 2-jets)
  - Take a jet algorithm, with resolution measure "Q", apply it to your events
  - At a very crude resolution, you find that everything is 2-jets

Resummation

“DLA” $\dfrac{\alpha\, s_{ab}}{s_{ai} s_{ib}}$

$d\sigma_X = \ldots$ 

$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}} \frac{ds_{1b}}{s_{1b}}$$

$$d\sigma_{X+2} \sim 2g^2 d\sigma_{X+1} \frac{ds_{a2}}{s_{a2}} \frac{ds_{2b}}{s_{2b}}$$

$$d\sigma_{X+3} \sim 2g^2 d\sigma_{X+2} \frac{ds_{a3}}{s_{a3}} \frac{ds_{3b}}{s_{3b}}$$

KLN

+ UNITARITY:

Virt = – Int(Tree) + F

(or: given a jet definition, an event has *either* 0, 1, 2, or *n* jets)

Interpretation: the structure evolves

$\sigma_{X+1}(Q) = \sigma_{X;incl} - \sigma_{X;excl}(Q)$

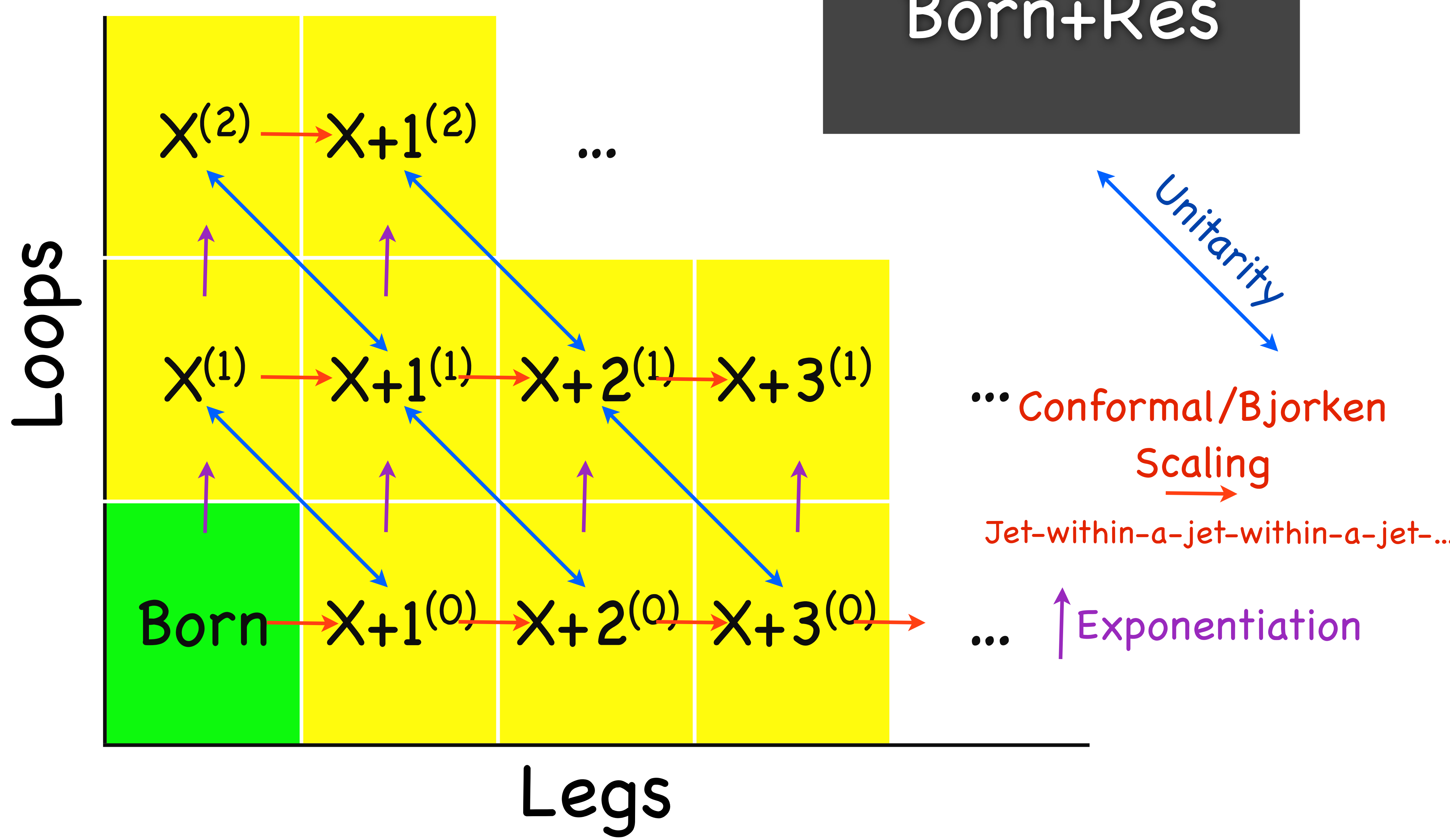
This includes both real and virtual corrections

