VII. Classical Tests of Cosmology

A cosmological model is specified by H_0 and q_0. For now, let us postpone measurement of H_0. Look at tests for q_0.

The principle behind measuring q_0 is to find a physical effect that depends on q_0. An alternative approach is to measure Ω_0. Each method has its advantages:

1. q_0: Pure test of geometry. Limitless possible ways to measure (just need standard candles of any sort). The major difficulty is that it is impossible to do locally with any practical test. Global measurements invariably become tied up with time evolution.
2. Ω_0: This is a very restricted kind of test. Must measure dynamical masses. Tough to do. However, it can be done purely locally, so no evolutionary concerns.

In practice, Ω_0 is easier to measure, although neither Ω_0 or q_0 is known with any high degree of reliability yet.

Tests for q_0

There are three practical tests that have been attempted over the years:

a) $m - z$ relation for standard candles
b) Angular diameter - z relation
c) Number counts

Magnitude-Redshift test. The classic standard candle is the first ranked galaxy in clusters. For reasons that are not clear, the luminosity of this galaxy is remarkably constant with an r.m.s. scatter of only 30% or so. The canonical application of the $m - z$ test is to plot the apparent mangitude m versus $\log z$ in a so-called Hubble diagram. If m measures the apparent bolometric magnitude of an object, then from Eq. 6.22, we have

$$\begin{aligned} m_{bol} &= -2.5 \log F_{bol} + \text{constant} \\ &= 5 \log z + 5 \log \left(1 + \frac{z(1-q_0)}{\sqrt{1+2q_0 z} + 1 + q_0 z} \right) + \text{constant}'. \end{aligned} \tag{7.1}$$

It will be understood that all logarithms are base 10. The constants include the unknown absolute luminosity of the galaxy and the uncertain Hubble constant. For small redshifts we find that $m \propto 5 \log z$ while at large redshifts the relation deviates in a way that depend on q_0. Fig. 7.1 shows the Hubble diagram relation for different values of q_0.

In practice, observations of first ranked galaxies require that several corrections be applied first.

a) Giant elliptical galaxies do not have particularly well defined total magnitudes. The reason is that they have extended envelopes with profiles that are nearly power laws with index -2, for which the total luminosity converges very slowly with increasing aperture size. The solution is to measure the galaxy luminosity inside some fixed metric radius. Other approaches that have been tried are to measure the luminosity inside a fixed isophotal radius (*i.e.*, the radius where the surface brightness falls to some predetermined value) or inside some characterstic scale radius (*e.g.*, a core radius) measured from the shape of the luminosity profile. In practice, neither of the latter two methods has proved superior to using a metric radius.

b) K-correction. Measurements of galaxy magnitudes almost never measure bolometric magnitudes, but instead usually measure the apparent magnitude within some fixed bandpass. The K-correction is defined to be the correction needed to convert the measured magnitude to a quantity that is proportional to the bolometric magnitude. If a galaxy had redshift z, then the frequency ν_0 that we measure on earth is equivalent to $\nu_0(1+z)$ at the galaxy. Also, the bandpass $\Delta\nu_0$ on earth is equivalent to $\Delta\nu_0(1+z)$ at the galaxy. Hence what we measure is

$$L'\Delta\nu_0 = L_{\nu_0(1+z)}\Delta\nu_0(1+z) = (1+z)\left(\frac{L_{\nu_0(1+z)}}{L_{\nu_0}}\right)(L_{\nu_0}\Delta\nu_0)\,. \qquad (7.2)$$

The bolometric luminosity is proportional to $L_{\nu_0}\Delta\nu_0$, so the K (or bolometric) correction is

$$K = -2.5\log\frac{L_{\nu_0}}{(1+z)L_{\nu_0(1+z)}}. \qquad (7.3)$$

Sometimes we work with $L_\lambda = L_\nu c/\lambda^2$. In this case

$$K = -2.5\log\frac{(1+z)L_{\lambda_0}}{L_{\lambda_0/(1+z)}}. \qquad (7.4)$$

Normally the sense of the K-correction is to increase the apparent brightness of a galaxy. Note that the way K is defined, this is still not the flux that we would measure if the galaxy were brought to rest [there would still be an extra factor of $(1+z)^2$].

