Università di Ferrara – Dipartimento di Fisica e Scienze della Terra
High Energy Physics Laboratory – Accelerator Physics Module, May 2015

Problems

1. The Universe contains some superb accelerators. Cosmic ray protons can enter the top of the atmosphere with an energy of 1 J or more. They are not easily bent by extragalactic magnetic fields, and therefore their original direction is preserved. The Pierre Auger Observatory observed that the origin of some of these extremely energetic cosmic rays are active galactic nuclei (AGN), where giant black holes are probably located.

   Calculate (with at least two significant digits) the difference $c - v$ between the speed of light in vacuum c and the speed v of a 1-joule proton.

2. Copy and complete the following table:

Particle	Electron e^-	Proton p	Gold ion $^{197}\mathrm{Au}^{79+}$
Charge, q [e]	-1	+1	+79
Rest energy, mc^2	0.511 MeV	0.938 GeV	197 u
Kinetic energy, T	1 MeV	1 GeV	1 GeV/nucleon
Momentum, pc [GeV]			
Velocity parameter, $\beta = v/c$			
Magnetic rigidity, $(B\rho)$ [T m]			

3. The Fermilab Booster is a synchrotron used to accelerate protons from a kinetic energy of 400 MeV up to 8 GeV. Its circumference is 468 m. The accelerating radiofrequency (rf) cavities operate at the 84th harmonic of the revolution frequency. Calculate by how much the revolution period, revolution frequency, and rf frequency vary during the acceleration cycle.

4. This problem is about self fields and space-charge forces in particle beams. Consider a beam of protons, each of mass $m = 1.67 \times 10^{-27}$ kg and charge $q = 1.60 \times 10^{-19}$ C, all moving along the $\hat{\mathbf{z}}$ direction with the same velocity $v = 2.5 \times 10^8$ m/s.

   If the number of particles is very large, the charge density of the beam can be approximated by a continuous function ρ. In this case, the charge density is cylindrically symmetrical, it is Gaussian with standard deviation σ in the plane perpendicular to the direction of motion, and it is independent of z and time. In cylindrical coordinates,

$$\rho(r, \theta, z, t) = \rho(r) = \frac{q\lambda}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right),$$

   where $\lambda = 150$ m^{-1} is a constant representing the number of particles per unit length.

   (a) Write an expression for the current density $\mathbf{j}$ and the total beam current I generated by the beam. Calculate the numerical value of the beam current.

   (b) Derive an expression for the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ generated by the beam, specifying their direction.

   (c) Calculate the Lorentz force $\mathbf{F}$ acting on each particle by adding the electic part $q\mathbf{E}$ and the magnetic part $q\mathbf{v} \times \mathbf{B}$. Sketch a plot of the magnitude of the force as a function of r.

   (d) In beam physics, this force is called the 'space-charge' force. Show that it is proportional to r for $r \ll \sigma$ and to $1/r$ for $r \gg \sigma$. Show that this force decreases with beam energy by exposing its dependence on the relativistic factor $\gamma \equiv (1 - v^2/c^2)^{-1/2}$.

5. Using the collider data collected by the Particle Data Group in its 2014 Review of Particle Physics (`pdg.lbl.gov`, Reviews, High-energy Collider Parameters), make a plot of peak luminosity vs. center-of-mass energy.

6. Consider the operation cycle of a collider. Data is taken for a time t_s. During this time, due to beam life time, emittance growth, and other factors, the luminosity decays exponentially with time constant t_L. At the time t_s, the beam is dumped, the collider is ramped down, more beam is injected, the beam energy is ramped up again, and the beams are put back into collision. These operations require a time t_o, known as the turn-around time. At time $t_s + t_o$, data taking restarts and the cycle is repeated.

   (a) Find an expression for the average integrated luminosity.

   (b) Determine whether it is possible to optimize the data-taking time t_s in order to maximize the average integrated luminosity.

   (c) Test this optimization algorithm with a realistic numerical example: $t_L = 15$ h, $t_o = 6$ h.

7. In the Tevatron, protons and antiprotons were accelerated from a kinetic energy of 150 GeV to 980 GeV in 60 s. Its circumference was $L = 6.28$ km, and its transition energy was $\gamma_t = 18.7$. The radiofrequency cavities operated at 53.1 MHz. The maximum energy they could provide was $qV = 1.4$ MeV per revolution.

   (a) Calculate the synchronous phase ϕ_s (assumed to be constant during acceleration).

   (b) For both the injection energy and the maximum energy, calculate the synchrotron tune ν_s and the synchrotron frequency f_s. For small oscillation amplitudes, how many revolutions are necessary to complete a synchrotron oscillation?

8. Using a programming language of your choice with graphic capabilities (R, python, ROOT, ...), write a program to calculate and plot the phase space trajectories in the (x, p) plane of a dynamical system described by Chirikov's standard map:

$$\begin{cases} p_{n+1} &= p_n + K \sin x_n \\ x_{n+1} &= x_n + p_n + K \sin x_n = x_n + p_{n+1} \end{cases}$$

   for several different initial conditions and a few different values of the parameter K. You may consider both x and p modulo 2π and restrict the analysis to the square $0 \leq x/2\pi \leq 1$ and $0 \leq p/2\pi \leq 1$. Notice how the dynamics becomes chaotic when $K \simeq 0.97$. This standard map serves as a model for many physical systems.