Don Edwards, May 20, 2007

The power radiated by a particle undergoing centripetal acceleration a is given by the Larmor formula as extended by special relativity:

$$P = \frac{1}{6\pi\varepsilon_0}\frac{e^2a^2}{c^3}\gamma^4. \tag{1}$$

The photon spectrum is characterized by a critical energy $\hbar\omega_c$ where

$$\omega_c = \frac{3}{2}\gamma^3\frac{c}{\rho}. \tag{2}$$

If the particle moves through an angle χ on a circle of radius ρ, then the energy radiated can be expressed in terms of the effective number of transition energy photons according to

$$N_c = \left(\frac{2}{3}\right)^2 \alpha\gamma\chi \tag{3}$$

where α is the fine structure constant. Eq. 3 represents the number of photons if they were all at the critical energy arising from a single passage through the bend.

At A0, one of the dogleg magnets produces a bend of 22.5 degrees, with $\rho = 0.758$ m. At 15 MeV, $N_c = 0.037$. The critical energy $\hbar\omega_c$ is 0.97×10^{-21} J or 6×10^{-3} eV. If the radiation from a 1 nC bunch were incoherent, the radiated energy would be 3×10^{-12} J, and would be difficult to detect. If the radiation were fully coherent, the radiated would be some 20 mJ.

The critical wavelength, $\lambda_c = 2\pi c/\omega_c$, is about 125 μm, so full coherence is quite unlikely. But dectector capability extends far below the mJ level, so would be atractive to provide for observation of an infrared signal.

Given the number of variables in the longitudinal-transverse phase change experiment, I hesitate to go further in this short note though a couple other orders of magnitude may be interesting. The energy required to increase by 1 mm-mrad the transverse emittance of a 1 nC, 15 MeV, bunch is only of order 20 μJ. One candidate for such an effect is CSR.

The other is the near-field. An estimate based on a conventional space charge approach could go as follows. The radially directed force on a particle at a distance r from the center of a cylindrically symmetric beam is

$$F \approx \frac{e^2}{4\pi\varepsilon_0\gamma^2}\frac{dN}{ds}\frac{r}{\sigma^2} \tag{4}$$

where σ characterizes the radial distribution and $r^2 \ll 4\sigma^2$. If this force lasts for a time $\rho\chi/c$, if we take $r \approx \sigma/2$, and $ds = \sigma = 10^{-3}$m, then the near-field would contribute about 1 μJ to transverse emittance change for 1 nC.