$$\begin{aligned} \sigma_{X;\mathrm{excl}} &= \sigma_X - \sigma_{X+1} \\ &= \sigma_X - \sigma_{X+1;\mathrm{excl}} - \sigma_{X+2;\mathrm{excl}} - \ldots \end{aligned}$$

Loops and Legs

Resummation

Born+Res



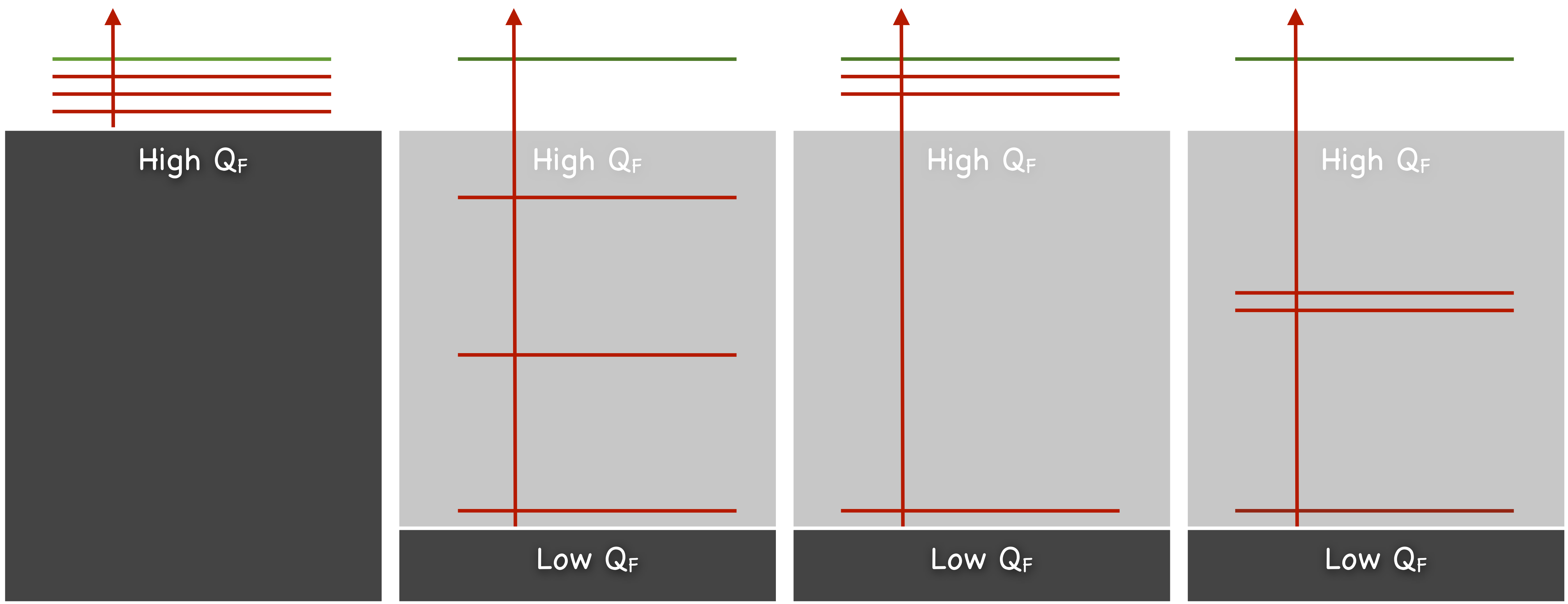
Structures in pQCD

X+3 hard jets

X+3 strongly ordered emissions

X+2 hard jets + ord. ems.

