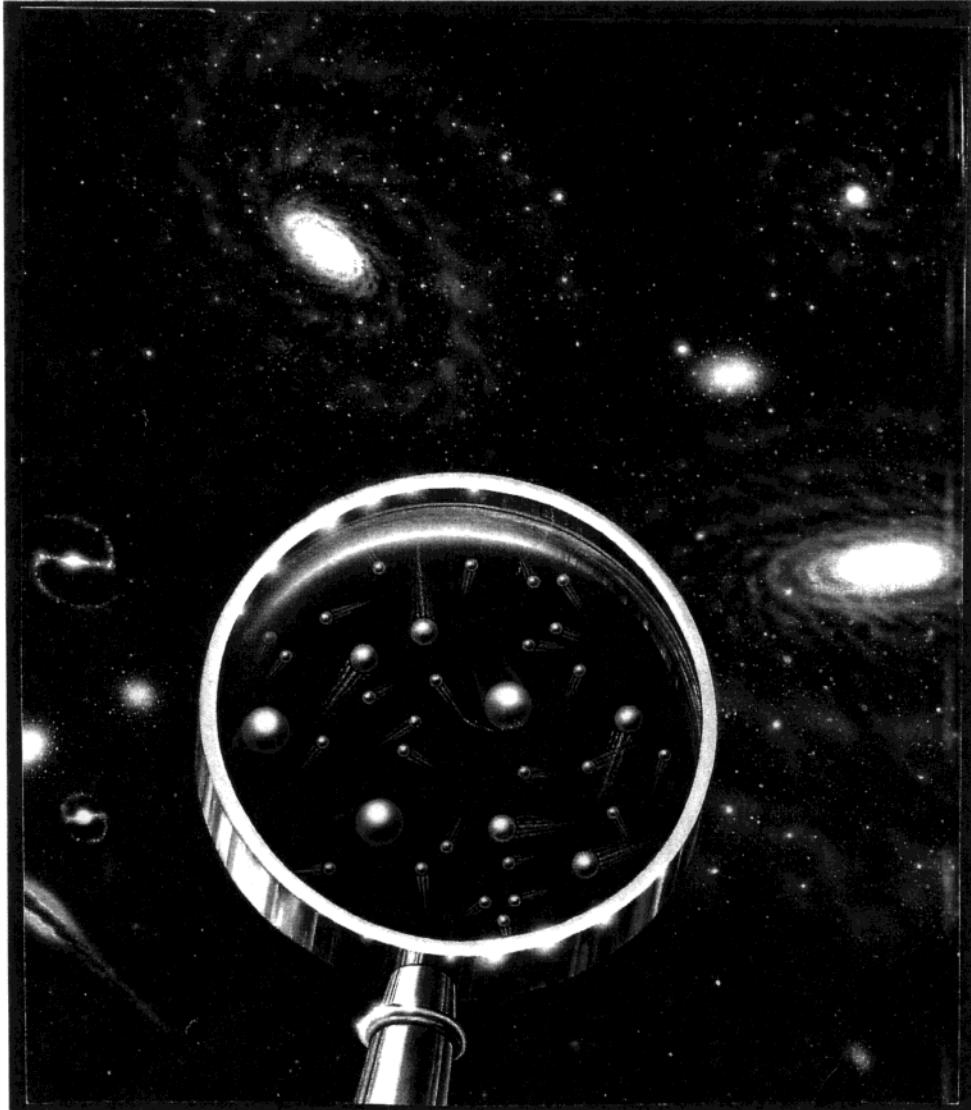


# *New Developments in Cosmology*



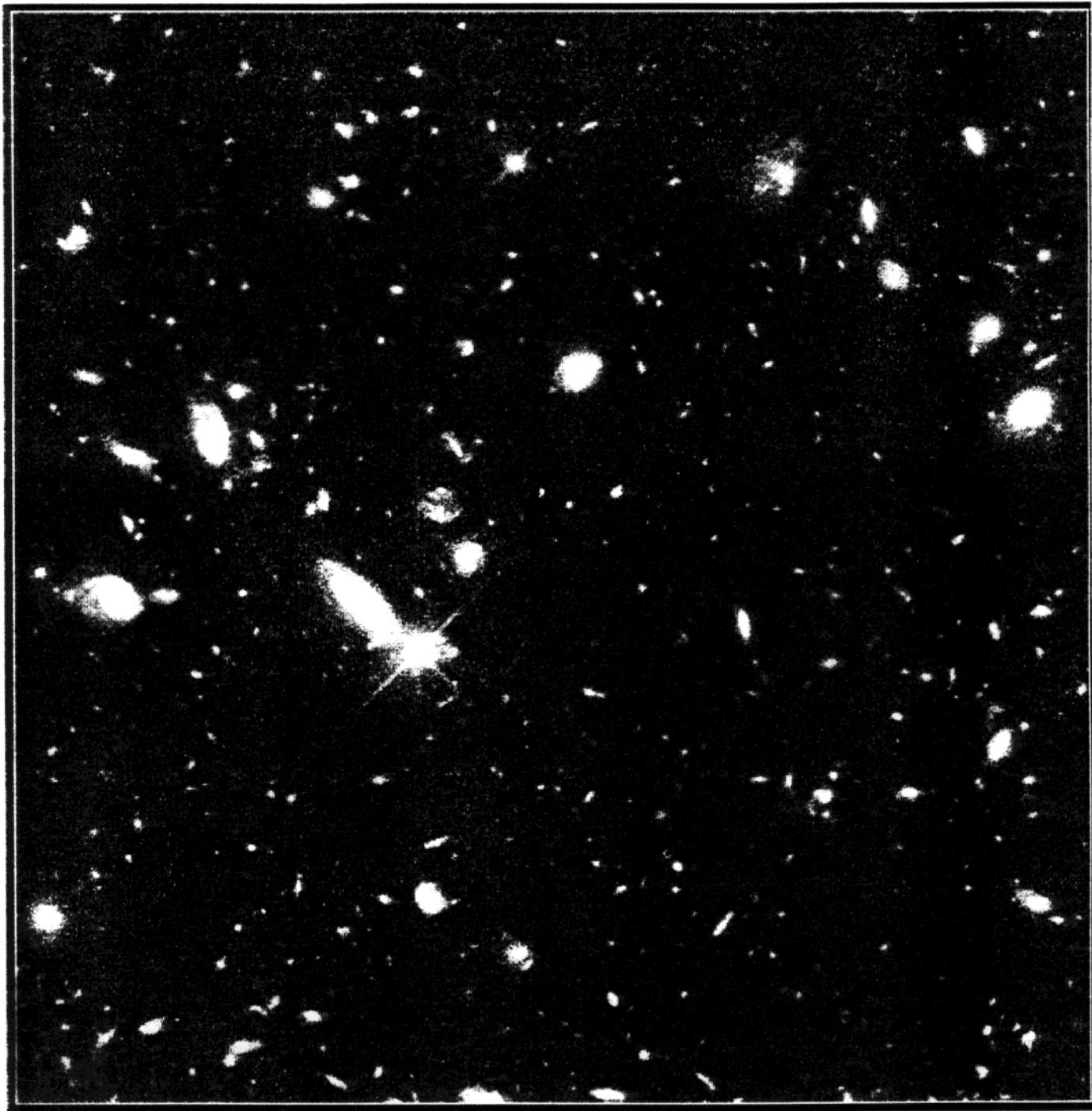
*... the particle connection*

**ICHEP'98**  
*Vancouver*

*Rocky Kolb*  
*Fermilab*

# *Observer's view*

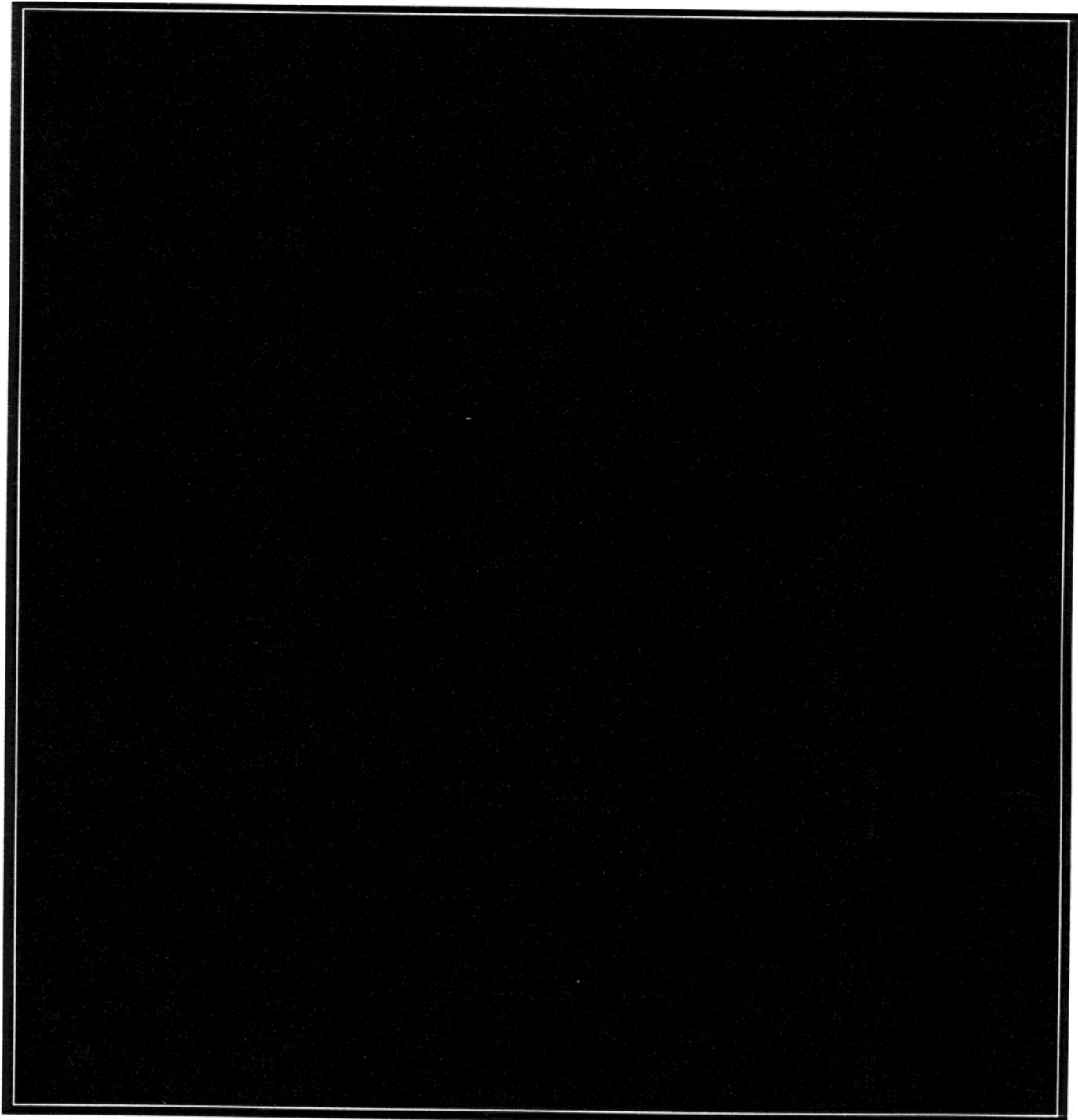
---



*structure  
(inhomogeneous)*

# *Theorist's view*

---



*smooth  
(homogeneous/isotropic)*

# Statistical Properties of the density field $\rho(\vec{x})$

Peebles

## *Density contrast*

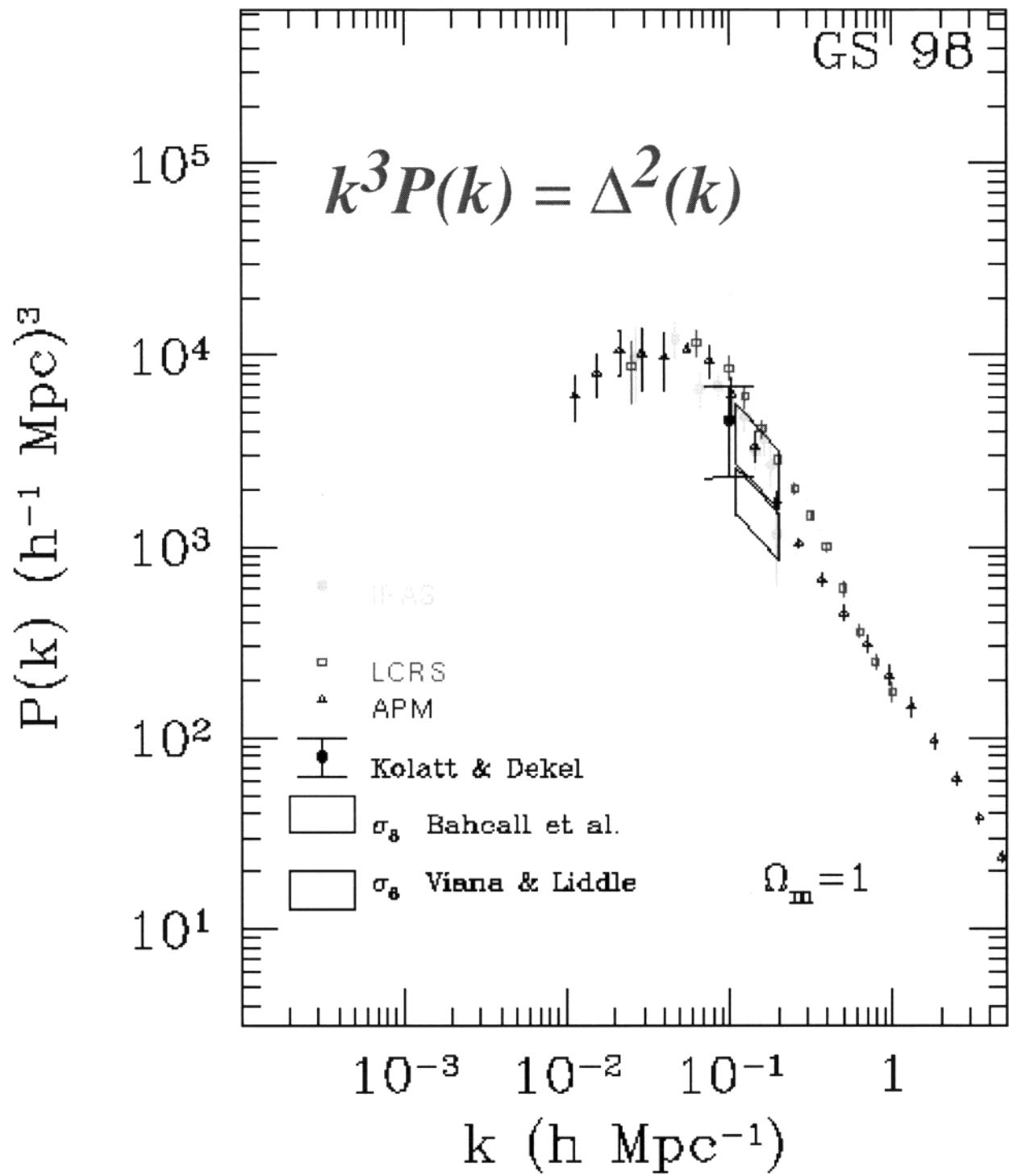
$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \langle \rho \rangle}{\langle \rho \rangle}$$

## *In Fourier space*

$$\begin{aligned} \delta(\vec{x}) &\equiv V \int d^3k \, \delta_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}} \\ \left( \frac{\delta\rho}{\rho} \right)^2 &\equiv \langle \delta(\vec{x}) \delta(\vec{x}) \rangle = A \int_0^\infty \frac{dk}{k} k^3 \left| \delta_{\vec{k}} \right|^2 \end{aligned}$$

## *The Power Spectrum*

$$\Delta^2(k) = k^3 \left| \delta_{\vec{k}} \right|^2 \quad \text{or} \quad P(k) = \left| \delta_{\vec{k}} \right|^2$$

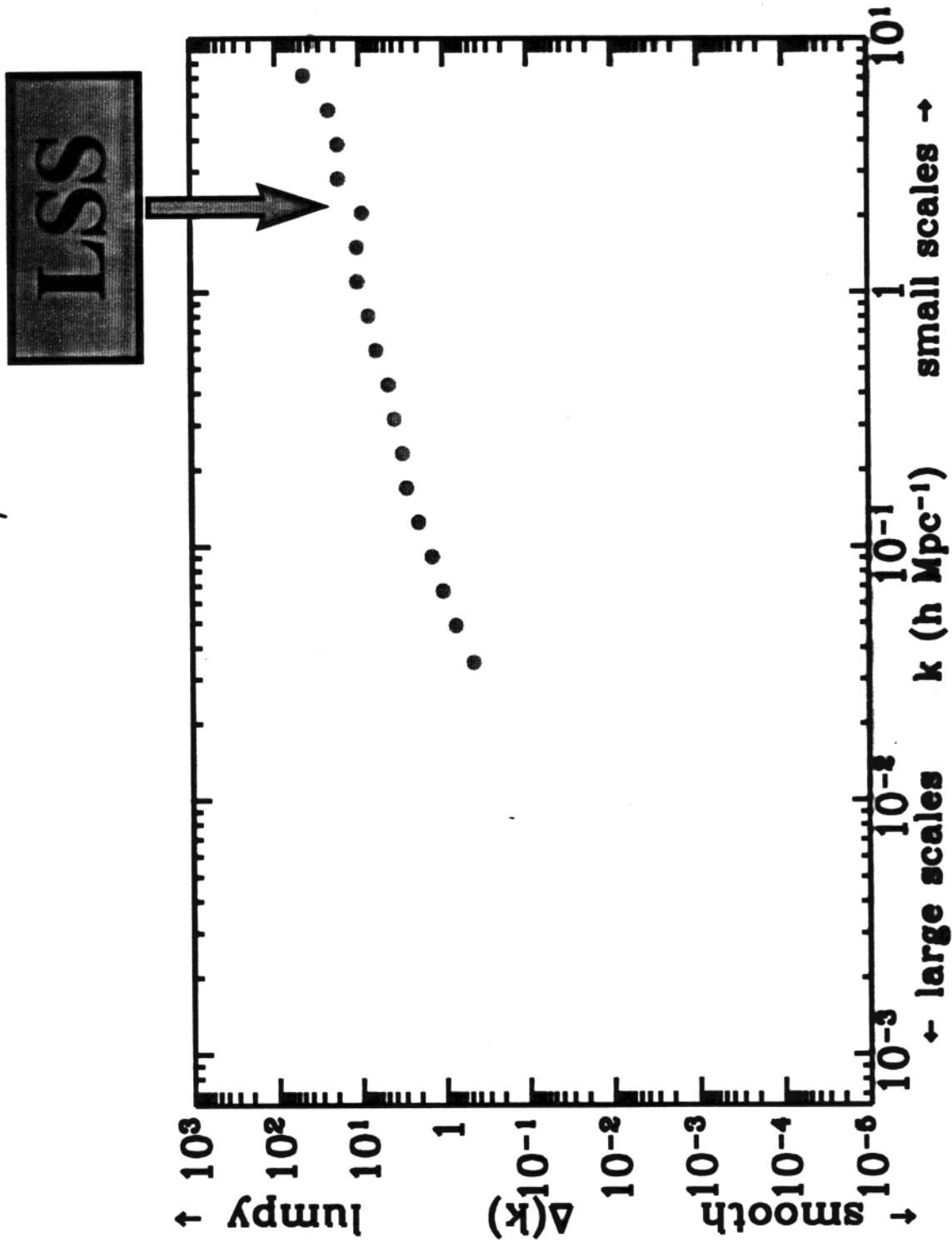


$$H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$$

**Mpc = 3.26 million light years**

## Power spectrum from large-scale structure

$$R \simeq 2\pi/k$$



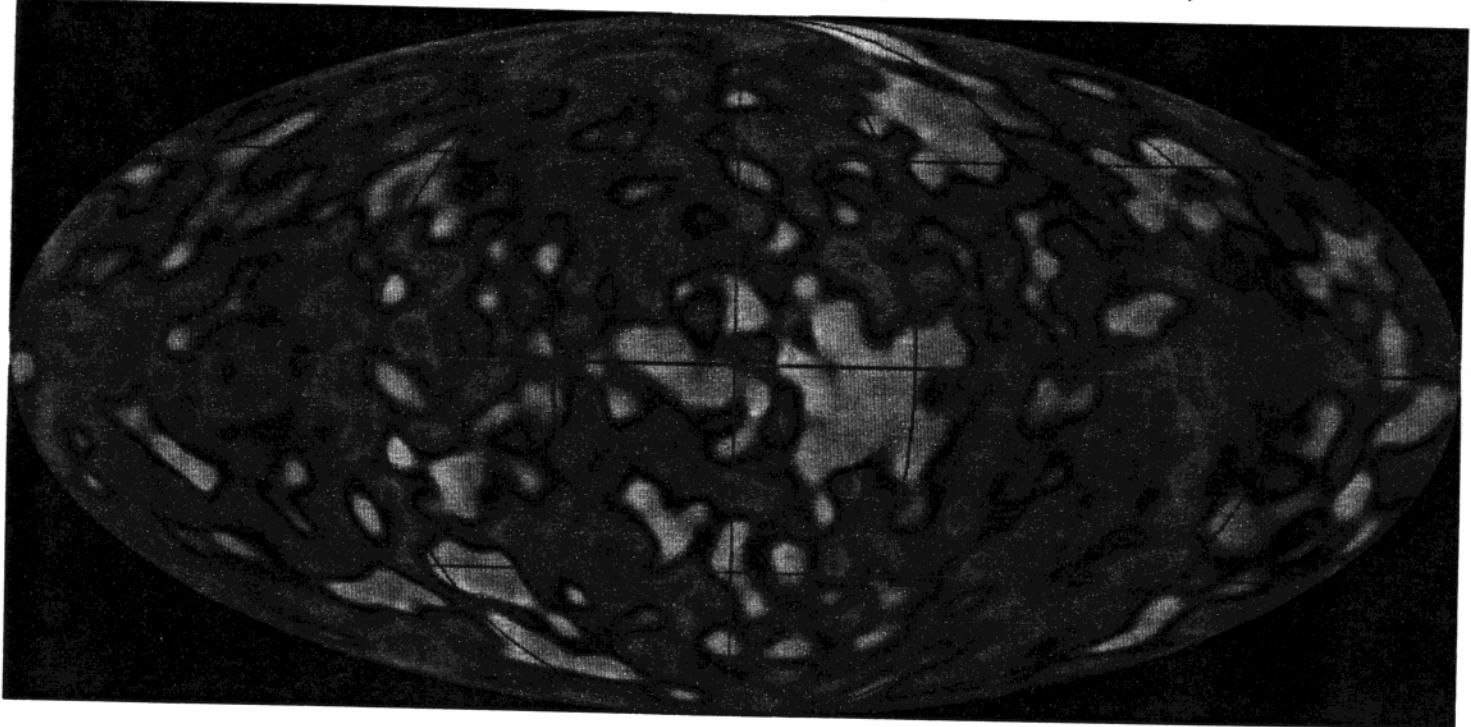
$$H_0 = 100h \text{ km s}^{-1}\text{Mpc}^{-1}$$

Mpc = 3.26 million light years

# Multipole Moments



(angular power spectrum)



