Idaho State University – Department of Physics
PHYS 499 / PHYS 630 Accelerator Physics — Spring 2009

Homework 7 (due Apr 1)

1. Consider a weak-focusing circular accelerator of radius R, where the vertical component of the magnetic field as a function of horizontal displacement x from the ideal orbit is given by the following expression:

$$B_y(x) = \frac{(B\rho)}{R}\frac{R^n}{(R+x)^n}.$$

The constant n is called *field index*; $(B\rho)$ is the magnetic rigidity of the ideal particle.

(a) Calculate the field gradient $B' \equiv \partial_x B_y$ on the ideal orbit ($x = y = 0$).

(b) Write the linearized equations of motion in the horizontal and vertical planes.

(c) Determine for which values of n transverse motion is stable in both the horizontal and vertical planes.

(d) For stable motion, find the horizontal and vertical betatron tunes ν_x and ν_y (i.e., the number of betatron oscillations per revolution) as a function of n.

2. The goal of this problem is to show that any given transport system is equivalent to a simple equivalent system consisting of 3 elements: drift, thin lens, drift; or thin lens, drift, thin lens. Assume the transport matrix of the given system is known:

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix},$$

with $\det M = 1$.

(a) Write the transport matrix (let us call it $M_{OFO'}$) describing a system consisting of a drift of length L_1, a thin lens with focal length f, and a drift of length L_2.

(b) If the given system is not telescopic ($m_{21} \neq 0$), determine f, L_1, and L_2 as a function of the elements of M so that $M_{OFO'} = M$.

(c) If the given system is telescopic ($m_{21} = 0$), write the transport matrix $M_{FOF'}$ describing a thin lens of focal length f_1, followed by a drift of length L, followed by a thin lens of focal length f_2. Find the values of L, f_1, and f_2 for which $M_{FOF'} = M$. Are there any other nontrivial special cases?

(d) Convince yourself that, in general, the total length of the equivalent system is different from the length of the given system.