X+2 unordered emissions + 1 ord.



$\alpha_X \times \alpha_s^3$
(X+3) (no hierarchy)
→ Order α_s^3 matrix elements + PDFs OK

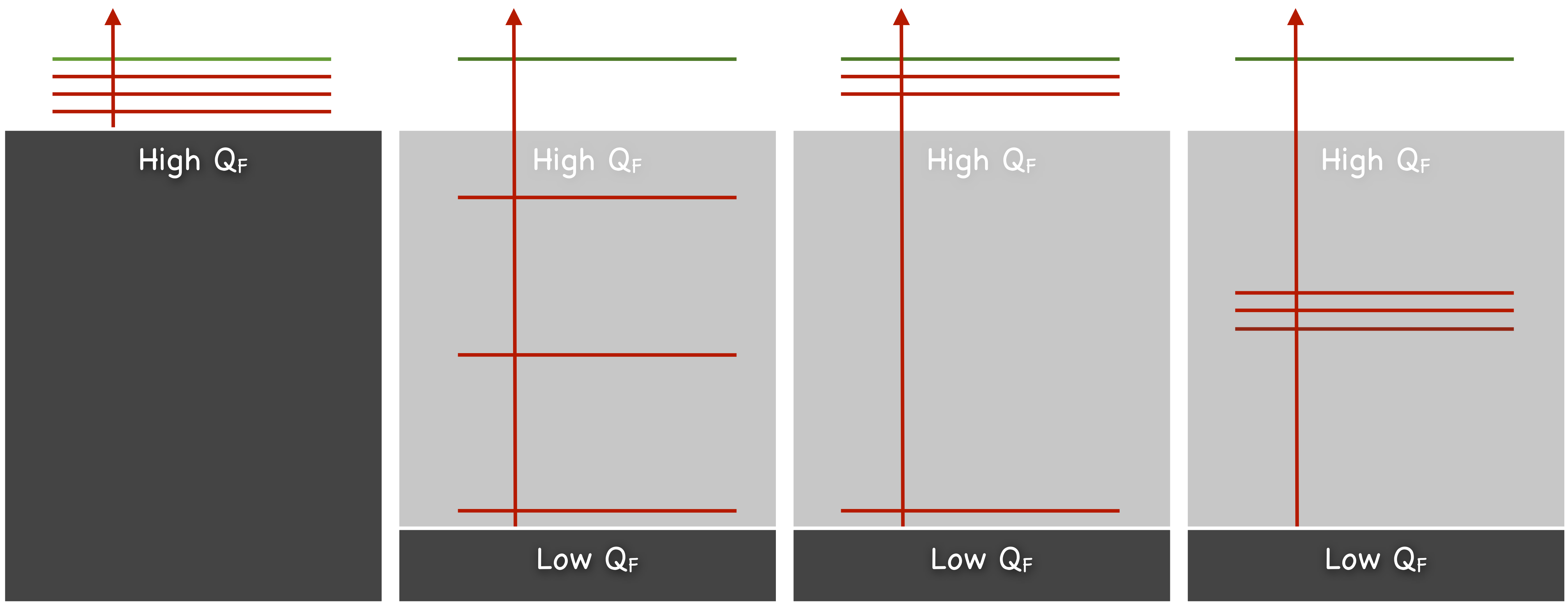
$\alpha_X \times \alpha_s^3$
X+1+1+1 hierarchy
(strongly ordered)
→ Born + LL = OK