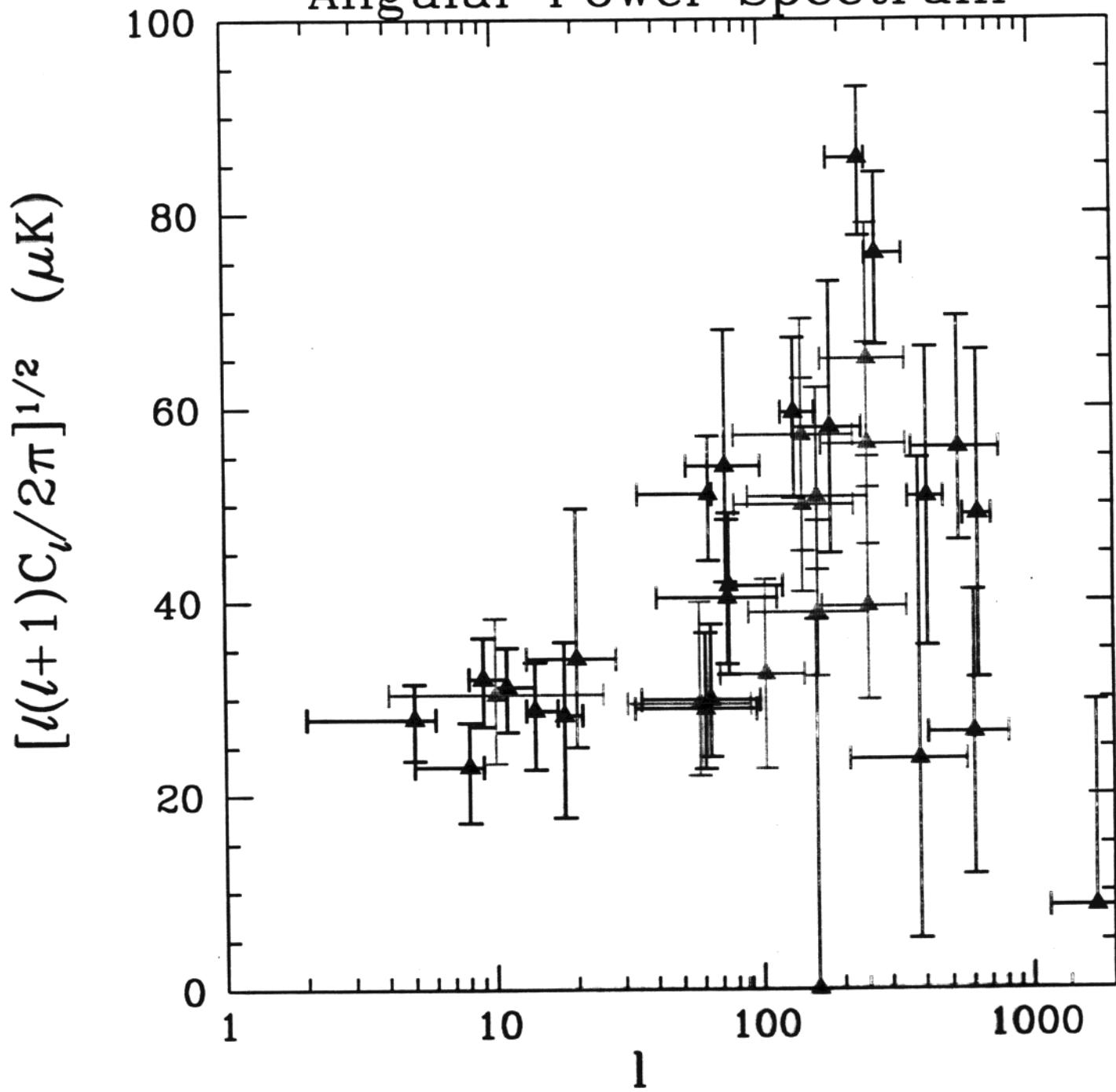
$$\frac{\Delta T(\theta, \phi)}{T} = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi)$$

$$\langle |a_{lm}|^2 \rangle \equiv C_l$$

# CBR Today

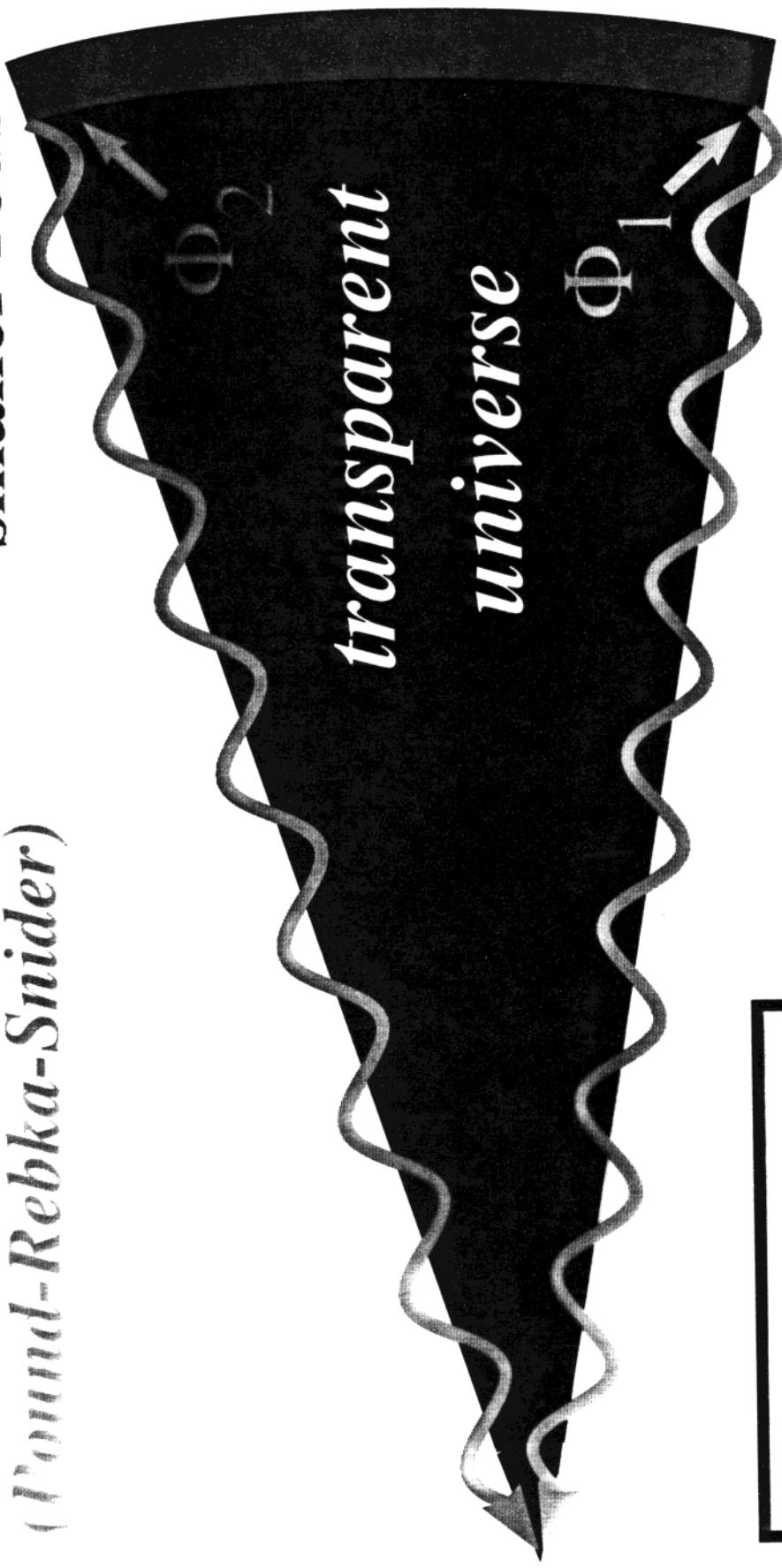
Bond and Knox 1997

## Angular Power Spectrum



# Sachs-Wolfe Effect (Vittorio-Rybka-Snider)

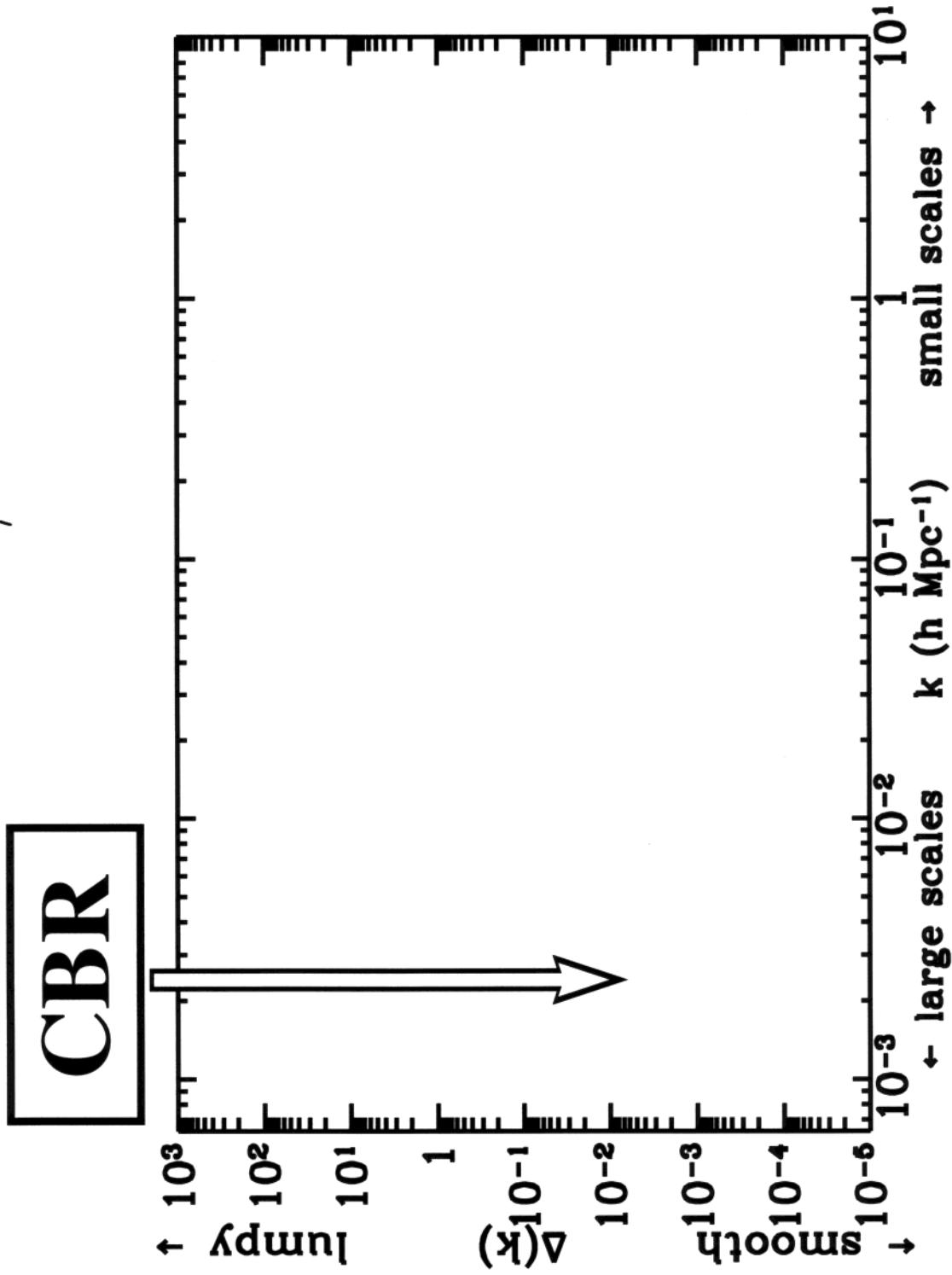
lower density  
smaller redshift



$$\frac{\Delta T}{T} \rightarrow -\frac{\Delta \Phi}{\Phi}$$

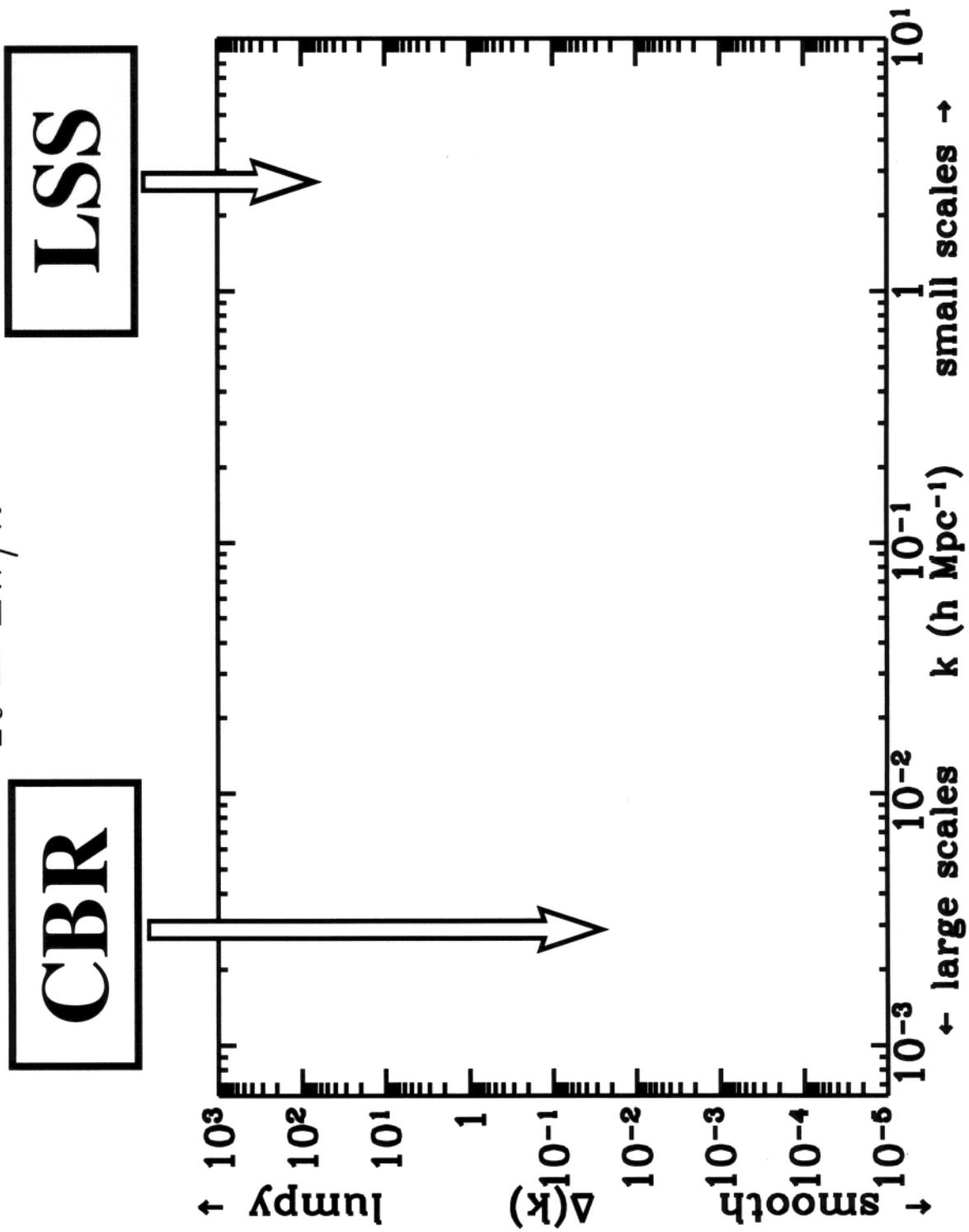
# Power spectrum from cbr fluctuations

$$R \simeq 2\pi/k$$

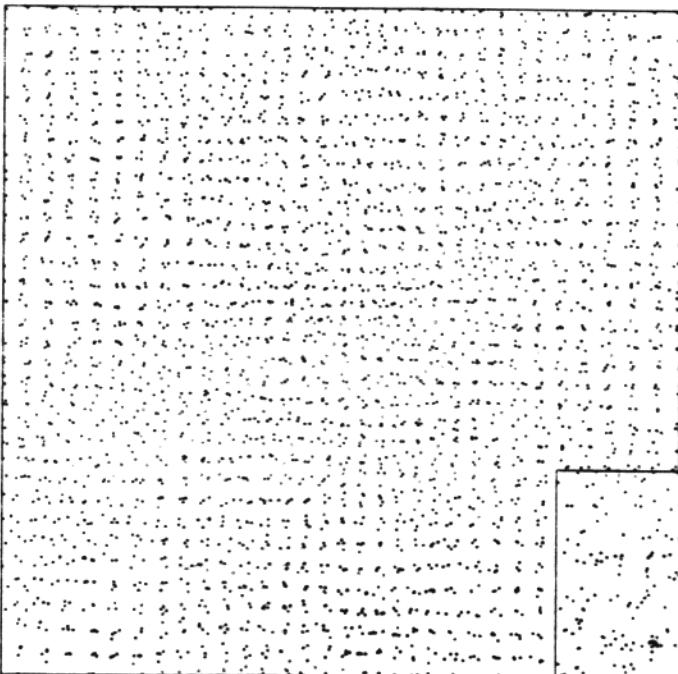


# Grand Unified Power Spectrum

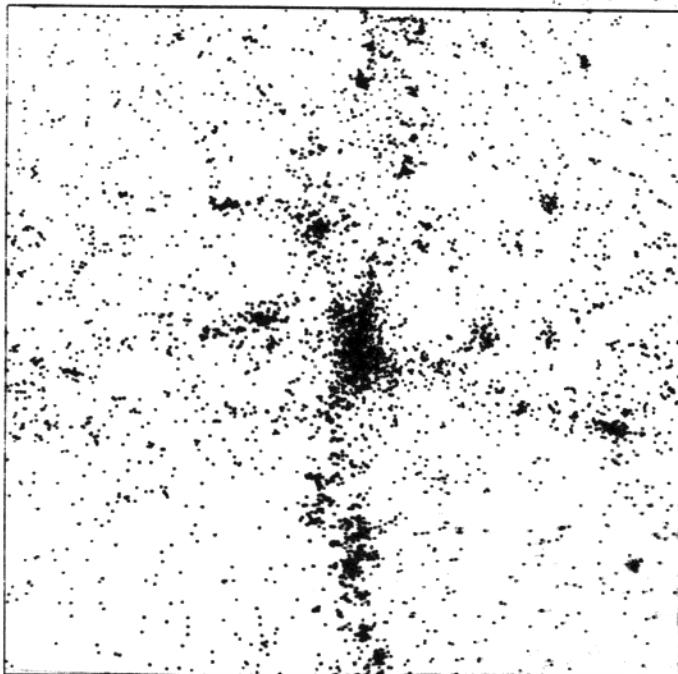
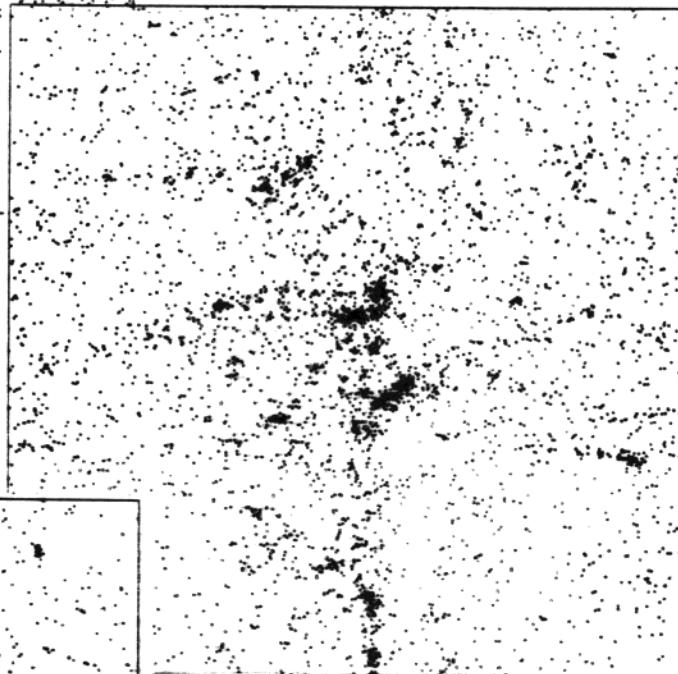
$$R \simeq 2\pi/k$$



# *Seeds of Structure*



*time*



## *Input from particle physics:*

---

### **1. matter content**

$$\Omega_B \quad \Omega_V \quad \Omega_\Lambda \quad \Omega_{\text{CDM}}$$

### **2. spectrum of perturbations**

$\Delta(k)$  (power spectrum)

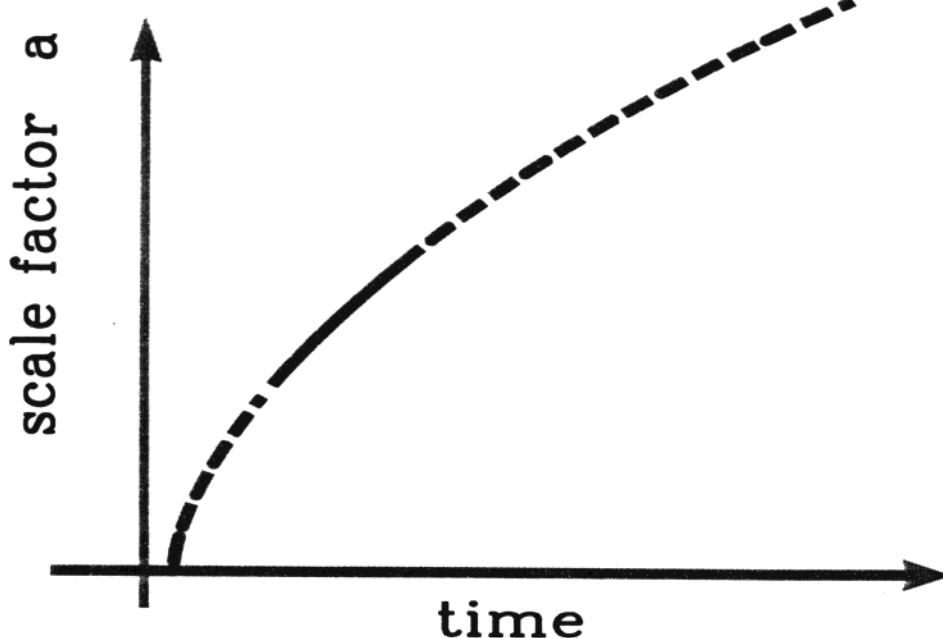
CMB correlations on scales  
larger than Hubble radius ( $\sim ct$ )