c) Aperture correction: Suppose we want to measure L insize a metric radius R (*e.g.*, 8 kpc for $H_0 = 100$). Then this radius corresponds to an angle $\delta\theta$ given by

$$\delta\theta = \frac{R(1+z)H_0}{Z}, \qquad (7.5)$$

where Z is the combination of z and q_0 defined in Chapter 6. But this leads to a dilemma: we need to know q_0 in order to determine the aperture size needed to measure the magnitudes from which we will derive q_0. Because the luminosity and aperture depend on q_0 in different ways, it is possible to resolve the dilemma (*e.g.* with an iterative solution), but the power of the Hubble diagram test for q_0 is weakened. The alternative choices for scale length each have their own problems: the isophotal radii need to be corrected for the $(1+z)^4$ of surface brightness on redshift. The only characteristic scale lengths that can be measured from galaxy profiles are core radii, and these become impossible to measure for galaxies at high redshift because atmospheric seeing blurs the galaxy images too much.

d) Evolutionary effects become important at large redshift. Galaxies at high redshift are younger than galaxies today, and galaxy evolution models predict that, in most cases, they had a higher luminosity in the past. Counteracting this, it has been suggested that first ranked galaxy luminosities increase in time due to accretion by dynamical friction of other cluster members. It is now recognized that evolutionary effects have a larger impact on the Hubble diagram than the q_0 dependence, and since the magnitude of the corrections is not well known, the Hubble diagram is actually more useful for studying galaxy evolution than it is for doing cosmological tests.

Angular Diameter Tests. The basis for this test is given by Eq. 7.5. Again, the idea is to find some object with a standard length and plot apparent diameter vs. redshift. Both first ranked galaxies and galaxy clusters have been suggested as objects having standard lengths. The measurements are difficult to do, and the assumption of constant length scale for clusters and galaxies is not yet well founded.

Number Counts. The easiest test to do is to count galaxies as a function of limiting magnitude. If $N(m)$ is the number of galaxies brighter than limiting magnitude m, then at low redshift, $\log N \propto 0.6m + \text{constant}$, but at high redshift the slope deviates from 0.6 due to the effects of cosmology. Classic integrated number count tests (whereby galaxies are counted in a fixed bandpass), however, suffer from the fact that the intrinsic luminosity function of galaxies is broad, unknown K corrections must be applied, and galaxy evolution is again likely to be important.

Number counts become more interesting if, by some method, we can measure the redshifts of objects. For this test, the redshifts need not be very accurate, and so-called photometric redshifts have been proposed as being viable. Photometric redshifts are based on the fact that multiband photometry of galaxies done using filters of intermediate width is effectively the same as very low resolution spectroscopy, and if some feature (for example, the HK break due to Ca II absorption lines around 3900 A) can be identified in the multiband photometry of a galaxy, then a rough redshift can be estimated.

The fundamental quantity that we would want to compute is dN/dz, where N is the total number of objects counted in some solid angle $\Delta\Omega$. First, let us write down the number of objects contained in a volume bounded by $\Delta\Omega$ in angle and Δu in comoving distance, where u is a function of z. Then

$$\Delta V = \Delta\Omega \Delta u R^2(z) S_k^2(z) R(z) \rho(z). \tag{7.6}$$

Aside from determining the relation between u and z (which will be dealt with imminently), the key relations that are needed are:

a) The comoving density of objects is assumed to be constant; hence

$$\rho(z) = \rho(0)(1+z)^3; \tag{7.7}$$

b)

$$R(z) = \frac{R_0}{(1+z)}. \tag{7.8}$$

Combining,

$$\Delta N = \rho_0 R_0^3 S_k^2(u)\Delta u \Delta\Omega \tag{7.9}$$

and so

$$\frac{dN}{dz} = \rho_0 R_0^3 S_k^2(u)\frac{du}{dz}. \tag{7.10}$$

Now we relate z to u. This is done in three steps:

a) From Eq. (6.12), $u = (\theta_0 - \theta_z)$. So

$$\frac{du}{dz} = -\left[\frac{d\theta}{dz}\right]\Bigg|_z. \tag{7.11}$$

b)

$$C_k(\theta) = \frac{1}{q} - 1; \tag{7.12}$$

c)

$$q = \frac{q_0(1+z)}{1+2q_0}. \tag{7.13}$$

Thus,

$$C_k(\theta) = \frac{1 - q_0 + q_0 z}{q_0(1+z)}, \tag{7.14}$$

$$-kS_k(\theta)\frac{d\theta}{dz} = \frac{dC_k(\theta)}{dz}. \tag{7.15}$$

and

$$\frac{dC_k(\theta)}{dz} = \frac{2q_0 - 1}{q_0(1+z)^2}, \tag{7.16}$$

$$S_k(\theta) = \sqrt{k(1 - C_k^2)} = \frac{\sqrt{k(2q_0-1)(2q_0 z+1)}}{q_0(1+z)}. \tag{7.17}$$