$\alpha_X \times \alpha_s^3$
(X+2)+1 hierarchy
→ Order α_s^2 ME + LL + PDFs = OK

$\alpha_X \times \alpha_s^3$
X+2+1 hierarchy
→ Born + NLL = OK

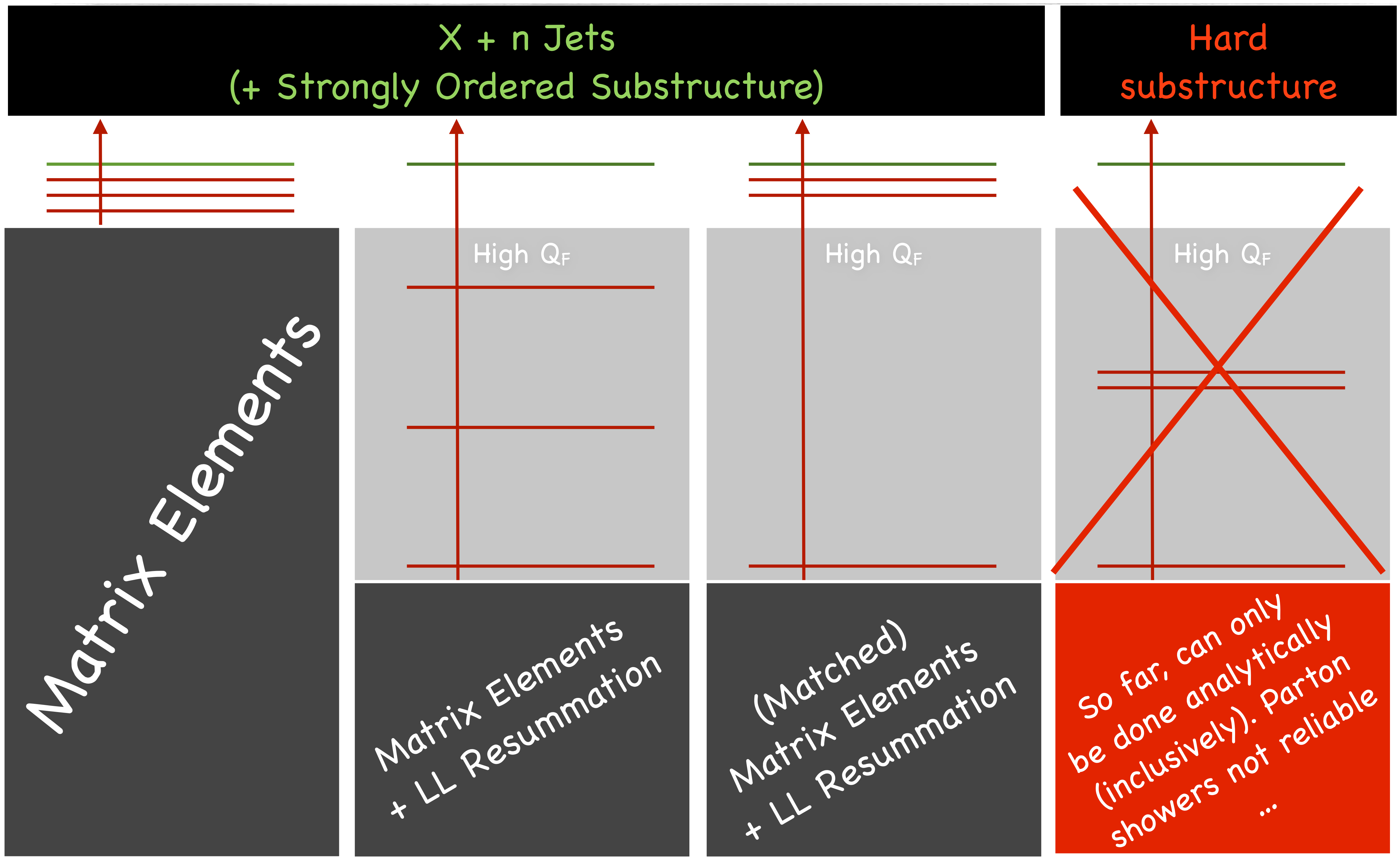
Structures in pQCD

X+3 hard jets	X+3 strongly ordered emissions	X+2 hard jets + ord. ems.	X+3 unordered emissions



$\alpha_X \times \alpha_s^3$	$\alpha_X \times \alpha_s^3$	$\alpha_X \times \alpha_s^3$	$\alpha_X \times \alpha_s^3$
(X+3) (no hierarchy) → Order α_s^3 matrix elements + PDFs OK	X+1+1+1 hierarchy (strongly ordered) → Born + LL = OK	(X+2)+1 hierarchy → Order α_s^2 ME + LL + PDFs = OK	X+3 hierarchy → Born + NNLL = OK

Structures in pQCD



Uncertainties

Uncalculated Orders

Can be large if we're in uncontrolled region

- e.g, "conformal" examples before
- How to know?