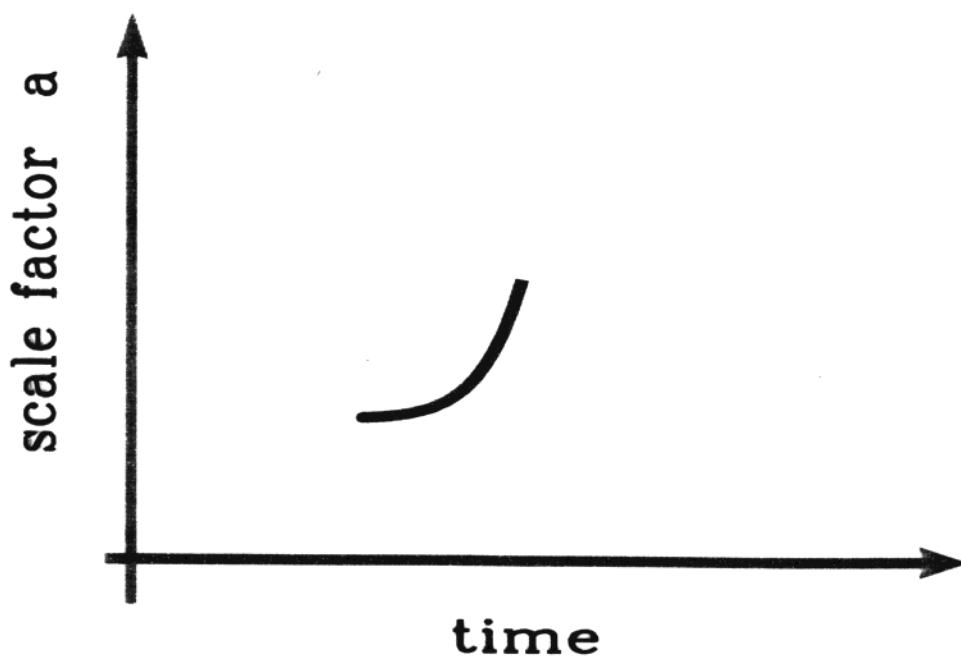
**INFLATION**  
(accelerated expansion)

Newton  
Einstein

$$\ddot{a} \sim -G(\rho + 3p)$$



**normal**  
 $\ddot{a} < 0$   
 $\rho + 3p > 0$



**accelerated**  
 $\ddot{a} > 0$   
 $\rho + 3p < 0$

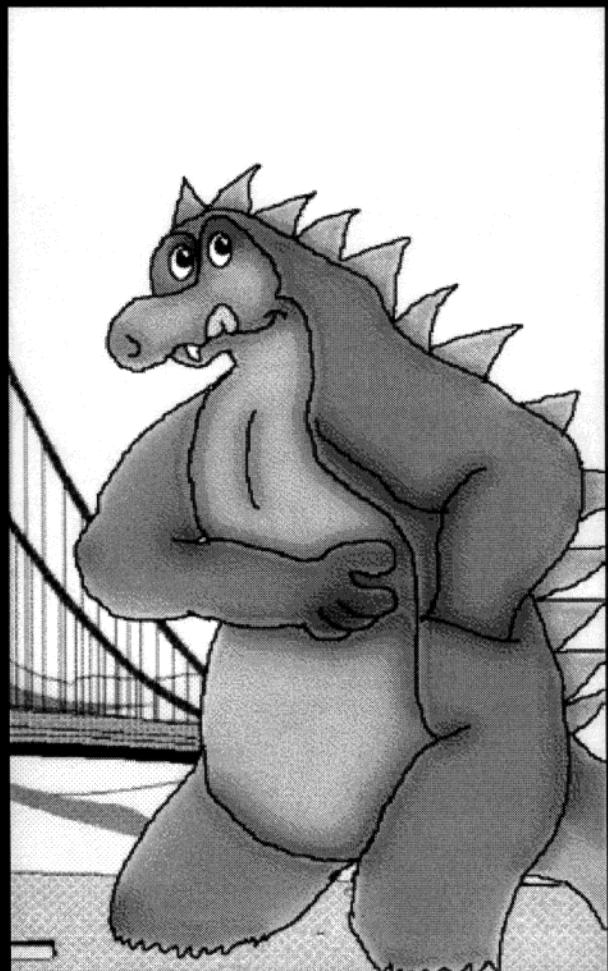
# *The Universal Symphony*



# **SUPERHEAVY DARK MATTER WIMPZILLA**

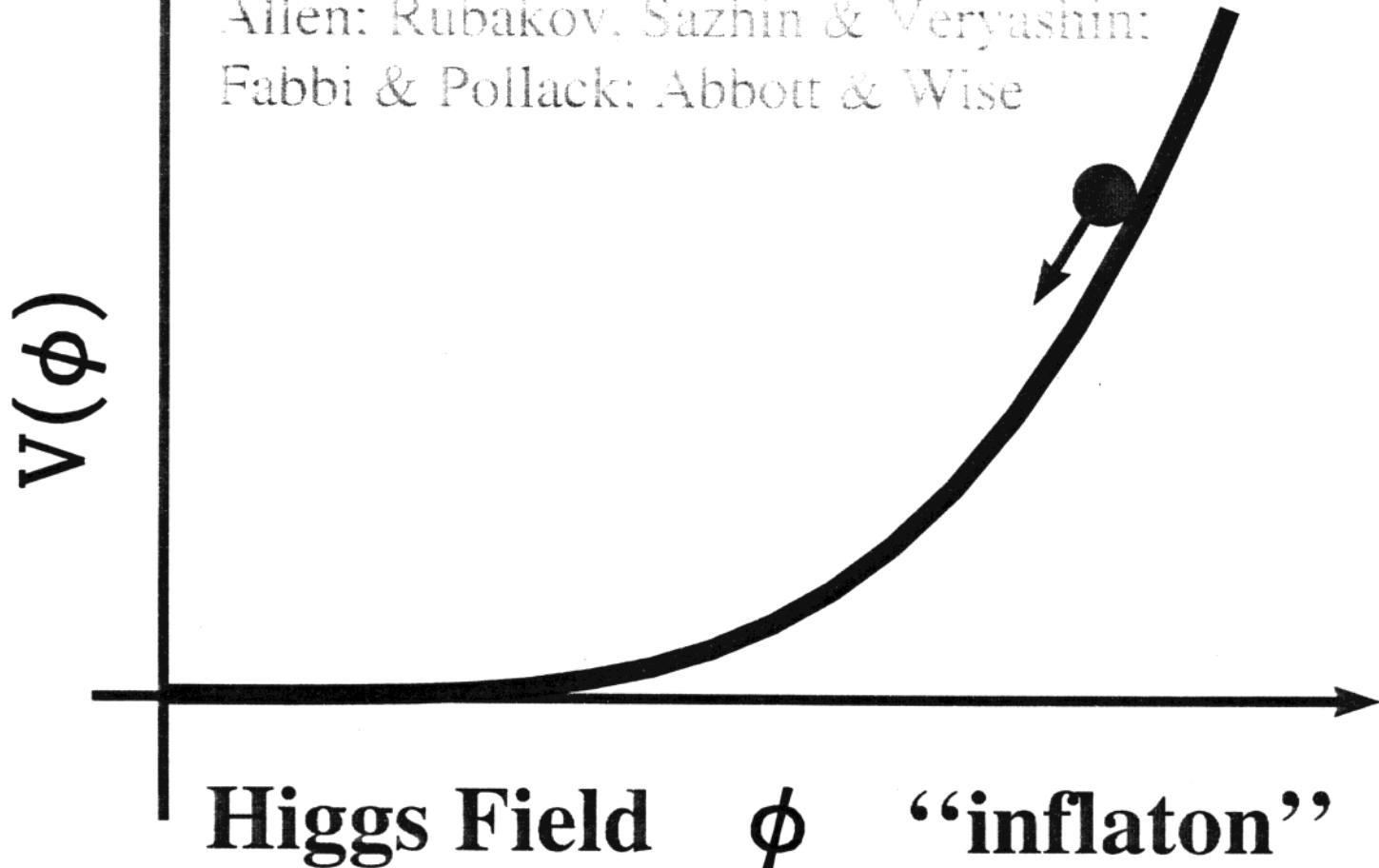
**SIZE  
DOES  
MATTER**

$M_X =$   
 $10^{12} \text{ GeV}$



**Chung, Kolb, Riotto**

ca. 1981-1983: Guth; Starobinski; Linde;  
Albrecht & Steinhardt; Guth & Pi; Hawking;  
Bardeen, Steinhardt & Turner; Starobinski;  
Allen; Rubakov, Sazhin & Veryashin;  
Fabbi & Pollack; Abbott & Wise



Higgs Field    $\phi$    “inflaton”

Classical equation of motion  
plus  
small quantum fluctuations

$$\delta\phi \longrightarrow \delta\rho \longrightarrow \delta T$$

# Comparison to Observation:

## ● Primordial Spectrum

Inflaton  
Potential



- *perturbation spectrum (n)*
- *tensor component (r)*

## ● Cosmological Parameters

$H_0$

- Hubble Constant

$\Omega_0$

- Total Density

$\Omega_B$

- Baryon Fraction

$\Omega_{DM}$

- Dark Matter Fraction

$\Lambda_0$

- Cosmological Constant

## ● Nature of Dark Matter

*cold*

- **slow** (*neutralinos, axions, ...*)

*warm*

- **not so slow** (*gravitinos*)

*hot*

- **pretty fast** (*neutrinos*)

# Baskin<sup>31</sup> Robbins<sup>®</sup>

## Flavor of the Month

**-CDM**

$H_0 = 50 \text{ km s}^{-1}\text{Mpc}^{-1}$ ;  $\Lambda = 0$ ;  
 $\Omega_B = 5\%$ ;  $\Omega_{CDM} = 95\%$ ;

$H = \text{const. during inflation}$

**-HDM**

$\Omega_B = 5\%$ ;  $\Omega_\nu = 95\%$

**-MDM**

$\Omega_\nu = 20\%$ ;  $\Omega_{CDM} = 70\%$ ;  $\Omega_B = 10\%$   
 $m_\nu = 5 \text{ eV}$

**-TCDM**

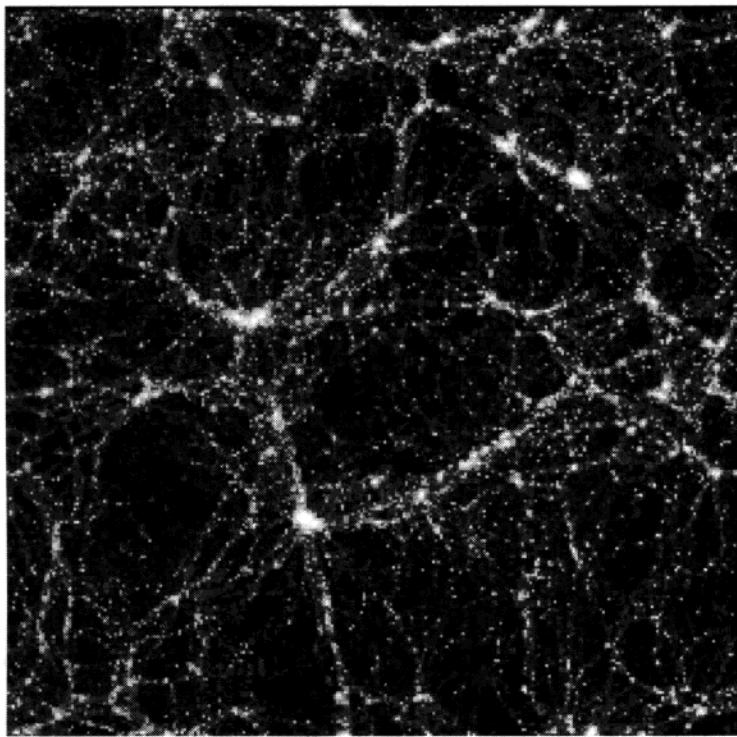
$H$  not const. during inflation

**- $\Lambda$ CDM**

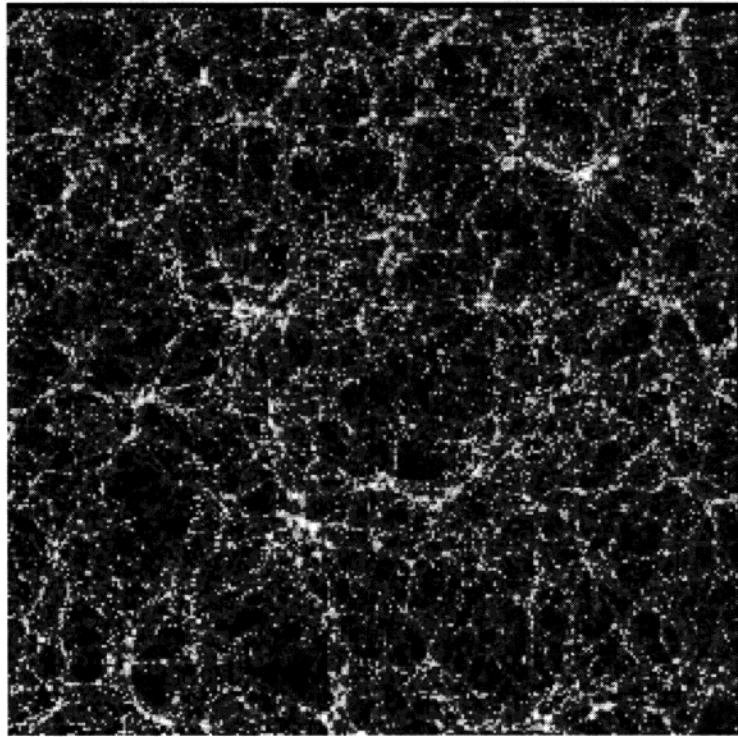
$\Omega_\Lambda = 50\%$ ;  $\Omega_{CDM} = 45\%$ ;  $\Omega_B = 5\%$