Combining,

$$\frac{dw}{dz} = -\frac{d\theta}{dz} = \frac{k(2q_0-1)}{\sqrt{k(2q_0-1)(2q_0+1)}(1+z)} \tag{7.18}$$

and

$$\frac{dN}{dz} = \rho_0 \frac{Z^2}{H_0^2} R_0 \frac{k(2q_0 - 1)}{\sqrt{k(2q_0-1)(2q_0+1)}(1+z)}. \tag{7.19}$$

Substituting

$$R_0 = \frac{1}{H_0\sqrt{k(2q_0-1)}} \tag{7.20}$$

finally gives

$$\frac{dN}{dz} = \frac{\rho_0 Z^2}{H_0^3(1+z)\sqrt{2q_0 z+1}}. \tag{7.21}$$

This equation is sometime written replacing Z with the luminosity distance $D_L = Z(1+z)/H_0$:

$$\frac{dN}{dz} = \frac{D_L^2}{H_0(1+z)^3\sqrt{2q_0 z+1}}. \tag{7.22}$$

The units are number of galaxies per unit solid angle.

For certain calculations (*e.g.*, the number density of QSO absorption line systems along any line-of-sight), it is useful to know the quantity ds/dz, where s is the linear path length. We have

$$\frac{ds}{dz} = \frac{Rdu}{dz} = \frac{R_0}{1+z}\frac{du}{dz}. \tag{7.23}$$

Upon substitution of du/dz, we find

$$\frac{ds}{dz} = \frac{1}{H_0(1+z)^2\sqrt{1+2q_0 z}}. \tag{7.24}$$

Returning to the number count test, we find, for example, that the difference in dN/dz between a $q_0 = 0$ and $q_0 = 1$ universe is a factor 1.28 at $z = 0.5$ and a factor 4.5 at $z = 1$. The sense is that there are more galaxies in a $q_0 = 0$ universe.

Practical application of the number count test must contend with several problems:

a) In any practical measurement, galaxies are selected on the basis of their apparent brightness. So

$$\frac{dN_{obs}}{dz} = \frac{dN}{dz}\int_{L(D_L)}^{\infty} \phi(L)dL, \tag{7.25}$$

ϕ being the luminosity function of galaxies.

b) Galaxy luminosity evolution affects dN/dz in a way that is difficult to disentangle from changes in q_0. There are at least two possible ways that evolution can be neutralized:

   i) Measure the luminosity function at each redshift. Provided that luminosity evolution is the same for all galaxies, then the number of galaxies brighter than some fiducial magnitude (say $L*$) will still be independent of the evolution. It is then necessary to measure the shape of the luminosity in order to extract the characteristic luminosity $L*$. If the shape of the luminosity function varies with redshift, then this approach will fail.

ii) Luminosity evolution affects the determination of q_0 in the opposite sense from the way that it does in the magnitude-redshift test: an increase in galaxy luminosity mimics a decrease in q_0 for number count tests. Hence the two tests could be combined in an appropriately weighted fashion to cancel the effects of evolution.

The only serious attempt to use number counts to measure q_0 has been by Loh and Spillar (198?), who find a formal value of $q_0 = 0.45 \pm 0.3$.

Tests of Ω

The measurement of Ω typically is two-step procedure: first one measures the masses of objects such as galaxies or galaxy clusters via dynamical means in order to determine a typical mass/light ratio, and then one measures the mean space density of those objects and derives a total mass density which is then compared to the critical density $3H_0^2/8\pi G$. A big complication in this procedure is that it appears that most of the mass in the universe is in a dark component that is only loosely bound to individual galaxies. Hence one is not yet sure what the true mass/light ratio of the universe is when averaged over a "fair volume". The mass/light ratios of individual galaxies (determined from rotation curves of spiral galaxies or virial analyses of elliptical galaxies) are typically in the range $6-12h$. [Note: dynamically determined masses are proportional to rv^2/G where r is some characteristic radius and v is some characteristic velocity. Luminosities are proportional to r^2. Hence M/L $\propto r^{-1}$. Since the Hubble constant is not yet well known, distances are usually written in units of h^{-1}, where $h = H_0/100$ km s^{-1} Mpc^{-1}.] [Spiral galaxies have flat rotation curves that imply mass increasing linearly with radius with no outer radius detected; hence M/L ratio is not particularly well defined. The values quoted here refer to the ratio measured inside some characteristic optical radius.] Masses of galaxy clusters are typically $300h$. The M/L ratio needed to close the universe is about $1200h$. Hence if the M/L ratio of clusters is typical of the entire universe, then $\Omega = 0.25$. Measuring M/L ratios on scale larger than galaxy clusters is quite difficult but can be attempted by measuring large-scale flows of galaxies. This subject will be examined more closely in Chapter 10.