How to estimate? (reliably?)

+ Non-Perturbative Effects

IR safety → as small as possible

IR safety → perturbative singularities cancel among themselves

+ Non-Factorizable Effects

Will get back to these tomorrow

Uncalculated Orders

Naively $O(\alpha_s)$ - True in e^+e^- !

$$\sigma_1(e^+e^- \to q\bar{q}(g)) = \sigma_0(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{CM})}{\pi} + O(\alpha_s^2)\right)$$

Generally larger in hadron collisions

- Typical "K" factor in pp (= σ_{NLO}/σ_{LO}) ≈ 1.5 ± 0.5
- Why is this? Many pseudoscientific explanations
  - Explosion of # of diagrams ($n_{Diagrams} \approx n!$)
  - New initial states contributing at higher orders (E.g., $gq \to Zq$)
  - Inclusion of low-x (non-DGLAP) enhancements
  - Bad (high) scale choices at Lower Orders, ...

Their's not to reason why // Their's but to do and die

***The Charge of the Light Brigade**, by Alfred, Lord Tennyson*

1. Changing the scale(s)

Why scale variation ~ uncertainty?

Scale dependence of calculated orders must be canceled by contribution from uncalculated ones (+ non-pert)

$$\alpha_s(Q^2) = \alpha_s(m_Z^2)\frac{1}{1 + b_0\,\alpha_s(m_Z)\ln\frac{Q^2}{m_Z^2} + \mathcal{O}(\alpha_s^2)}$$

$$b_0 = \frac{11N_C - 2n_f}{12\pi}$$

→ $\alpha_s(Q'^2)\,|M|^2 - \alpha_s(Q^2)\,|M|^2 \approx \alpha_s^2(Q^2)\,|M|^2 + \ldots$

→ Generates terms of higher order, but proportional to what you already have → a first naive* way to estimate uncertainty

*warning: some theorists believe it is the only way … but be agnostic! There are other things than scale dependence …

Dangers

$p_{\perp 1}$= 50 GeV
$p_{\perp 2}$= 50 GeV
$p_{\perp 3}$= 50 GeV

Complicated final states

Intrinsically Multi-Scale problems

with Many powers of α_s

Hardest imaginable scale

E.g., W + 3 jets in pp

6 8

$$\alpha_s^3(m_W^2) < \alpha_s^3\left(m_W^2 + \langle p_\perp^2 \rangle\right) < \alpha_s^3\left(m_W^2 + \sum_i p_{\perp i}^2\right)$$