**-OCDM**

$\Omega_{CDM} = 45\%$ ;  $\Omega_B = 5\%$

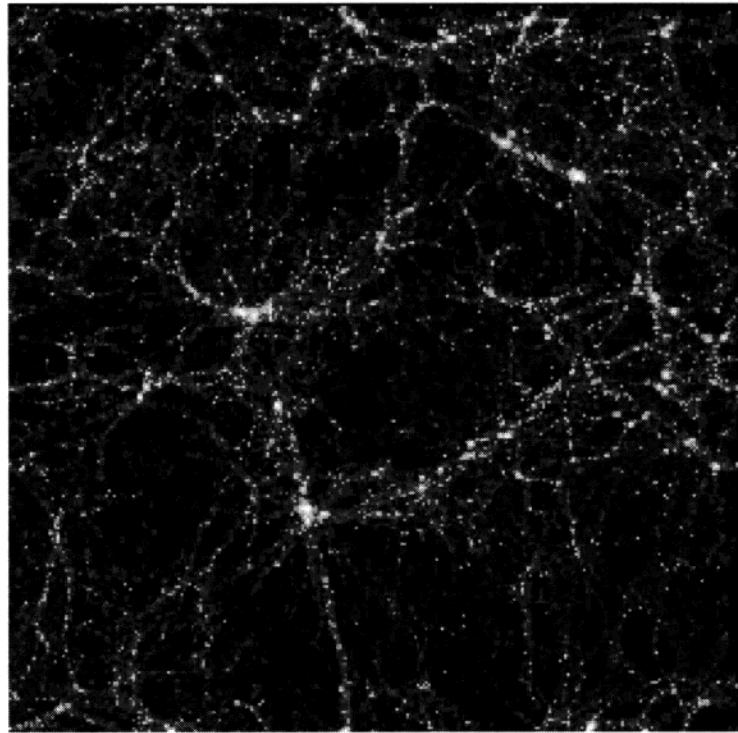


$\Lambda$ CDM



SCDM

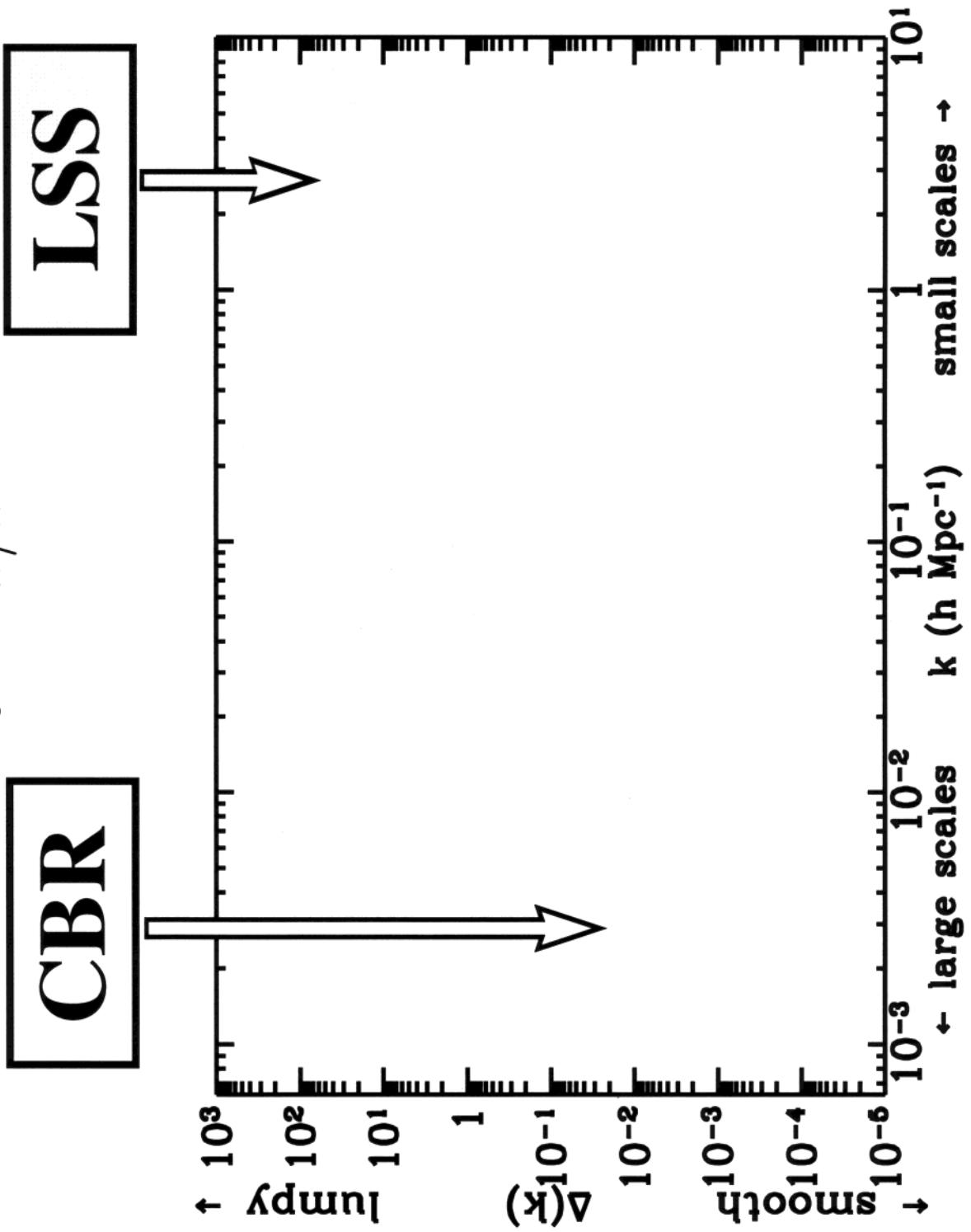
The VIRGO  
Collaboration 1996

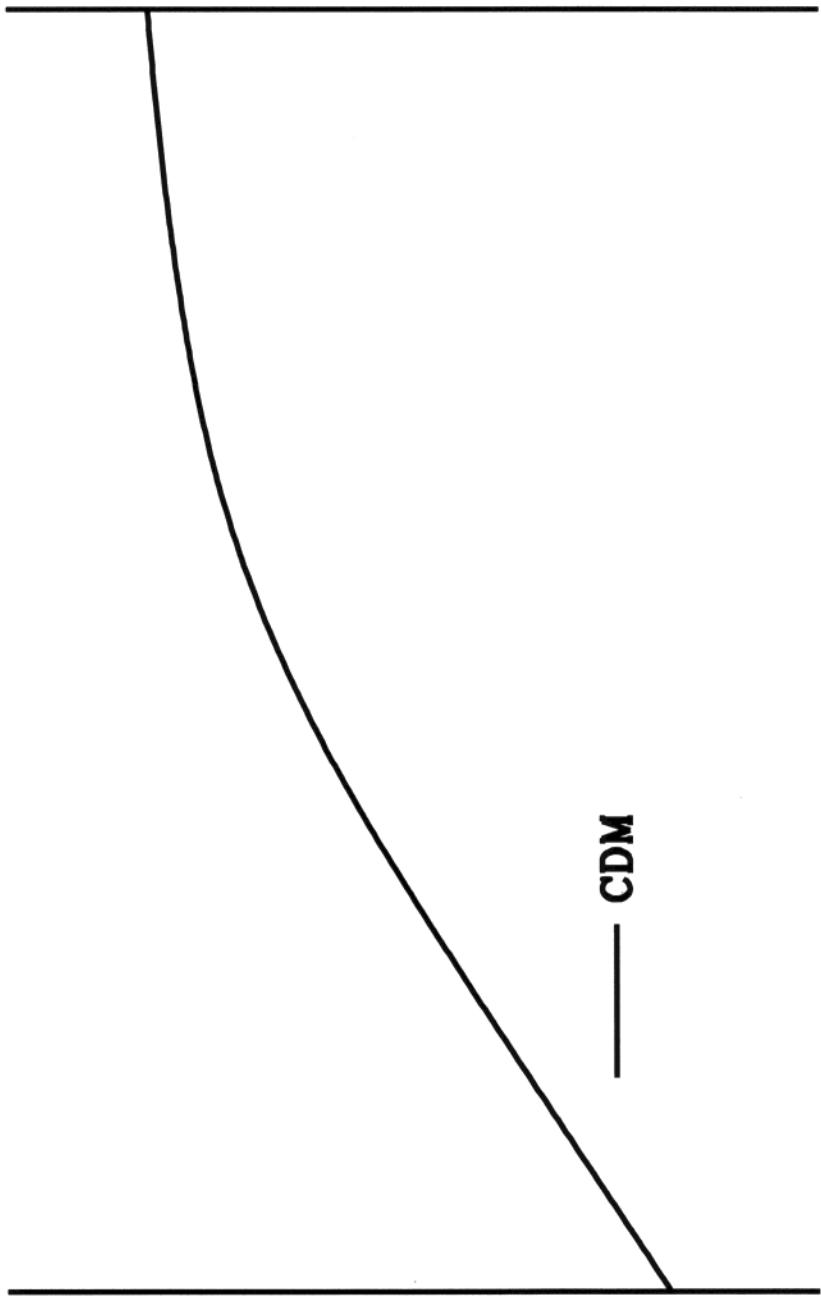


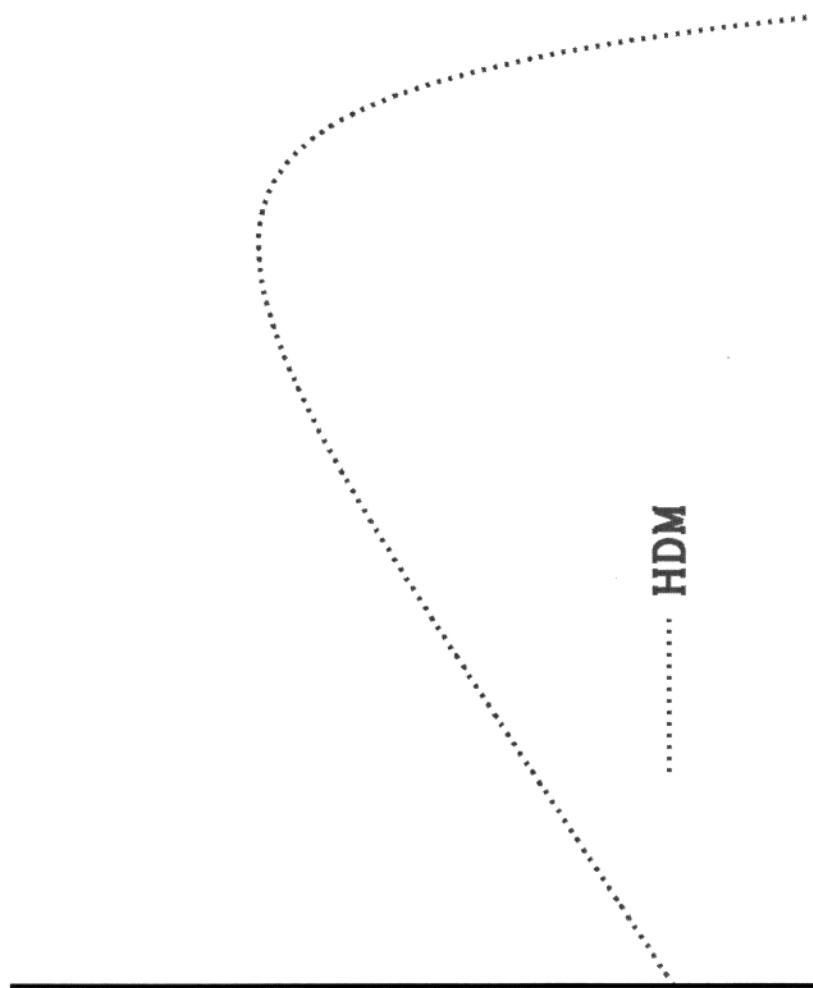
OCDM

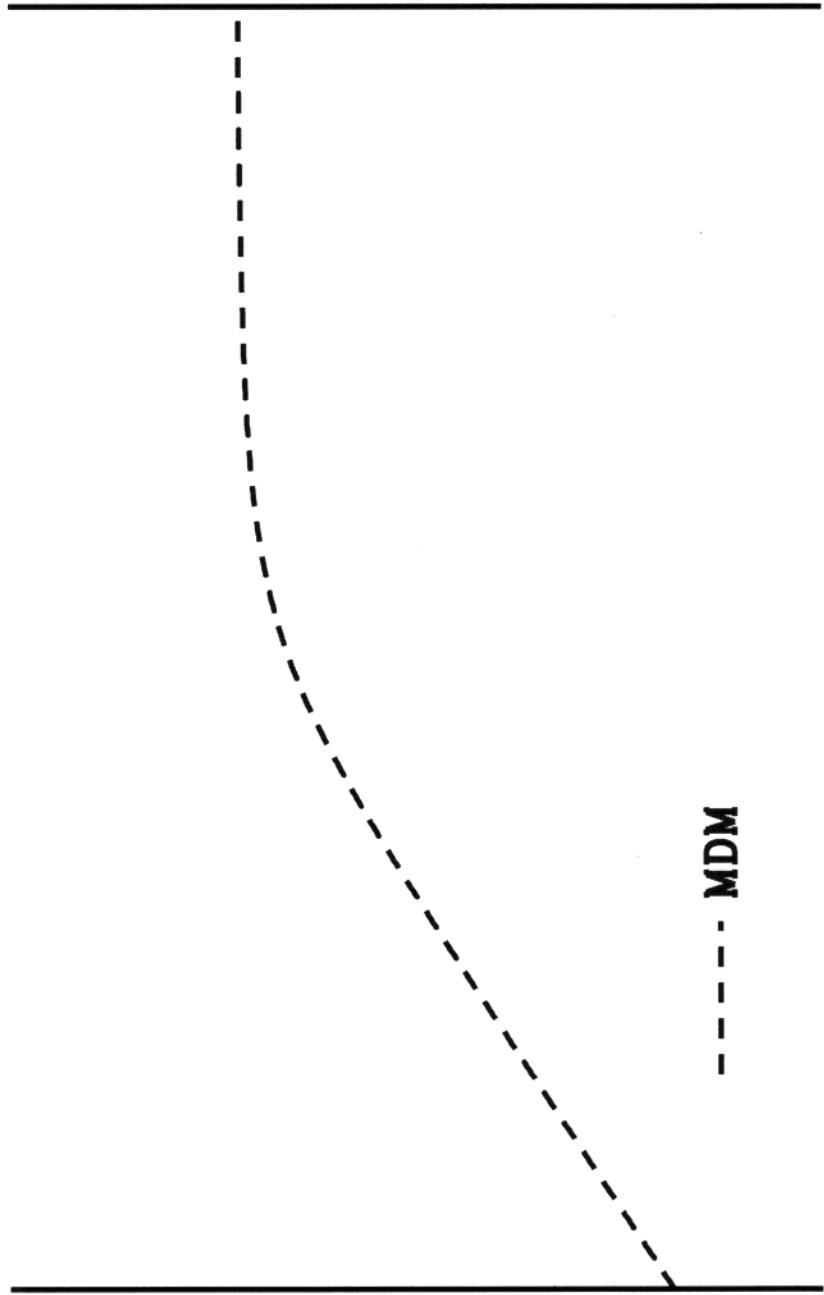
# Grand Unified Power Spectrum

$$R \simeq 2\pi/k$$



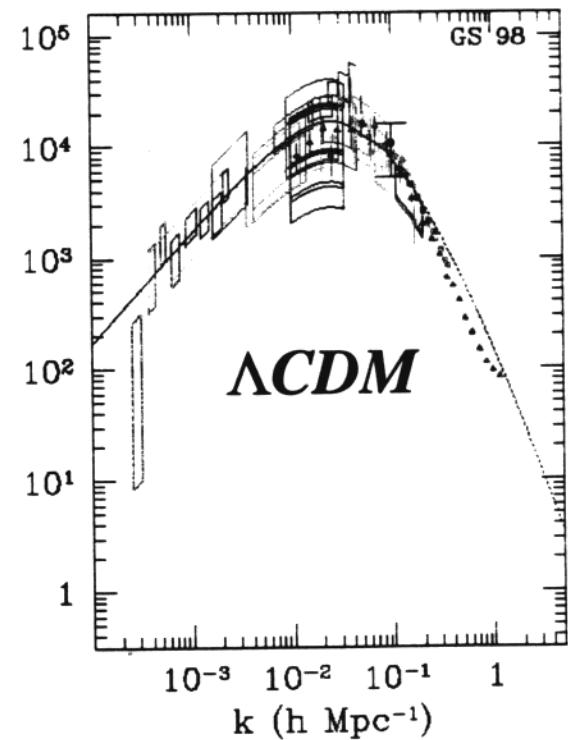
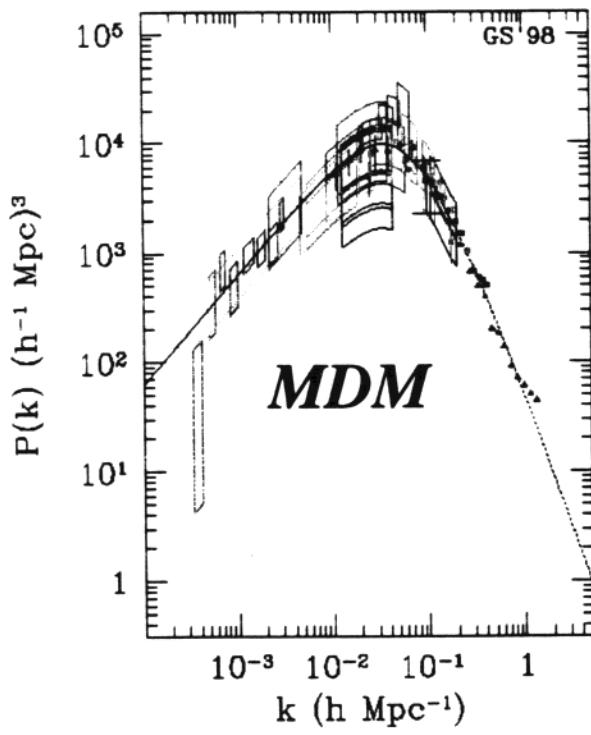
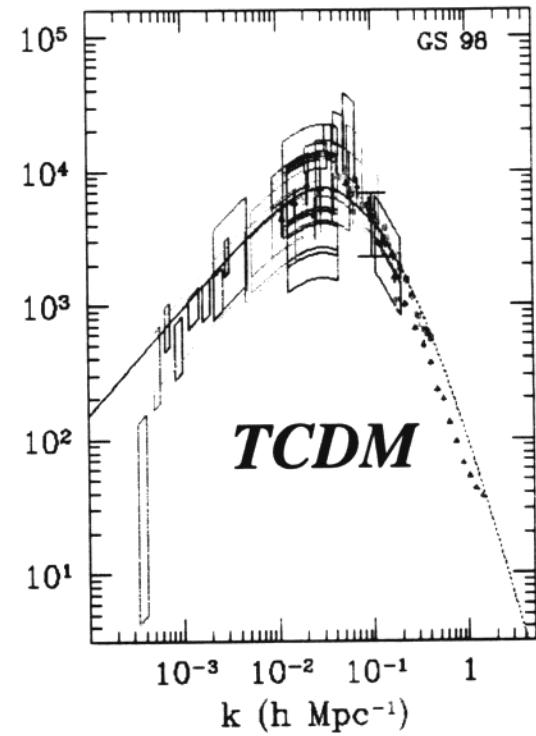
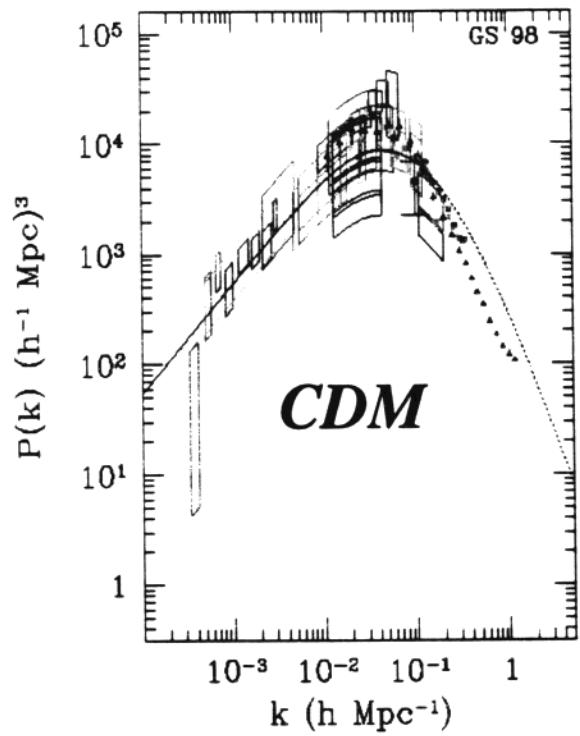








$$\kappa^2 P(k) = \Delta^2(k)$$



# *One analysis:*

*(Gawiser & Silk, Science 1998)*

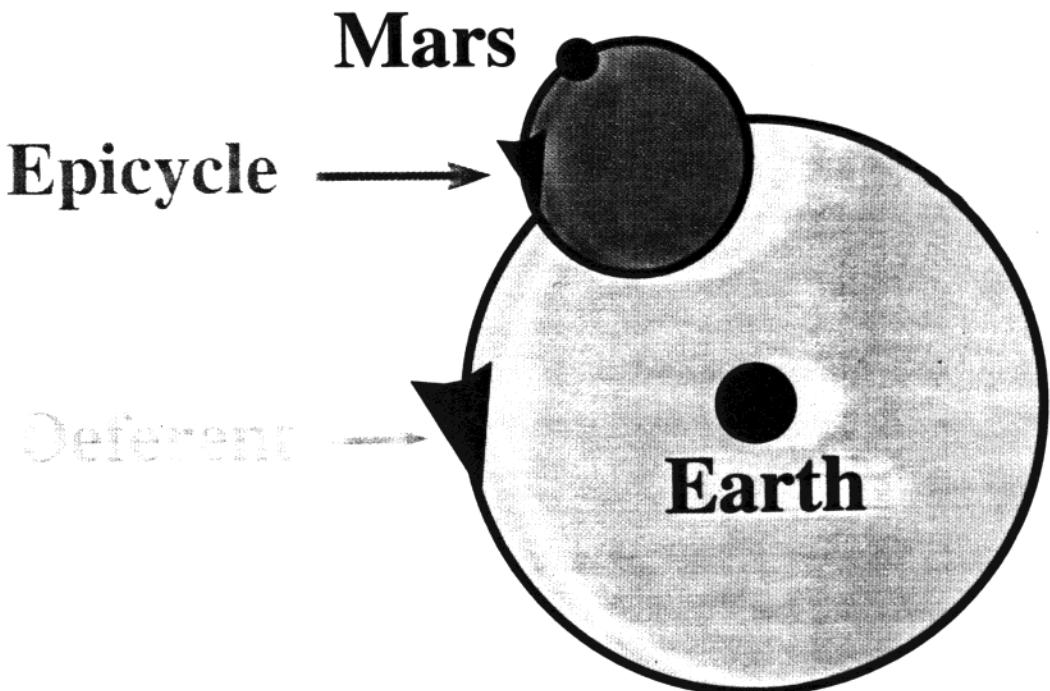
<i>Model</i>	$\chi^2/dof^*$
<b><i>SCDM</i></b>	<b>3.8</b>
<b><i>TCDM</i></b>	<b>2.1</b>
<b><i><math>\Lambda</math>CDM</i></b>	<b>1.9</b>
<b><i>OCDM</i></b>	<b>1.8</b>
<b><i>MDM</i></b>	<b>1.2</b>

\* Could add tensor mode, etc.

# The Best Fit Cosmology:

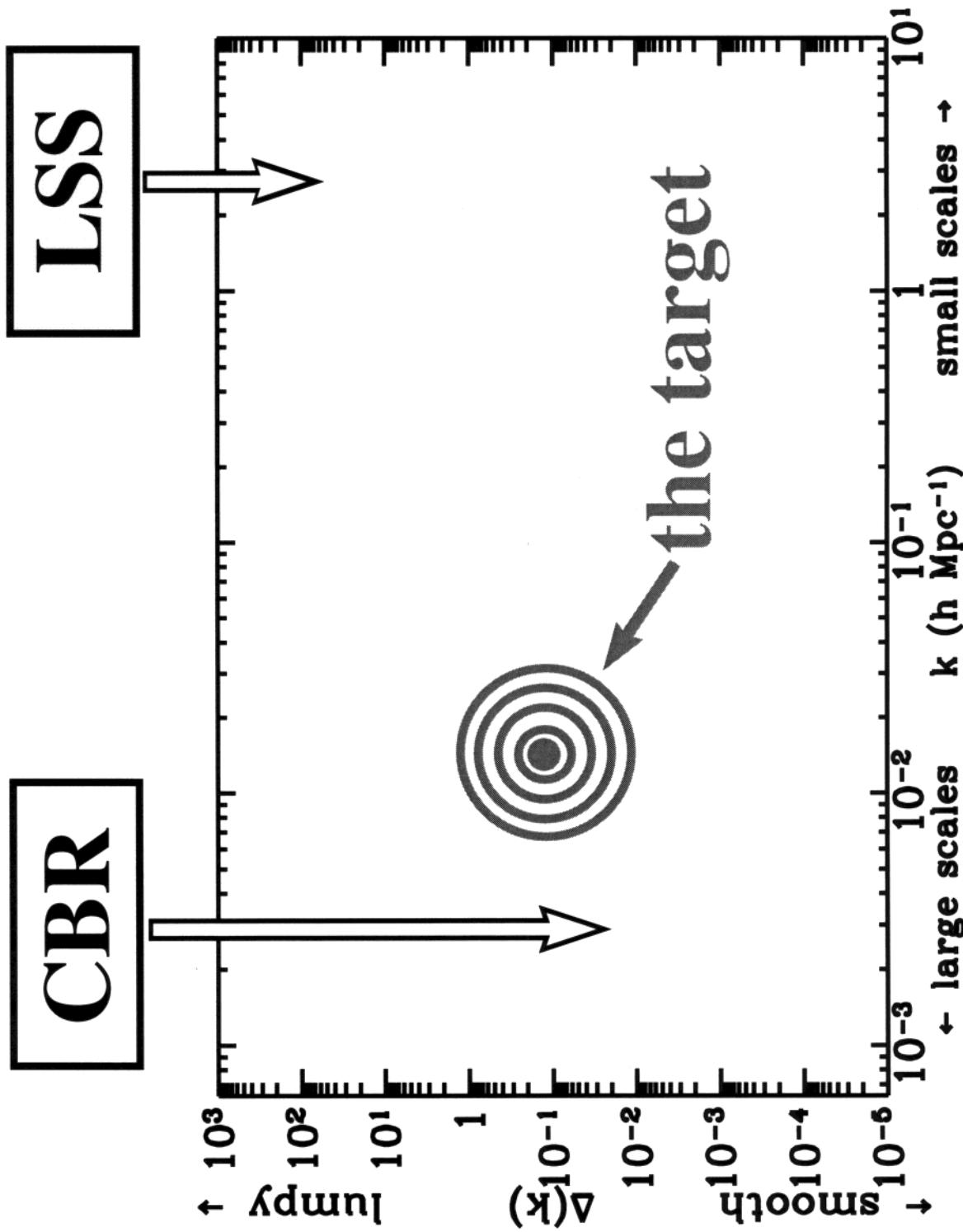
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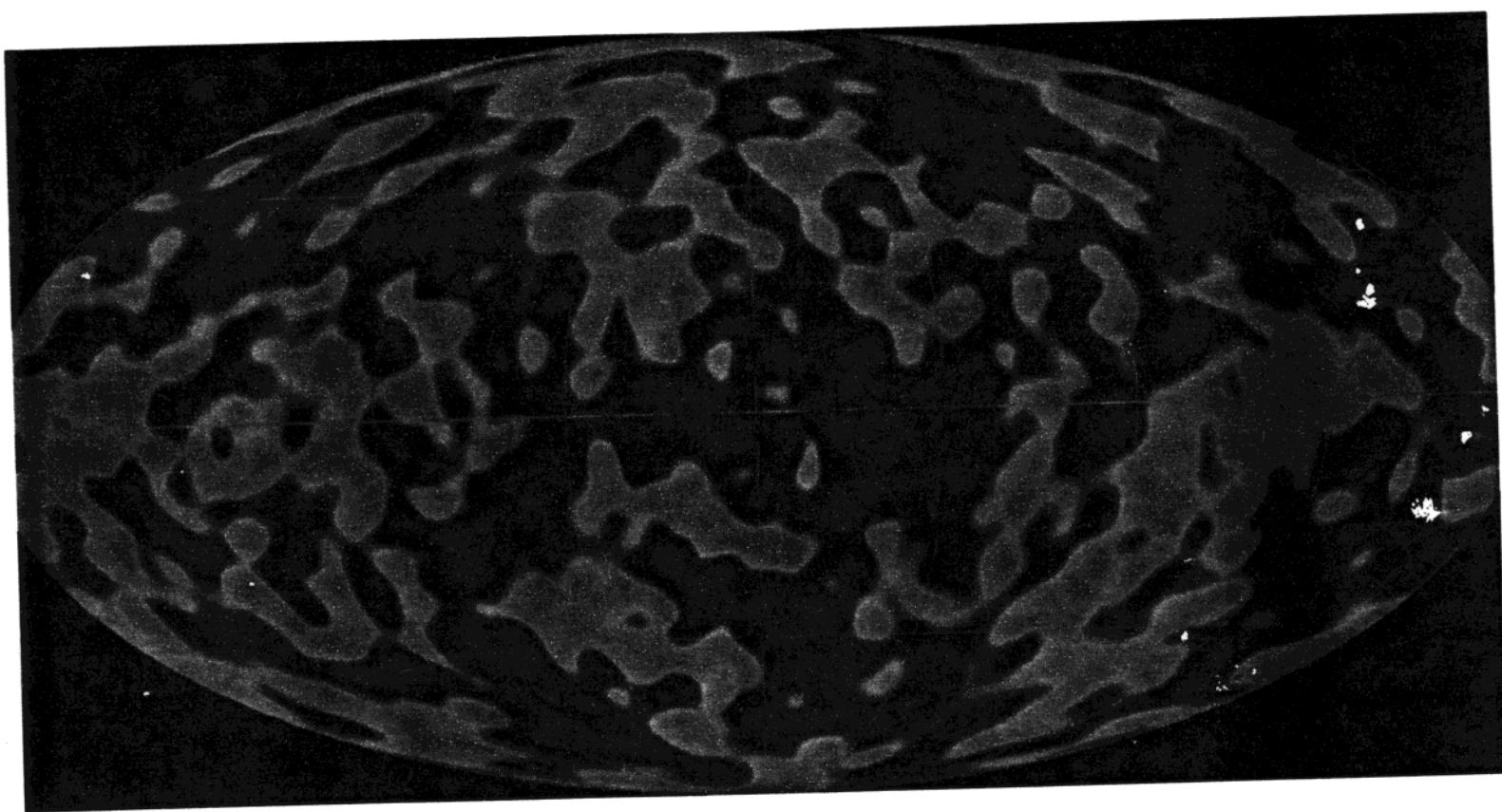
baryons  
cold dark matter  
hot dark matter  
(each in types of  
of equal mass)  
cosmological constant  
warm dark matter  
and season with a little



Era of precision cosmology:

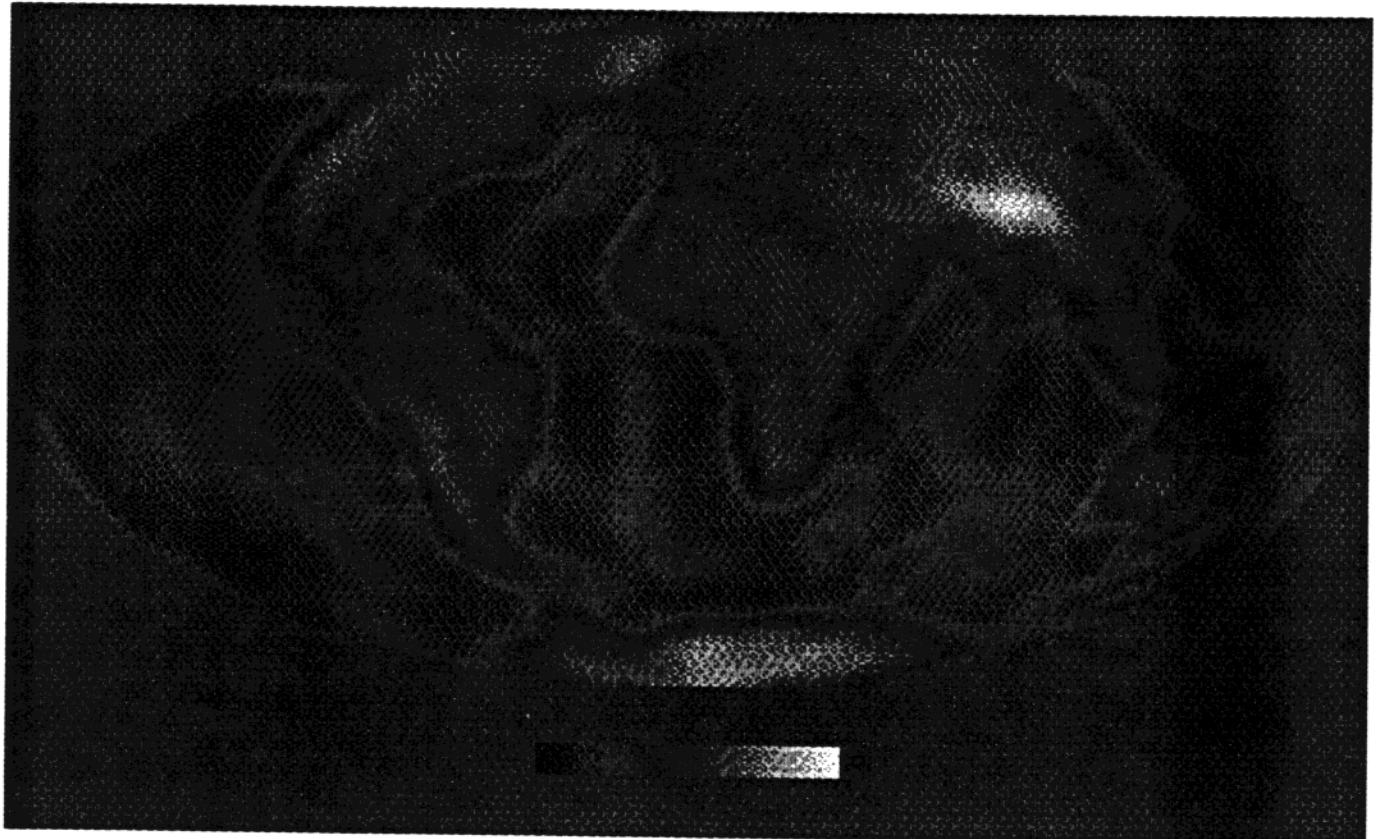
CBR on finer angular scales  
LSS on larger distance scales