7

Global Scaling: jets don't care about m_W

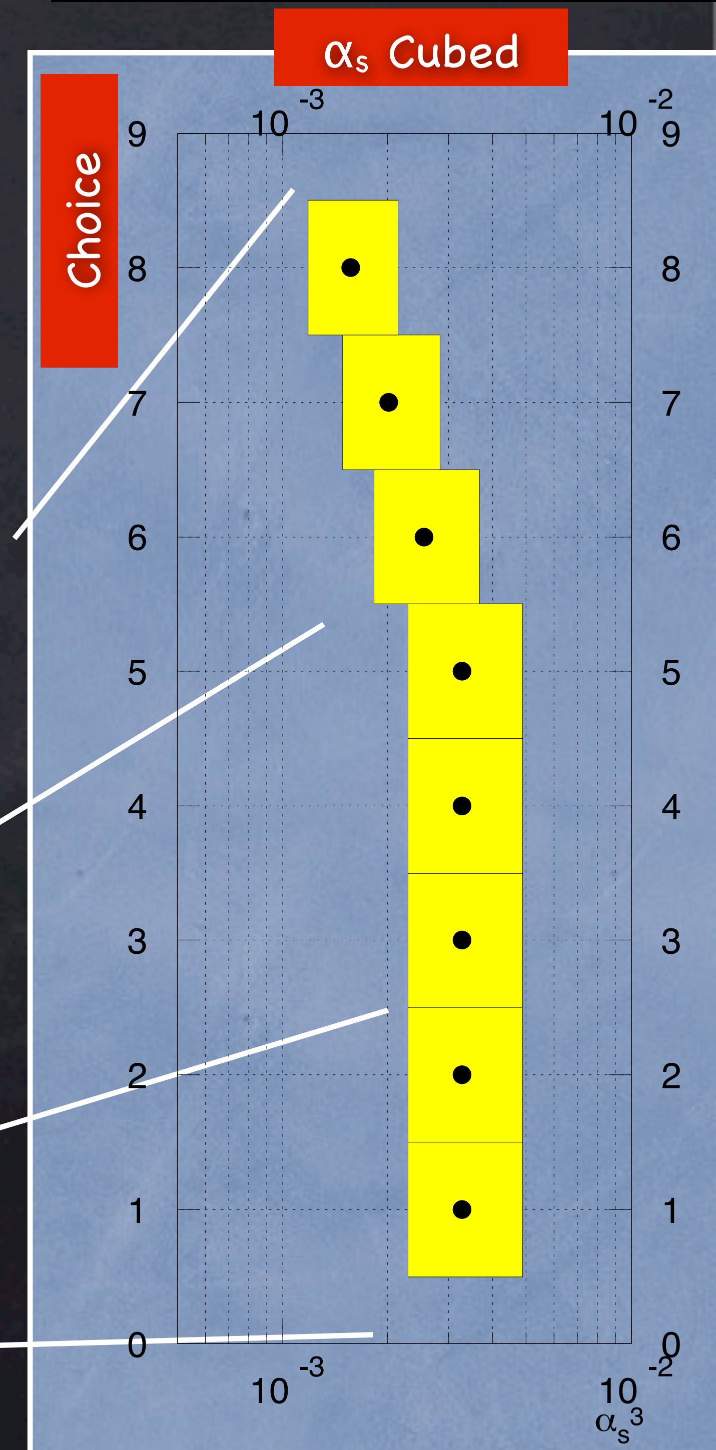
4

3 $$\alpha_s^3(\min[p_\perp^2]) < \alpha_s^3(\langle p_\perp^2 \rangle) < \alpha_s^3(\max[p_\perp^2])$$ 5

MC parton showers: "Local scaling"

$$\alpha_s(p_{\perp 1})\alpha_s(p_{\perp 2})\alpha_s(p_{\perp 3}) \sim \alpha_s^3\left(\langle p_\perp^2 \rangle_{\mathrm{geom}}\right)$$

2 1



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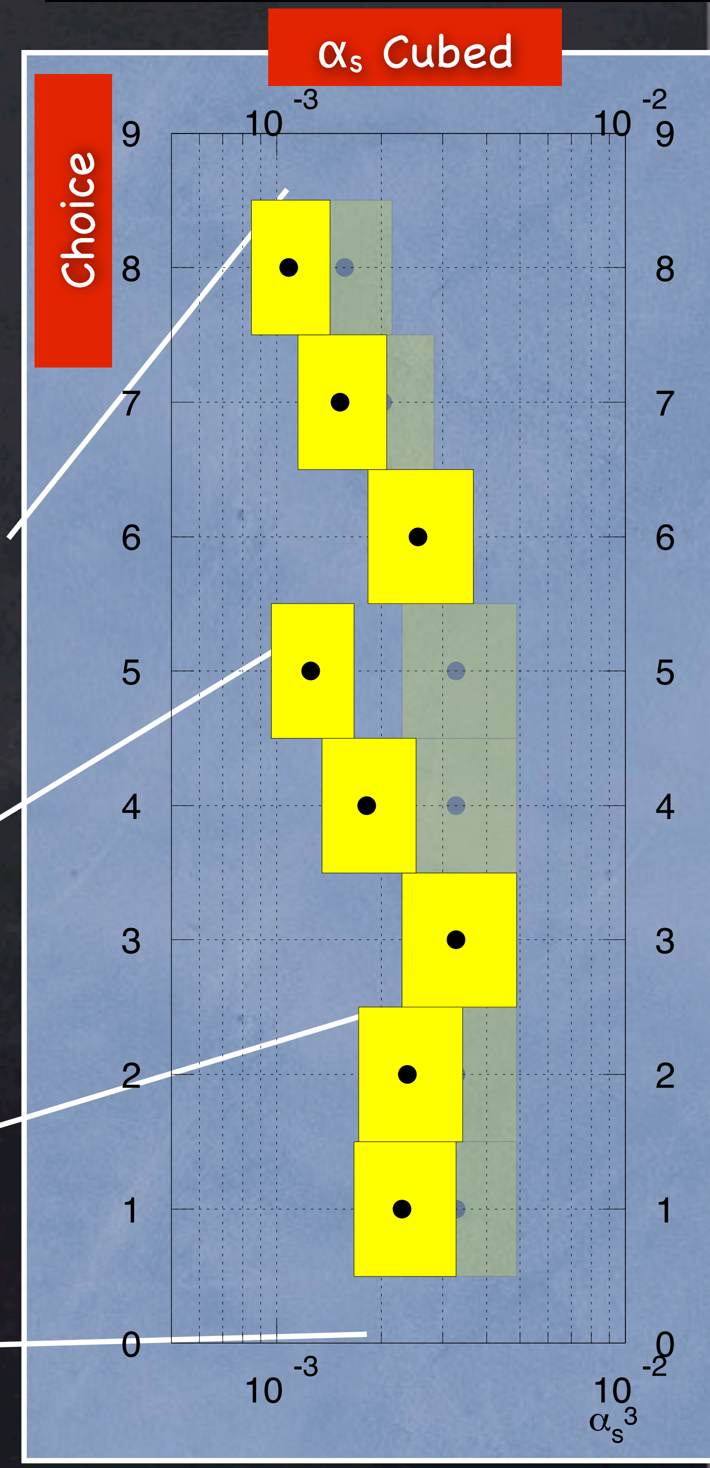