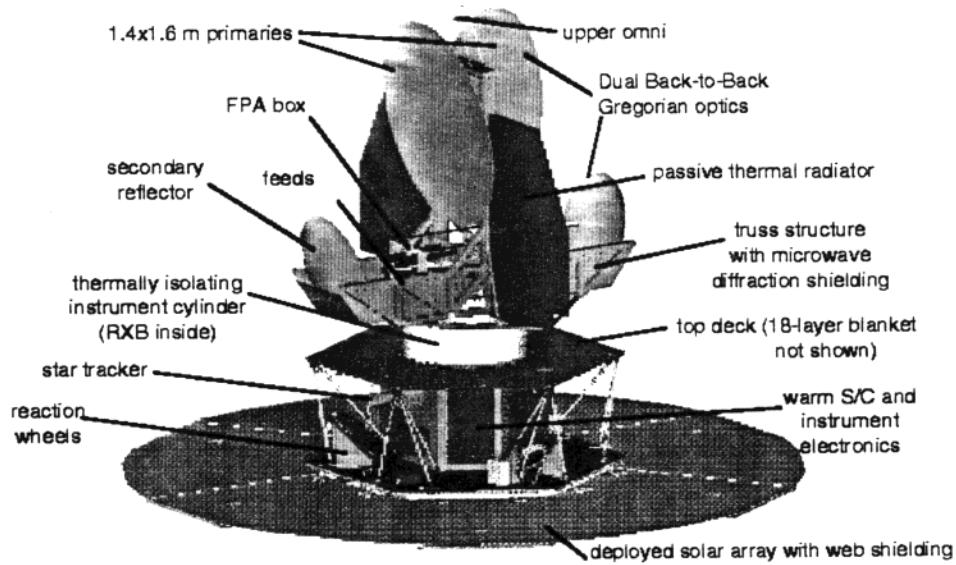


COBE

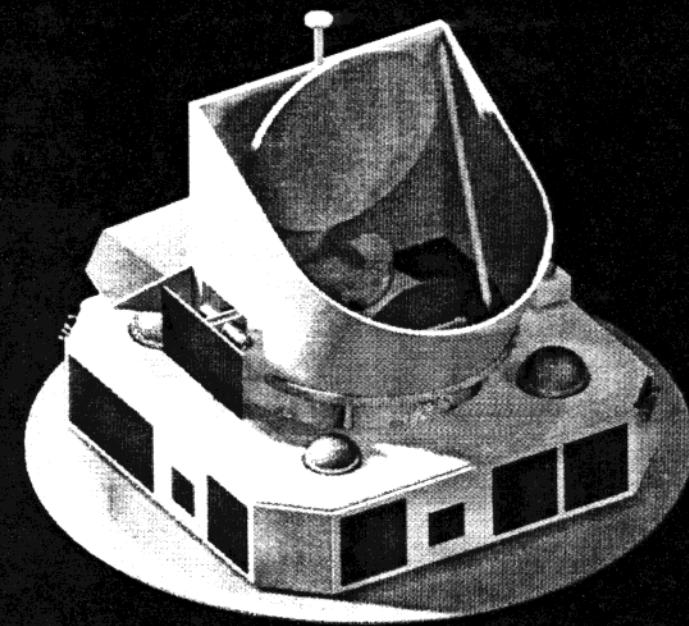
*Earth at  
COBE resolution  
(10°)*



# MAP

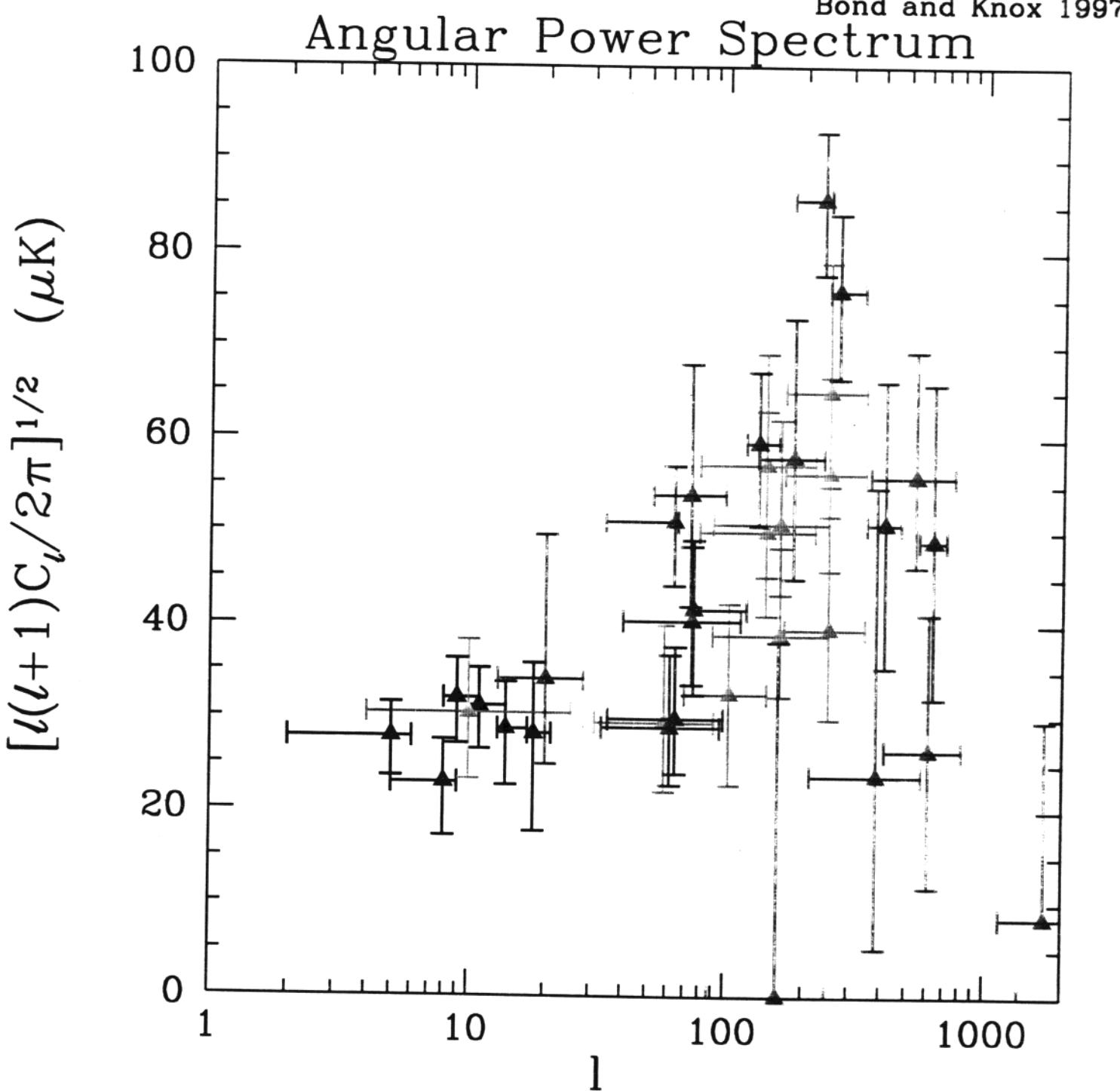


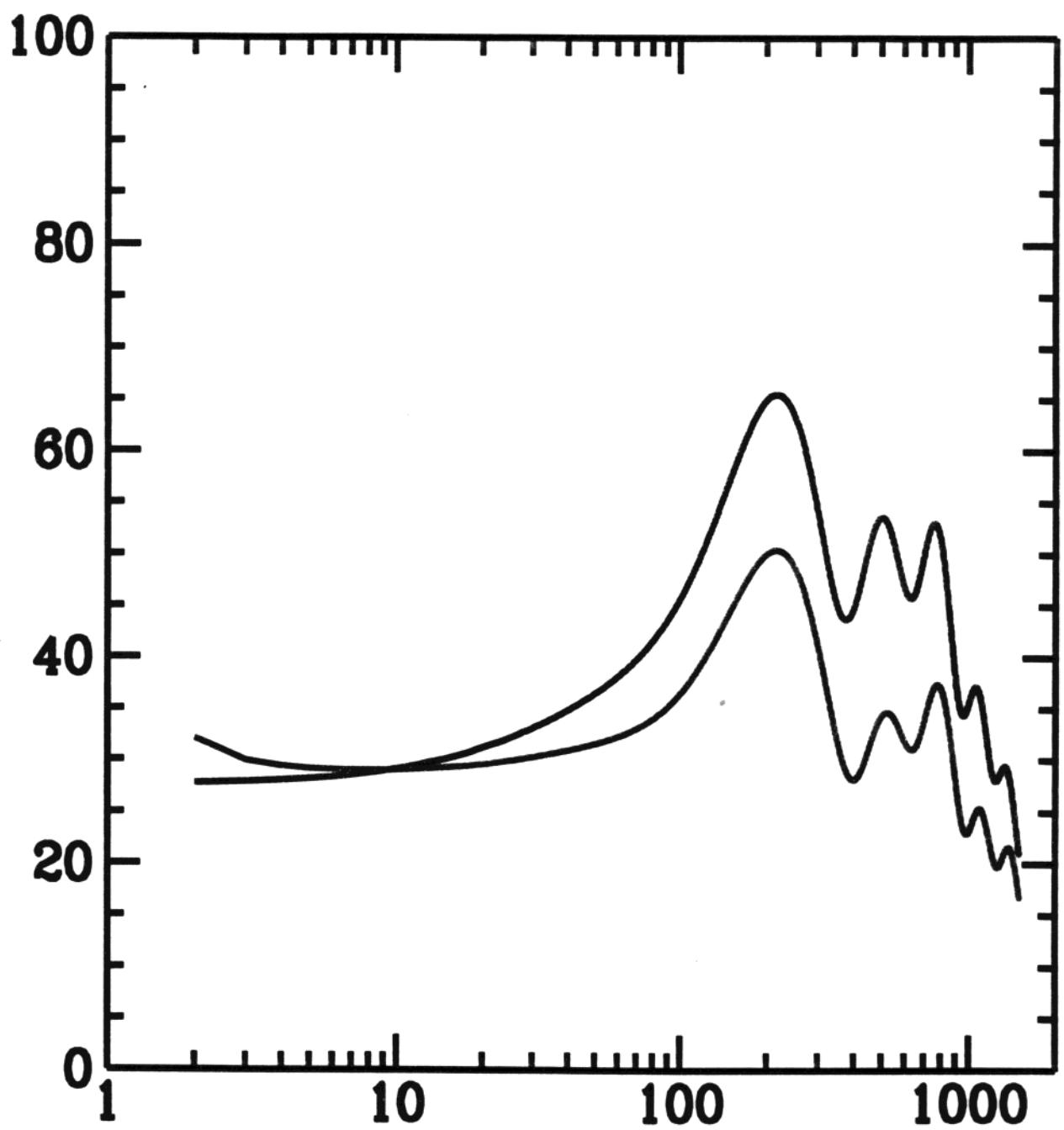
# PLANCK



# CBR Today

Bond and Knox 1997





**vanilla**  
**SCDM**

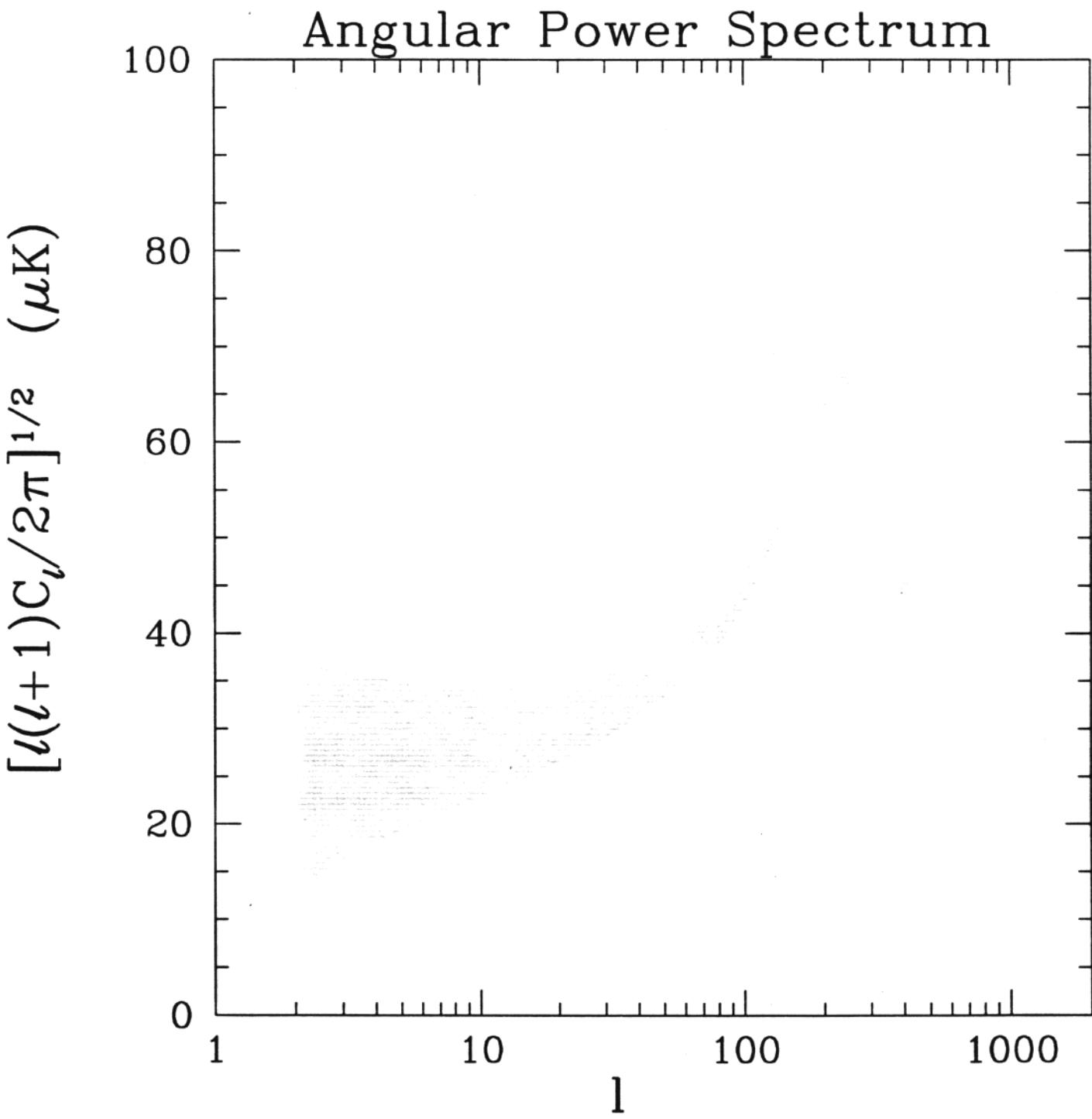
$n=1; r=0$

**Rocky**  
**Road**

$n=0.9; r=0.7$

# In the Post-Planck Era

Bond and Knox



# *Mapping the Universe*

**know the position of an object by**

*location on the sky*    **2 dimensions**

*distance*                        **1 dimension**

*(spectrum/redshift)*

---

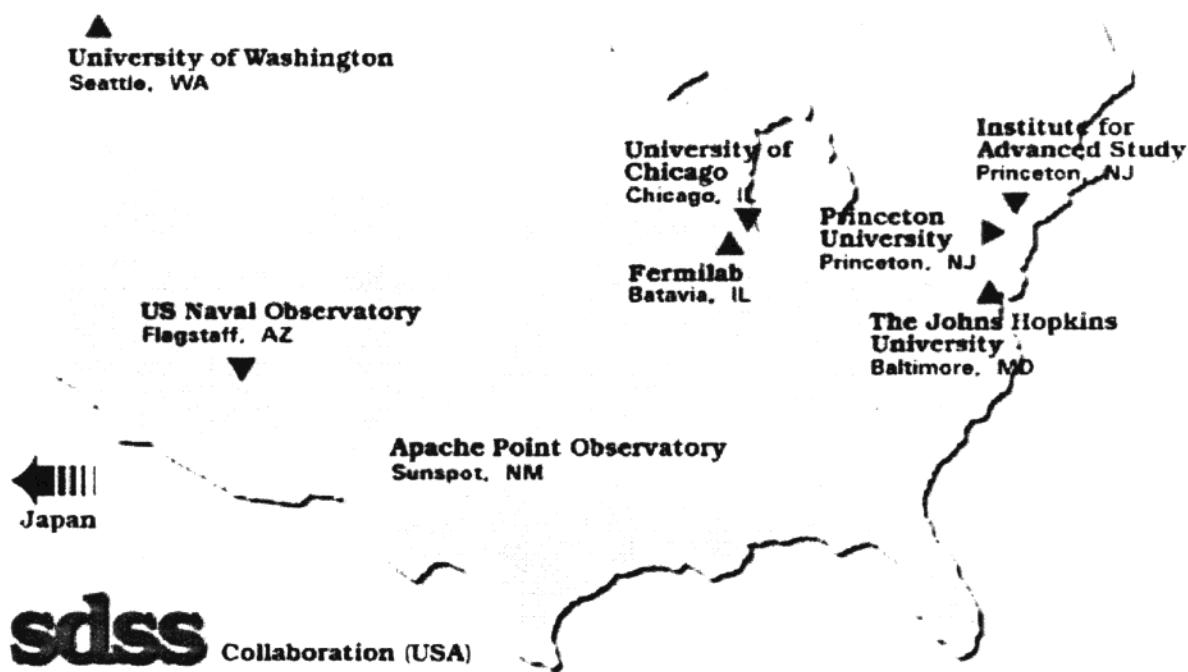
**3 dimensions**

**today:**                        *30,000 galaxies*

**1999:**                        *100,000 galaxies*

**200N:**                        *1,000,000 galaxies*  
**SDSS**                            *150,000 quasars*

# Sloan Digital Sky Survey



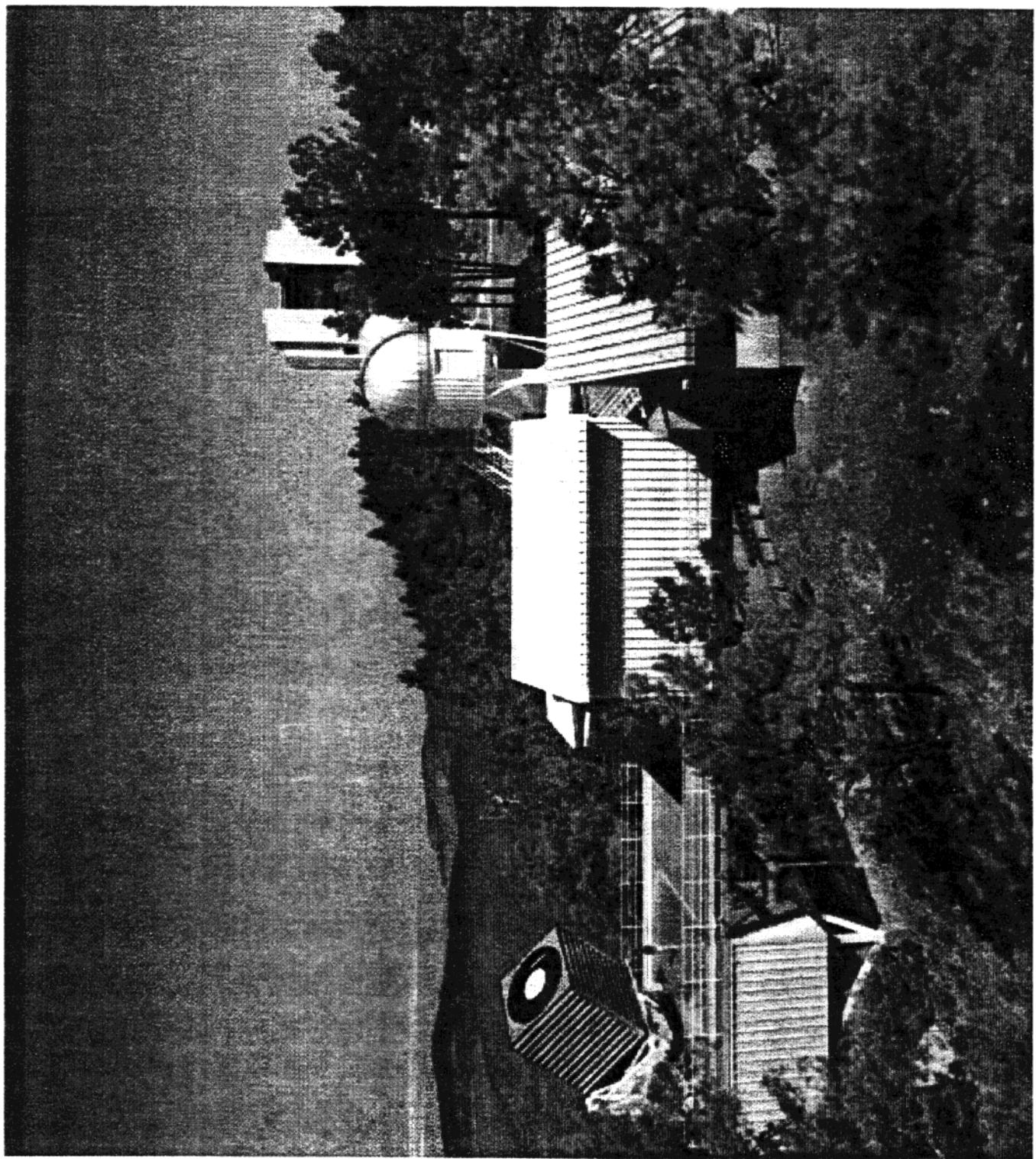
## Map of the Universe

*Construct a 2.5 meter telescope*

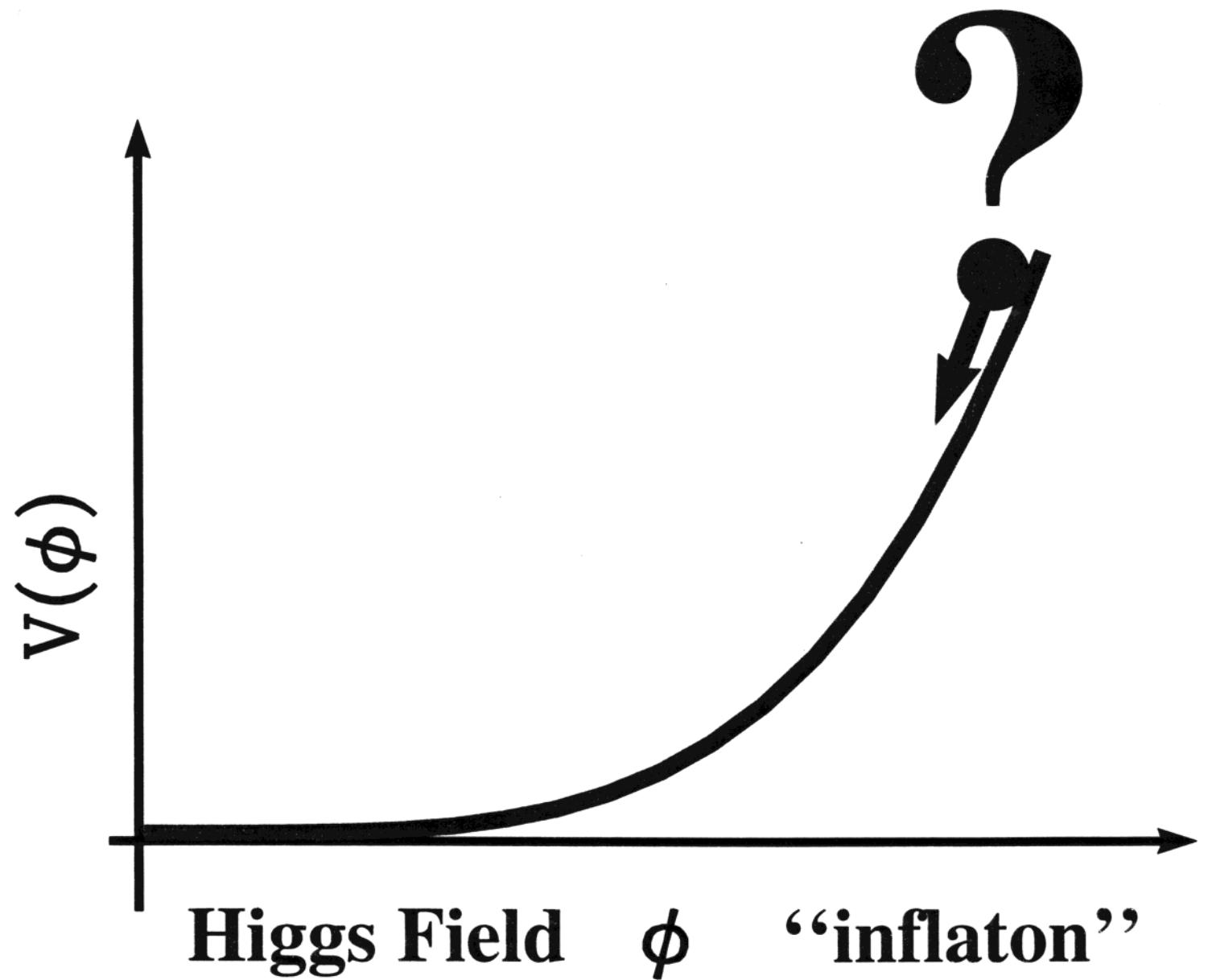
*Build Special Purpose Instruments*

*Image the sky---identify galaxies and QSOs*

*Map the 3-D location of a million galaxies  
(and a hundred thousand quasars)*



# *Who is the Inflaton?*



# **Models of Inflation**

**old, new, middle-aged,  
chaotic, quixotic, ergodic,  
exotic, heterotic, autoerotic,  
natural, supernatural, *au natural*,  
one field, two field, home field,  
modulus, modulo, moduli,  
self-reproducing, self-promoting,  
hybrid, low-bred, white-bread,  
first-order, second-order, new-world order,  
pre-big-bang, no-big-bang, post-big-bang,  
*D*-term, *F*-term, winter-term,  
supersymmetric, superstring, superstitious,  
extended, hyperextended, overextended,  
*D*-brane, *p*-brane, *No*-brain,  
dilaton, axion, crouton, .....**

# Classification

**Type I:** *single-field, slow-roll models  
(or models that can be  
expressed as such)*

**Ia:** *large-field models*

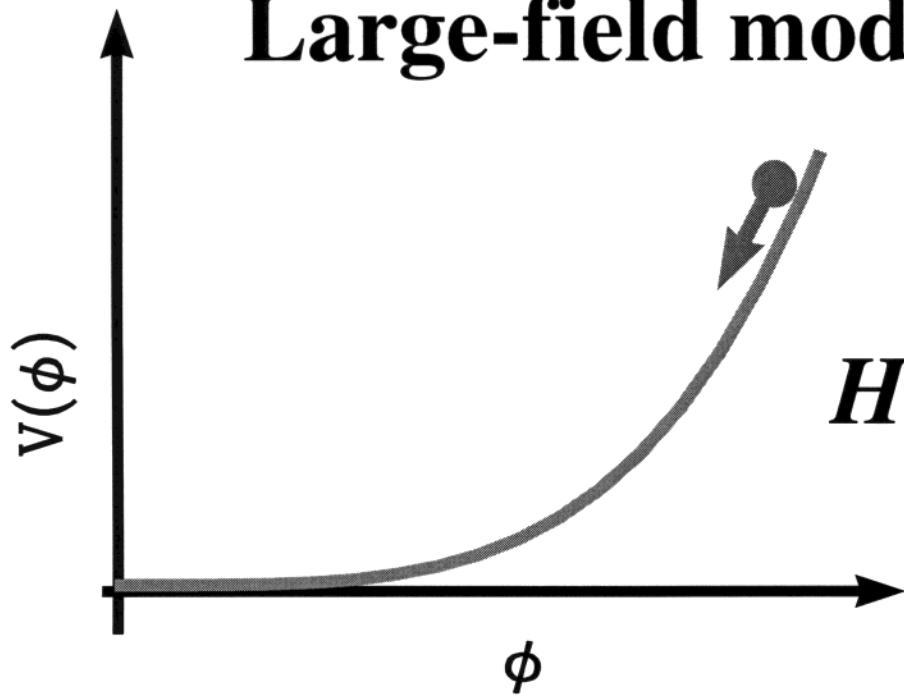
**Ib:** *small-field models*

**Type II:** *anything else*

**No M theory?**

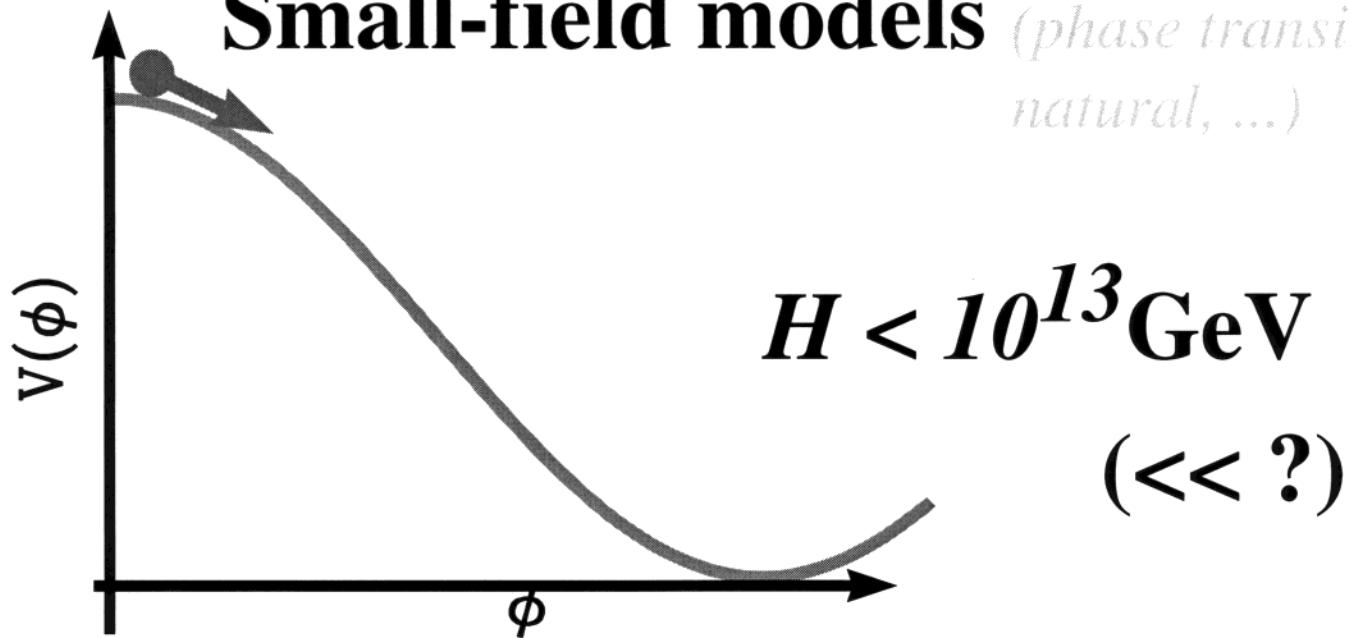
# Large-field models

(chaotic,  
power-law, ...)



# Small-field models

(phase transitions,  
natural, ...)



Tensor pert's proportional to  $H$

$$\delta(G_{\mu\nu}) = 8\pi G \delta(T_{\mu\nu})$$

Bardeen (1980)

## *Reference Spacetime: (Flat FRW)*

$$ds^2 = a^2(\tau) \left\{ d\tau^2 - \delta_{ij} dx^i dx^j \right\}$$

$$\tau = \text{conformal time} \quad a^2(\tau) d\tau^2 = dt^2$$

## *Perturbed Spacetime: (Scalar)*

$$ds^2 = a^2(\tau) \left\{ 2A d\tau^2 - 2\partial_i B dx^i d\tau - [-2\psi \delta_{ij} + 2\partial_i \partial_j E] dx^i dx^j \right\}$$

## *Perturbed Spacetime: (Tensor)*

$$ds^2 = a^2(\tau) h_{ij} dx^i dx^j$$

$$h_{ij} \left\{ \begin{array}{l} \text{transverse, traceless tensor} \\ \text{(gravity waves)} \end{array} \right.$$

# Variational Formalism for Quantization

Mukhanov

## Action:

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_{Pl}^2}{16\pi} R + \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right]$$

$$\begin{aligned} g_{\mu\nu} &= g_{\mu\nu}^{FRW}(t) + \delta g_{\mu\nu}(\vec{x}, t) \\ \phi(\vec{x}, t) &= \phi_0(t) + \delta\phi(\vec{x}, t) \end{aligned}$$

...ADM variables...expand to 2<sup>nd</sup> order in pert'ns...  
use background field eqns...integrate by parts...

$$\delta_2 S = \int d^4x \left[ \frac{1}{2} \partial_\mu u \partial^\mu u - \frac{1}{2} \left( \frac{1}{z} \frac{d^2 z}{d\tau^2} \right) u^2 \right]$$

Minkowski scalar field  $u$       with mass<sup>2</sup> =  $-z^{-1} d^2 z/d\tau^2$

$$u = a\delta\phi + z\dot{\delta g}_{\mu\nu} \qquad \qquad z = a\dot{\phi}/H$$

(proportional to  
curvature perturbation)

(depends only on  
background field evolution)

$$u_k \rightarrow \mathcal{R}_k \rightarrow \Delta(k)$$

$$\frac{d^2 u_k}{d\tau^2} + \left( k^2 - \frac{1}{z} \frac{d^2 z}{d\tau^2} \right) u_k = 0$$

$$z = a\dot{\phi}/H$$

"Mass" is complicated  
model-dependent function  
of *inflaton potential*  $V(\phi)$ .

# Slow-Roll Parameters

- *cosmological model*

$$V(\phi), V'(\phi), V''(\phi), V'''(\phi)$$

- $\epsilon; \eta; \xi; \dots$  *slow-roll parameters*

$$\epsilon \sim V' \quad \eta \sim V'' \quad \xi \sim V''' + \dots$$

Look  
At This!

- *gives mass as function of time*

$$-m_u^2 = 2a^2 H^2 \left[ 1 + \epsilon - \frac{3}{2}\eta + \epsilon^2 - 2\epsilon\eta + \frac{1}{2}\eta^2 + \frac{1}{2}\xi^2 \right]$$

# Amplitudes and Spectra

Stewart & Lyth

Scalar:

$$\frac{A_S(k)}{A_S(k)} = \frac{2}{5\sqrt{\pi}} \frac{H(\phi)}{m_{Pl}} \frac{1}{\sqrt{\epsilon(\phi)}} [1 - 0.46\epsilon(\phi) - 0.73\eta(\phi)]$$

$$n(k) \equiv \frac{d \ln A_S^2}{d \ln k} = 1 - 4\epsilon(\phi) - 2\eta(\phi) + 2.16\epsilon^2(\phi) + 1.3\epsilon(\phi)\eta(\phi) - 1.46\eta^2(\phi)$$

Tensor:

$$\frac{A_T(k)}{A_S(k)} = \frac{2}{5\sqrt{\pi}} \frac{H(\phi)}{m_{Pl}} [1 - 0.27\epsilon(\phi)]$$

$$n_T(k) \equiv \frac{d \ln A_T^2}{d \ln k} = -2\epsilon(\phi) - 3.08\epsilon^2(\phi) + 1.08\epsilon(\phi)\eta(\phi)$$

CDM: $n = 1$	$n_T = 0$	$A_T = 0$
--------------	-----------	-----------

# Consistency Relation

Copeland, Kolb, Liddle, Lidsey

$$n_T = -2 \frac{A_T^2}{A_S^2} \left[ 1 - \frac{A_T^2}{A_S^2} + (1 - n) \right]$$

**Consistency relation is a generic feature of Type I inflation models**

**There is NO general relation between  $r$  and  $1-n$  !!!!!!!!**

# Model Space

$$[\varepsilon, \eta]$$

$$\varepsilon = \frac{m_{PL}^2}{16\pi} \left( \frac{V'}{V} \right)^2$$

$$\eta = \frac{m_{PL}^2}{8\pi} \frac{V''}{V} - \varepsilon$$

# CBR Space

$$[n, r]$$

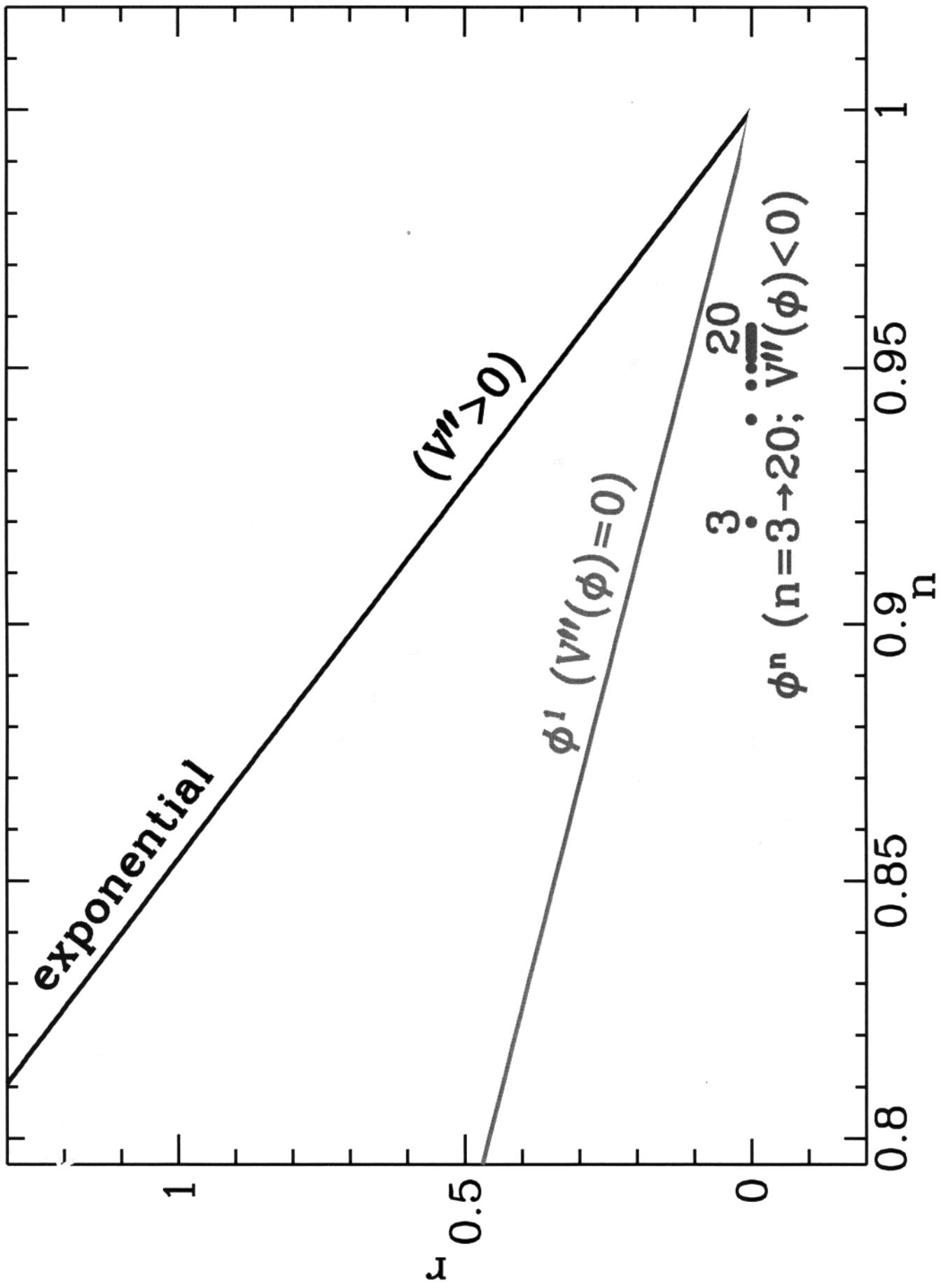
*scalar*  
*spectral*  
*index*

$$r = \left( \frac{\text{tensor}}{\text{scalar}} \right)_{l=2}$$

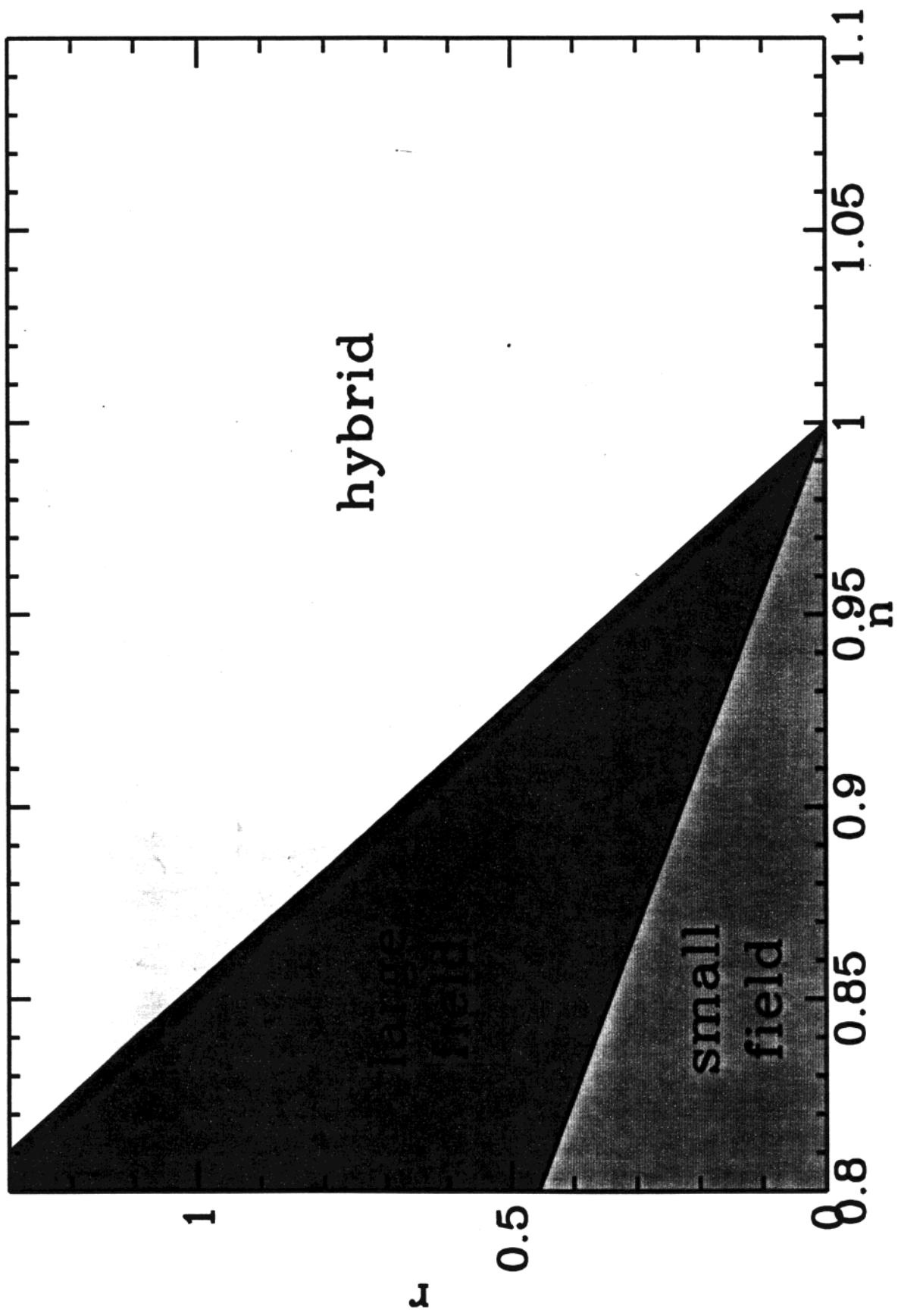
$$V'(\phi) \& V''(\phi) \leftrightarrow [\varepsilon, \eta] \leftrightarrow [n, r]$$

# Well studied inflation models (to lowest order).

model	$n - 1$	$r$
large field	$(2\eta - 4\epsilon)$	$(13.7\epsilon)$
small field	$-\frac{2p+4}{p+200}$	$\frac{13.7p}{p+200}$
au natural	$-\frac{p-1}{25(p-2)}$	$\simeq 0$
linear	$-\frac{m_P^2 l}{2\pi\mu^2}$	$\simeq 0$
power law	$\Lambda^4 \exp \sqrt{\frac{16\pi\phi^2}{pm_P^2}}$	$-2p^{-1}$
hybrid	$\Lambda^4 [1 + (\phi/\mu)^p]$	$13.7p^{-1}$
		?????