Whatever they might tell you

If you have multiple QCD scales

→ variation of μ_R by factor 2 in each direction not good enough! (nor is × 3, nor × 4)

Need to vary also functional dependence on each scale!

2. Infrared Safety

Definition

An observable is infrared safe if it is **insensitive** to

SOFT radiation:

Adding any number of infinitely soft particles should not change the value of the observable

COLLINEAR radiation:

Splitting an existing particle up into two comoving particles each with half the original momentum should not change the value of the observable

(Not accidentally, these are the two singular limits from before)

IR Safety

Theorem:

For all "IR Safe Observables", hadronization corrections (non-perturbative corrections) are POWER SUPPRESSED

$$\text{IR Safe Corrections} \quad \propto \quad \frac{Q_{\text{IR}}^2}{Q_{\text{UV}}^2}$$

All "non-IR Safe Observables" receive logarithmically divergent pQCD corrections in the IR, which must be canceled by logarithmically divergent hadronization corrections → VERY sensitive to UV→IR transition

$$\text{IR Sensitive Corrections} \quad \propto \quad \alpha_s^n \log^m \left(\frac{Q_{\text{UV}}^2}{Q_{\text{IR}}^2} \right) \quad , \quad m \leq 2n$$

IR Safety

Compare an IR safe and unsafe Jet

- May look pretty similar in experimental environment
  - (proof that nature has no trouble canceling all divergencies, no matter what the observable)

So what's the trouble?

- It's not nice to your theory friends ...
  - If they use a truncation of the theory (i.e., pQCD)
    - pQCD badly divergent if IR unsafe, but only power corrections if IR safe
  - Even if they have a hadronization model
    - Dependence on hadronization model → larger uncertainty

Stereo Vision

Use IR Safe algorithms

To study short-distance physics

These days, as fast as IR unsafe algos and widely implemented (e.g., FASTJET), including

> "Cone-like": SiSCone, Anti-kT, …
>
> "Recombination-like": kT, Cambridge/Aachen, …

Then use IR Sensitive observables

E.g., number of tracks, identified particles, …

To explicitly check hadronization and other IR models

More about IR in next lecture …

Ultraviolet – Summary

Your friends

Factorization

Allows you to do meaningful calculations in pQCD

And allows you to make universal fits of non-pQCD to data (e.g., PDFs, fragmentation functions)

Infrared Safety

Allows you to minimize the sensitivity to the non-pQCD corrections (and do meaningful comparisons to pure pQCD)

Unitarity

Allows you to "guess" virtual corrections from real ones

→ enables you to "resum" parts of pQCD to all orders!