$r = (\text{tensor/scalar})_{l=2}$

$n = \text{scalar spectral index}$



# RECONSTRUCTION

*Hamilton-Jacobi Approach:*  $H(\phi)$

$$\left(\frac{dH}{d\phi}\right)^2 - \frac{12\pi}{m_{Pl}^2} H^2 = -\frac{32\pi^2}{m_{Pl}^4} V(\phi)$$

$$V = \frac{m_{Pl}^2 H^2}{8\pi} (3 - \epsilon)$$

$$V' = -\frac{m_{Pl}^2}{\sqrt{4\pi}} H^2 \epsilon^{1/2} (3 - \eta)$$

$$V'' = H^2 \left( 3\epsilon + 3\eta - \eta^2 + \xi^2 \right)$$

Lidsey, Liddle, Kolb, Copeland, Barriero, Abney  
*Reviews of Modern Physics, 4/97*

Observables needed to reconstruct a given derivative of the potential to a given order.

	lowest-order	next-order	next-to-next-order
$V$	$A_T^2$	$A_T^2, A_S^2$	$A_T^2, A_S^2, n$
$V'$	$A_T^2, A_S^2$	$A_T^2, A_S^2, n$	$A_T^2, A_S^2, n, dn/d \ln k$
$V''$	$A_T^2, A_S^2, n$	$A_T^2, A_S^2, n, dn/d \ln k$	
$V'''$	$A_T^2, A_S^2, n, dn/d \ln k$		

# Reconstruction Strategy

## I) From CBR anisotropy

1) Find tensor mode

$$r = (\text{tensor/scalar})_{l=2}$$

2) Fit scalar spectrum

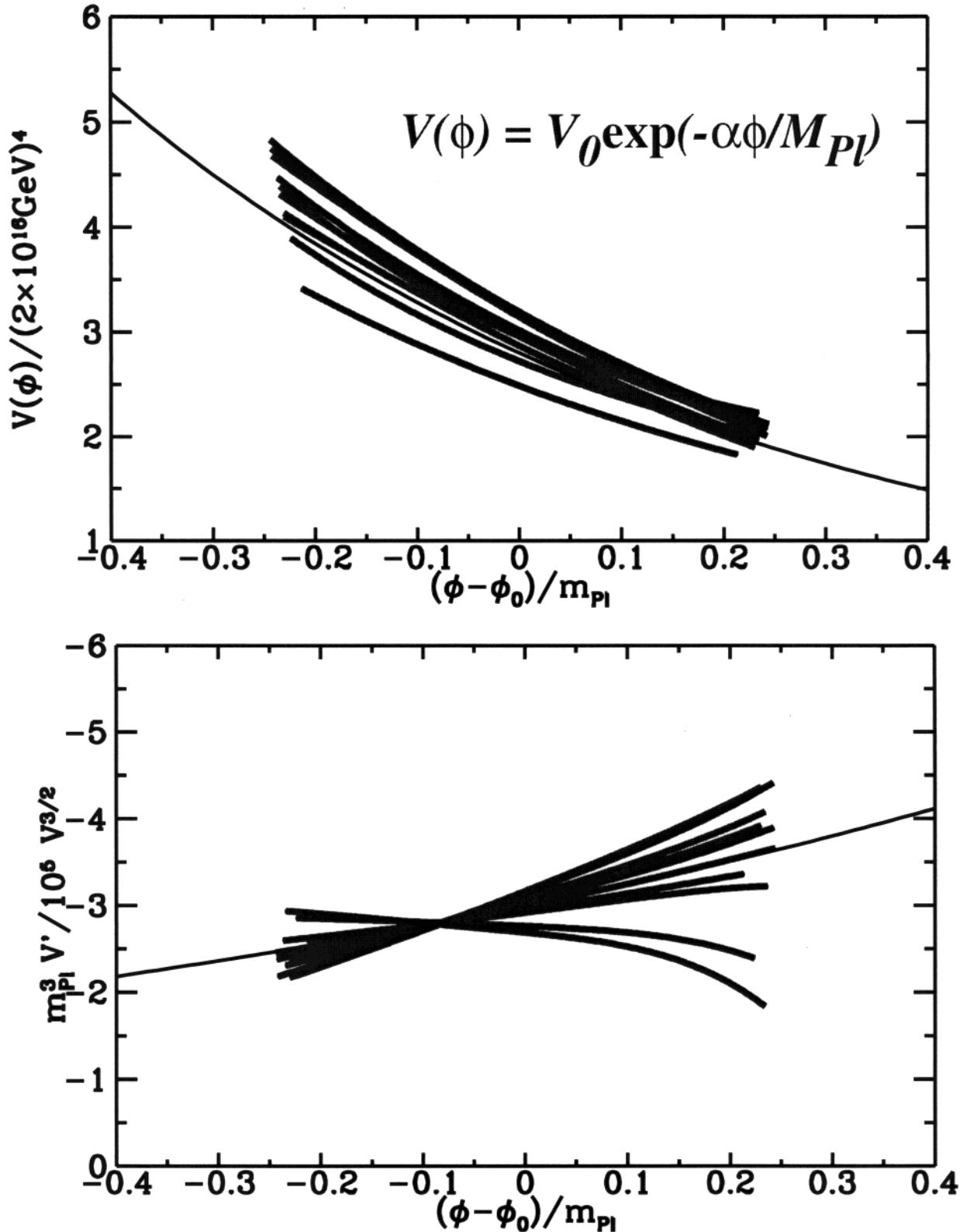
$$(A_S^2(k_*) , n_* , dn/dk|_*, \dots)$$

II) Express  $V(\phi_*)$ ,  $V'(\phi_*)$ , ..., and  
 $(\phi - \phi_*)$  in terms of observables

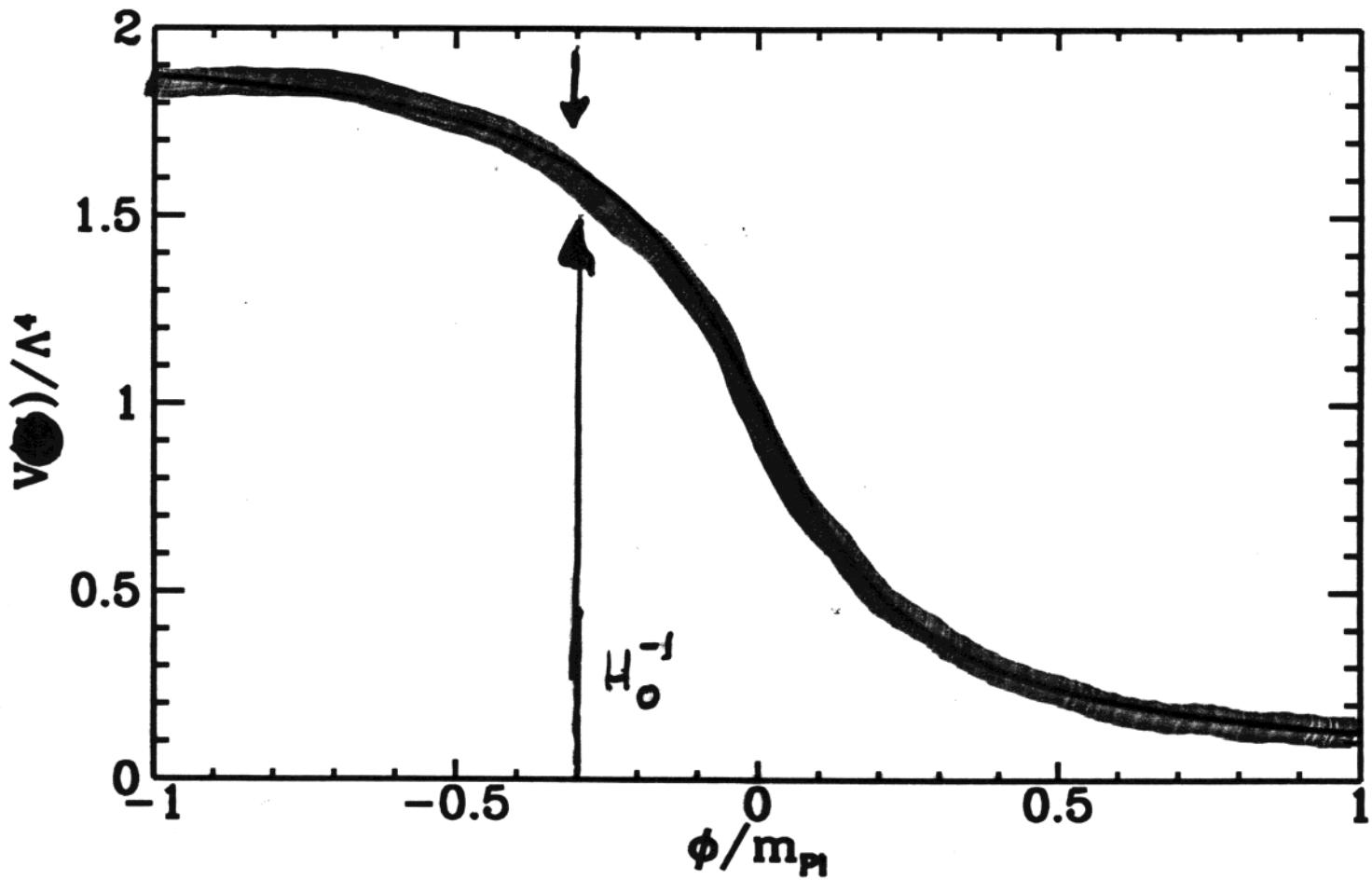
III) Taylor series for  $V(\phi)$ :

$$V(\phi) = V(\phi_*) + V'(\phi_*) (\phi - \phi_*) + \dots$$

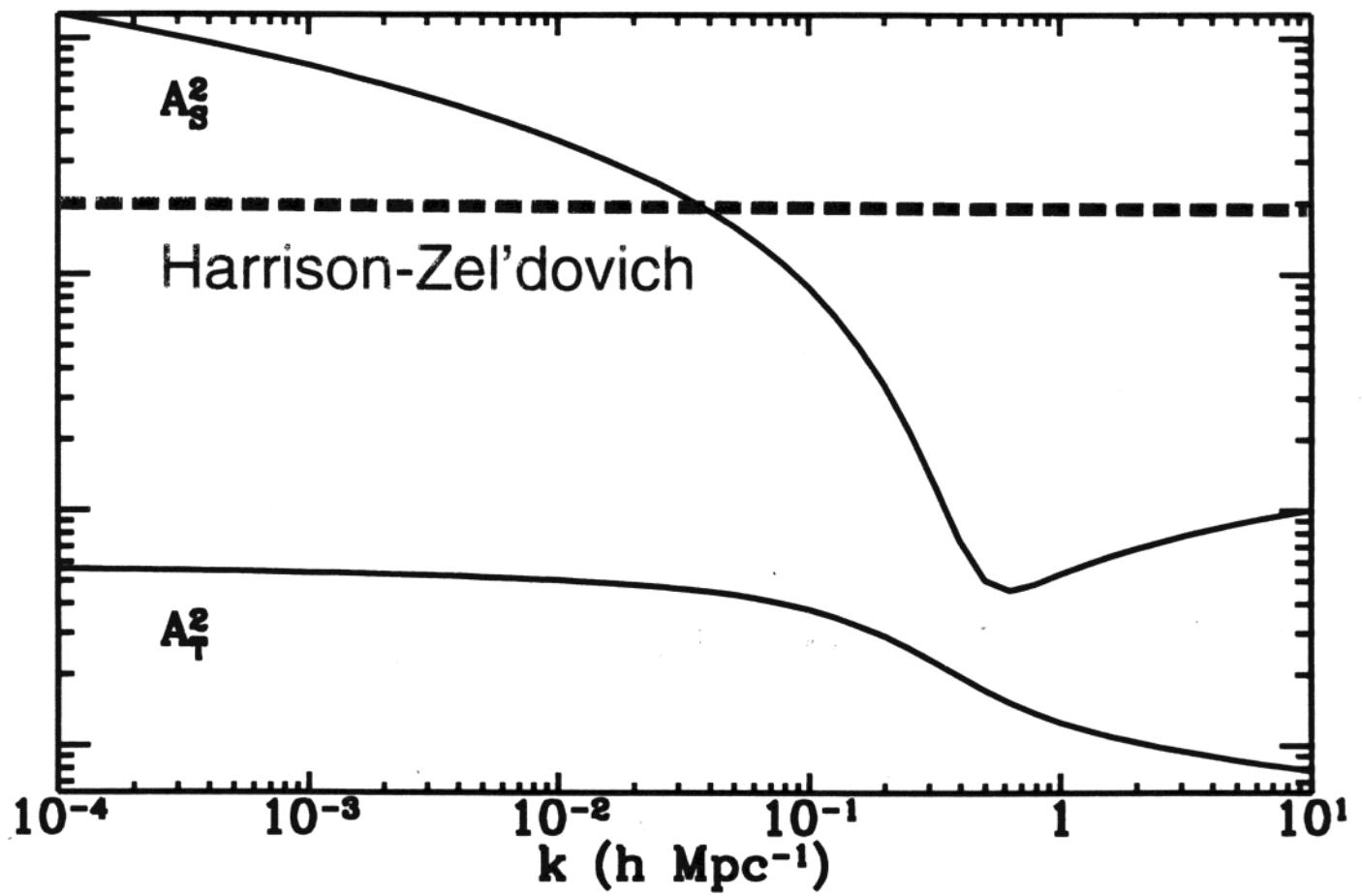
# *No distinguishing characteristic*



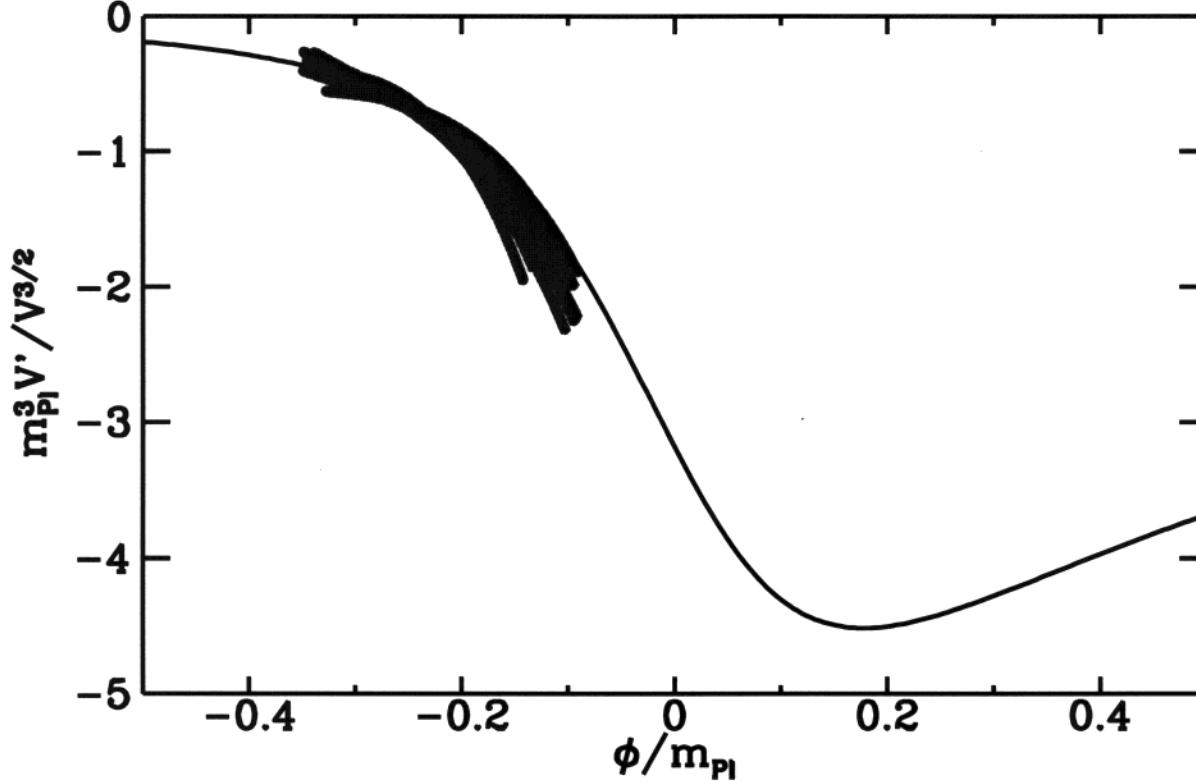
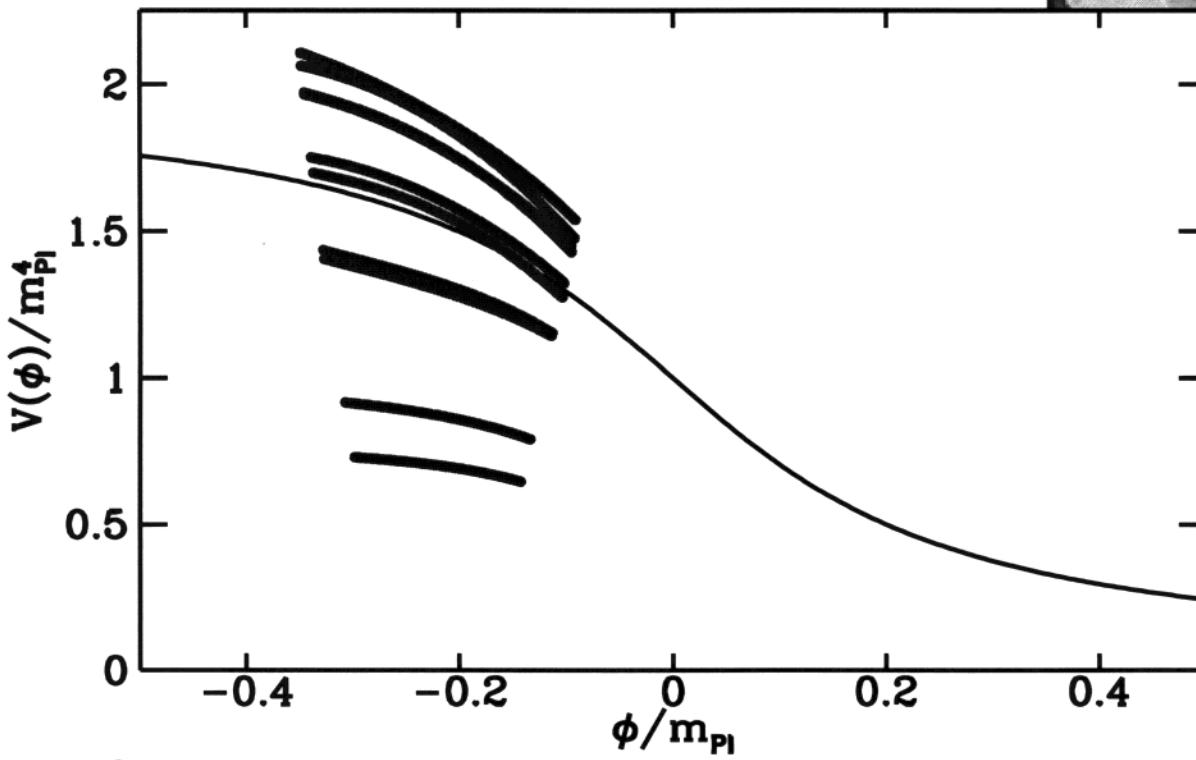
$$V(\phi) = \Lambda^4 \left[ 1 - \frac{2}{\pi} \tan^{-1} \frac{5\phi}{m_P} \right]$$



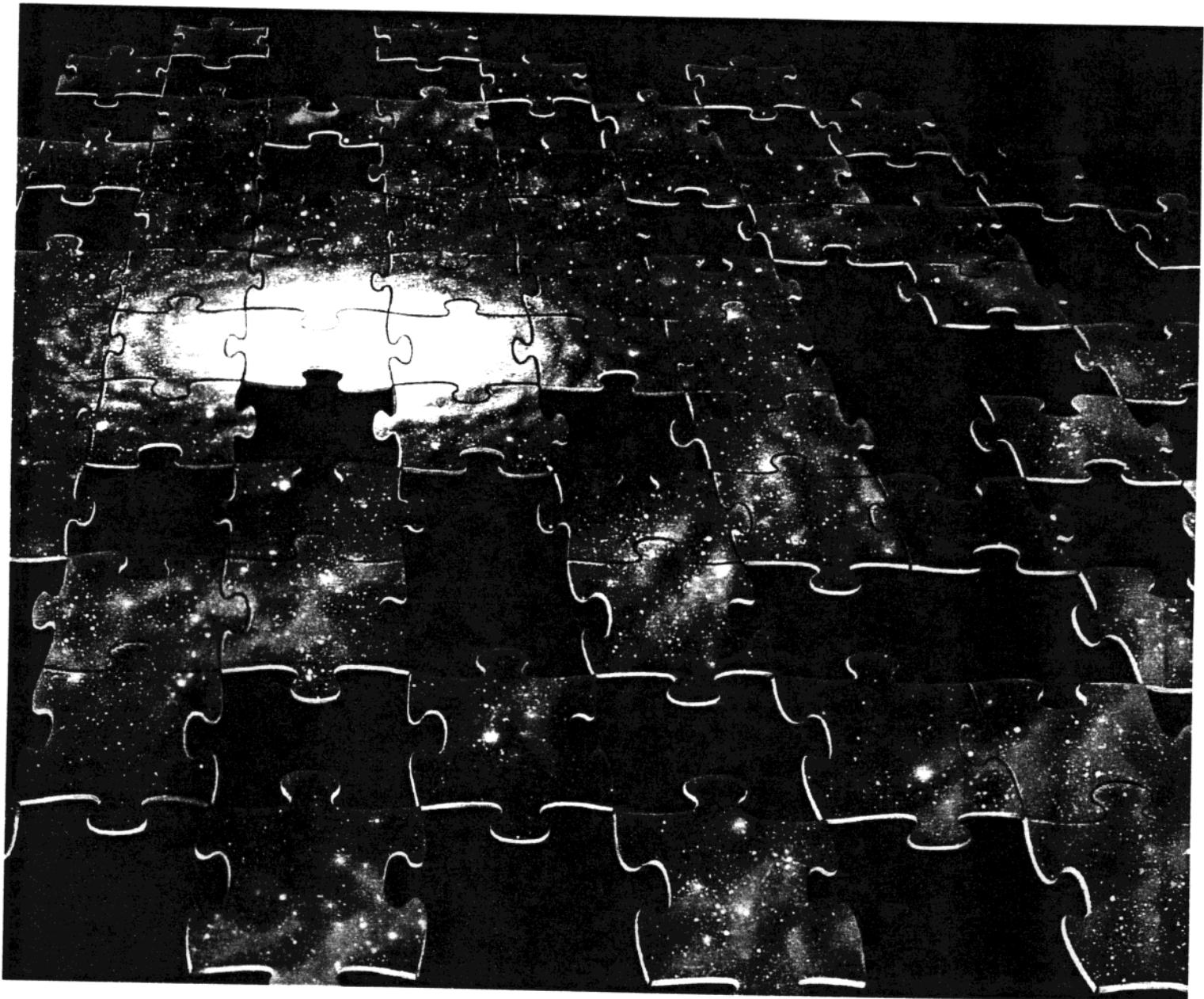
WANG, MUKHANOV, STEINHARDT



# *distinguishing characteristics?*



Pieces



DARK MATTER

# *90% of the Universe* ***is DARK!!***

- The Weight of Space?
- Planets?
- Mass disadvantaged stars of color?
  - brown dwarfs
  - red dwarfs
  - white dwarfs**
- Black holes?
- Fossil remnant of the *Big Bang*?

**WIMP:** *weakly interacting massive particle*

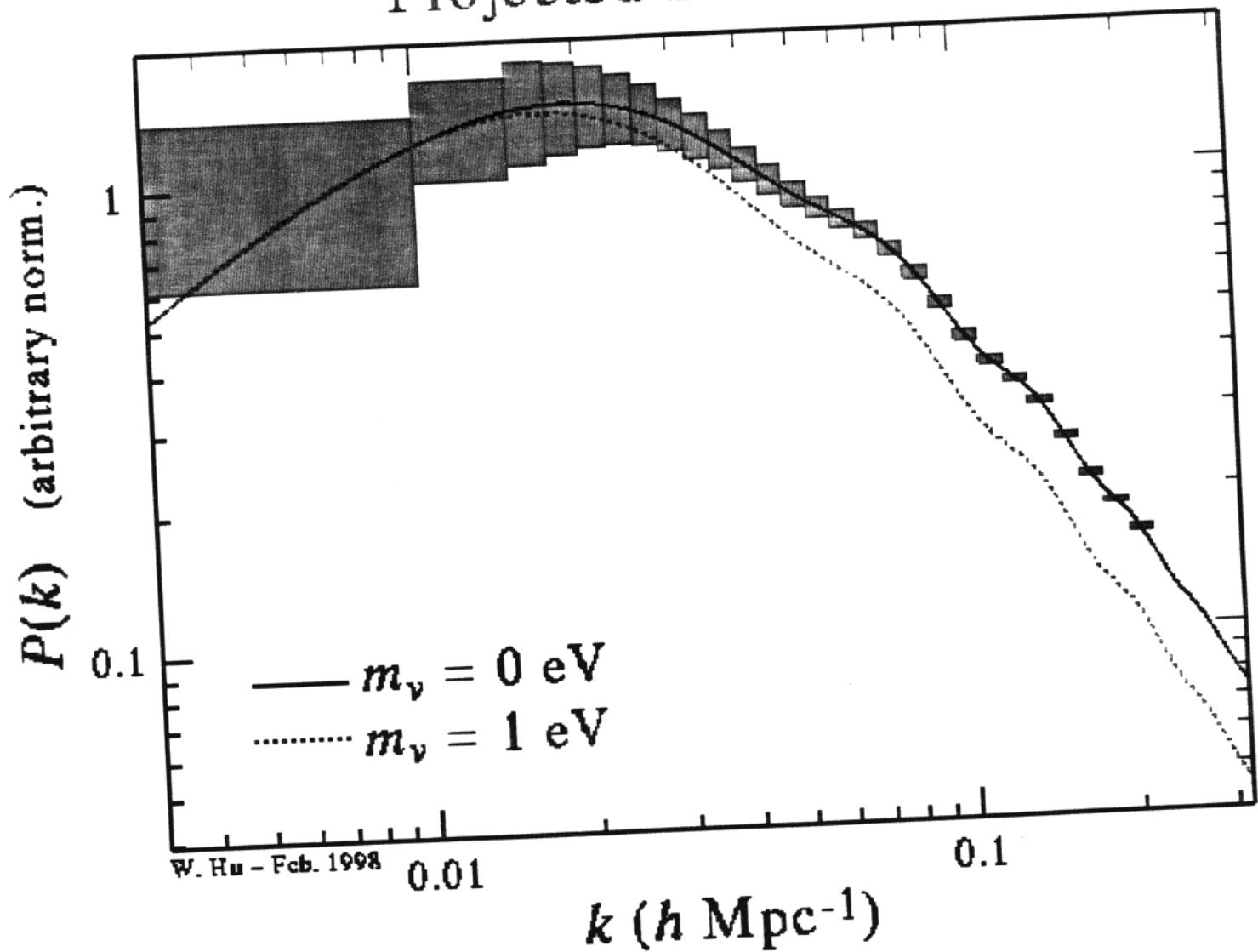
*neutrino, axion, neutralino,  
sneutrino, gravitino,  
magnetic monopoles, .....*



# neutrino mass from SDSS

(J.D. Eisenstein, Technion)

Projected SDSS BRG



$$\Delta^2(k) = k^3 P(k)$$

## *Challenges for particle physics:*

---

**1.  $\Omega_{MATTER} = \Omega_{CLUSTERED} \sim 0.3$**

(open universe unspeakably ugly)

→  **$\Omega_{UNCLUSTERED} \sim 0.7$**

**2. acceleration:  $\ddot{a} > 0$**

(cosmological constant conceptually clumsy)

→  $\rho + 3p < 0$

**3. timing (why about equal now?)**

(feeble anthropic principle?)

→  **$\Omega_{UNCLUSTERED}$  scales as  $\Omega_{CLUSTERED}$**

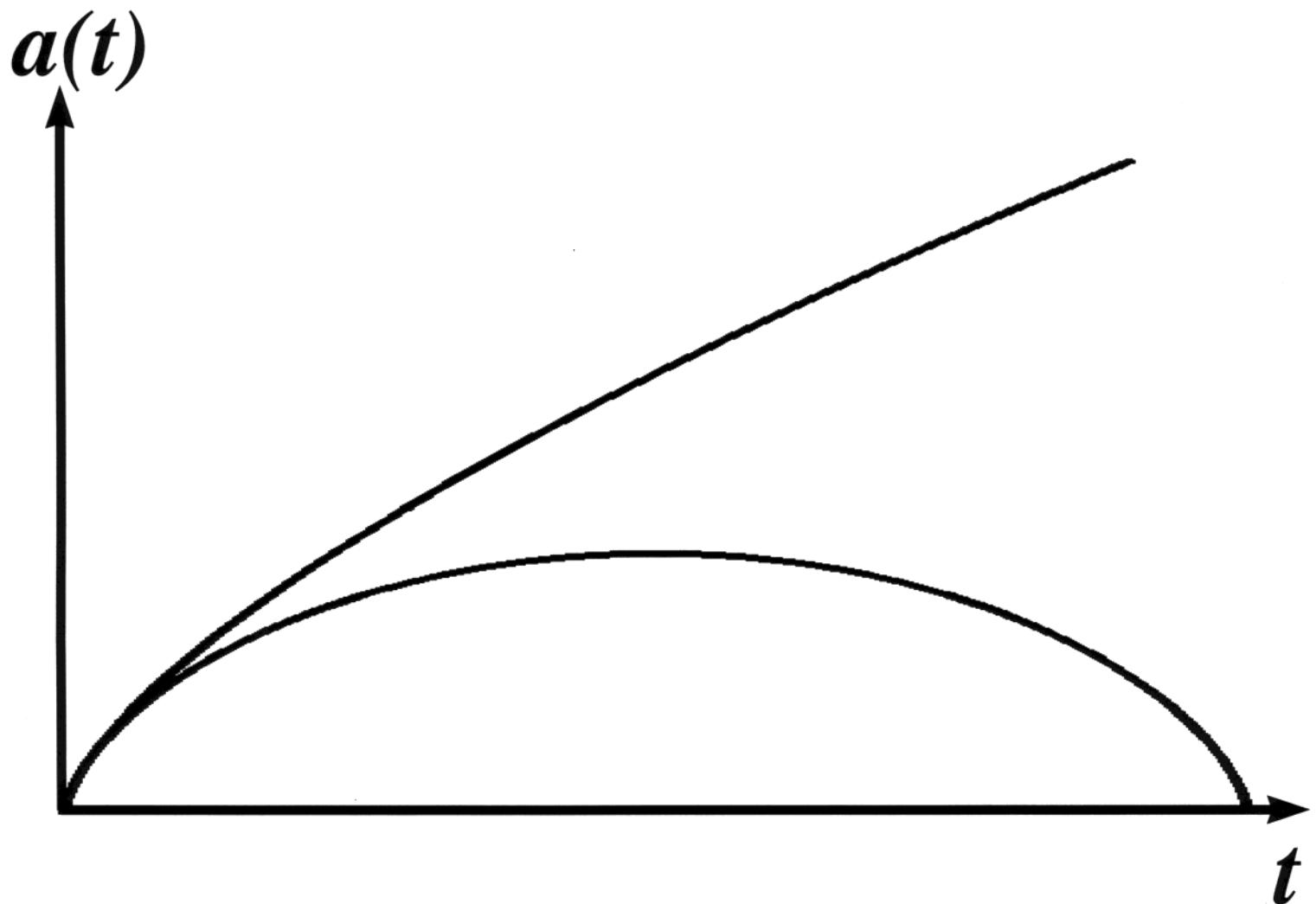
## *Possible Solution to 1+2 (not 3?):*

---

**"Dynamical- $\Lambda$ " models**

**“The conditions for a closed cosmos are much more natural than the inconvenient, uncomfortable boundary conditions for the infinity in an unlimited universe.”**

*Albert Einstein  
1954*



$$a(t) = a(t_0) + \dot{a}(t_0)(t - t_0) + \ddot{a}(t_0)(t - t_0)^2 + \dots$$

$$\frac{\dot{a}(t_0)}{a(t_0)} \equiv H_0$$

$$-\frac{\ddot{a}(t_0)}{a(t_0)H_0^2} \equiv q_0$$

**Hubble constant**

**deceleration parameter**

**Hubble's law:**  $H_0 d_L = z + \frac{1}{2}(1 - q_0)z^2 + \dots$

# Searching for distant supernovae

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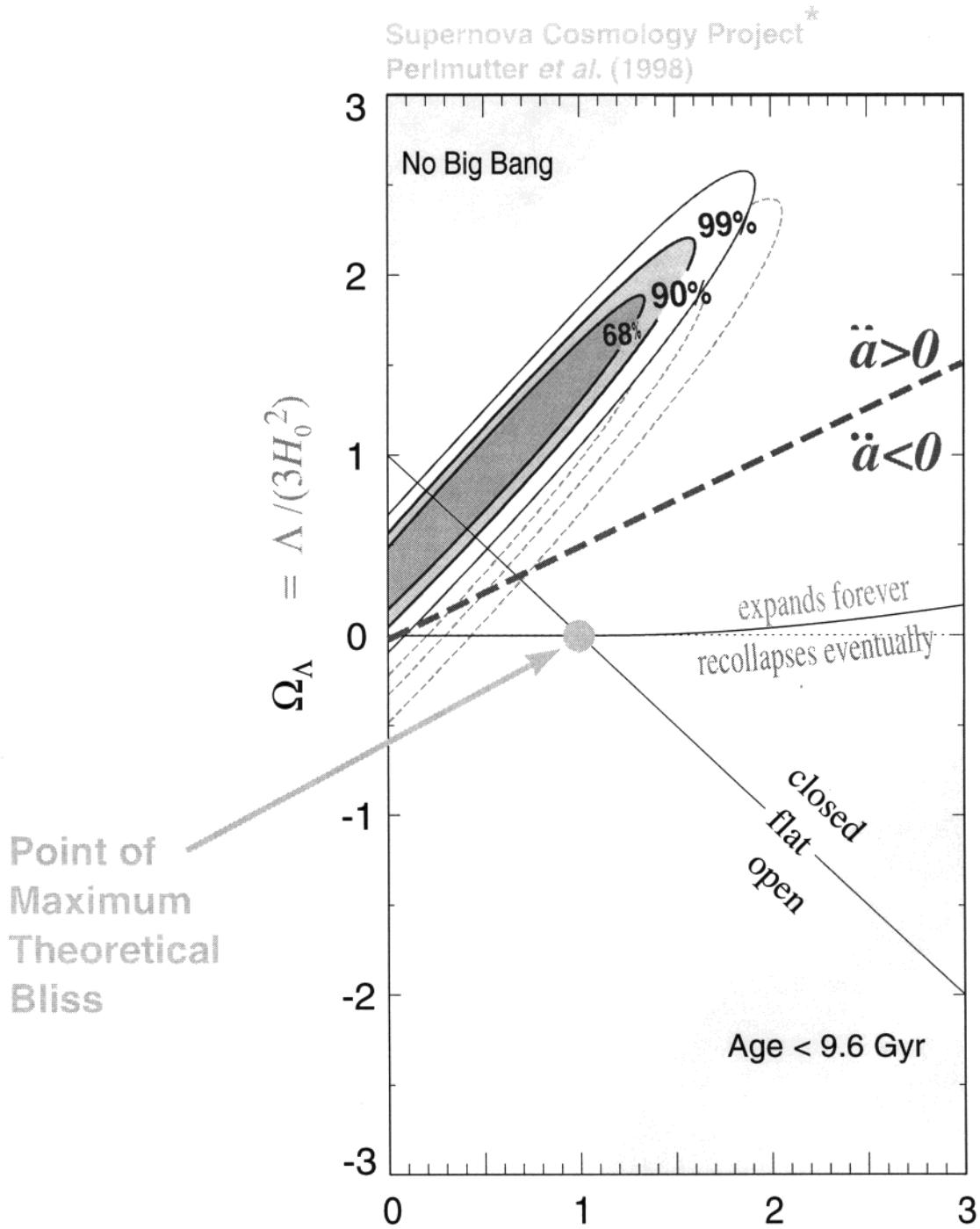
## The High-z Supernova Team

B. Schmidt (Mt. Stromlo),  
M. Phillips, N. Suntzeff, R. Schommer, (CTIO).  
A. Clocchiatti (Universidad Catolica Chile)  
M. Hamuy (Steward Obs.).  
R. Kirshner, P. Garnavich, and P. Challis, (Harvard).  
C. Hogan, C. Stubbs, A. Diercks and D. Reiss (Univ. Washington).  
A. Filippenko and A. Riess (Berkeley).  
R. Gilliland (STScI),  
J. Tonry (Hawaii),  
C. Smith (Michigan),  
J. Spyromilio, P. Woudt, G. Contardo and B. Leibundgut (ESO)

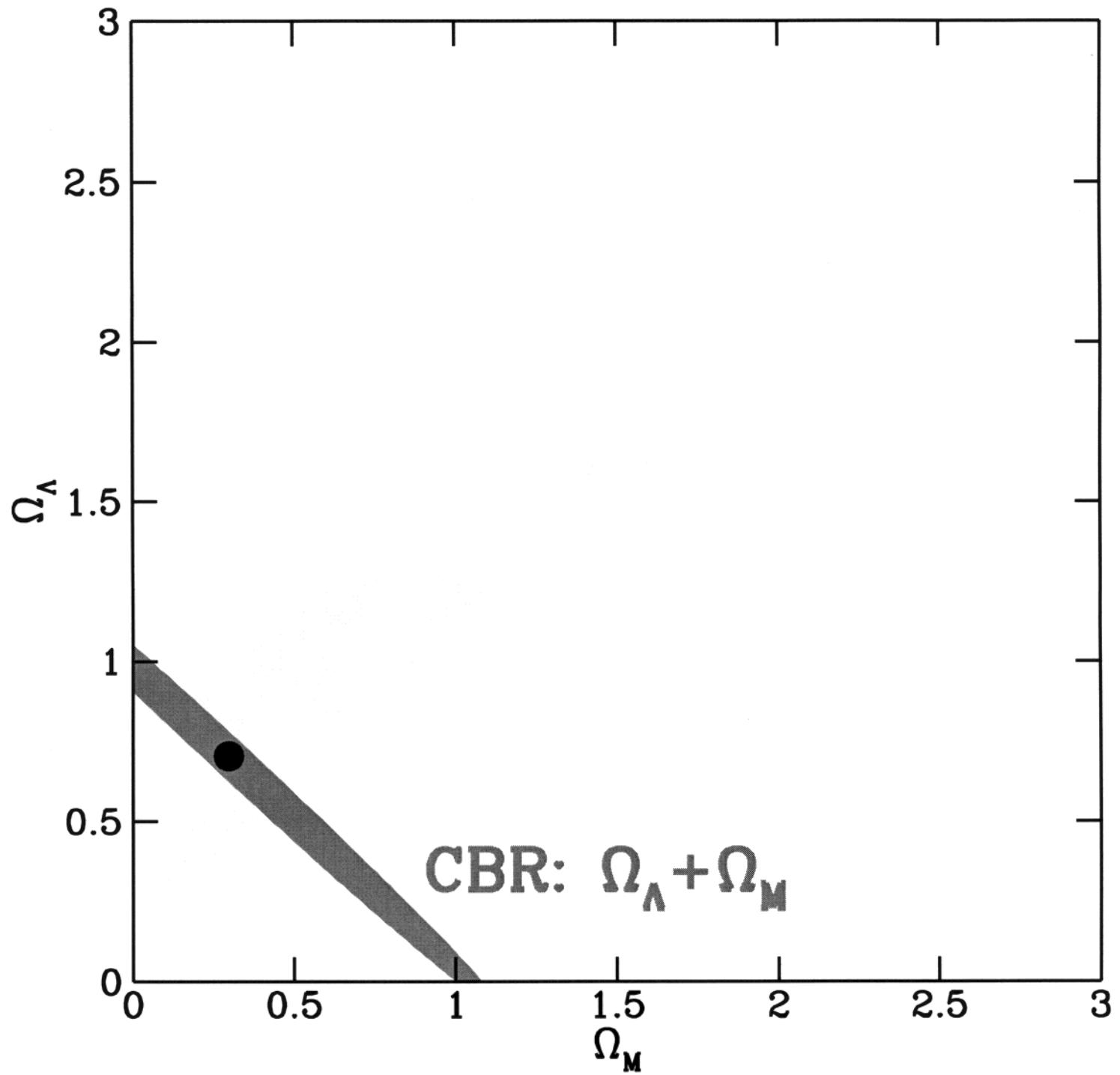
## The Berkeley Supernova Cosmology Project

S. Perlmutter, G. Goldhaber, C. Pennypacker, S. Deustua, G. Aldering, D. Groom, P. Nugent, M. Kim, I. Small (Berkeley).  
A. Goobar (Univ. of Stockholm),  
R. Pain, A. Kim (Paris),  
R. Ellis and R. McMahon (Cambridge),  
P. Bunclark, D. Carter, J. Lee, and M. Irwin (Cambridge).  
B. Boyle, and K. Glazebrook (AAT),  
H. Newberg (Fermi National Laboratory),  
B. Schaefer (Yale),  
A. Fruchter and N. Panagia (STScI),  
I. Hook, C. Lidman, J. Danziger, S. Benetti (ESO).  
M. Della Valle (Padova),  
P. Ruiz-Lapuente (Barcelona),  
N. Walton (La Palma),  
and W. Couch (Univ. New South Wales)

$$\Omega_\Lambda = 0.7 \quad \Omega_M = 0.3$$



\*Similar results from  
High-z supernova team



If cosmological constant--

Fundamental energy scale:

$$\rho_{\Lambda}^{1/4} = 2 \times 10^{-3} \text{ eV}$$

Fundamental length scale:

$$10^{-2} \text{ cm}$$

W

Wetterich

Ratra &amp; Peebles

Ferreira &amp; Joyce

Copeland, Liddle &amp; Wands

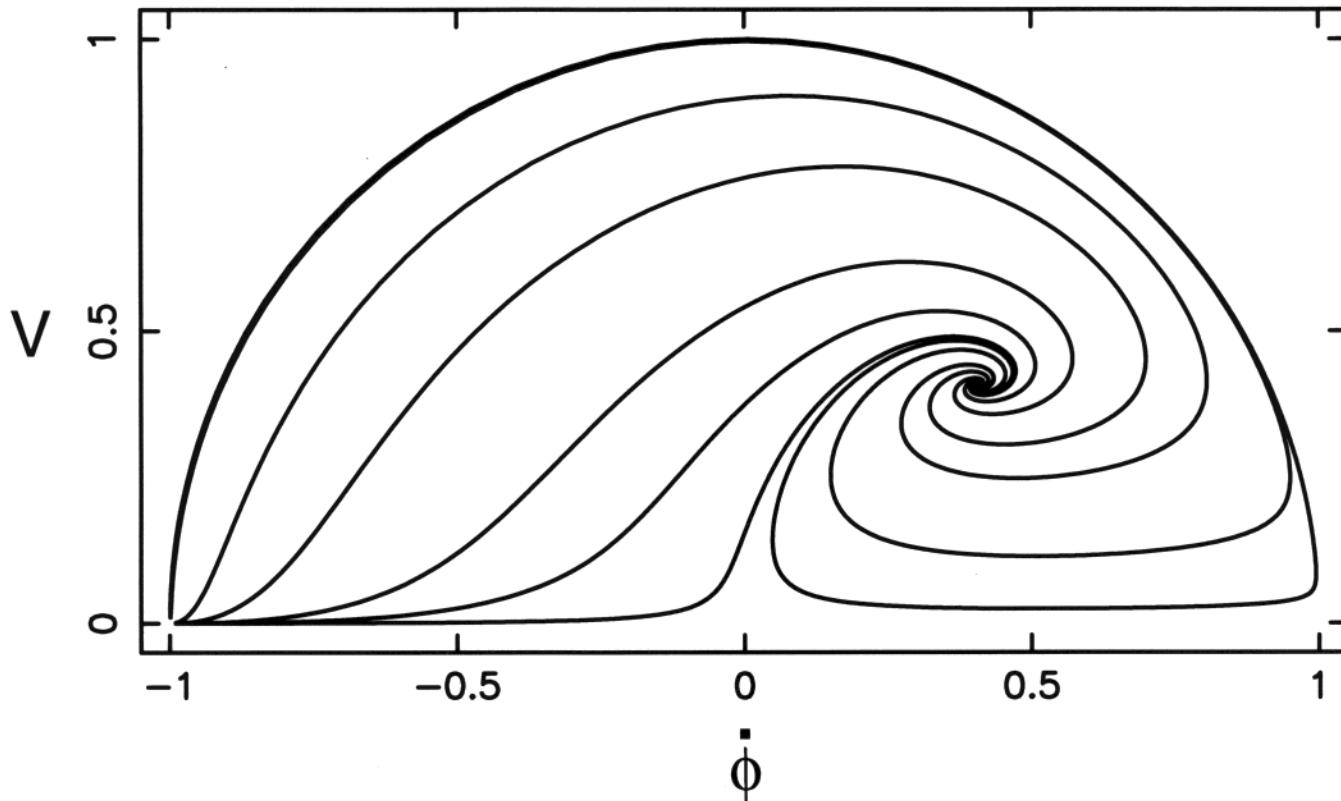
# INTERESTING FAILURE\*

## Scalar field energy

$$V(\phi) = V_0 \exp(-\alpha\phi/M_{Pl})$$

*attractor scaling (snailing?) solution:*

Copeland, Liddle, Wands (98)



$$\frac{\rho_\phi}{\rho_R + \rho_M} \rightarrow \text{constant} \sim \alpha^{-2}$$

\* BBN  $\rightarrow \Omega_\phi$  too small;  $\ddot{a} < 0$

$$V(\phi) = V_0 \exp(-\alpha\phi/M_{Pl})$$

## from classical dilatation-invariant scalar-tensor gravity models

- Quantum effects break classical dilatation invariance
- Scale dependence introduced by quantum effects
- Subleading  $\phi$ -dependence from anomalous dimension

$$\frac{\rho_\phi}{\rho_R + \rho_M} \longrightarrow \begin{array}{l} \textit{grows slowly} \\ \textit{(logarithmically?) with} \\ \textit{time????} \end{array}$$

Stay tuned (Kolb & Wetterich)

# New Era of Precision Cosmology:

*without a standard model  
for structure formation*

## Precision Tests May Reveal:

*nature of dark matter  
 $m_\nu$  as small as 1eV from SDSS  
the weight of space  
information about inflaton  
large field/small field  
reconstruction  
probe energy scale  $\sim 10^{16}$ GeV  
dynamics of many fields?*

## Precision Tests May Lead to:

*"new physics!"*

**“Theory, like mist on  
eyeglasses, obscures  
vision.”**

***C. Chan***